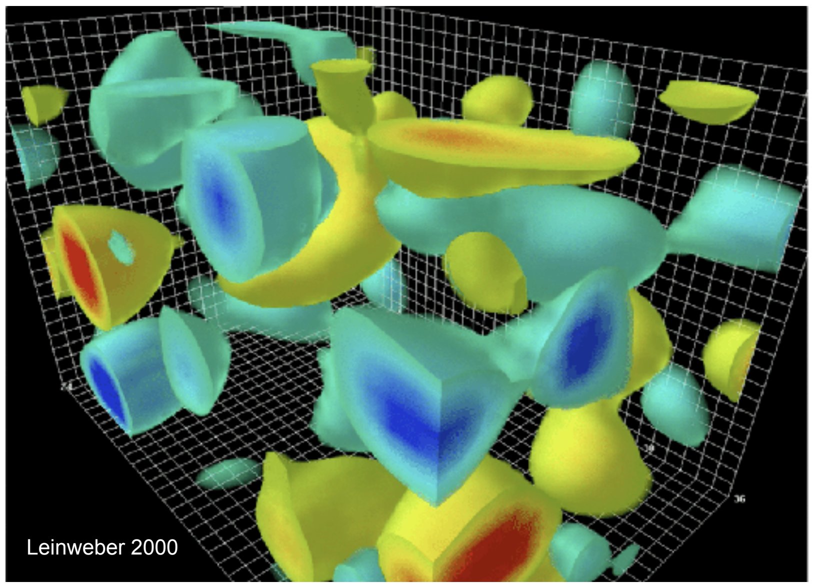
Quark orbital angular momentum from QCD instantons

C. Weiss (JLab) [weiss@jlab.org],
Towards improved hadron femtography, U. Virginia, 20-24 July 2026 [Webpage]



Explore connection: Chiral symmetry breaking
→ effective spin-flavor dynamics → quark OAM

Employ instanton vacuum: ChSB from topological fluctuations of gauge fields

Explain flavor-nonsinglet $L_{u-d} < 0$
Obtain interpretation as "chiral magnetic effect"

Based on:
J-Y Kim, H-Y Won, C Weiss, 2605.23125 [INSPIRE],
J-Y Kim, C Weiss, PLB 848 (2024) 138387 [INSPIRE],

Angular momentum in QCD

Energy-momentum tensor form factors

Spin and orbital angular momentum

Effective dynamics from ChSB

Instanton vacuum

Effective spin-flavor dynamics from ChSB

Nucleon as mean-field solution

Orbital angular momentum in nucleon

Instanton effect in twist-3 EMT

Large flavor-nonsinglet $L_{u-d} < 0$ in nucleon

Interpretation as "chiral magnetic effect"

Angular momentum: Operators

$$T^{\mu\nu}(x) = \sum_f T_f^{\mu\nu} + T_g^{\mu\nu}$$

$$T_f^{\mu\nu} = \bar{\psi}_f \, \gamma^\mu \, \overleftrightarrow{\nabla}^\nu \, \psi_f$$

$$\overleftrightarrow{\nabla}^\nu \equiv \overleftrightarrow{\partial} - igA$$

$T_f^{\{\mu\nu\}}$ – trace   twist-2

$T_f^{[\mu\nu]}$ twist-3

$$J_f^{\mu\alpha\beta} = x^\alpha T_f^{\mu\beta} - x^\beta T_f^{\mu\alpha}$$

Energy-momentum tensor

Conserved current $\partial_\mu T^{\mu\nu} = 0$

Measures total energy/momentum/stress carried by QCD fields

Contribution of quark fields (flavor f)

Not conserved individually

Contains symmetric/antisymmetric parts with twist-2 and 3

Angular momentum of quark fields

Contains orbital and spin angular momentum

Spin-orbital separation gauge-invariant for quarks, not for gluons

Lorce, Leader 2014; Lorce, Mantovani, Pasquini 2017

Angular momentum: Nucleon

$$\langle N' | T_f^{\mu\nu} | N \rangle = \bar{u}' \left[\frac{P^\mu P^\nu}{M_N} A_f(t) + \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{4M_N} D_f(t) \right.$$
$$\left. + \frac{P^{\{\mu} i\sigma^{\nu\}\lambda} \Delta_\lambda}{2M_N} J_f(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda} \Delta_\lambda}{2M_N} S_f(t) + M_N g^{\mu\nu} \bar{C}_f(t) \right] u$$

$$\langle T_f^{0k} \rangle (\mathbf{x}) \equiv \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\mathbf{x}\cdot\boldsymbol{\Delta}} \frac{\langle N' | T_f^{0k}(0) | N \rangle}{2P^0}$$

$$L_f \equiv \int d^3x\, \epsilon^{3jk} x^j \langle T_f^{0k} \rangle (\mathbf{x}) \qquad = J_f(0) - S_f(0)$$

$$S_f \equiv -\tfrac{1}{2} \int d^3x\, \epsilon^{3jk} x^j \langle T_f^{[0k]} \rangle (\mathbf{x}) \qquad = S_f(0)$$

$$J_f \equiv \tfrac{1}{2} \int d^3x\, \epsilon^{3jk} x^j \langle T_f^{\{0k\}} \rangle (\mathbf{x}) \qquad = J_f(0)$$

$$J_f = S_f + L_f$$

Nucleon matrix element of quark EMT

Covariant parametrization $P \equiv \frac{1}{2}(p' + p),\ \Delta \equiv p' - p$

Invariant form factors $A_f, D_f, J_f, S_f(t) \leftrightarrow$ multipoles

Interpretation in special frames

Here: Use Breit frame $\mathbf{P} = 0,\ \Delta^0 \neq 0$

Polyakov 2003: Mechanical properties

Alt: Use light-front $\mathbf{P}_T = 0,\ \Delta^+ = 0$

Correspondence understood. E.g. Lorce, Moutarde, Trawinski 2019

Angular momentum in nucleon

Orbital, spin and total AM, natural decomposition

Defined in terms of EMT invariant form factors,
only interpretation refers to frame

Here: Accept definitions, explore dynamical content

Quark spin as axial charge

$$T_f^{[\mu\nu]} = \bar{\psi}_f \, \gamma^{[\mu} \overleftrightarrow{\nabla}^{\nu]} \psi_f$$

$$= -\frac{1}{4} \epsilon^{\mu\nu\alpha\beta} \partial_\alpha [\bar{\psi}_f(x) \gamma_\beta \gamma_5 \psi_f(x)]$$

QCD equations of motion: Antisymmetric part of quark EMT operator equal to total derivative of axial current

$$\langle N \,|\, \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f \,|\, N \rangle = \bar{u} \gamma^\mu \gamma_5 u \; g_{A,f}$$

Nucleon matrix element of axial current (flavor f)

$$S_f = \frac{1}{2} g_{A,f}$$

Quark spin as axial charge

Orbital angular momentum

$$T_f^{0i} \;=\; \underbrace{\frac{1}{2} T_f^{\{0i\}}}_{\text{twist-2}} \;+\; \underbrace{\frac{1}{2} T_f^{[\mu\nu]}}_{\text{twist-3}}$$

Non-symmetric quark EMT in OAM involves twist-2 and twist-3 parts of tensor

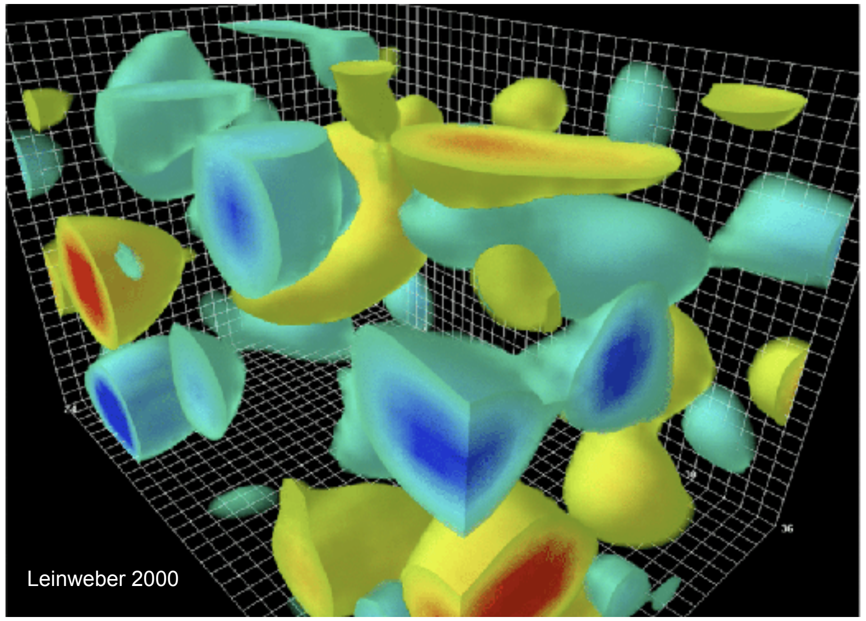
OAM sensitive to interactions - dynamics!

Angular momentum: This study

Goals

- Compute L_f, S_f, J_f in effective dynamics resulting from chiral symmetry breaking
- Explain interaction effects in $L_f \leftrightarrow$ twist-3
- Understand nucleon spin decomposition in flavor-nonsinglet sector $u - d$

Effective dynamics: Topological gauge fields



Yellow/blue: Top. charge density positive/negative

QCD vacuum populated by localized gauge fields with topological charge $\frac{1}{32\pi^2}\int F\tilde{F} \approx \pm 1$

Instantons: Classical solutions of YM equations, self-dual fields $\tilde{F} = \pm F$, tunneling trajectories

Typical size $\bar{\rho} \approx$ 0.3 fm, separation $\bar{R} \approx$ 1 fm

Induce localized zero modes of fermion field with definite chirality

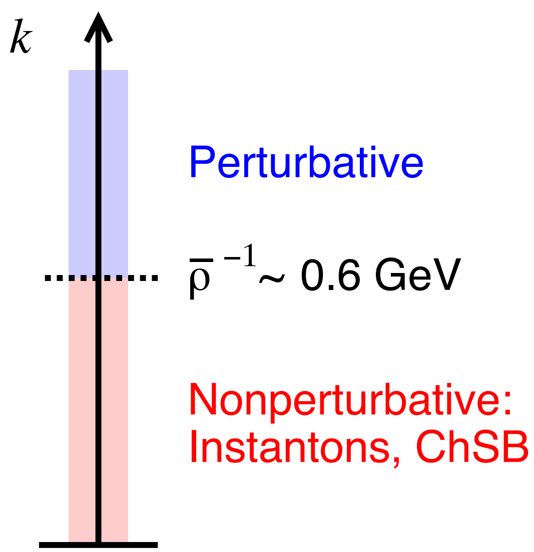
Chiral symmetry breaking: Condensate, mass generation, effective spin-flavor dynamics

Observed in LQCD: Smooth gauge field configurations from cooling or gradient flow; instanton effects in correlation functions

Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al. 1993; DeGrand et al 1997; de Forcrand et al 1997, …, Athenodorou et al 2018; Alexandrou et al 2020

Effective description: Instanton vacuum

Shuryak 1982; Diakonov, Petrov 1984



Separation of modes

$k > \bar{\rho}^{-1}$: Integrate perturbatively: Renormalization

$k < \bar{\rho}^{-1}$: Integrate nonperturbatively: Instantons, ChSB

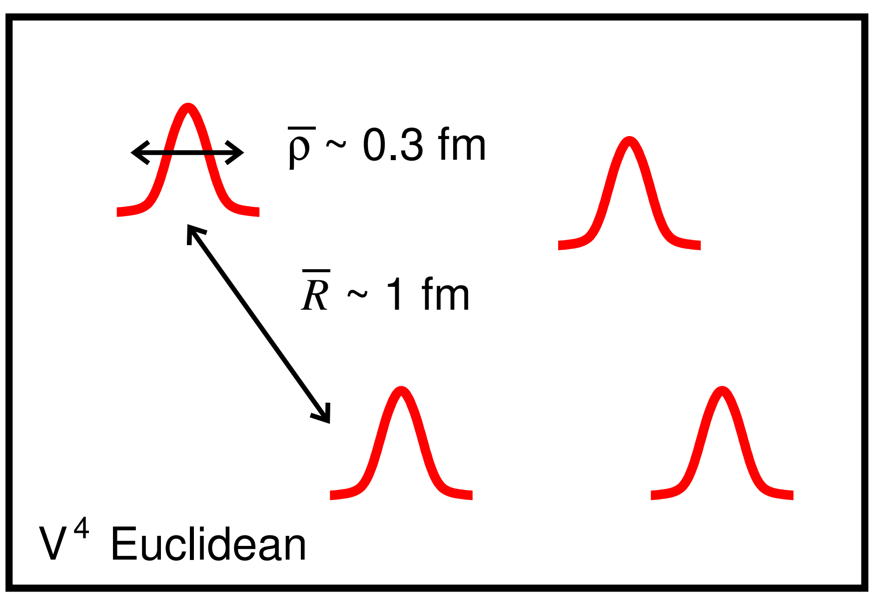
Nonperturbative integration

$$A(x) = \sum_I A_I(x \,|\, z_I, \rho_I, O_I) + \sum_{\bar{I}} A_{\bar{I}}(\,..\,)$$

gauge potential → classical top. fields

$$\int [DA] \rightarrow \int \prod_{I,\bar{I}} dz_I \, d\rho_I \, dO_I \, d_0(\rho_I)$$

functional integral → collective coordinates

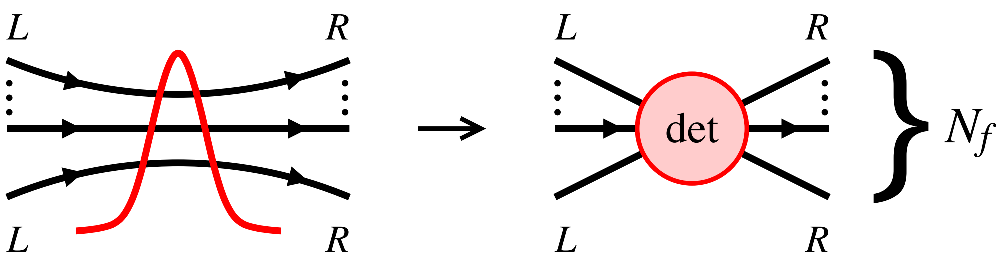


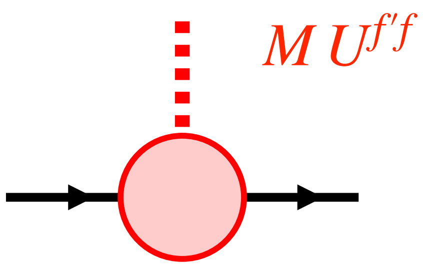
Variational approximation to QCD functional integral

Shuryak 1982; Diakonov, Petrov 1984. Many considerations/elements

Small parameter: Packing fraction $\pi^2 \bar{\rho}^4 / \bar{R}^4 \approx 0.1$

Effective dynamics: Quarks





$$Z_{\text{eff}} = \int DU \int D\bar{\psi} D\psi \, \exp\left(i \int d^4x \, \mathscr{L}_{\text{eff}} \right)$$

$$\mathscr{L}_{\text{eff}} = \bar{\psi}(x) \left[i\gamma\partial - M\, U^{\gamma_5}(x) \right] \psi(x)$$

$$U^{\gamma_5} \equiv \frac{1+\gamma_5}{2} U + \frac{1-\gamma_5}{2} U^\dagger$$

Chiral symmetry breaking

Single instanton: Multifermion vertex $\propto$ det $\bar{\psi}^{f'}_L \psi^f_R$

Finite density: Dynamical quark mass $M \sim$ 0.3-0.4 GeV

Active for momenta $k \lesssim \bar{\rho}^{-1} \approx$ 0.6 GeV

Large-N_c limit and bosonization

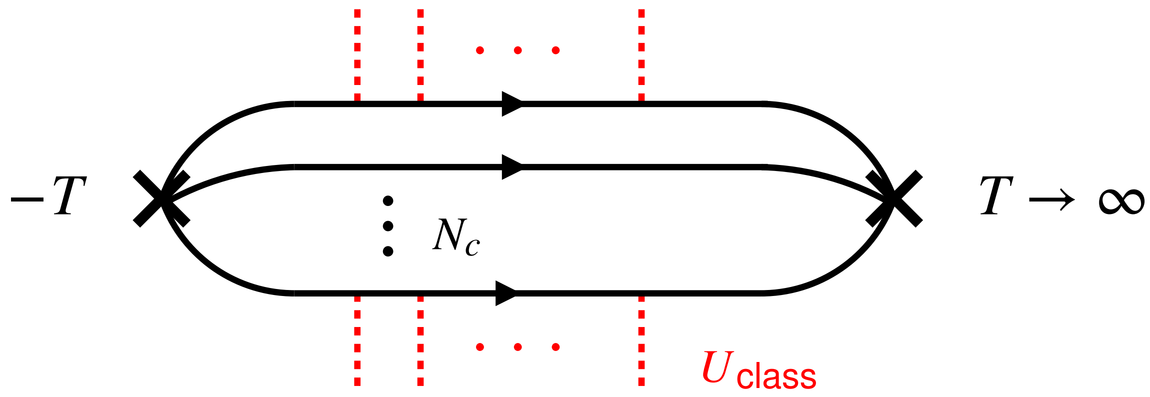
Fermion integral computed in saddle point approximation
Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; ...

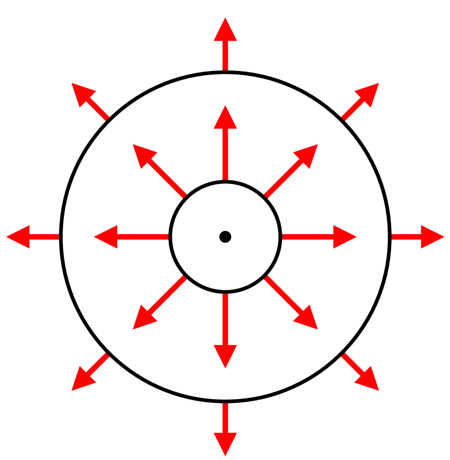
$M\, \bar{\psi}^{f'}_L U^{f'f} \psi^f_R$, $\quad U^{f'f} \equiv \langle \text{det}'_{f'f}\, \bar{\psi}_L \psi_R \rangle$ chiral boson field

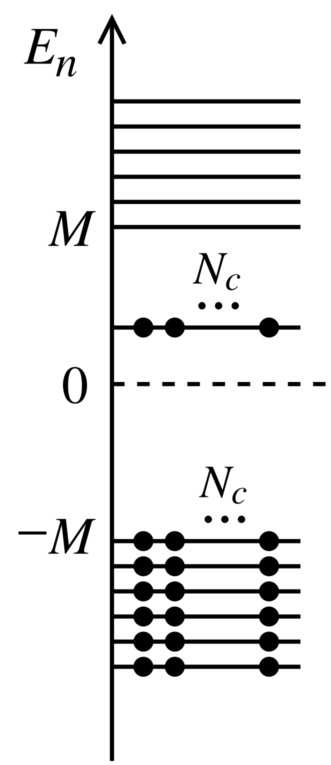
Spin-flavor interactions of quarks with chiral boson field

Meson correlation functions: Pion pole, η' mass, ...

Effective dynamics: Nucleon







$U_{\text{class}}(\mathbf{x}) = \exp[i\tau^a \hat{x}^a P(r)]$

Realization of general mean-field picture of baryons in large-N_c QCD Witten 1983

Baryon correlation function

Develops classical chiral field $U_{\text{class}} \neq 1$ (“soliton”)

Isospin along spatial direction ("hedgehog")

Mean-field state

Quarks move in single-particle orbits

Hamiltonian $H = \gamma^0(\gamma \mathbf{p} + M U_{\text{class}}^{\gamma_5})$

Wave functions $H\Phi_n = E_n \Phi_n$

Spectrum contains discrete level and continuum

Self-consistent system
Kahana, Ripka1984; Diakonov, Petrov, Pobylitsa 1988

Baryon states

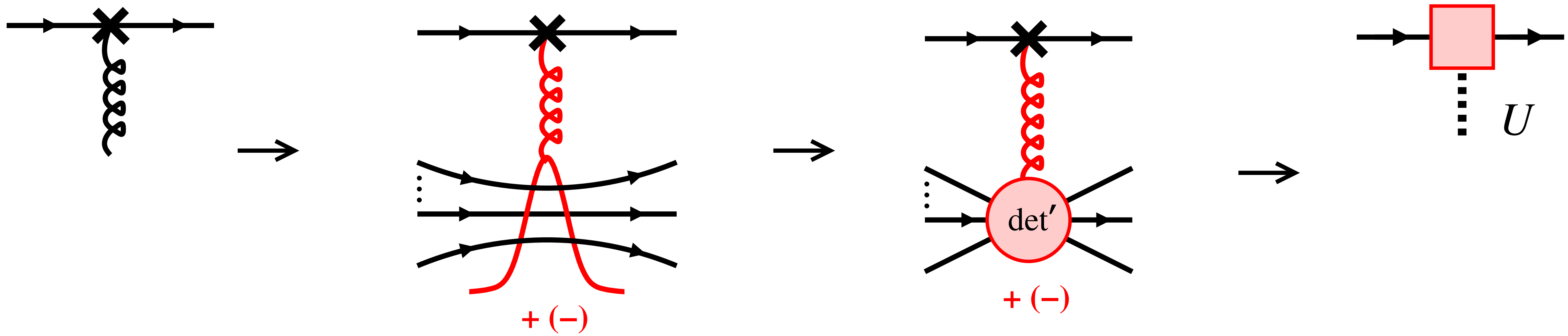
Spin/isospin states from collective rotations

Matrix elements computed in $1/N_c$ expansion

OAM: Evaluating QCD operators

$\bar{\psi}\,\mathscr{F}[A]\,\psi$

$\bar{\psi}\,\mathscr{F}_{\text{eff}}[U]\,\psi$



QCD operator
with gauge fields

(at scale $\mu = \rho^{-1}$)

Evaluated in instanton
gauge field

Effective operator with
multifermion interaction

Effective operator in
bosonized form

Effective operators: Systematic method, expansion in instanton packing fraction and $1/N_c$
Diakonov, Polyakov, Weiss, 1995

QCD gauge interactions → chiral spin-flavor interactions

OAM: Twist-3 quark EMT

$$T^{[\alpha\beta]}_{u-d}(x) \;=\; \bar{\psi}(x)\,\gamma^{[\alpha}\nabla^{\beta]}\tau^3\,\psi(x)$$

QCD twist-3 quark EMT,
flavor-nonsinglet τ^3

$$\downarrow$$

$$T^{[\alpha\beta]}_{u-d,\,\mathrm{eff}}(x) \;=\; \bar{\psi}(x)\,\left(\gamma^{[\alpha}\partial^{\beta]}\tau + \tfrac{i}{4}M\sigma^{\alpha\beta}[U^{\gamma_5}(x),\tau^3]\right)\psi(x)$$

Effective operator with
chiral spin-flavor interaction

$$= \bar{\psi}(x)\left(\;\ldots\; - \tfrac{i}{2}M\epsilon^{3bc}\pi^b(x)\,\sigma^{\mu\nu}\gamma_5\,\tau^c\right)\psi(x)$$

Pion field couples to quark spin:
Chiral-odd, isovector

$$U^{\gamma_5}(x) = \sigma(x) + i\gamma_5\tau^a\pi^a(x)$$

Spin-flavor dependence: Instanton effect specific to twist-3 flavor-nonsinglet EMT,
absent in flavor-singlet or twist-2 EMT

Coefficient: Given by dynamical quark mass, effect of ChSB, large

J-Y Kim, Weiss 2024

$$T^{[\alpha\beta]}_{u-d,\,\mathrm{eff}}(x) \;=\; \bar\psi(x)\,\left(\gamma^{[\alpha}\partial^{\beta]}\tau + \frac{i}{4} M\sigma^{\alpha\beta}[U^{\gamma_5}(x), \tau^3]\right)\psi(x)$$

$$= \; -\frac{1}{4}\,\epsilon^{\alpha\beta\gamma\delta}\,\partial_\gamma[\bar\psi(x)\,\gamma_\delta\gamma_5\tau\,\psi(x)]_{\mathrm{eff}}$$

axial current operator in effective theory

Equation-of-motion relation

Twist-3 effective operator obeys same equation-of-motion relation as QCD operator

Now realized through effective EOM of massive quark field $[i\gamma\partial - MU^{\gamma_5}]\,\psi = 0$

Result of consistent treatment of effective dynamics and effective operators

J-Y Kim, Weiss 2024

OAM: Angular momentum decomposition

$T^{0i}_{u-d}\,,$ $\quad U(x) \to U_{\rm cl}(\mathbf{x})$ classical chiral field in nucleon

$$L_{u-d} = -\frac{N_c}{3} \sum_{n\,\mathrm{occ}} \int d^3x\, \Phi^\dagger_n(\mathbf{x}) \left[L^3 \tau^3 - \tfrac{1}{2} M \epsilon^{3jk} \epsilon^{3bc} x^j \pi^b_{\rm cl}(\mathbf{x}) \gamma^0 \Sigma^k \tau^c \right] \Phi_n(\mathbf{x})$$

nucleon matrix element of effective operator as sum over quark levels

S_{u-d}, J_{u-d} similar expressions

quarks interact with classical chiral field

AM decomposition

J_{u-d} 0.24

S_{u-d} 0.49

L_{u-d} $-0.25 = -0.04 - 0.21$

Kim, Won, Weiss 2026

Large $L_{u-d} < 0$ obtained due to chiral interaction

Explains LQCD results for J_{u-d} and S_{u-d}

EMTC, χQCD: $L_{u-d} \sim -0.48$ (scale $\mu = 2$ GeV)

Spin sum rule $S_{u-d} + L_{u-d} = J_{u-d}$ satisfied in $1/N_c$ expansion

NLO corrections increase S_{u-d}, make L_{u-d} more negative

$$L_{u-d}[\text{int}] = \int d^3x\ \epsilon^{3jk}\epsilon^{3bc} B^{jb}(\mathbf{x})\, S^{kc}(\mathbf{x})$$

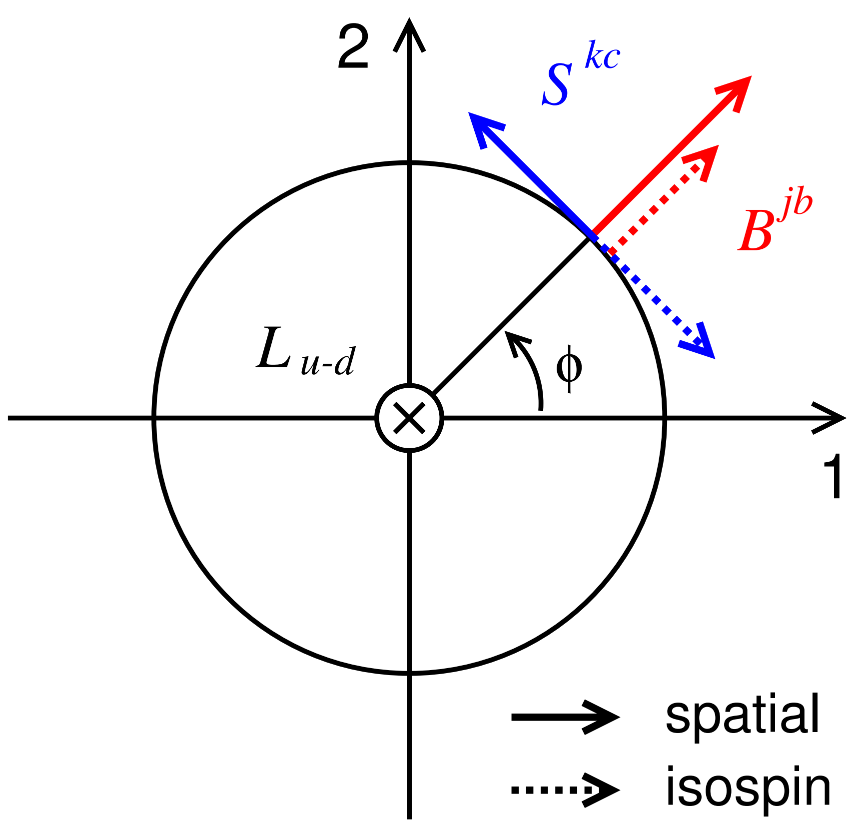
Alt. form of chiral interaction term

$$B^{jb}(\mathbf{x}) \equiv M\, x^j \pi^b_{\text{cl}}(\mathbf{x})$$

"Chiral magnetic field" formed from classical pion field and radius vector

$$S^{kc}(\mathbf{x}) \equiv \frac{N_c}{3} \sum_{n\,\text{occ}} \Phi^\dagger_n(\mathbf{x}) \left[\tfrac{1}{2}\gamma^0 \Sigma^k \tau^c\right] \Phi_n(\mathbf{x})$$

Quark spin-isospin current in large-N_c nucleon



Interpretation

L_{u-d} as result of "spin precession" of quarks caused by chiral magnetic field

Kim, Won, Weiss 2026

Direction: Quark spin/isospin tangential in 1,2 plane
Magnetic field radial in 1,2 plane. L_{u-d} in 3-direction

Sign: Quark spin and isospin antialigned, $L_{u-d} < 0$

Magnitude: Large effect, not relativistic correction

Analogy with chiral magnetic effect in hot/dense matter?
Kharzeev 2006; Fukushima, Kharzeev, Warringa 2008

OAM: Summary

- Instantons induce chiral spin-flavor interaction in flavor-nonsinglet twist-3 EMT:
  Quark spin/isospin couples to pion field

- Effective twist-3 EMT operator obeys same equation-of-motion relation with axial current as original QCD operator

- Interaction of quarks with classical chiral field in nucleon generates large $L_{u-d} < 0$

- Effect explains large difference between J_{u-d} and S_{u-d} observed in LQCD

- Effect can be interpreted as "chiral magnetic effect"

Quark spin-orbit correlations in nucleon

$T^{\mu\nu}_{5,f} = \bar{\psi}_f \gamma^\mu \gamma_5 \overleftrightarrow{\nabla}^\nu \psi_f$ parity-odd analog of EMT — Lorce 2014

Large instanton effect in twist-3 tensor operator — J-Y Kim, H-C Kim, H-Y Won, C Weiss Phys.Rev.D 110 (2024)

t-dependent gravitational form factors and densities

Nucleon form factors in large-N_c mean field picture — Goeke, Grabis, Ossmann, Polyakov, Schweitzer, 2007

$\bar{C}$ form factor of quark and gluon EMT — Polyakov, Son 2018

Gluonic GFFs from instanton-antiinstanton molecules — Liu, Shuryak, Zahed 2024+

Trace anomaly from instanton density fluctuations — Diakonov, Polyakov, Weiss 1996; Liu, Shuryak, Weiss, Zahed 2024

Nucleon matrix elements of QCD operators

Higher-twist operators in DIS polarized/unpolarized — Balla, Polyakov, Weiss 1998; Dressler, Maul, Weiss, 2000

Higher-dimensional operators in BSM physics — Weiss 2021

Recent review: Weiss, *Acta Phys.Polon.B* 56, 3-A7 (2025) [INSPIRE]

Extensions

Instanton vacuum from lattice QCD

Gradient flow: Controlled resolution scale, RNG-type

Lüscher, Weisz 2011

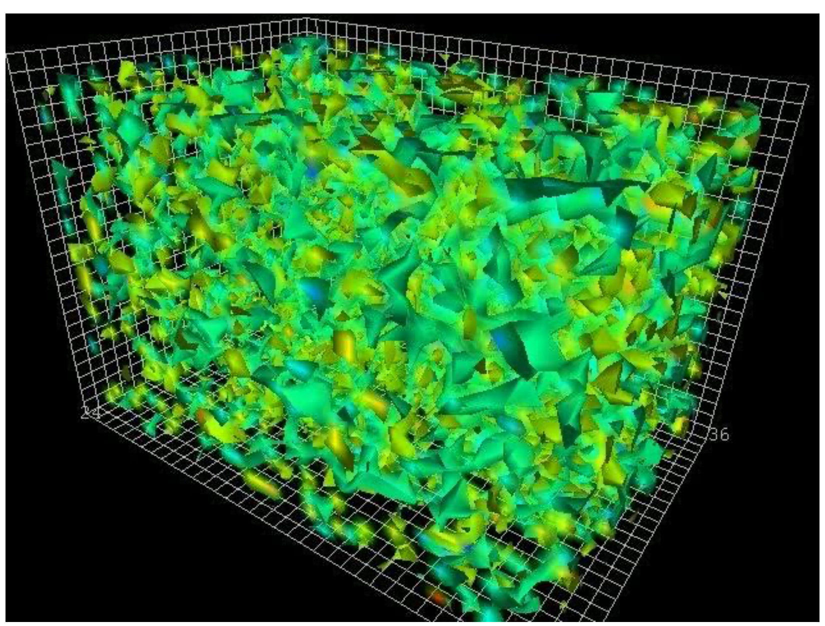
Analyze "smooth" configurations, identify instantons, study distributions...

Athenodorou et al 2018; Alexandrou et al 2020, ...

Instanton vacuum as semiclassical description of "low-resolution" QCD

Scale evolution, instanton-antiinstanton molecules at intermediate scales: A. Sturzu, Wei-Yang Liu, CW; study in progress

Many opportunities!



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