

Hadronic Matrix Elements of the QCD EMT

—Lattice validation of hadron mass and trace-anomaly sum rules

Xiang Gao

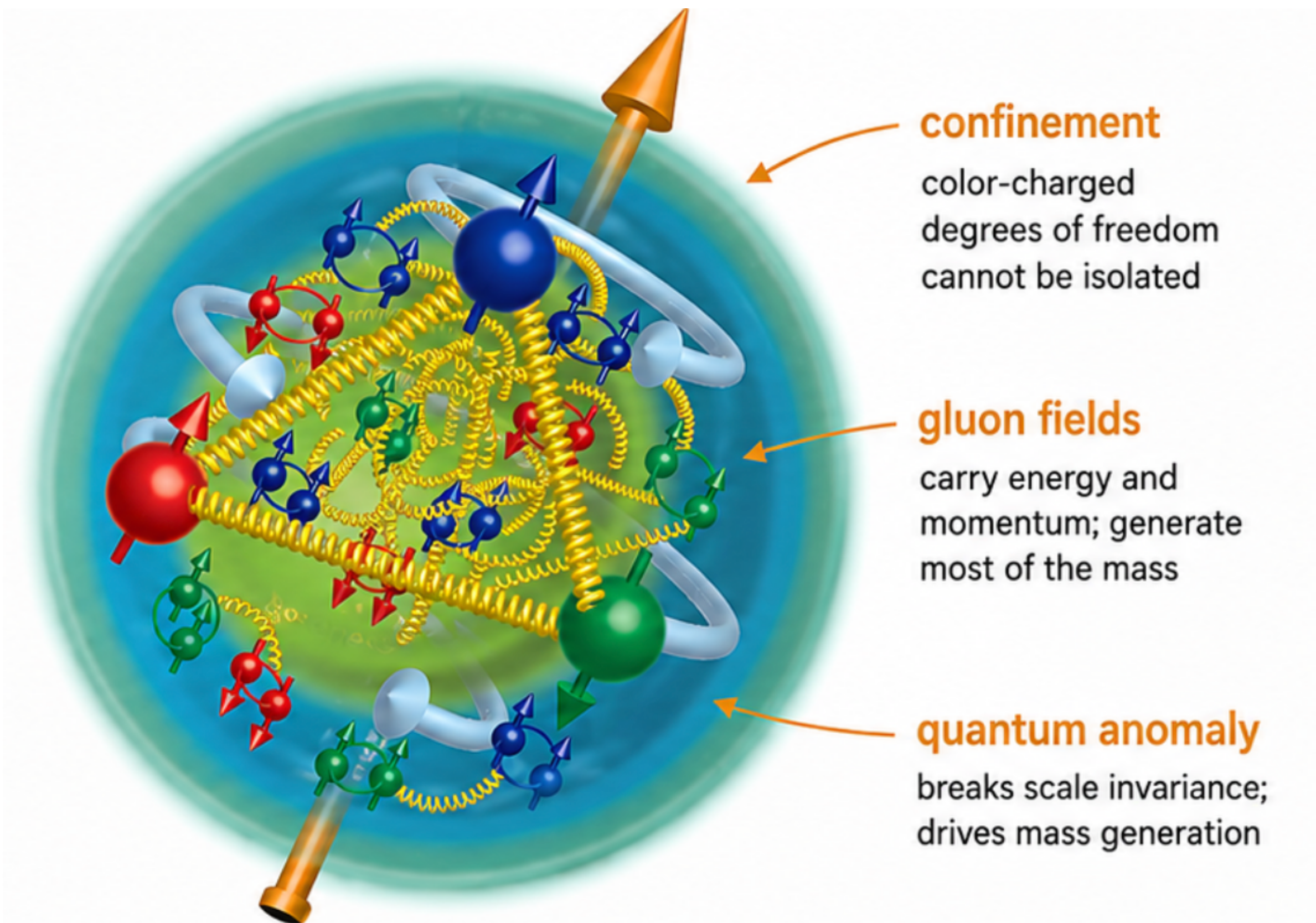
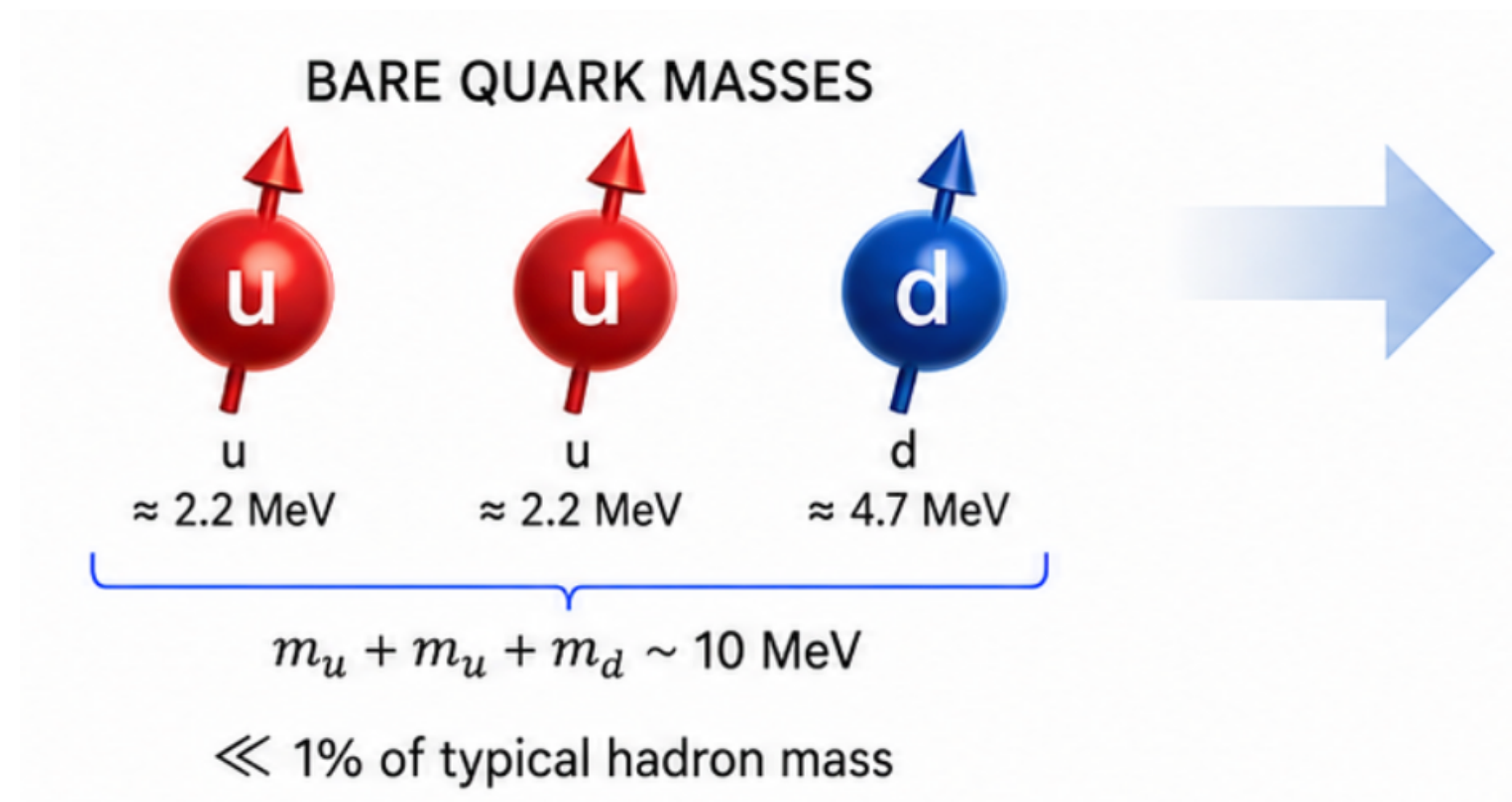
Based on arXiv: [2601.13070](https://arxiv.org/abs/2601.13070)

in collaboration with D. Bollweg, H.-T. Ding, R. Luo, S. Mukherjee

Towards improved hadron femtography with hard exclusive reactions, edition V,
University of Virginia, Jul 20 – 24, 2026

Hadron mass is a QCD question

Most visible mass is generated by **QCD dynamics**, **not** by bare quark masses.



How to understand the hadron mass in terms of quark and gluon fields?

The QCD EMT: from quark-gluon fields to hadron mass

To understand hadron mass in terms of **quarks** and **gluons**, we need the operator that measures **energy** and **momentum** in QCD.

Noether current of
spacetime translations

$$T_{\text{QCD}}^{\mu\nu} = T_{q,R}^{\mu\nu} + T_{g,R}^{\mu\nu}$$

$$P^\mu = \int d^3x T^{0\mu}$$

$$H_{\text{QCD}} = \int d^3x T^{00}$$

Hadron matrix elements

$$\langle H | T_{q,g}^{\mu\nu} | H \rangle$$

$\langle p' | T^{\mu\nu} | p \rangle \longrightarrow$ *gravitational form factors*

$\langle p | T^{00} | p \rangle \longrightarrow$ *energy / rest-frame mass*

$\langle p | T^\mu_\mu | p \rangle \longrightarrow$ *scale breaking / trace anomaly*

The EMT connects quark-gluon dynamics to hadron mass, GFFs, and the trace anomaly.

How do we weigh a hadron?

$$T^{\mu\nu} = T_{q,R}^{\mu\nu}(\mu) + T_{g,R}^{\mu\nu}(\mu), \quad \partial_\mu T^{\mu\nu} = 0$$

$T^{\mu\nu}$ is conserved current; q/g components are scale and scheme dependent.

$$M_H = \frac{1}{2M_H} \langle H | T^{00} | H \rangle$$

Rest-frame Hamiltonian definition of mass

● Two-terms

$$T^{00} = T_{q,R}^{00} + T_{g,R}^{00}$$

$$M = U_q + U_g$$

Quark and gluon
energy contributions

● Three-terms

$$M = (U_q - \sigma) + U_g + \sigma$$

$$\sigma \equiv \frac{1}{2M_H} \langle H | \sum_f m_f \bar{\psi}_f(x) \psi_f(x) | H \rangle$$

Explicit quark-mass
contribution isolated

● Ji' four-term decomposition

$$T^{00} = \bar{T}_{q,R}^{00} + \bar{T}_{g,R}^{00} + \frac{1}{4} T^\mu_\mu$$

$$M = M_q + M_g + M_a + \sigma$$

Traceless and trace
parts separated

How does QCD acquire a scale?

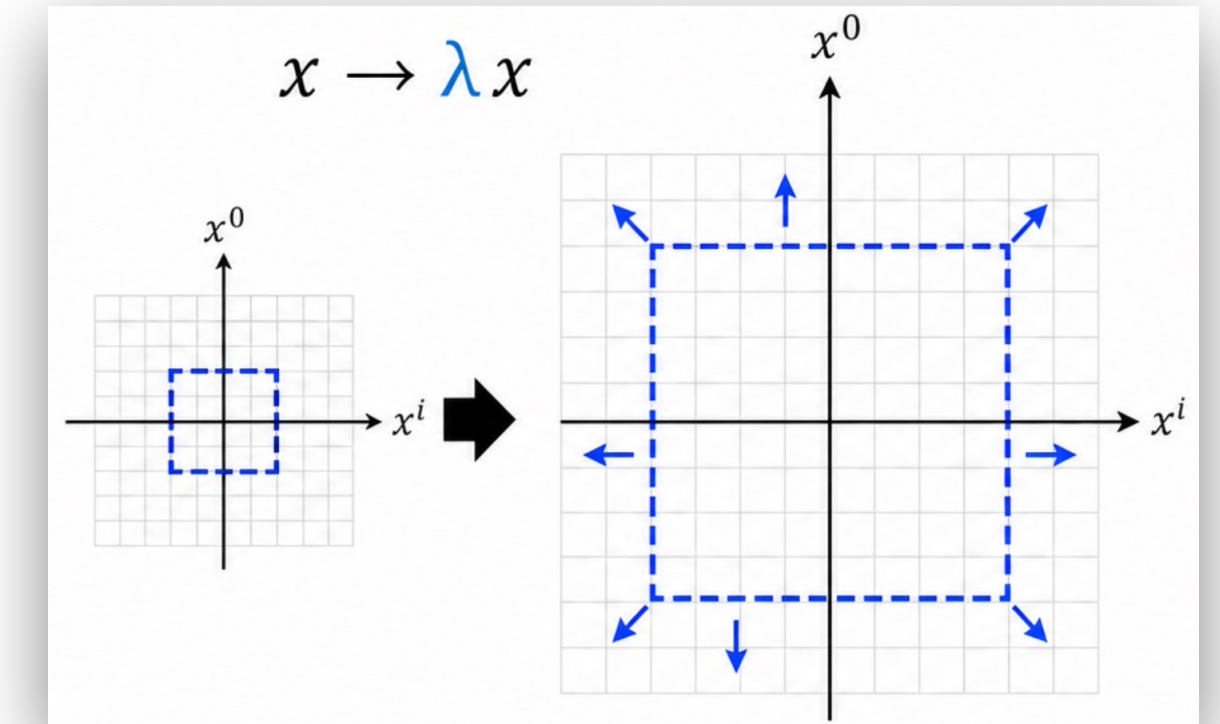
● Massless classical QCD is scale invariant

$$D^\mu = x_\nu T^{\mu\nu}$$

$$\partial_\mu D^\mu = T^\mu_\mu$$

$m_q = 0 \rightarrow T^\mu_\mu = m_q \bar{\psi}\psi = 0$: no intrinsic dimensional parameter in the theory

dilatation symmetry



● Quantum + explicit symmetry breaking

$$T^\mu_\mu = \frac{\beta(g)}{2g} (F^2)_R + (1 + \gamma_m) (m_q \bar{\psi}\psi)$$

Anomalous (quantum) scale breaking, survives at $m_q = 0$

Explicit scale breaking



● Hadron mass scale

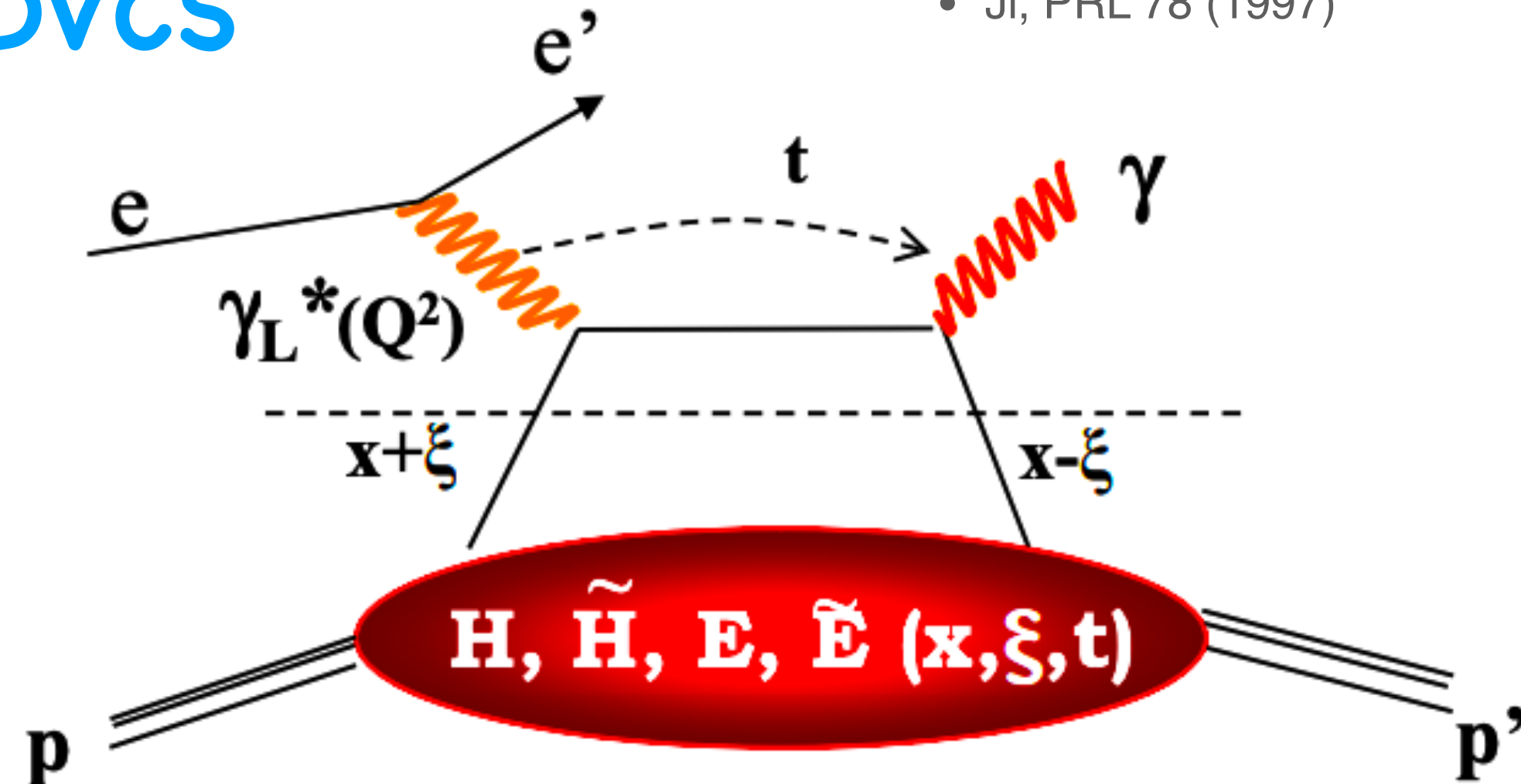
$$\langle P | T^\mu_\mu | P \rangle = 2P^\mu P_\mu = 2M^2$$

$$M = \frac{1}{2M} \left(\langle (T_{q,R})^\mu_\mu \rangle + \langle (T_{g,R})^\mu_\mu \rangle \right)$$

Experimental access: GFFs from exclusive processes

DVCS

• Ji, PRL 78 (1997)



Leading-twist GPD moments:

$$\int_{-1}^1 dx x H^q(x, \xi, t) = A^q(t) + \xi^2 C^q(t)$$

$$\int_{-1}^1 dx x E^q(x, \xi, t) = B^q(t) - \xi^2 C^q(t)$$

$$\langle p', s' | T_{q,g}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i \sigma^{\nu)\rho} \Delta_\rho}{2M} \right. \\ \left. + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{C}_{q,g}(t) M g^{\mu\nu} \right] u(p, s).$$

$$P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2. \quad \text{not fixed by twist-2 GPD moments}$$

Exclusive processes constrain the symmetric-traceless EMT sector

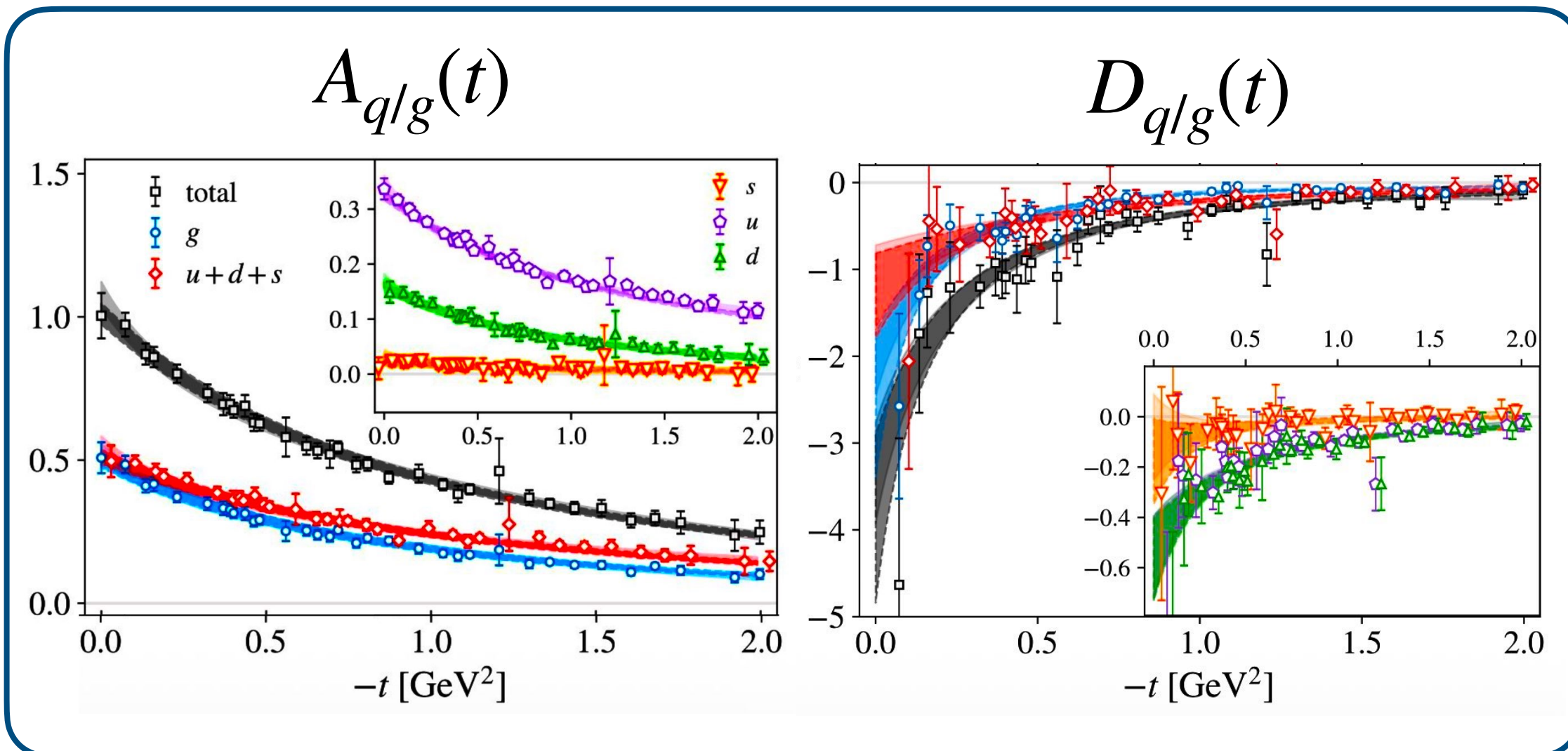
Lattice QCD access: local and nonlocal routes

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y. Zhao, et al, RMP 93 (2021).
- A. Radyushkin, PRD 100 (2019)
- A. Radyushkin, Int.J.Mod.Phys.A 2020

Local twist-2 operators

$$O_q^{\{\mu\nu\}} = \bar{\psi} \gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} \psi - \text{traces}$$

$$\langle p' | T_{q/g}^{\mu\nu} | p \rangle \longrightarrow A(t), B(t), D(t)$$

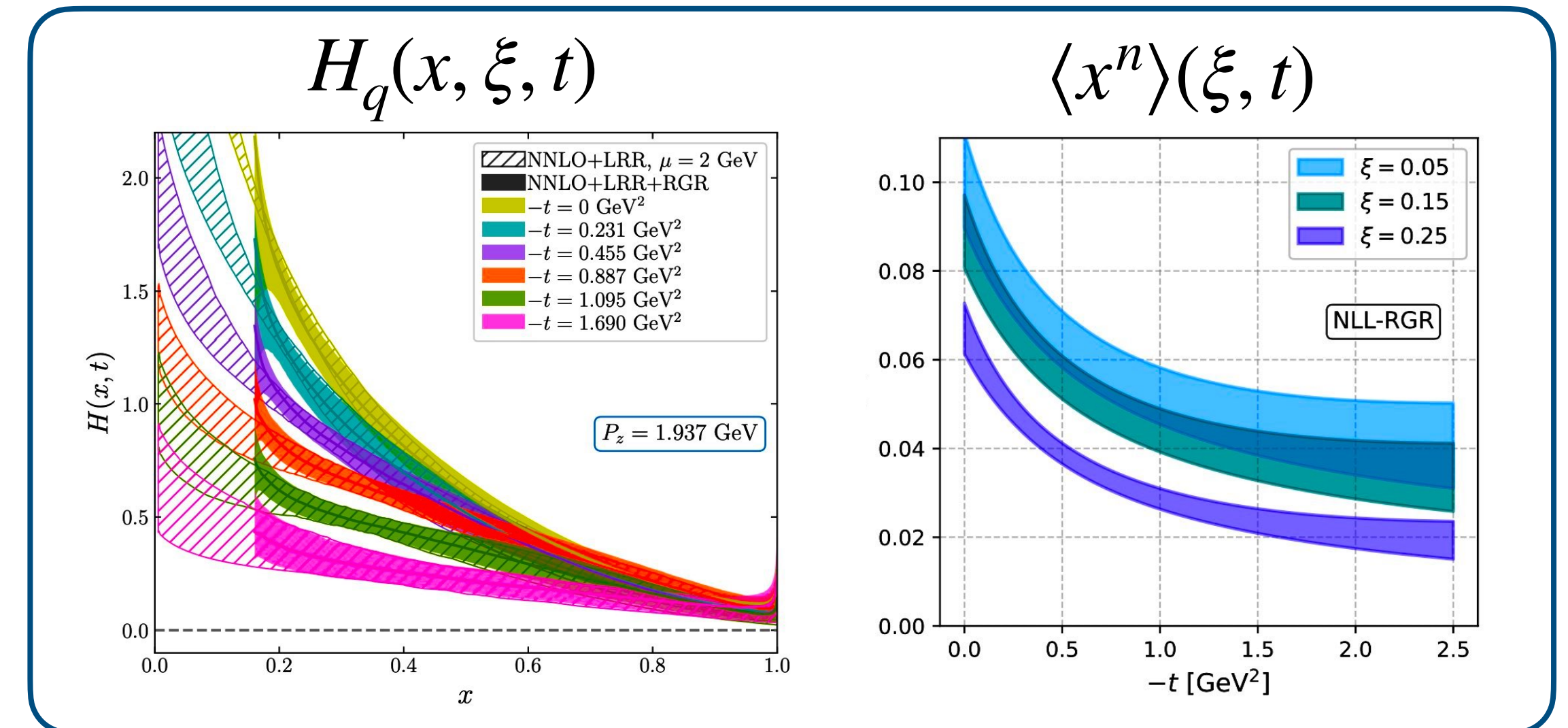


• MIT, PRL 132 (2024) 25, 251904

Nonlocal operators

$$\tilde{h}_F(z, P, \Delta) = \langle p' | \bar{\psi}(0) \Gamma \mathcal{W}(0, z) \psi(z) | p \rangle$$

Euclidean spatial correlator $\longrightarrow$ LaMET / SDF matching $\longrightarrow$ **x -dependent GPDs / GPD moments**



- H.-T. Ding, XG, Qi Shi et. al., JHEP 02 (2025) 056
- XG, F. Yao et. al., PRD 113 (2026) 1, 014505

What remains missing? The scalar trace

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_\alpha \right)}_{\bar{T}^{\mu\nu} : \text{symmetric traceless, twist-2}} \underbrace{\frac{1}{4} g^{\mu\nu} T^\alpha_\alpha}_{\text{scalar trace, twist-4}}$$

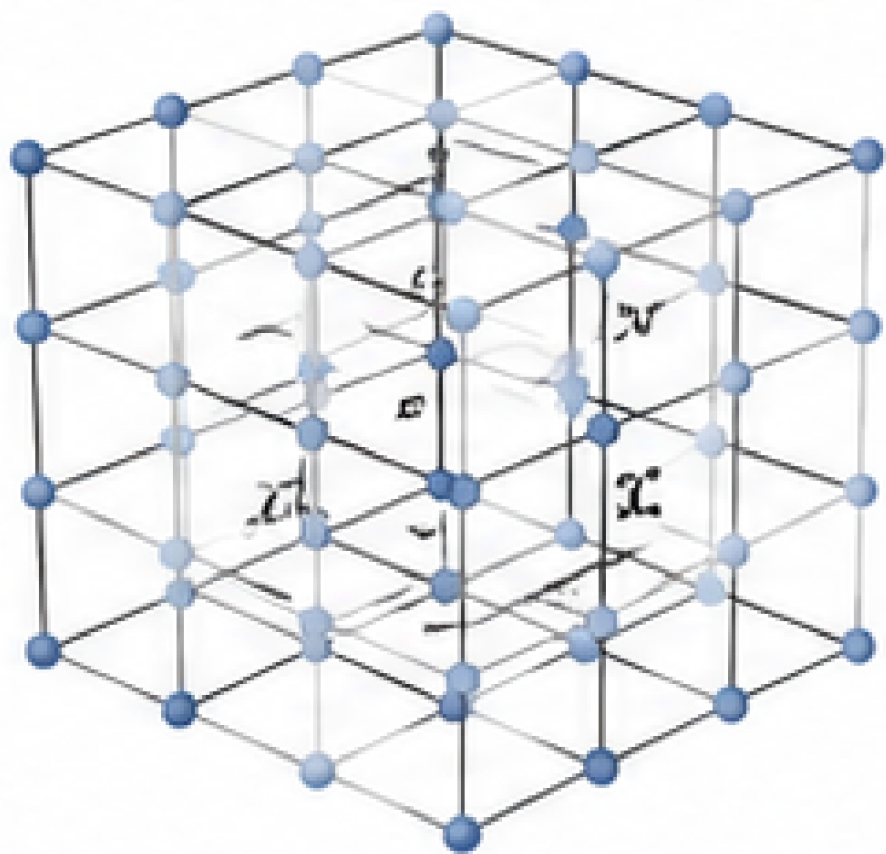
- GPDs from **twist expansion** gives access to the traceless EMT: the **trace sector is absent**; trace-related $\bar{C}_{q/g}$ form factors are not constrained.
- **No independent test** of mass/trace-anomaly sum rules.

Full EMT matrix elements from lattice QCD: why hard?

$$T^{\mu\nu} = O_{1,\mu\nu} - \frac{1}{4}O_{2,\mu\nu} + \frac{1}{4}O_{3,\mu\nu} = \underbrace{-F_a^{\mu\alpha}F_{a,\alpha}^{\nu} + \frac{1}{4}g^{\mu\nu}F_a^{\alpha\beta}F_{a,\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum_f i\bar{\psi}_f \gamma^{\{\mu} D^{\nu\}} \psi_f}_{T_q^{\mu\nu}}$$

Bare lattice EMT

$$T_{\mu\nu}^{\text{bare}}(x)$$



- **Lattice discretization breaks $O(4)$ to $H(4)$:** only traceless $T^{\mu\nu}$ components in $H(4)$ irreps are safe.

- Trace mix with lower-dimensional operators with a^{-n} divergence, e.g.,

$$\langle \bar{\psi} \gamma_\mu \overleftrightarrow{D}^\mu \psi \rangle^R \sim \lim_{a \rightarrow 0} \left[\langle \bar{\psi} \gamma_\mu \overleftrightarrow{D}^\mu \psi \rangle + \frac{1}{a} \langle \bar{\psi} \Gamma \psi \rangle + \dots \right]$$

No continuum limit!

- **No direct access to trace sector.**

Renormalized full EMT including trace

$$T_{\mu\nu}^R(x)$$



Gradient flow as finite-resolution UV filter

- M. Luscher, Commun.Math.Phys. 293 (2010)
- M. Luscher, JHEP 08 (2010) 071
- M. Luscher, P. Weisz, JHEP 02 (2011) 051

Gauge-covariant diffusion in four dimensions:

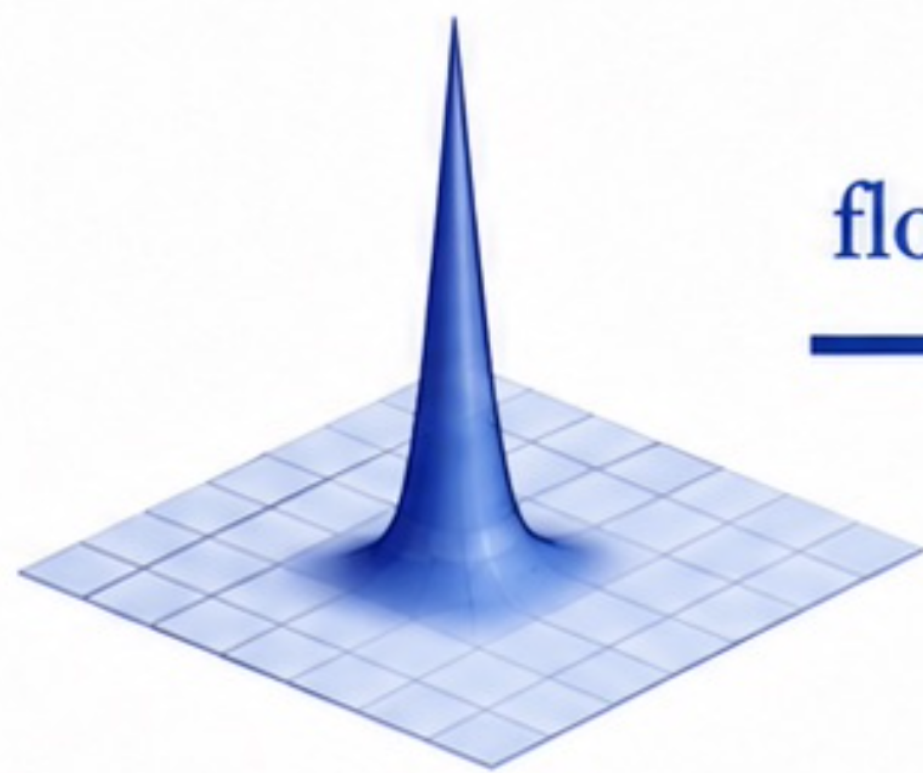
$$\partial_{t_f} B_\mu(t_f, x) = D_\nu F_{\nu\mu}(t_f, x), \quad B_\mu(0, x) = A_\mu(x)$$

Local QCD field

Flowed field/operator

$$A_\mu(x)$$

$$t_f = 0$$

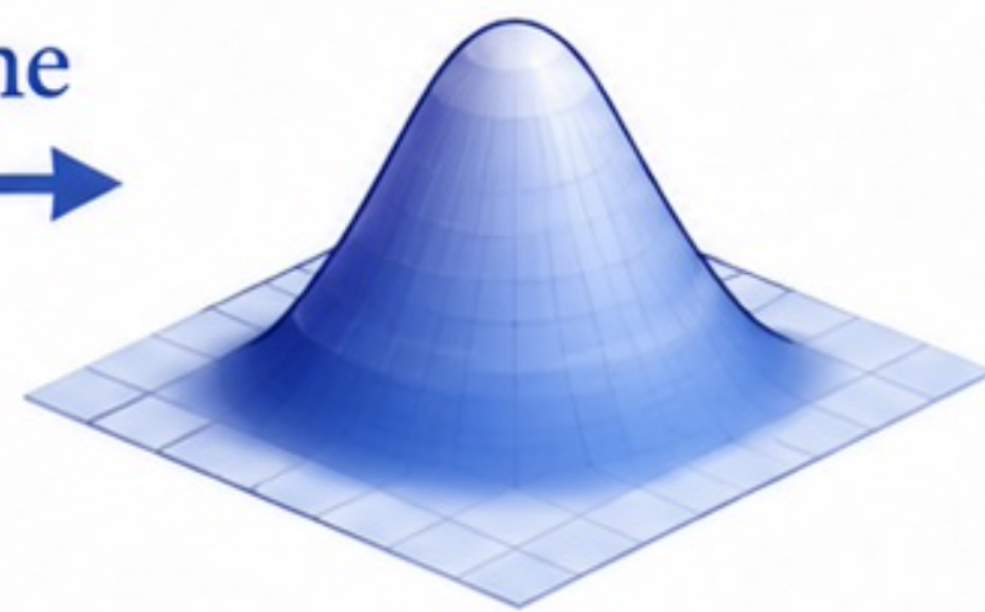


Local operator
pointlike

flow time
→

$$B_\mu(t_f, x)$$

$$t_f > 0$$



Smearred over
 $r_{\text{flow}} \simeq \sqrt{8t_f}$

$$B_\mu(t_f, p) \simeq e^{-t_f p^2} A_\mu(p)$$

$$r_{\text{flow}} \simeq \sqrt{8t_f}, \quad \mu_t \sim \frac{1}{\sqrt{8t_f}}$$

- UV modes are exponentially suppressed.
- At finite t (dimension **[length²]**), the operator has a **physical resolution scale**.
- The flowed operator is a smeared probe, **not a local QCD operator**.

Gradient flow smooths the operator over a radius $r_{\text{flow}} \simeq \sqrt{8t_f}$

At fixed flow time: a UV-finite continuum operator basis

● Flowed composite operator

$$\tilde{O}_i(t_f, x) = O_i \left[B_\mu(t_f, x), \chi(t_f, x), \tilde{\chi}(t_f, x) \right]$$

● Continuum limit at fixed physical flow time

$$\tilde{O}_i(t_f, a) = \underbrace{\tilde{O}_i(t_f, 0)}_{\text{Continuum limit (UV finite)}} + \underbrace{c_1 \frac{a^2}{t_f} + c_2 a^2 + \dots}_{\text{Cut-off effects}}$$

Continuum limit
(UV finite)

Cut-off effects

- Gauge fields and ringed fermion fields define **UV-finite** flowed EMT operators $\tilde{O}_i(t_f, x)$ at $t_f > 0$.
- **Regulator independent**: holds for any underlying regulator (lattice, DR, ...).
- **Safe** to take continuum limit $a \rightarrow 0$ at finite t_f .

Positive flow time provides a UV-finite operator basis with a controlled continuum limit

From flowed probes to the local EMT at $t_f = 0$ boundary

$t_f = 0$ (boundary):

local 4D QCD EMT and scale Ward identity

$$T^\mu_\mu = \frac{\beta(g)}{2g} (F^2)_R + (1 + \gamma_m) (m\bar{\psi}\psi)_R$$

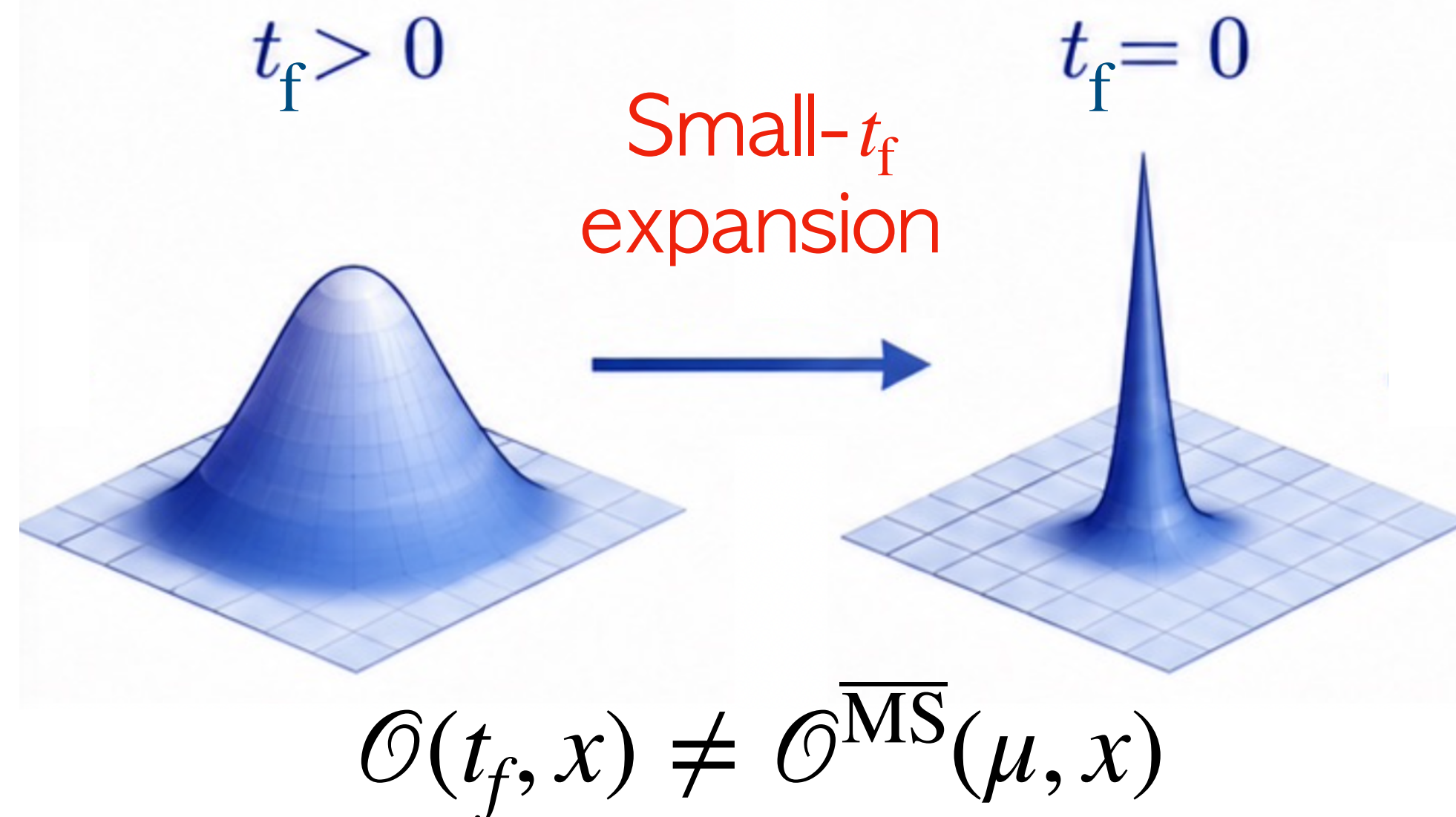
trace anomaly of 4D QCD

flow time t

bulk
($t_f > 0$)

$t_f > 0$

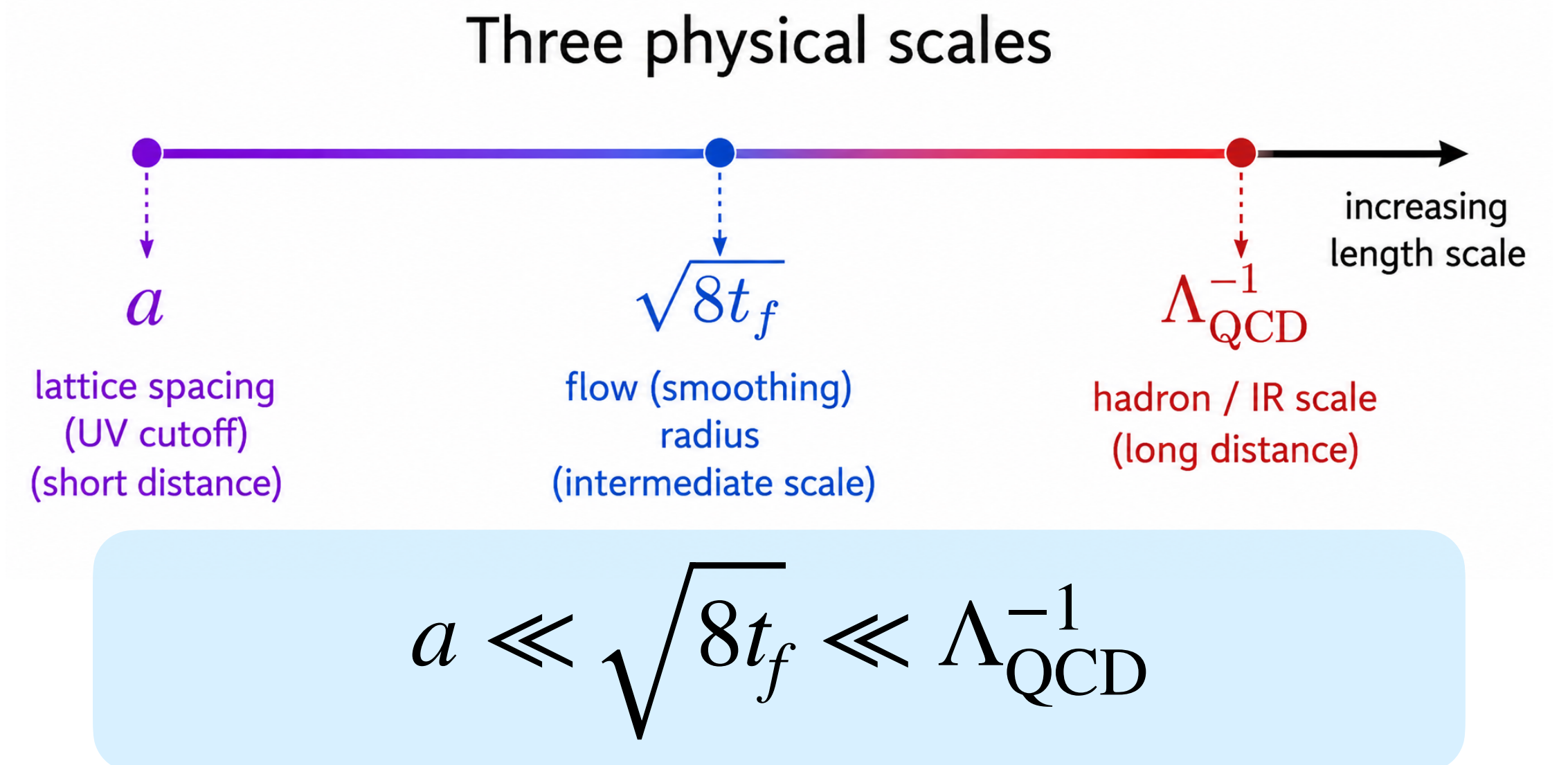
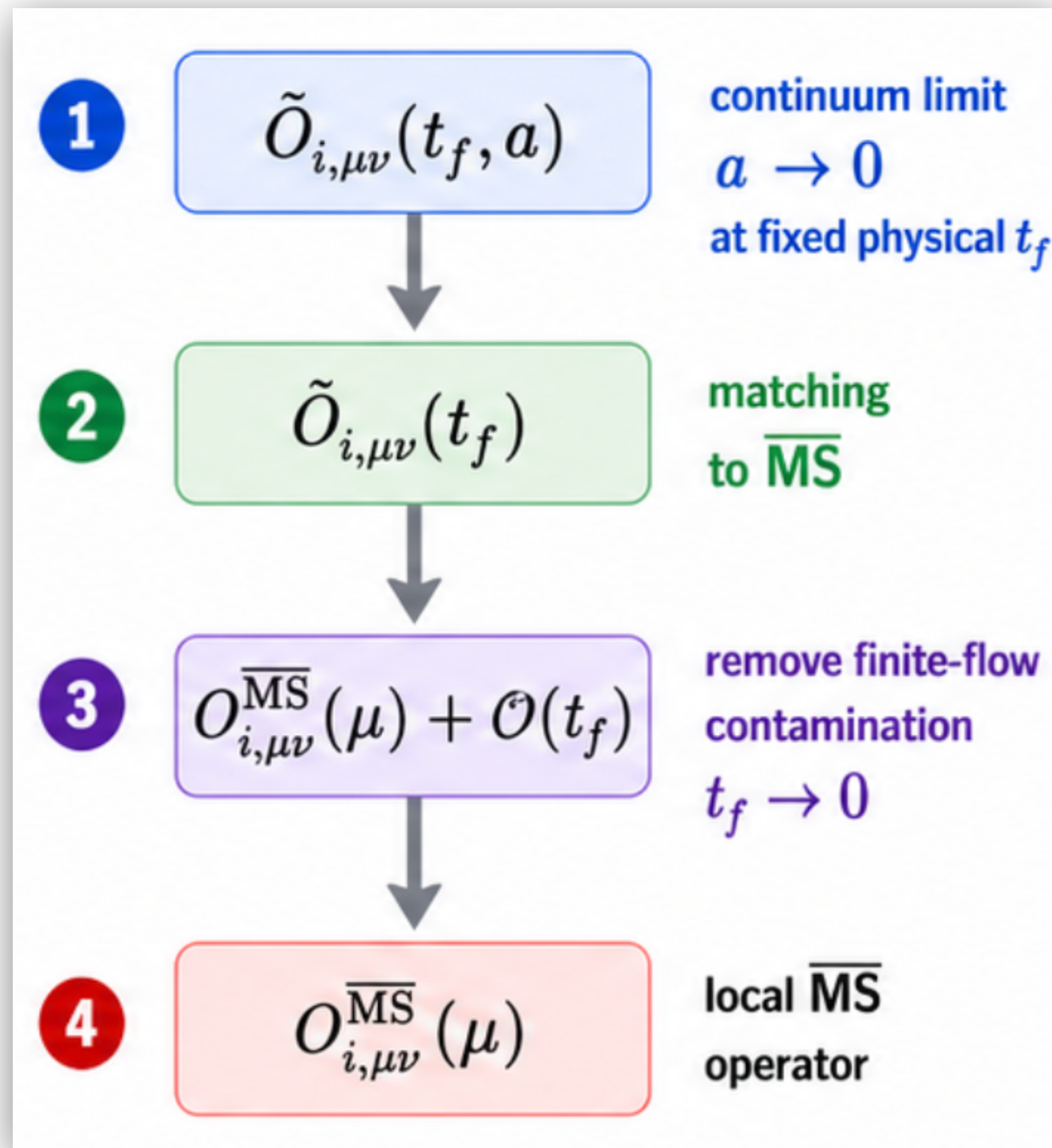
The underlying **4D QCD** theory is **unchanged**, but the flowed operators are not yet the local conserved EMT.



$$\mathcal{O}_i(t_f, x) = \sum_j C_{ij}(t_f, \mu) \mathcal{O}_j^{\overline{\text{MS}}}(\mu, x) + \mathcal{O}(t_f)$$

- The local EMT is **recovered after matching** and $t_f \rightarrow 0$ limit.
- The **boundary trace anomaly** is encoded in the matching coefficients $C_{ij}(t_f, \mu)$ and the flow-time dependence.

A controlled path to the full local EMT



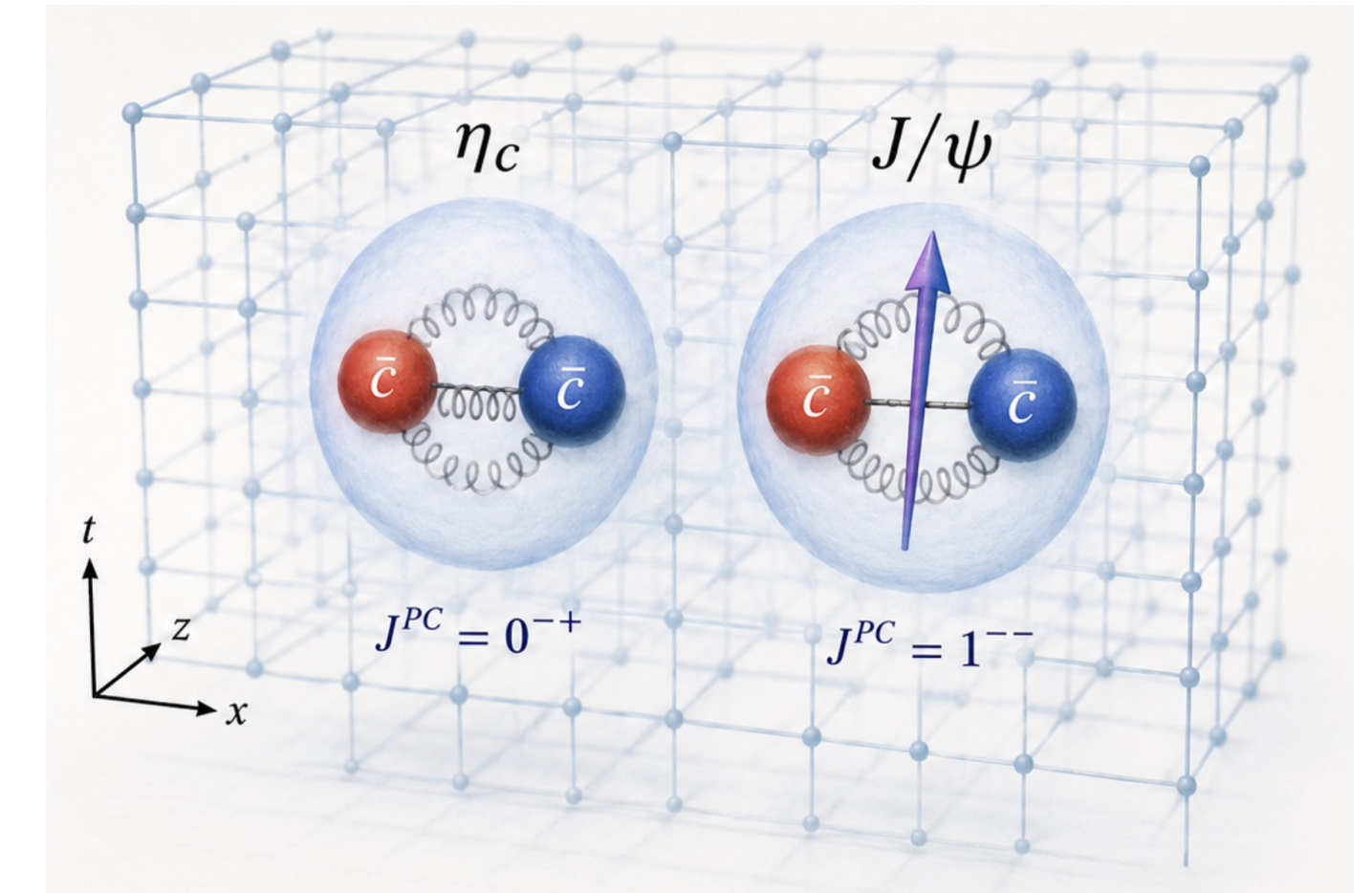
- t_f too small: **cutoff** artifacts dominate ($\sim a^2/t_f, a^2, \dots$).
- t_f too large: **oversmearing**, loss of short-distance information.

All quark and gluon EMT components, including the trace, are derived in one common $\overline{\text{MS}}$ scheme and scale

Lattice calculation: charmonium states as a demonstration

- η_c and J/ψ states: **three ensembles** with physical η_c and J/ψ meson masses; Light quark contribution ignored.

a [fm]	N_s	N_t	$m_s^{\text{sea}}/m_l^{\text{sea}}$	M_π^{sea} [MeV]	c_{sw}	am_c^{val}
0.06	48	64	20	160	1.0336	0.306
0.05	64				1.030934	0.236
0.04	64				1.02868	0.167



- Start with the rest frame: **$P = 0$** :

$$\frac{1}{2M} \langle P = 0 | T_{q/g}^{\mu\nu} | P = 0 \rangle = M \begin{pmatrix} A_{q/g}(0) + \bar{C}_{q/g}(0) & 0 & 0 & 0 \\ 0 & -\bar{C}_{q/g}(0) & 0 & 0 \\ 0 & 0 & -\bar{C}_{q/g}(0) & 0 \\ 0 & 0 & 0 & -\bar{C}_{q/g}(0) \end{pmatrix}$$

Bare EMT matrix elements from flowed operators

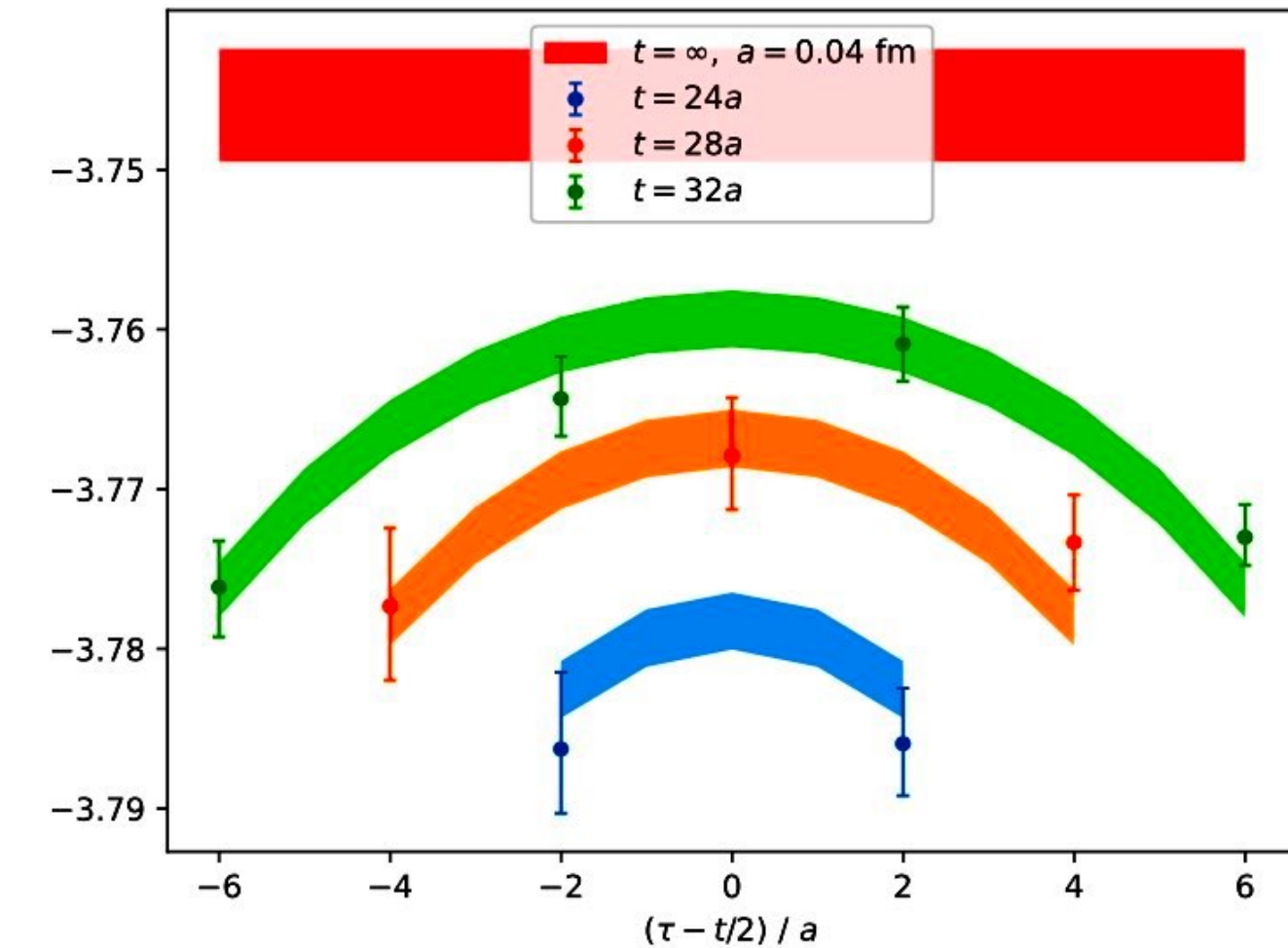
- Three-point function of flowed operator:

$$C_{i,\mu\nu}^{3\text{pt}}(t_f; t_s, \tau) = \sum_{x,y} \left\langle N(y, t_s) \tilde{O}_{i,\mu\nu}(t_f; x, \tau) N^\dagger(0,0) \right\rangle - \left\langle C_\Gamma^{2\text{pt}}(t_s) \right\rangle \sum_x \left\langle \tilde{O}_{i,\mu\nu}(t_f; x, \tau) \right\rangle$$

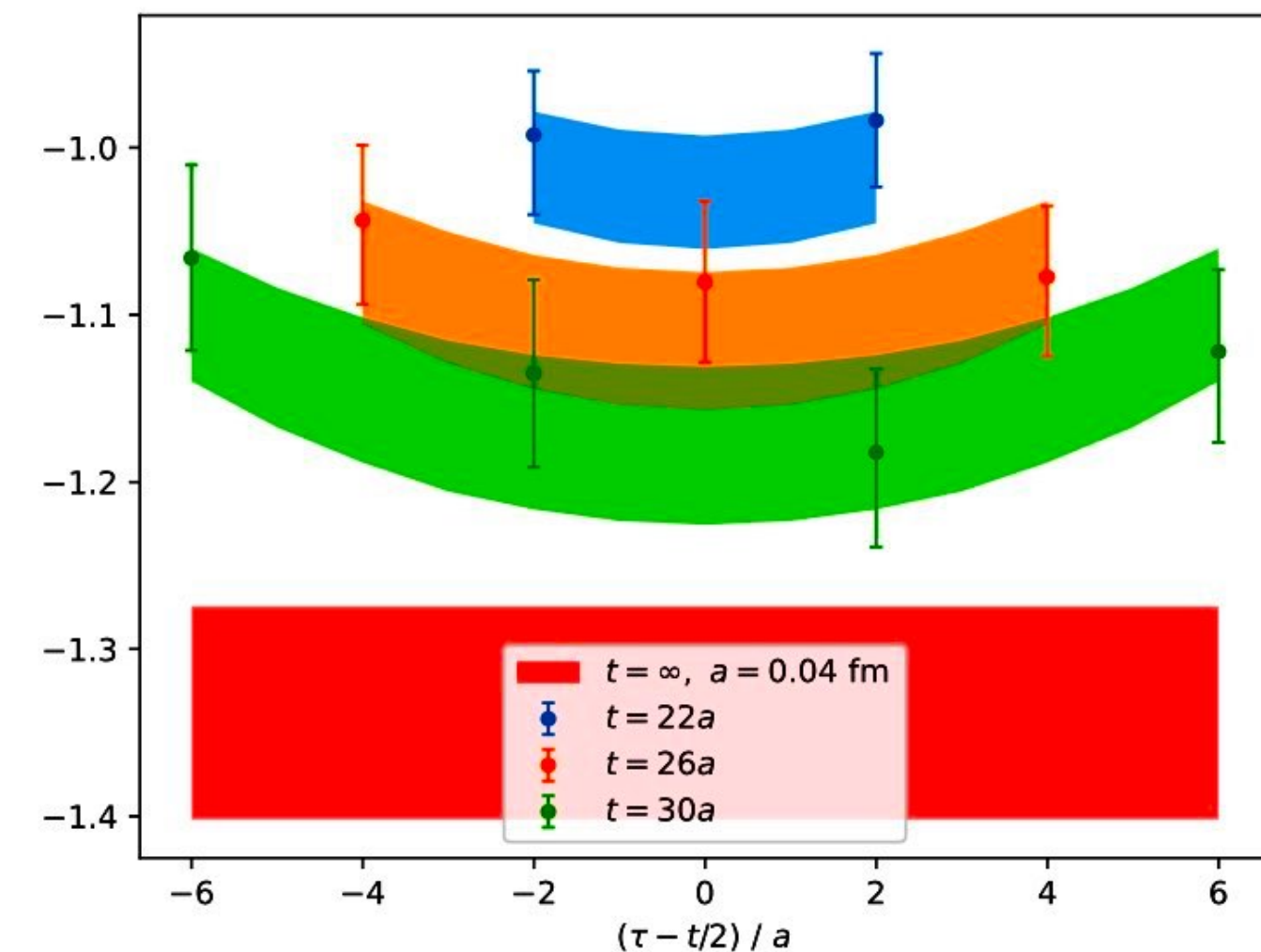
- Bare matrix elements from two-state fit:

$$C_{i,\mu\nu}^{3\text{pt}}(t_f; t_s, \tau) = \sum_{m,n} Z_m^* Z_n \frac{\langle m | \tilde{O}_{i,\mu\nu}(t_f) | n \rangle}{2M} e^{-\tau E_n} e^{-(t-\tau)E_m}$$

quark EMT $\langle \eta_c | \tilde{O}_{3,\mu\nu}(t_f) | \eta_c \rangle$



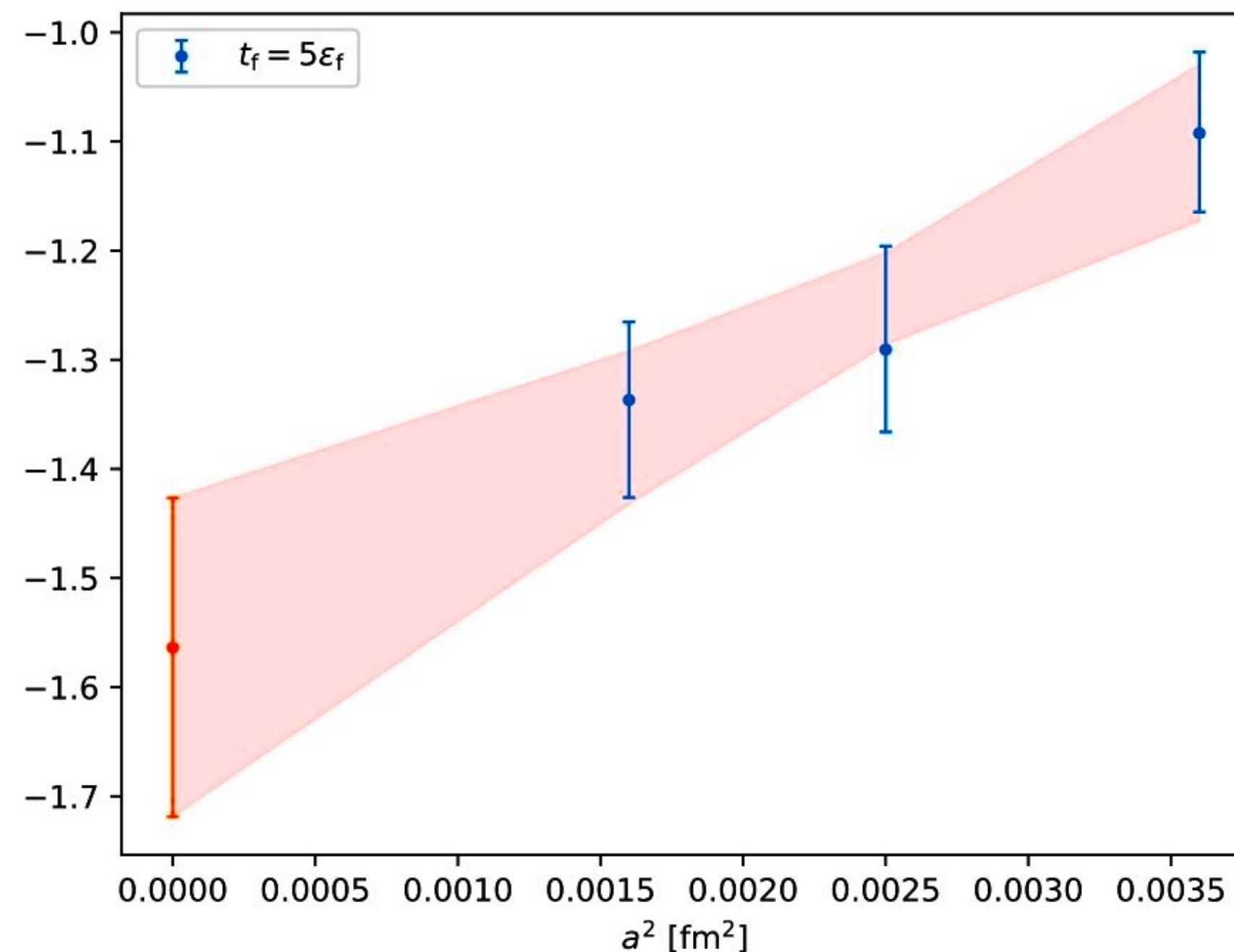
gluon EMT $\langle \eta_c | \tilde{O}_{1,\mu\nu}(t_f) | \eta_c \rangle$



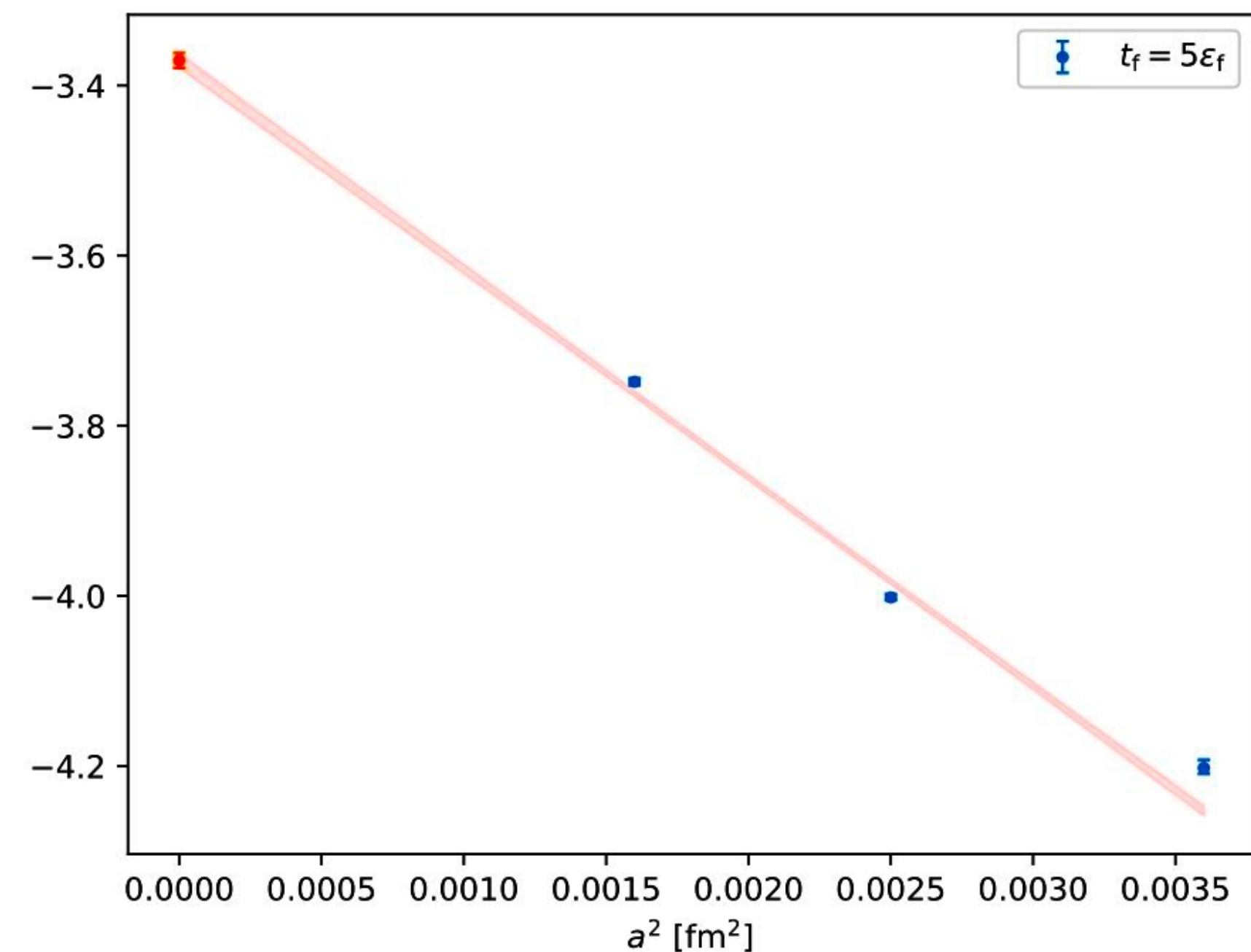
Continuum extrapolation at fixed flow time

$$\langle \eta_c | \tilde{O}_{i,\mu\nu}(t_f, a) | \eta_c \rangle = \langle \eta_c | \tilde{O}_{i,\mu\nu}(t_f) | \eta_c \rangle + a^2 X_{i,\mu\nu}(t_f)$$

gluon EMT $\langle \eta_c | \tilde{O}_{1,\mu\nu}(t_f) | \eta_c \rangle$



quark EMT $\langle \eta_c | \tilde{O}_{3,\mu\nu}(t_f) | \eta_c \rangle$

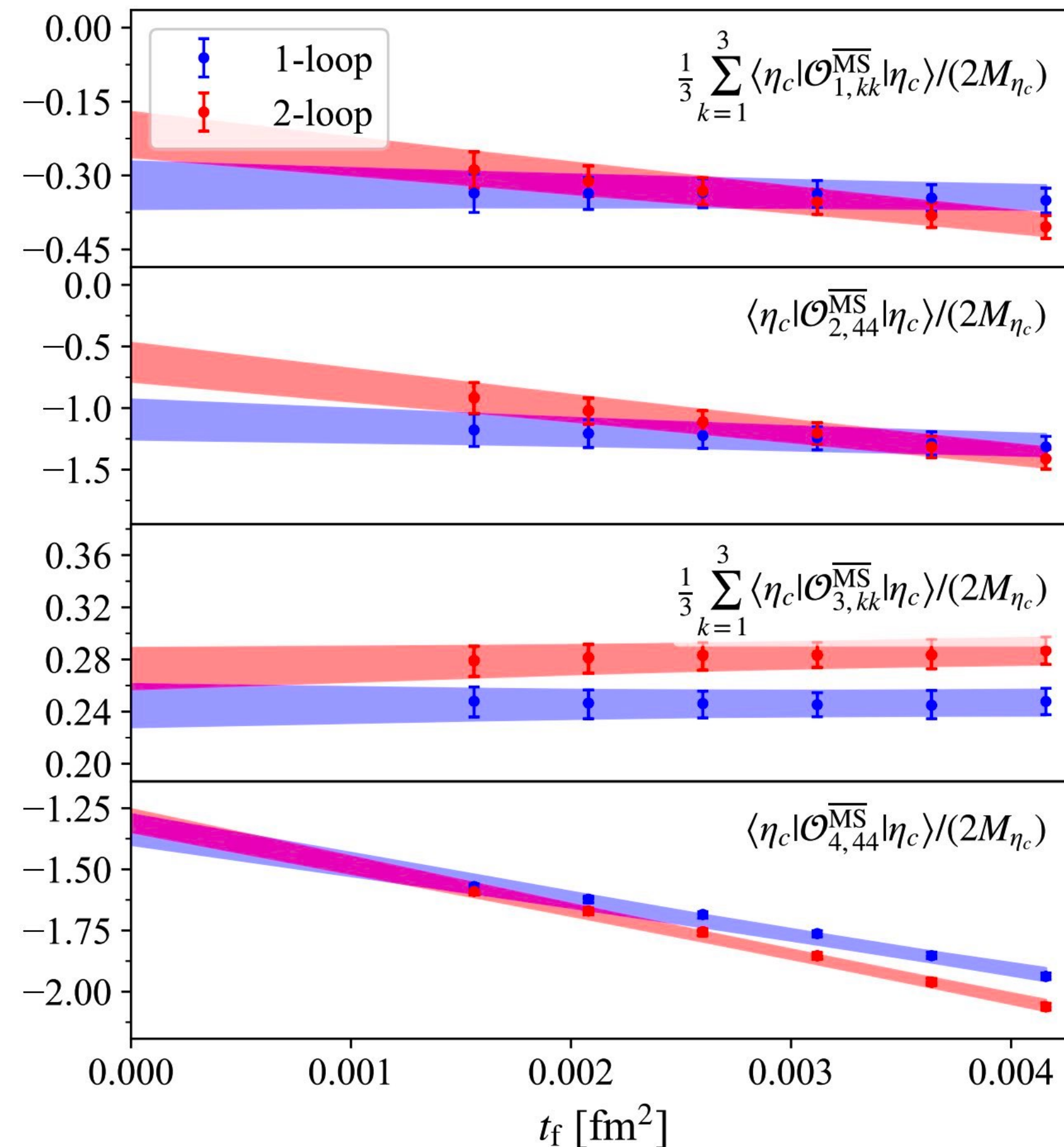


At fixed physical t_f , the continuum extrapolation is well controlled due to UV finiteness

Matching to $\overline{\text{MS}}$ and extrapolation to zero flow time

$$O_{i,\mu\nu}^{\overline{\text{MS}}}(\mu) = M_{ij}^{\overline{\text{MS}} \leftarrow \text{Flow}}(\mu, t_f) \tilde{O}_{j,\mu\nu}(t_f) + \mathcal{O}(t_f \Lambda^2)$$

- H. Suzuki, PTEP 2013 (2013) 083B03
- H. Makino, H. Suzuki, PTEP 2014 (2014) 063B02
- R. Harlander, Y. Kluth, F. Lange, Eur.Phys.J.C 78 (2018) 11, 944



- One- and two-loop matching are close.
- **small-flow-time** window is under control.
- the final result is the **local $\overline{\text{MS}}$** EMT matrix element.

GFFs from EMT matrix elements (rest frame $P=0$)

- η_c (spin-0)

$$\langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0)$$

- J/ψ (spin-1): denote the polarization by γ_λ

$$\begin{aligned} \langle P, \gamma_\lambda | T_i^{\mu\nu} | P, \gamma_\lambda \rangle &= 2P^\mu P^\nu A_{0,i}(0) \\ &+ \left(2\delta^{\mu\lambda} \delta^{\nu\lambda} + \frac{1}{2} g^{\mu\nu} \right) M^2 \bar{f}_i(0) \\ &- g^{\mu\nu} M^2 \bar{C}_{0,i}(0). \end{aligned}$$

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
gluon(g)	0.084(45)	0.058(18)	0.077(35)	-0.146(29)	-0.012(32)
charm(c)	0.918(17)	-0.068(4)	0.926(34)	0.176(87)	0.023(5)
total	1.000(46)	-0.010(18)	0.997(43)	0.030(30)	0.010(32)

- $A_{q/g}$ probes the **momentum fraction** $\langle x \rangle_{q/g}$.
- $\bar{C}_{q/g}$ probes the **trace-related** channel.
- The quark and gluon sectors combine into the **conserved total EMT**:
 $A_g(0) + A_q(0) \approx 1, \bar{C}_g(0) + \bar{C}_q(0) \approx 0$

The sigma term from the QCD equation of motion

- σ term: quantifies the **explicit quark-mass contribution** to the hadron mass and is scale and scheme invariant.

$$\sigma \equiv \frac{1}{2M} \langle P | \sum_f m_f \bar{\psi}_f(x) \psi_f(x) | P \rangle$$

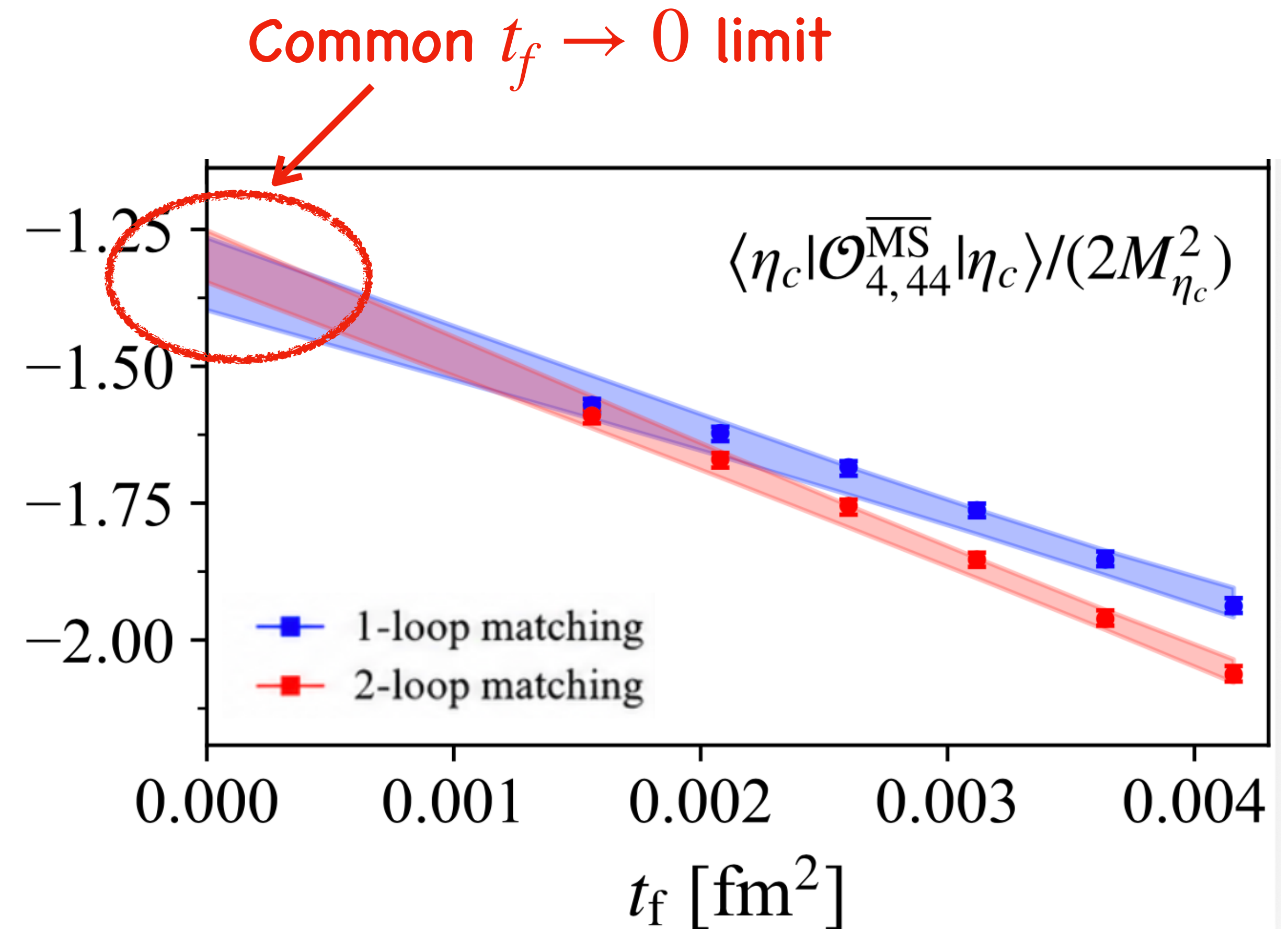
- For physical-state matrix elements, the **QCD EOM** gives,

$$\sum_f \bar{\psi}_f(x) \overset{\leftrightarrow}{D} \psi_f(x) = -2 m_f \bar{\psi}_f(x) \psi_f(x)$$

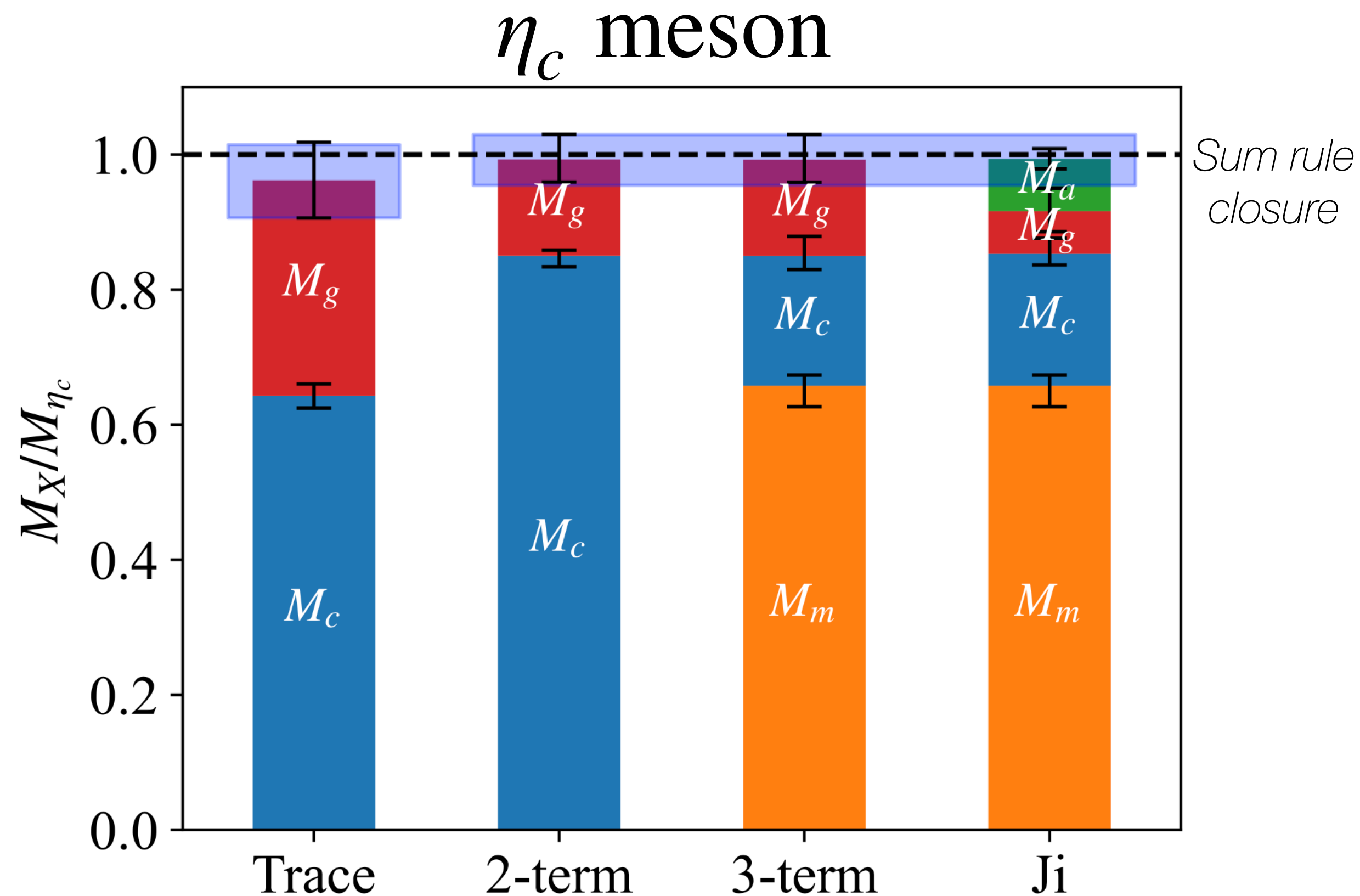
$$\sigma = -\frac{1}{4M} \langle P | \sum_f \bar{\psi}_f(x) \overset{\leftrightarrow}{D} \psi_f(x) | P \rangle$$

$$= -\frac{1}{4M} \langle P | O_{4,\mu\nu}^{\overline{\text{MS}}} | P \rangle$$

Trace operator: power divergent
mixing avoided through GF



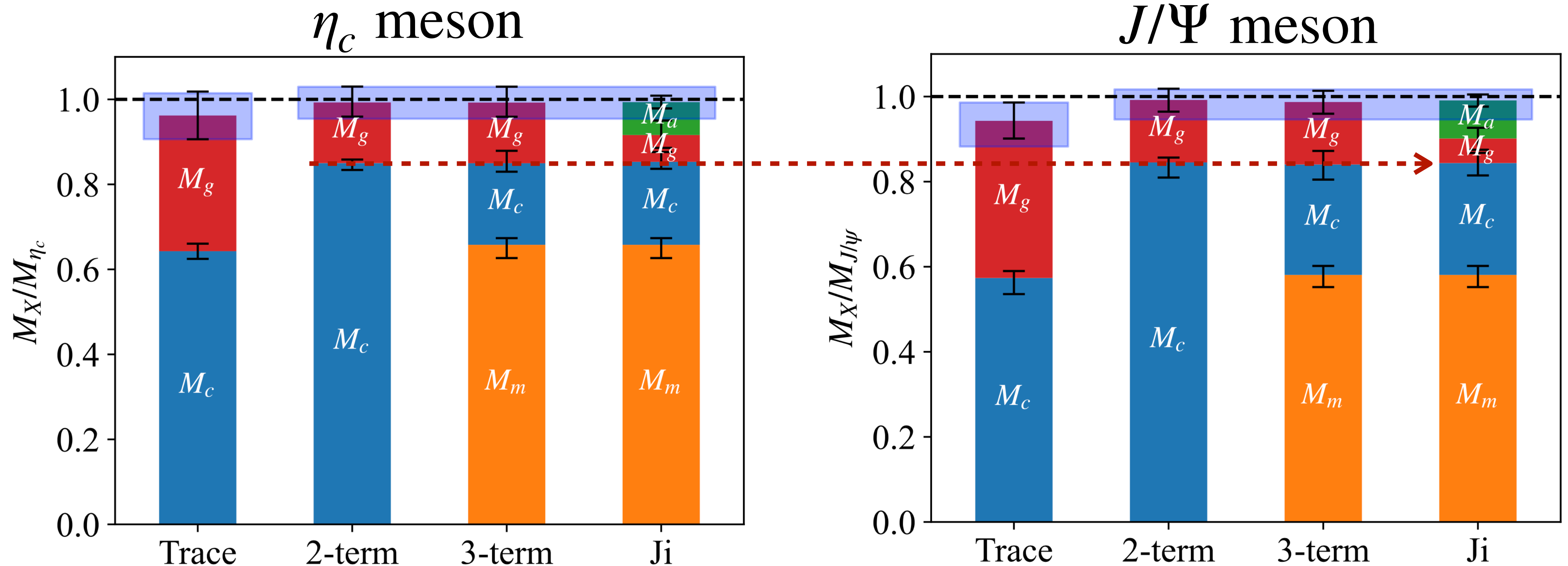
Quark-gluon decomposition of mass and trace anomaly



All ingredients are determined in a common $\overline{\text{MS}}$ framework.

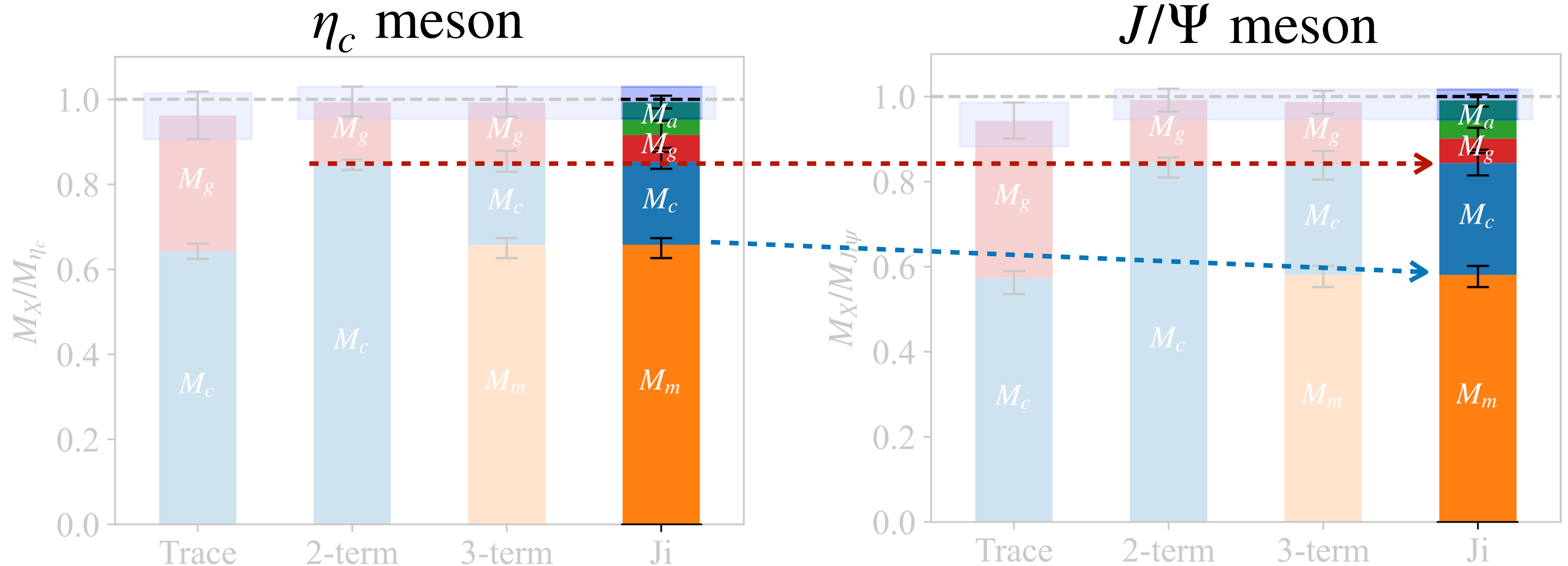
- Trace, two-term, three-term, and Ji decompositions **all close**.
- **First simultaneous lattice validation** in a common scheme.
- The **gluon** sector remains substantial even in heavy quarkonium,

Quark-gluon decomposition of mass and trace anomaly



- **Gluon** contributions are **remarkably similar** between η_c and J/ψ .
- Changing heavy-quark spin primarily **reorganizes** the charm **charm energy/sigma contribution**.

Quark-gluon decomposition of mass and trace anomaly



- **Gluon** contributions are **remarkably similar** between η_c and J/ψ .
- Changing heavy-quark spin primarily **reorganizes** the charm **charm energy/sigma contribution**.

Summary

- Gradient flow provides a UV-finite and regulator-independent definition of flowed EMT operators, which admits a controlled continuum limit and avoids power-divergent mixing.
- Combined with perturbative matching to $\overline{\text{MS}}$ and a $t_f \rightarrow 0$ extrapolation, this gives us access to the full EMT including the trace.
- In charmonium, we extract quark and gluon EMT matrix elements and determine forward-limit GFFs, including the trace-related $\bar{C}_i$.
- We directly validate the momentum, trace-anomaly, and mass/energy decomposition sum rules simultaneously in a common scheme.
- The framework is general and can be straightforwardly adopted for mass and spin decompositions as well as GFFs of other hadrons and nuclei.

Thanks for your attention!