

Any-spin Form Factors in the chiral representation

A systematic covariant basis for hadron femtography

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$$\int dx x^{n-1} \text{GPD} \longleftrightarrow \langle p_f, \lambda_f | O^{\mu_1 \dots \mu_r} | p_i, \lambda_i \rangle,$$

$$\langle O_R \rangle = \sum_{a=1}^{N_R(j)} \mathcal{B}_{a,R} F_{a,R}(t).$$

One basis, several communities

- hard exclusive amplitudes and GPD moments;
- lattice local currents and Mellin moments;
- nonlocal quasi-/pseudo-distributions after matching or taking moments;
- electromagnetic, axial, and energy-momentum currents;

The practical question

How do we know that the tensors $\mathcal{B}_{a,R}$ are complete and independent for any target spin j ?

Introduction

Hidden traces

A relation that is automatic at low spin can remove a legitimate open-index multipole at larger j .

Mixed symmetries

Reduced exchange $\mathcal{X}$:

swap initial and final states, send $\Delta \rightarrow -\Delta$, restore the standard leg order, and re-expand. It is not physical T or crossed C .

Guessed reductions

A familiar Gordon (on-shell) identity does not prove that a higher-spin covariant is redundant.

We need a finite mother list, an exact quotient/reduction, and an independent count.

The Lorentz algebra

- Algebra for Generators of the Lorentz group

$$[\mathbb{J}_I, \mathbb{J}_m] = i\epsilon_{lmn}\mathbb{J}_n, \quad [\mathbb{J}_I, \mathbb{K}_m] = i\epsilon_{lmn}\mathbb{K}_n, \quad [\mathbb{K}_I, \mathbb{K}_m] = -i\epsilon_{lmn}\mathbb{J}_n$$

- Defining: $\mathbb{A}_m = \frac{1}{2}(\mathbb{J}_m + i\mathbb{K}_m)$, $\mathbb{B}_m = \frac{1}{2}(\mathbb{J}_m - i\mathbb{K}_m)$

Two independent $\mathfrak{su}(2)$ subalgebras: (j_A, j_B)

$$[\mathbb{A}_I, \mathbb{B}_m] = 0$$

$$[\mathbb{A}_I, \mathbb{A}_m] = i\epsilon_{lmn}\mathbb{A}_n$$

$$[\mathbb{B}_I, \mathbb{B}_m] = i\epsilon_{lmn}\mathbb{B}_n$$

j	chiral pair	familiar case
$\frac{1}{2}$	$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$	Dirac spinor
1	$(1, 0) \oplus (0, 1)$	spin-1 parity doublet
general	$(j, 0) \oplus (0, j)$	any-spin carrier

Parity exchange irreps $(j, 0) \xleftrightarrow{P} (0, j)$. It leads to consider: $(j, 0) \oplus (0, j)$

Parity turns the minimal chiral pair into Dirac-like spinors

$$U(p, \lambda) = \begin{pmatrix} u_L(p, \lambda) \\ u_R(p, \lambda) \end{pmatrix} \in (j, 0) \oplus (0, j),$$

$$\beta = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix},$$

$$\gamma^{\mu_1 \dots \mu_{2j}} = \begin{pmatrix} 0 & t^{\mu_1 \dots \mu_{2j}} \\ \bar{t}^{\mu_1 \dots \mu_{2j}} & 0 \end{pmatrix},$$

$$(\not{p}^{(j)} - M^{2j}) U(p, \lambda) = 0.$$

$$\bar{U}_f \Gamma U_i = \frac{1}{2M^{2j}} \bar{U}_f \left(\{ \not{\mathcal{P}}^{(j)}, \Gamma \} + \frac{1}{2} [\not{\mathcal{D}}^{(j)}, \Gamma] \right) U_i$$

$$0 = \bar{U}_f \left(\frac{1}{2} \{ \not{\mathcal{P}}^{(j)}, \Gamma \} + [\not{\mathcal{D}}^{(j)}, \Gamma] \right) U_i$$

$\mathcal{P}$ and $\mathcal{D}$ are the average and difference products of $2j$ external momenta

Weyl convention is used

$$t^{\mu_1 \dots \mu_{2j}} \propto \text{ST}_{\mu_1 \dots \mu_{2j}} [J^{\mu_1} \dots J^{\mu_{2j}}]$$

$$t^{0 \dots 0} = \bar{t}^{0 \dots 0} = \mathbf{1}$$

Each chiral block contains exactly one rotational spin (exact d.o.f):

$$\dim(j, 0) = \dim(0, j) = 2j + 1.$$

For every spin j : Parity exchanges the blocks through β , whereas γ_5 assigns opposite chiral signs.

No auxiliary conditions

No tensor-spinor, transversality constraint, or Rarita–Schwinger subsidiary condition is introduced.

Generalized gamma matrices give an any-spin Dirac calculus

$$\gamma^{\mu_1 \cdots \mu_{2j}} = \begin{pmatrix} 0 & t^{\mu_1 \cdots \mu_{2j}} \\ \bar{t}^{\mu_1 \cdots \mu_{2j}} & 0 \end{pmatrix}, \quad \gamma^{\mu_1 \cdots \mu_{2j}} \gamma^{\nu_1 \cdots \nu_{2j}} = \begin{pmatrix} t^{\mu_1 \cdots \mu_{2j}} \bar{t}^{\nu_1 \cdots \nu_{2j}} & 0 \\ 0 & \bar{t}^{\mu_1 \cdots \mu_{2j}} t^{\nu_1 \cdots \nu_{2j}} \end{pmatrix}$$

Central role of the covariant t -tensors (spinors, boosts, propagators, gamma matrices)

Alternating products

Bilinear calculus involve $t\bar{t}t\cdots$ and $\bar{t}t\bar{t}\cdots$.
Cubic and Quadratic products are reducible.

$$t^{\mu_1 \cdots \mu_{2j}} \bar{t}^{(\rho)} t^{(\sigma)} = \frac{1}{[(2j)!]^3} \mathcal{S}_{\{(\mu)(\rho)(\sigma)\}} \left(\prod_{l=1}^{2j} C^{\mu_l \rho_l \sigma_l \alpha_l} \right) t_{(\alpha)}$$

$$C^{\mu\rho\sigma\alpha} = g^{\mu\rho} g^{\sigma\alpha} - g^{\mu\sigma} g^{\rho\alpha} + g^{\mu\alpha} g^{\rho\sigma} + i\epsilon^{\mu\rho\sigma\alpha}$$

Closure of the algebra

t -tensors span the $(2j+1)$ -dimensional hermitian matrices, $su(2j+1)$ algebra.
Alternating products can be linearized back onto a single t or $\bar{t}$ tensor.

$$t^{\mu_1 \cdots \mu_{2j}} \bar{t}^{(\rho)} = \frac{1}{[(2j)!]^2} \mathcal{S}_{\{(\mu)(\rho)\}} \left(\prod_{l=1}^{2j} Q^{\mu_l \rho_l \alpha_l} \right) t_{(\alpha)}$$

$$\begin{aligned} Q^{\mu\rho\alpha} &= C^{\mu\rho\sigma\alpha} \eta_\sigma \\ &= -g^{\rho\alpha} \eta^\mu + g^{\mu\alpha} \eta^\rho + i\epsilon^{\mu\rho\sigma\alpha} \eta_\sigma \end{aligned}$$

$$\eta^\mu = (1, 0, 0, 0)$$

The Q_{red} intertwiners turn spin matrices into Lorentz tensors

$$Q_{\text{red}}^{\mu\rho a} = -g^{\rho\alpha}\eta^\mu + g^{\mu\alpha}\eta^\rho + i\epsilon^{\mu\rho\sigma\alpha}\eta_\sigma,$$

$$\mathcal{T}_m^{\mu_1\rho_1,\dots,\mu_m\rho_m} = Q_{\text{red}}^{\mu_1\rho_1 a_1} \dots Q_{\text{red}}^{\mu_m\rho_m a_m} t_{a_1\dots a_m 0\dots 0}, \quad 0 \leq m \leq 2j$$

$$T_m = \mathcal{T}_m - (\text{all lower-order } C_{\text{red}}\text{-traces})$$

$$\mathcal{M}_m^{\mu_1\rho_1,\dots,\mu_m\rho_m} = \frac{1}{m!} \mathcal{S}_{\{(\mu\rho)\}} \prod_{\ell=1}^m \mathbb{M}^{\mu_\ell\rho_\ell} - C_{\text{red}}\text{-traces} = \frac{i^m}{2^m} m! \binom{2j}{m} T_m$$

A finite raw operator tower

The components $t_{a_1\dots a_m 0\dots 0}$ span the Hermitian matrices on the $(2j+1)$ -dimensional chiral spin space. Each Q_{red} converts one Cartesian multipole index into an antisymmetric Lorentz pair.

First member is the Lorentz generator

$$\frac{1}{j} \mathbb{M}^{\mu\rho} = i Q_{\text{red}}^{\mu\rho a} t_{a 0\dots 0} = i \mathcal{T}_1^{\mu\rho}.$$

The commutator with $\mathbb{M}^{\alpha\beta}$ rotates every pair of $\mathcal{T}_m$ covariantly.

Cosyn, Vera, JHEP 12 (2025) 068.

Covariant Multipoles & Dirac basis

$$T_m = Q_{\text{red}}^{\mu_1 \rho_1 a_1} \cdots Q_{\text{red}}^{\mu_m \rho_m a_m} t_{a_1 \cdots a_m 0 \cdots 0} - (\text{all lower-order } C_{\text{red}}\text{-traces}),$$

$$G_m^{\mu_1 \rho_1, \dots, \mu_m \rho_m} = \begin{pmatrix} T_m^{\mu_1 \rho_1, \dots, \mu_m \rho_m} & 0 \\ 0 & \overline{T}_m^{\mu_1 \rho_1, \dots, \mu_m \rho_m} \end{pmatrix},$$

$\text{TL}_{C_{\text{red}}}$ is the C_{red} -traceless projection

- antisymmetric inside every pair $(\mu_a \rho_a)$;
- symmetric under exchange of Lorentz pairs;
- C_{red} -traceless between any two different pairs;
- mutually orthogonal orders $m = 0, 1, \dots, 2j$ are the monopole, dipole, quadrupole, ...

Complete Dirac basis

$$\gamma^{\mu_1 \cdots \mu_{2j}}, \gamma^{\mu_1 \cdots \mu_{2j}} \gamma_5,$$

$$G_0 = \mathbf{1}, \dots, G_{2j}^{\mu_1 \rho_1, \dots, \mu_{2j} \rho_{2j}}, \gamma_5$$

Operator covariants retain their target-multipole label

$$\langle p_f, \lambda_f | O_R^{\mu_1 \cdots \mu_r} | p_i, \lambda_i \rangle = \sum_{m=0}^{2j} \sum_a \underbrace{\mathcal{B}_{m,a}^{\mu_1 \cdots \mu_r}(P, \Delta; G_m)}_{\substack{\text{operator geometry} \\ \times \text{ target multipole } m}} F_{m,a}^R(t).$$

What the label m tells us

- $m = 0, 1, 2, \dots$ is target monopole, dipole, quadrupole, $\dots$;
- the operator rank determines how many Lorentz indices remain open;
- different contractions at fixed m are different operator geometries, not different target multipoles.

Arbitrary-spin EMT

$$\langle T^{\mu\nu} \rangle = \sum_{m,a} \mathcal{B}_{m,a}^{\mu\nu}(G_m) F_{m,a}^{\text{EMT}}(t).$$

The symmetric rank-two operator couples systematically to every allowed target multipole through $m \leq 2j$.

Equivalent bases may exist; this basis keeps the physical multipole label explicit.

Four families generate every current of rank at most two

Discrete-symmetry blocks

$$\mathbb{D}_m = i^m \bar{U}_f G_m U_i, \quad \tilde{\mathbb{D}}_m = i^m \bar{U}_f G_m \gamma_5 U_i.$$

$$PT+ = (P+, T+) \oplus (P-, T-),$$

$$\bar{U}_f U_i, i \bar{U}_f \gamma_5 U_i \in PT+.$$

Elastic Lorentz alphabet

$$P = \frac{1}{2}(p_f + p_i), \quad \Delta = p_f - p_i,$$

$$t = \Delta^2, \quad P \cdot \Delta = 0, \quad P^2 + \frac{1}{4} \Delta^2 = M^2,$$

$$\Omega^{\mu\nu} = P^\mu \Delta^\nu - P^\nu \Delta^\mu.$$

Every closed multipole pair contracts with Ω .

family	definition	open multipole indices
S_m	$\Omega_{A_1} \cdots \Omega_{A_m} \mathbb{D}_m^{A_1 \cdots A_m}$	none
$V_m^\mu[X]$	$\Omega^{m-1} X_\rho \mathbb{D}_m^{\cdots, \rho\mu}$	one
$Z_m^{\mu\nu}$	$\Omega^{m-1} \mathbb{D}_m^{\cdots, \mu\nu}$	one Lorentz pair
$Q_m^{\mu\nu}[X, Y]$	$\Omega^{m-2} X_\rho Y_\sigma \mathbb{D}_m^{\cdots, \rho\mu, \sigma\nu}$	two Lorentz pairs

$X, Y \in \{P, \Delta\}$; pseudo families use $\tilde{\mathbb{D}}_m$, and Hodge duals reduce by chiral self-duality and Schouten identities.

Generate exhaustively; the scalar sector example

Ordered mother-list grammar

S	S_m
V	$P^\mu S_{2r}, V_m^\mu[\Delta], \Delta^\mu S_{2r}, V_m^\mu[P]$
A	$X^{[\mu} V_m^{\nu]}[Y], P^{[\mu} \Delta^{\nu]} S_{2r}, Z_m^{\mu\nu}, Q_m^{[\mu\nu]}[P, \Delta]$
ST	$TL[X^\mu Y^\nu S_{2r}], TL[X^{(\mu} V_m^{\nu)}[Y]], TL[Q_m^{\mu\nu}[X, Y]]$

Mother means exhaustive

Scalar example

mother list module on-shell null relations:

$\{S_0, S_1, \dots, S_{2j}\}$ /on-shell relations

$$\mathcal{B}_S^{P+} = \{S_{2r}\}_{r=0}^{[j]}$$

$$N_S^{P+} = [j] + 1$$

Even orders already saturates the scalar rank,

$$S_{2r+1} = \sum_{s=0}^{[j]} \kappa_{2r+1,s}(t, M^2) S_{2s}.$$

True PT requires the complete covariant and a real-linear projector

Five distinct operations

$\mathcal{P}, \mathcal{X}$	complex-linear: parity/chiral exchange; leg exchange with $\Delta \mapsto -\Delta$
$\mathcal{H}$	anti-linear: exchange plus conjugation
$\mathcal{T}$	antiunitary: momentum, spin, and Lorentz reversal plus conjugation
$\mathcal{C}$	particle–antiparticle exchange after crossing

Full-covariant grades: external spinors transform; powers of Δ alone do not fix the grade.

Realified Hermitian space

$$\begin{aligned} f = x + iy &\mapsto \mathfrak{r}(f) = \begin{pmatrix} x \\ y \end{pmatrix}, \\ \mathfrak{H}_H &= \ker_{\mathbb{R}}(\mathcal{H}_{\mathbb{R}} - 1), \\ \Pi_{P,+} &= \frac{1}{2}(1 + \mathcal{P}), \\ \Pi_{PT,+} &= \frac{1}{2}(1 + \mathcal{PT}), \\ \Pi_{P+,T+} &= \frac{1}{4}(1 + \mathcal{P} + \mathcal{T} + \mathcal{PT}). \end{aligned}$$

True PT

$$PT+ = (P+, T+) \oplus (P-, T-).$$

The pseudo block supplies $(-, -)$.

Independence is tested at generic on-shell kinematics

1. Candidates

Start from the ordered mother list after Hermiticity and the requested P or PT projection.

2. Sample

Choose one nonzero, nonthreshold spacelike $t = t_0$; include all λ_i, λ_f and several nonspecial momentum directions.

3. Rank

Evaluate every candidate on the same stack. The rank is the number of independent covariants—and hence of FFs—in that sector.

One relation must work everywhere in the stack

A proposed linear dependence uses one set of coefficients for every spin transition and every sampled direction; coefficients may not be refitted point by point.

The fixed t_0 selects convenient basis labels; the final tensors and FFs retain their full t dependence.

Stable pivots turn the numerical rank into reproducible FF labels

$$A\Pi = Q \begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix},$$

$$\|R_{22}\| \ll \|R_{11}\|,$$

$$A_J = A_I X, \quad X = R_{11}^{-1} R_{12}.$$

A rank-revealing QR factorization (RRQR), checked by a singular-value decomposition (SVD), fixes N . A deterministic scan then keeps the earliest independent columns in the physically preselected mother-list order.

Stable pivot set means that the selected analytic span is unchanged under new directions, new t_0 , tolerance scans, QR/SVD comparison, and hold-out substitutions.

Finite pivot map

$$f_{\text{red}} = f_I + X f_J,$$

$$K = \Pi \begin{pmatrix} -X \\ \mathbf{1} \end{pmatrix}, \quad AK = 0.$$

Nonpivot FF contributions are transferred to pivot FFs; they are not silently discarded.

A finite algorithm works at every fixed spin

1. Represent

Build G_m and canonical chiral spinors for $0 \leq m \leq 2j$.

2. Generate

Create every index-compatible mother contraction; add the γ_5 block when P -odd or true- PT sectors are needed.

3. Reduce

Impose Hermiticity and the requested projector, then the chiral trace, Schouten, on-shell, and Gordon relations.

4. Certify

Evaluate generic points, compare RRQR/SVD, select ordered pivots, reconstruct exact nulls, and check the independent count.

The basis is rebuilt at the requested spin; it is never extrapolated from a lower-spin closed list.

Crossing counts the amplitudes; CPT identifies the direct sector

For the crossed particle-antiparticle pair,

$$\begin{aligned} S &= 0, 1, \dots, 2j, & L &= |J - S|, \dots, J + S, \\ P_{P\bar{P}} &= \varsigma_j (-1)^L, & C_{P\bar{P}} &= (-1)^{L+S}, \\ \varsigma_j &= (-1)^{2j}. \end{aligned}$$

$$N_R^{P,C}(j) = \sum_{(J,\pi) \in \mathcal{R}_R} \sum_{S=0}^{2j} \sum_{L=|J-S|}^{J+S} \delta_{\pi, \varsigma_j} (-1)^L \delta_{\xi_R, (-1)^{L+S}}.$$

Each allowed (S, L) wave contributes one reduced amplitude.

CPT dictionary

$$c = \chi_R p t, \quad \chi_S = \chi_A = \chi_{ST} = +1,$$

$$\chi_V = -1.$$

sector	interpretation
conventional P, C	$(p, t) = (+, +)$ and $c = \chi_R$
direct $P+$	$p = +$ and both $c = \pm \chi_R$
true $PT+$	$(+, +) \oplus (-, -)$ and $c = \chi_R$

The crossed count is basis independent; agreement with the on-shell rank certifies the construction.

The closed formulas expose exactly which symmetry sector is counted

Set $n = \lfloor j \rfloor$ and $\epsilon_j = 2(j - n) \in \{0, 1\}$. For $j > 0$:

current sector	conventional P, C	direct $P+$	direct true $PT+$
scalar	$n + 1$	$n + 1$	$2j + 1$
vector	$2j + 1$	$4n + 2 + \epsilon_j$	$4j + 1$
antisymmetric	$2j$	$6j + 1$	$4j$
symmetric-traceless	$5n + 2 + \epsilon_j$	$9n + 2 + 3\epsilon_j$	$10n + 1 + 5\epsilon_j$
all rank-two components	$8n + 3 + 2\epsilon_j$	$16n + 4 + 6\epsilon_j$	$16n + 2 + 8\epsilon_j$

The three columns answer different symmetry questions

The worked ranks count independent FFs in four current sectors

Each tuple is $N = (N_S, N_V, N_A, N_{ST})$: the numbers of independent scalar, vector, antisymmetric, and symmetric-traceless GFFs.

j	highest multipole	crossed P, C	direct $P+$	direct true $PT+$
$\frac{1}{2}$	G_1	(1, 2, 1, 3)	(1, 3, 4, 5)	(2, 3, 2, 6)
1	G_2	(2, 3, 2, 7)	(2, 6, 7, 11)	(3, 5, 4, 11)
$\frac{3}{2}$	G_3	(2, 4, 3, 8)	(2, 7, 10, 14)	(4, 7, 6, 16)
2	G_4	(3, 5, 4, 12)	(3, 10, 13, 20)	(5, 9, 8, 21)

Example: for $j = 1$, (3, 5, 4, 11) means 3 scalar + 5 vector + 4 antisymmetric + 11 ST GFFs in the true- PT sector.

Independent numerical checks

Closed formulas, parity-reduced sums, and explicit *JSL* enumeration

On-shell rank checks

All four spins pass QR/SVD, multiple- t , tolerance, and hold-out tests; the largest closure residual is 4.3×10^{-15} .

Each list item is one independent projected bilinear

$$\langle p', \lambda' | \hat{N}(0) | p, \lambda \rangle$$

$$(P, T) = (+, +)$$

$j = \frac{1}{2}$: monopole G_0 and dipole G_1

$$S : \bar{u}(p', \lambda') u(p, \lambda) F_0^S(t)$$

$$V : \bar{u}(p', \lambda') [2P^\mu F_0^V(t) + i\sigma^{\mu\Delta} F_1^V(t)] u(p, \lambda)$$

$$A : \bar{u}(p', \lambda') \left[P^{[\mu} \frac{i}{2} \sigma^{\nu]\rho} \Delta_\rho F_1^A(t) \right] u(p, \lambda)$$

$$ST : \bar{u}(p', \lambda') \left[2P^\mu P^\nu F_{0,0}^T(t) + 2(\Delta^\mu \Delta^\nu) F_{1,0}^T(t) \right. \\ \left. + P^{\{\mu} \frac{i}{2} \sigma^{\nu\}\rho} \Delta_\rho F_{1,1}^T(t) \right] u(p, \lambda)$$

$$Tr : \bar{u}(p', \lambda') [2M^2 g^{\mu\nu} F_1^{Tr}(t)] u(p, \lambda)$$

$j = 1$: add the quadrupole G_2

$$S : \bar{u}(p', \lambda') (F_0^S(t) G_0 + F_2^S(t) G_2^{P\Delta, P\Delta}) u(p, \lambda)$$

$$V : \bar{u}(p', \lambda') \left[2P^\mu (F_0^V(t) G_0 + F_2^V(t) G_2^{P\Delta, P\Delta}) \right. \\ \left. + iG_1^{\mu\Delta} F_1^V(t) \right] u(p, \lambda)$$

$$A : \bar{u}(p', \lambda') \left[P^{[\mu} \frac{i}{2} G_1^{\nu]\Delta} F_{0,1}^A(t) + \Delta^{[\mu} \frac{i}{2} G_1^{\nu]\Delta} F_{1,1}^A(t) \right] u(p, \lambda)$$

$$ST : \bar{u}(p', \lambda') \left[2P^\mu P^\nu (F_{1,0}^T(t) G_0 + F_{1,2}^T(t) G_2^{P\Delta, P\Delta}) \right. \\ + 2\Delta^\mu \Delta^\nu (F_{2,0}^T(t) G_0 + F_{2,2}^T(t) G_2^{P\Delta, P\Delta}) + P^{\{\mu} \frac{i}{2} G_1^{\nu\}\Delta} F_{1,1}^T(t) \\ \left. + \Delta^{\{\mu} \frac{i}{2} G_1^{\nu\}\Delta} F_{1,1}^T(t) + G_2^{\mu\Delta, \nu\Delta} F_{1,2}^T(t) \right] u(p, \lambda)$$

$$Tr : \bar{u}(p', \lambda') [2M^2 g^{\mu\nu} (F_{1,0}^T(t) G_0 + F_{1,2}^T(t) G_2^{P\Delta, P\Delta})] u(p, \lambda)$$

Applications: local moments, nonlocal programs, and arbitrary-spin EMT

What the basis directly organizes

- electromagnetic, axial, and energy-momentum matrix elements;
- gravitational form factors of spin-1, $\frac{3}{2}$, 2, and higher targets;
- an explicit target-multipole label for every operator covariant.

$G_0, \dots, G_{2j}$ make the target multipole content explicit while the finite quotient guarantees a complete, nonredundant FF basis.

Discussion: which target and current should we build next?

Spin-1 nuclei

Which vector, axial, or EMT basis best interfaces with deuteron GPD?

Resonances

Can the same stable-pivot workflow simplify transition matrix elements involving spin- $\frac{3}{2}$ states?

Analysis software

Which basis metadata should be standardized so phenomenology and lattice calculations exchange FFs unambiguously?

Questions and discussion

Full bilinear transformation laws

For $M_{\lambda_f \lambda_i}^{\mu_1 \dots \mu_r} = \bar{U}_f \Gamma^{\mu_1 \dots \mu_r} U_i$ and $\tilde{p} = (p^0, -\mathbf{p})$,

$$\begin{aligned} [\mathcal{P}M]_{\lambda_f \lambda_i}^{\mu_1 \dots \mu_r} &= \eta_P \prod_{k=1}^r P^{\mu_k}_{\alpha_k} M_{\lambda_f \lambda_i}^{\alpha_1 \dots \alpha_r}(\tilde{p}_f, \tilde{p}_i), \\ [\mathcal{T}M]_{\lambda_f \lambda_i}^{\mu_1 \dots \mu_r} &= \eta_T \prod_{k=1}^r P^{\mu_k}_{\alpha_k} \sum_{a,b} (\Theta_j)_{a\lambda_f} (\Theta_j)_{b\lambda_i} [M_{ab}^{\alpha_1 \dots \alpha_r}(\tilde{p}_f, \tilde{p}_i)]^*, \\ [\mathcal{PT}M]_{\lambda_f \lambda_i}^{\mu_1 \dots \mu_r} &= \eta_P \eta_T \sum_{a,b} (\Theta_j)_{a\lambda_f} (\Theta_j)_{b\lambda_i} [M_{ab}^{\mu_1 \dots \mu_r}(p_f, p_i)]^*. \end{aligned}$$

The two spacetime parity matrices cancel in the PT line; the Wigner rotations and complex conjugation do not.

Hermitian bilinears have distinct P , T , and PT grades

$$\mathbb{D}_m = i^m \bar{U}_f G_m U_i,$$

$$\tilde{\mathbb{D}}_m = i^m \bar{U}_f G_m \gamma_5 U_i.$$

Hermitian building block	P	physical T	PT
$\mathbb{D}_m$	proper	$(-1)^m$	$(-1)^m$
$\tilde{\mathbb{D}}_m$	pseudo	$(-1)^m$	$(-1)^{m+1}$
$\bar{U}_f U_i$	+	+	+
$i\bar{U}_f \gamma_5 U_i$	−	−	+

True PT is not the same selection as parity

The Hermitian scalar and pseudoscalar are both PT even. Hence

$PT+ = (P+, T+) \oplus (P-, T-)$, whereas a $P+$ projection removes the second summand.

The null space contains four logically distinct mechanisms

- 1 the single C_{red} trace;
- 2 four-dimensional Schouten identities;
- 3 on-shell relations, including $P \cdot \Delta = 0$;
- 4 the generalized Gordon system.

Define generalized elastic tensors $\mathcal{P}$ and $\mathcal{D}$ from the symmetrized and differenced products of $2j$ external momenta. Then

$$\begin{aligned}\overline{U}_f \Gamma U_i &= \frac{1}{2M^{2j}} \overline{U}_f \left(\{ \mathcal{P}^{(j)}, \Gamma \} + \frac{1}{2} [\mathcal{D}^{(j)}, \Gamma] \right) U_i, \\ 0 &= \overline{U}_f \left(\frac{1}{2} \{ \mathcal{P}^{(j)}, \Gamma \} + [\mathcal{D}^{(j)}, \Gamma] \right) U_i.\end{aligned}$$

Numerics certify rank; algebra certifies identities

A new numerical null vector is quoted as exact only after rational reconstruction and symbolic verification at independent on-shell points.

Formal quotient and fixed- t evaluation matrix

$$\mathfrak{B}_R^\Sigma(j) = \mathfrak{F}_R^\Sigma(j) / \mathfrak{N}_R^\Sigma(j),$$
$$A_R^\Sigma(j; t_0) = \bigoplus_{k, \lambda_f, \lambda_i} \mathcal{C}_{a,R}^\Sigma(P_k, \Delta_k; \lambda_f, \lambda_i).$$

$$N_R^\Sigma(j) = \text{rank } A_R^\Sigma(j; t_0),$$

Spaces

$\mathfrak{F}$ is the projected finite mother space. $\mathfrak{N}$ is its null subspace from C_{red} traces, Schouten identities, on-shell kinematics, and generalized Gordon relations. $\mathfrak{B}$ is the space of equivalence classes modulo those nulls.

Ordered evaluation

Column a is candidate $\mathcal{C}_a$; rows stack the kinematic point k and all spin transitions (λ_f, λ_i) . One common coefficient vector lies in $\ker A$ exactly when it is an on-shell null direction.

Parity-reduced one-dimensional J^{PC} count

For fixed J, S , the number of L values with parity $\ell = \pm 1$ is

$$\mathcal{L}_{J,S}(\ell) = \min(J, S) + \frac{1 + \ell(-1)^{J+S}}{2}.$$

The crossed count becomes

$$N_R^{P,C}(j) = \sum_{(J,\pi) \in \mathcal{R}_R} \sum_{S=0}^{2j} \frac{1 + \xi_R \varsigma_j \pi (-1)^S}{2} \left[\min(J, S) + \frac{1 + \varsigma_j \pi (-1)^{J+S}}{2} \right].$$

Why this form is useful

It removes the explicit L loop and provides an implementation independent of both the triple sum and the covariant-rank calculation.

Spin- $\frac{1}{2}$: true PT in the scalar sector

$$\begin{aligned}\langle O \rangle_{PT+} &= \bar{U}_f U_i F_S^{++}(t) + i \bar{U}_f \gamma_5 U_i F_S^{--}(t), \\ \langle O^\mu \rangle_{P+, \mathcal{X}+} &= P^\mu S_0 F_0^{V,e}(t) + V_1^\mu[\Delta] F_0^{V,o}(t).\end{aligned}$$

- Parity alone keeps only the first scalar term.
- True PT keeps both Hermitian terms because $(-1) \times (-1) = +1$.
- Current conservation, when appropriate, is imposed after the complete kinematic basis is constructed.

Mother sizes and reduced dimensions in the worked examples

Each entry is $M_R \rightarrow N_R^{P+}$; parentheses give $2M_R \rightarrow N_R^{PT+}$.

j	scalar	vector	antisymmetric	symmetric-traceless
$\frac{1}{2}$	$1 \rightarrow 1 (2 \rightarrow 2)$	$4 \rightarrow 3 (8 \rightarrow 3)$	$6 \rightarrow 4 (12 \rightarrow 2)$	$7 \rightarrow 5 (14 \rightarrow 6)$
1	$2 \rightarrow 2 (4 \rightarrow 3)$	$8 \rightarrow 6 (16 \rightarrow 5)$	$13 \rightarrow 7 (26 \rightarrow 4)$	$17 \rightarrow 11 (34 \rightarrow 11)$
$\frac{3}{2}$	$2 \rightarrow 2 (4 \rightarrow 4)$	$10 \rightarrow 7 (20 \rightarrow 7)$	$19 \rightarrow 10 (38 \rightarrow 6)$	$24 \rightarrow 14 (48 \rightarrow 16)$
2	$3 \rightarrow 3 (6 \rightarrow 5)$	$14 \rightarrow 10 (28 \rightarrow 9)$	$26 \rightarrow 13 (52 \rightarrow 8)$	$34 \rightarrow 20 (68 \rightarrow 21)$

The mother list grows, but the reduction remains a finite matrix problem at every fixed j .

Each list item is one independent projected bilinear

How to read the notation

$s_r = \mathcal{S}_r$ closes every multipole pair; $p_r = P\mathcal{S}_r$ carries a vector index with P ; $v_m^X = V_m[X]$ opens one multipole pair; $a_m^{XY} = X \wedge V_m[Y]$ is antisymmetric; and t, w, q are symmetric-traceless combinations with zero, one, or two opened pairs. A tilde replaces $\mathbb{D}_m$ by the γ_5 bilinear $\tilde{\mathbb{D}}_m$.

$j = \frac{1}{2}$: monopole G_0 and dipole G_1

$S : (s_0 \mid \tilde{s}_0),$
 $V : (p_0, v_1^\Delta \mid \tilde{p}_0),$
 $A : (a_1^{P\Delta} \mid \tilde{a}_1^{\Delta P}),$
ST : 3 proper $\mid$ 3 pseudo.

$$N^{PT+} = (2, 3, 2, 6)$$

$j = 1$: add the quadrupole G_2

The same families acquire s_1 , t_1 , and the two-open-pair term q_2 . The proper|pseudo splits are

$S : 2 \mid 1,$ $V : 3 \mid 2,$
 $A : 2 \mid 2,$ ST : 7 $\mid$ 4.

$$N^{PT+} = (3, 5, 4, 11)$$

The bar separates proper $(P, T) = (+, +)$ from pseudo $(-, -)$ bilinears; their union is true- PT even.

Numerical validation protocol

- construct G_m directly from Cartesian STF spin multipoles and Q_{red} ;
- test parity, physical T , the i^m phase law, and Hermiticity;
- use all $(2j+1)^2$ spin transitions and several momentum directions at fixed t ;
- compare column-pivoted QR with SVD over $5 \times 10^{-10} \leq r_{\text{tol}} \leq 5 \times 10^{-9}$;
- repeat at $q_0 = 0.49, 0.73, 1.07$ and append hold-out directions;
- verify the crossed triple sum, parity-reduced sum, and closed formulas for $0 \leq j \leq 20$.

test	maximum relative residual
parity covariance	0
physical- T covariance and phase law	4.005×10^{-16}
Hermiticity	1.154×10^{-15}
odd scalar on the even tower	7.564×10^{-16}



W. Cosyn and F. Vera, "Spinor representations for fields with any spin: Lorentz tensor basis for operators and covariant multipole decomposition," *JHEP* **12** (2025) 068, arXiv:2503.22046.



F. Vera, "Observables for targets with any spin," POETIC 2025, 23 February 2025.



W. Cosyn and F. Vera, "Chiral construction of generalized form factors for massive states of arbitrary spin," working manuscript and numerical verification, July 2026.



H. Sun, T. Tan and J.-H. Yu, "Covariant Construction of Generalized Form Factors," arXiv:2604.25771v1 [hep-ph] (2026).



S. Cotogno, C. Lorcé, P. Lowdon and M. Morales, "Covariant multipole expansion of local currents for massive states of any spin," *Phys. Rev. D* **101** (2020) 056016.



G. S. Bali et al., "Nucleon generalized form factors from two-flavor lattice QCD," arXiv:1812.08256.



S. Bhattacharya et al., "Generalized parton distributions from the pseudo-distribution approach on the lattice," arXiv:2405.04414.



Towards Improved Hadron Femtography with Hard Exclusive Reactions, edition V, University of Virginia, 20–24 July 2026, <https://indico.jlab.org/event/1059/overview>.