QED Corrections to Hard Exclusive Processes

Workshop
TOWARDS IMPROVED HADRON FEMTOGRAPHY WITH HARD EXCLUSIVE REACTIONS
University of Virginia
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- Introduction
- QED corrections due to photon radiation; electroproduction of pions and eta-mesons
- Two-Photon Exchange
- Conclusions and Outlook

Radiative Corrections for Exclusive Processes

- Photon emission is a part of any charged-lepton scattering process - accelerated charges radiate - and should be included in data analyses
- Typical magnitudes for QED corrections for exclusive processed are around -20-30% due to large log enhancement factor $\log \frac{Q^2}{m_e^2}$ and kinematic cuts
- Two-photon exchange corrections - a hadronic stucture-dependent part of QED corrections - are angular-dependent and estimated at a few per cent; can be directly measured via positron vs electron scattering
- Exclusive electron scattering processes such as p(e,e'h_1)h_2 are actually inclusive p(e,e'h_1)h_2 nγ, where an infinite number of low-energy photons can be generated
- Low-energy photons do not affect polarization observables, thanks to Low theorem

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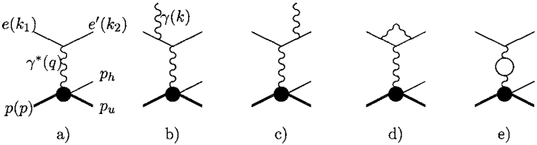
QED Corrections for Electroproduction of Pions

Afanasev, Akushevich, Burkert, Joo, Phys.Rev.D**66**, 074004 (2002)

- Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; extension to DVMP is straightforward;
- Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e'p)n (EXCLURAD code)



- Fortran code EXCLURAD is available at www.jlab.org/RC
- Used for data analysis at JLab, COMPASS, MAMI,...

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QED Corrections for (Exclusive) Electroproduction of Pions

Sample results from EXCLURAD Afanasev, Akushevich, Burkert, Joo, Phys.Rev.D**66**, 074004 (2002)

- QED corrections to unpolarized cross sections reach tens of per cent
- Corrections are dependent on both polar and azimuthal angles of outgoing hadron (pion), which affects extraction of resonance parameters in the resonance region and GPDs in the deep-virtual region

AFANASEV *et al.*

PHYSICAL REVIEW D **66**, 074004 (2002)

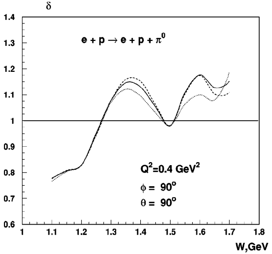

FIG. 3. *W* dependence of RC to the cross section of neutral pion

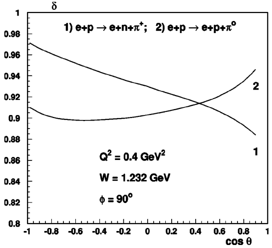

FIG. 5. RC to the cross section as a function of cos θ.

- QED corrections due to real-photon emission are smaller for spin polarization asymmetries

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Upgrade of EXCLURAD to include eta-electroproduction on protons, Illari, Afanasev, Briscoe, Kashevarov, Schmidt, Strakovsky, arXiv:2604.22943, Accepted in PRD July 2026 (a part of Isabella Illari's PhD project)

- Amplitude-Based Analysis of QED Radiative Corrections to Electroproduction of η-Mesons on Protons
- A formalism for radiative correction calculations in exclusive η electroproduction on the proton is presented, extending the treatment developed for the pion channel. The EXCLURAD code is used in the radiative correction procedure with EtaMAID-2023 multipole amplitudes. The cross-section correction factor δ varies by up to $\sim$30% across the resonance region $W = 1.49$–2.0 GeV at $E_{\rm beam} = 6.535$ GeV, with a local maximum near $W \simeq 1.66$ GeV driven by the $S_{11}(1535)$ and $S_{11}(1650)$ resonances. The beam-spin asymmetry is suppressed by 15–25% at the same kinematics. Numerical results covering $Q^2 = 0.3$–4.0 GeV^2 and the full angular range are provided for kinematics relevant to CLAS12 experiments at Jefferson Lab.

QED Corrections for (Exclusive) Electroproduction of η-mesons

Illari et al, arXiv:2604.22943

- QED corrections to unpolarized cross sections reach 30%, beam spin asymmetry is suppressed by 15-25% (relative)

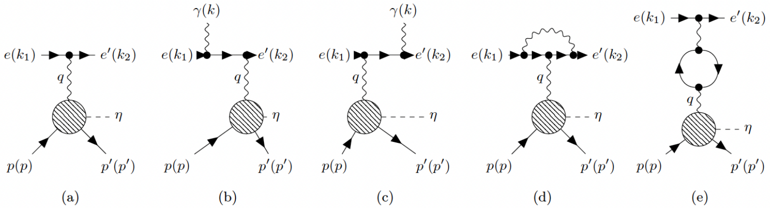

FIG. 2. Feynman diagrams contributing to the Born and $\mathcal{O}(\alpha)$ electroproduction cross sections. (a) Born process; (b) initial-state bremsstrahlung off the incoming electron; (c) final-state bremsstrahlung off the outgoing electron; (d) one-loop electron–photon vertex correction; and (e) vacuum polarization insertion on the virtual photon propagator. The hatched blob represents all hadronic structure (EtaMAID-2023); $q = k_1 - k_2$ is the virtual photon four-momentum. Diagrams produced with Ti*k*Z-Feynman [30].

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QED Corrections for (Exclusive) Electroproduction of η-mesons

Illari et al, arXiv:2604.22943 (Phys Rev D (2026))

- etaMAID-2023 was used for modeling partial wave amplitudes for electroproduction of η

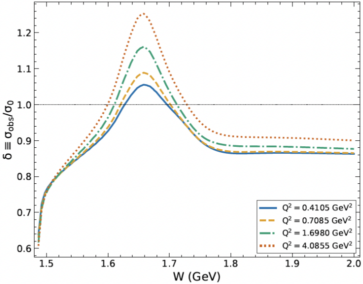

FIG. 4. Cross-section RC factor $\delta \equiv \sigma_{\rm obs}/\sigma_0$ vs. W at $\cos\theta^* = -0.75$, $\phi^* = 90^\circ$, $v_{\rm cut} = 0.166$ GeV^2. Curves: $Q^2 = 0.4105, 0.7085, 1.6980, 4.0855$ GeV^2 (see legend). The horizontal line marks $\delta = 1$. All curves rise from $\delta \approx 0.61 - 0.64$ at threshold to a local maximum at $W \simeq 1.655$ GeV (peak $\delta \approx 1.05 - 1.25$, increasing with Q^2), then fall and flatten to $\delta \approx 0.86 - 0.90$ for $W \gtrsim 1.85$ GeV.

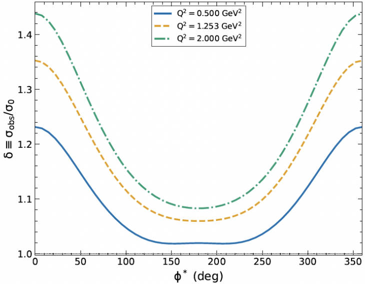

FIG. 6. $\delta \equiv \sigma_{\rm obs}/\sigma_0$ vs. ϕ^* at $W = 1.660$ GeV, $\cos\theta^* = -0.750$, $v_{\rm cut} = 0.166$ GeV^2, for $Q^2 = 0.500, 1.253, 2.000$ GeV^2 (see legend). Cosine-like modulation with $\delta > 1$ throughout; both amplitude and offset increase with Q^2. At $Q^2 = 0.500$ GeV^2, $\delta \in [1.02, 1.23]$; at $Q^2 = 2.000$ GeV^2, $\delta \in [1.08, 1.44]$. The horizontal dotted line marks $\delta = 1$.

QED Corrections for (Exclusive) Electroproduction of η-mesons

Illari et al, arXiv:2604.22943 (Phys Rev D (2026))

- Spin asymmetries are also affected by QED corrections; depend on kinemaic cuts, tighter missing-mass (inelasticity) cuts yield smaller corrections

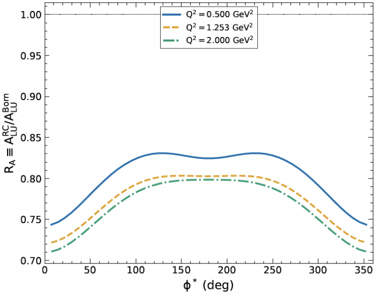

FIG. 7. Asymmetry ratio $R_A \equiv A_{LU}^{\rm RC}/A_{LU}^{\rm Born}$ vs. ϕ^* at $W = 1.660$ GeV, $\cos\theta^* = -0.750$, $v_{\rm cut} = 0.166$ GeV^2, for $Q^2 = 0.500, 1.253, 2.000$ GeV^2 (see legend). The dotted horizontal line marks $R_A = 1$ (no RC effect on the asymmetry). R_A is undefined near $\phi^* = 0°$ and $180°$ where $A_{LU}^{\rm Born} \to 0$. All curves lie below unity; suppression deepens with Q^2: $R_A \in [0.74, 0.83]$ at $Q^2 = 0.500$ GeV^2 and $R_A \in [0.71, 0.80]$ at $Q^2 = 2.000$ GeV^2. The weak ϕ^* modulation, peaking near $\phi^* \approx 90°$, indicates that the $\sin\phi^*$ shape of A_{LU} is preserved

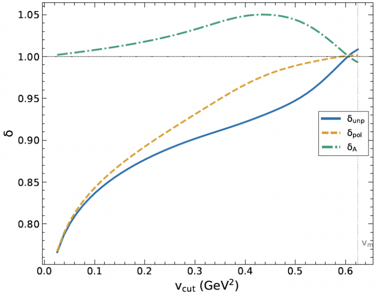

FIG. 8. RC factors vs. inelasticity cut $v_{\rm cut}$ for η electroproduction at $W = 1.900$ GeV, $Q^2 = 0.4105$ GeV^2, $\cos\theta^* = -0.750$, $\phi^* = 90°$. $\delta_{\rm unp}$: RC to the unpolarized cross section; $\delta_{\rm pol}$: RC to the polarized cross section; $\delta_A = \delta_{\rm unp}/\delta_{\rm pol}$: RC to the beam polarization asymmetry. Kinematic upper limit $v_m \approx 0.625$ GeV^2 (Eq. (4)). EtaMAID was used for structure functions.

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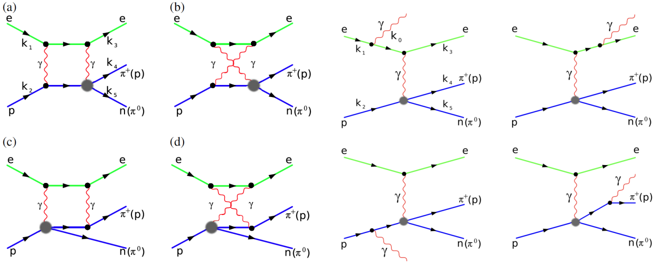
- Ge/Gm polarization vs Rosenbluth discrepancy is understood to be partly due to two-photon exchange (resulting from about 5 per cent missing systematic correction at high momentum transfers (see review A Afanasev, PG Blunden, D Hasell, BA Raue, Prog. Part. Nucl. Phys., 2017)
- JLab experiment Katich et al., Phys.Rev.Lett. 113 (2014)022502 reveals about 5 per cent polarization asymmetries in DIS on 3He that are zero in one-photon exchange approximation
- Proposed positron beamline at JLab will provide a direct probe for two-photon effects via measurements of electron-hadron vs positron-hadron scattering asymmetries

Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Calculated previously neglected QED corrections from two-photon exchange
- Used a soft-photon approximation, results expressed in terms of Passarino-Veltman integrals



- Computed corrections result in about 5 per cent variation of cross section from backward to forward scattering angles
- Conclusion: Important for the analysis of angular dependences, $\cos(\phi)$ moments in particular

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Assumptions & Calculations

Using soft-photon approximation (SPT [1]) by neglecting the momentum for one of the photon while calculating the amplitude, such that

$$
\begin{aligned}
M^{2\gamma} =& M^{1\gamma} \cdot \sum_{l} [\frac{-e^2}{2\pi} \cdot \sum_{i,j} (2k_i \cdot k_j) \\
& \cdot C_0(\{k_i, m_i\}, \{\mp k_j, m_j\}) \qquad (1) \\
=& \sum_{l=N,q',s} \sum_{i=a,b,c} M^{1\gamma} M_{l,i,box}, \qquad (2)
\end{aligned}
$$

where the Passarino-Veltman three-point scalar integral

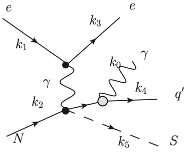
$$
\begin{aligned}
& C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \\
& \frac{1}{i\pi^2} \int d^4q \frac{1}{q^4} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}. \qquad (3)
\end{aligned}
$$

The correction

$$
\delta_{box} = \frac{2Re[M^{2\gamma} M^{1\gamma\dagger}]}{|M^{1\gamma}|^2} = 2Re[\sum_{l;i} M_{l,i,box}]. \qquad (4)
$$

Assumptions & Calculations

Infrared divergence of two-photon exchange is canceled by interference between emission from electron lines and hadron lines (hadron Bremsstrahlung)



One of the possibilities for the hadron Bremsstrahlung process [2].

[2] Afanasev et al. Phys. Rev. D 88, 053008 (2013)

Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Angular dependencies of two-photon corrections affect σ_L/σ_T extraction

AFANASEV, ALEKSEJEVS, AND BARKANOVA PHYSICAL REVIEW D **88,** 053008 (2013)

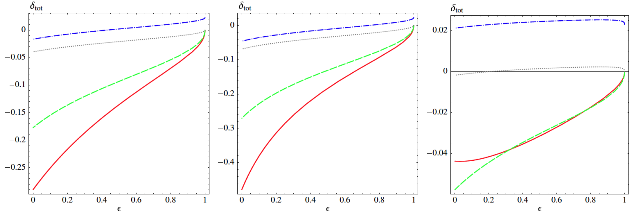
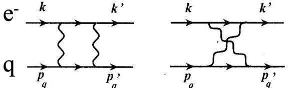


FIG. 5 (color online). π^0 electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter ϵ for momentum transfers $Q^2 = 3.0$ GeV^2 (left plot), $Q^2 = 7.0$ GeV^2 (middle plot), and $Q^2 = 0.4$ GeV^2 (right plot). All plots are given for $\phi_4 = 90°$ and $\theta_4 = 90°$ and $W = 1.232$ GeV. Dot-dashed curve, SPT; dotted curve, SPT with $\alpha\pi$ subtracted; dashed curve, SPMT; solid curve, FM approach.

- These effects can be directly measured with proposed positron beamline at JLab
- Two-photon correction x2 = electron-positron scattering asymmetry

Next Step: Hard Two-Photon Exchange

- Hard TPE on a parton
- Remove Soft-Photon Exchange at parton level and include it at hadronic level following AA, Brodsky, Carlson, Chen, Vanderhaeghen, PRL 93:122301,2004; PRD 92:013008,2005
- Two-photon amplitude for a (massless) quark



$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V^e_\mu \otimes V^q_\mu \times f_V + A^e_\mu \otimes A^q_\mu \times f_A),$$

$$V^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu u_{e,q},\ A^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2-s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2-s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

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Theory Challenges

- Both soft and hard photons in the loop integral are present
- Soft photons do not resolve the quark/parton structure
- Soft/hard scale separation is necessary
- We used Grammer-Yennie procedure for soft/hard separation - as in AA, Brodsky, Carlson, Chen, Vanderhaeghen, PRD72, 013008 (2005)
- The results become dependent on soft-hard separation scheme, QED and QCD have to be consistently combined
- Not all of the contributions are factorizable in terms of GPDs
- Addition of "hard" TPE appears to reduce the total effect for both the cross section and azimuthal asymmetries
- For SIDIS results, see Lee and Afanasev, arXiv:2504.17123 and PRD **111**, 113008 (2025)

Conclusion

- QED radiative effects are essential for extracting physics information from data, with 20-30% differences between Born and full calculations
- New calculations reported for exclusive electroproduction of η for CLAS analysis
- Extension of QED correction calculations for electroproduction of other mesons $\rho, \omega, \phi, \ldots$ requires realistic modeling of reaction amplitudes
- TPE effects alter angular dependence of cross sections in hard exclusive reactions and SIDIS
- Setting constraints for TPE and accounting for QED radiation is necessary for extracting GPDs, TMDPDFs and FFs, can be done with *positron beams* at CEBAF
  - Addition of hard two-photon exchange at a parton level *reduces* the total correction
  - Next steps: "Hard" TPE for (exclusive) DVMP, spin asymmetries and spin-density matrices of electroproduced vector mesons

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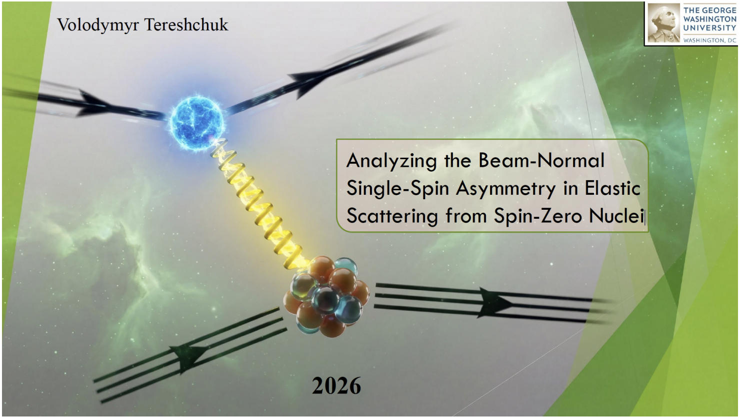
Ad

16:50 Thursday talk by Volodymyr Tereshchuk (GWU):
Is the *"PREX Asymmetry Puzzle"* resolved?!



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