GPDs, Their Interpretations, and How to Find Them

- **Generalized Parton Distributions (GPDs)**
  - Definition and ambiguity
- **Their interpretations**
  - Matching partons to hadron properties
- **How to find them?**
  - Single diffractive, hard exclusive processes
- **Summary and Outlook**

In collaboration with
N. Sato, Z. Yu, Y. Zeng,
…

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Jefferson Lab

U.S. DEPARTMENT of ENERGY

Generalized Parton Distributions (GPDs)

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

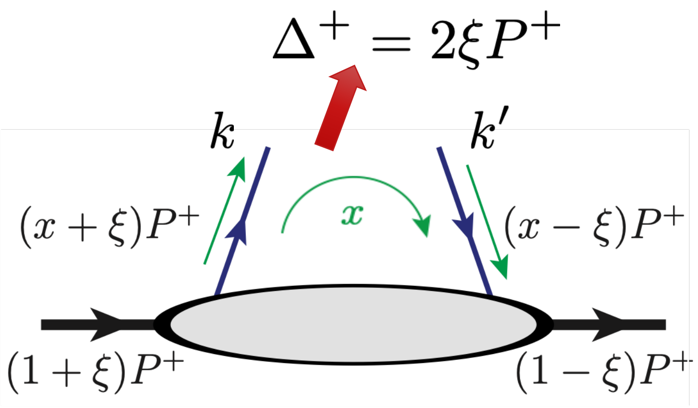
❑ Definition:

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$

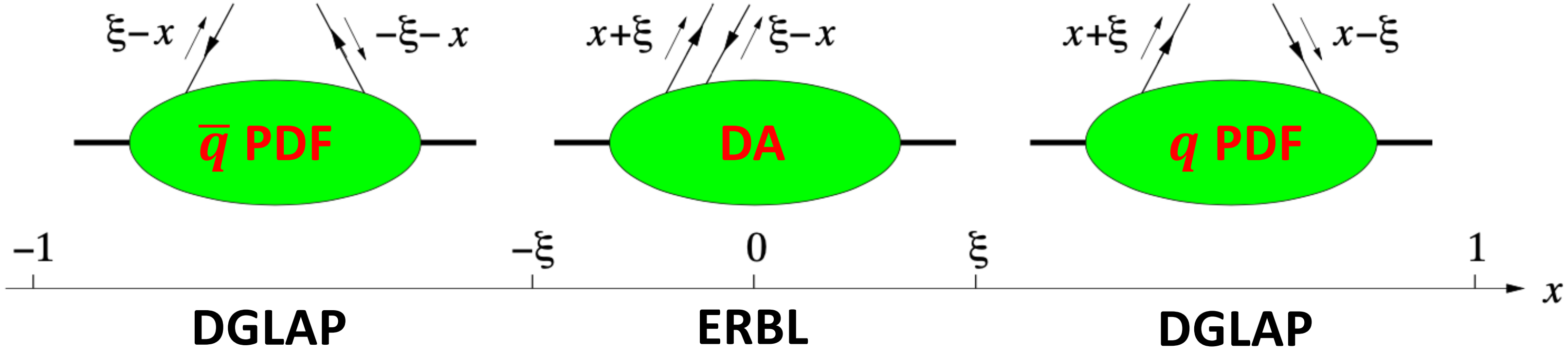


$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

❑ Combine *PDFs (p'=p)* and *Form Factors (z=0)* :

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



Similar definition for gluon GPDs

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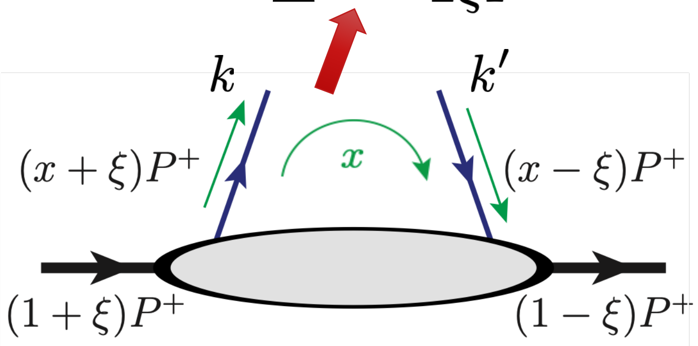
$$\Delta^+ = 2\xi P^+$$



$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

❑ GPDs depend on the Choice of "+" for given p and p' :

- Need to choose a light-cone vector: $n^\mu = (n^+, n^-, \mathbf{n}^\perp) = (0, 1, \mathbf{0}^\perp)$

  with $n^2 = 0,\ n\cdot v = v^+$ for any 4-vector $v^\mu = (v^+, v^-, \mathbf{v}^\perp)$

- For a single vector, *p,* there is a unique light-cone vector ``n'' to pick up the largest component of *p,* as *p*$^+$.

- When $p \neq p'$, we have infinite choices for the light-cone vector ``n''!

  If $\sqrt{|\Delta^2|} = \sqrt{|(p-p')^2|} \ll p^0 \sim p'^0$, GPDs with different choice of ``n'' differ by powers of $|\Delta^2|/\hat{s}$

  with the hard scale $\hat{s}$, e.g., $\hat{s} \sim Q^2$ for DVCS.

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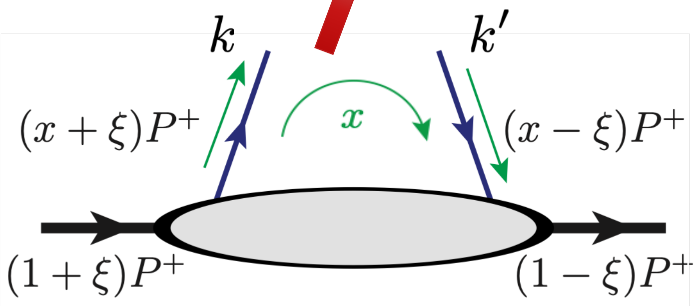


$$P^+ = \frac{p^+ + p'^+}{2}$$
$$\Delta = p - p' \qquad t = \Delta^2$$

❑ Variables of GPDs – *x* , ξ , *t* :

- $t = \Delta^2 = (p-p')^2$ is uniquely fixed by the momentum of scattered hadron *p'* .
- $\xi = (p-p')\cdot n/(p+p')\cdot n$ is uniquely fixed once we make a choice of the light-cone vector ``n''
- $x = (k+k')\cdot n/(p+p')\cdot n$ carries partonic information and need to be probed by the hard probe!

Questions:

- **For extracting universal GPDs via QCD global analysis, is there an "ideal" choice of the light-cone vector "n" ?**
- **What kind of hard probes have better access to the *x*-dependence of GPDs ?**

Properties of GPDs – Hadronic = Various Moments of GPDs

C. Lorce, "Hadron structure with GPDs", CNF lecture series, July13-17, 2026, JLab

❑ QCD energy-momentum tensor:

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q\, i\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\,\psi_q - g^{\mu\nu}\bar{\psi}_q\left(i\gamma\cdot\overleftrightarrow{D} - m_q\right)\psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta}F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4}g^{\mu\nu}\left(F^a_{\rho\eta}\right)^2$$

Ji, PRL78, 1997

V. D. Burkert, et al. RMP 95 (2023) 041002

❑ "Gravitational" form factors:

$$\langle p'\,|T_i^{\mu\nu}|\,p\rangle = \bar{u}(p')\left[A_i(t)\frac{P^\mu P^\nu}{m} + J_i(t)\frac{iP^{(\mu}\sigma^{\nu)\Delta}}{2m} + D_i(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m} + m\,\bar{c}_i(t)\,g^{\mu\nu}\right]u(p)$$

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx\, x\, F_i(x,\xi,t) \propto \langle p'|T_i^{++}|p\rangle \quad \propto \quad \bar{u}(p')\left[\underbrace{\left(A_i + \xi^2 D_i\right)}_{\int_{-1}^{1} dx\, x\, H_i(x,\xi,t)}\gamma^+ + \underbrace{\left(B_i - \xi^2 D_i\right)}_{\int_{-1}^{1} dx\, x\, E_i(x,\xi,t)}\frac{i\sigma^{+\Delta}}{2m}\right]u(p)$$

$C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)

Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Angular momentum sum rule:

$$J_i = \lim_{t\to 0}\int_{-1}^{1} dx\, x\left[H_i(x,\xi,t) + E_i(x,\xi,t)\right] \qquad i = q, g$$

→ **3D tomography**
Relation to GFFs
Angular Momentum → ***NEEDs x-dependence of GPDs!***

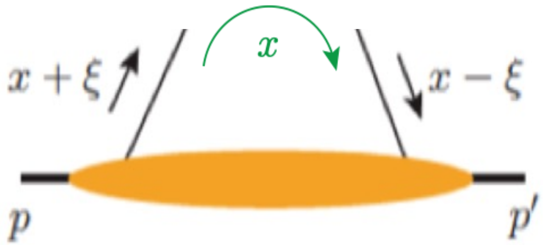
Need to know the x-dependence of GPDs to construct the proper moments!

Properties of GPDs – Partonic

❑ Impact parameter dependent parton density distribution:

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

➡ Quark density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$



Measurement of p' fixes (t, ξ)

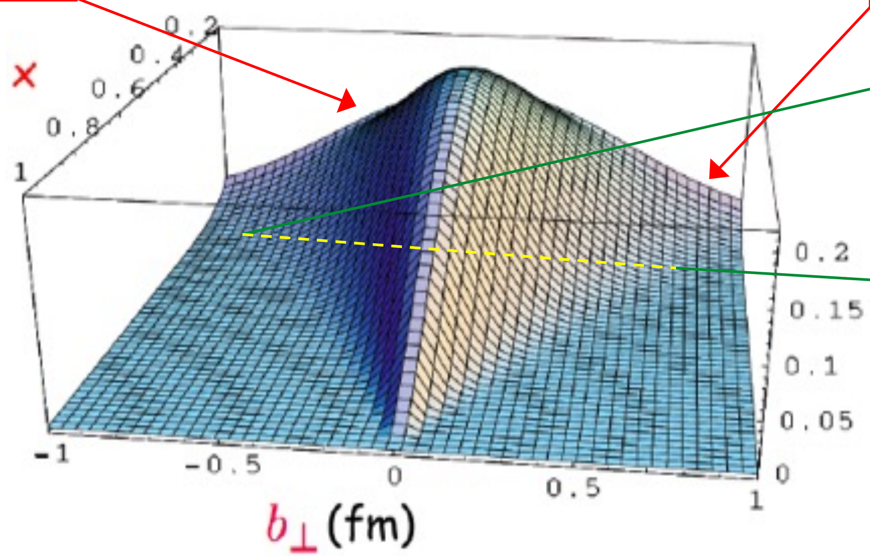
x = momentum flow between the pair

How fast does glue density fall?

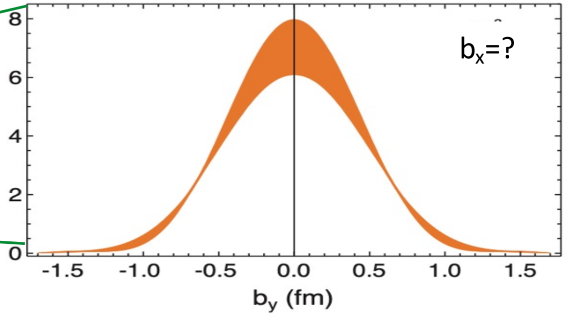
Tomographic image of hadron in slice of x

How far does glue density spread?

➡



Modeled by M. Burkdart, PRD 2000



Slice in (x,Q)

$$\langle q_\perp^N \rangle \equiv \int db_\perp b_\perp^N q(x, b_\perp, Q)$$

- Should $r_q(x)$ > $r_g(x)$, or vice versa?
- How do they compare with known radius (EM charge radius, mass radius, …), & why?

➡ **Proton radii, $r_q(x)$ & $r_g(x)$, from *q* & *g* spatial density**

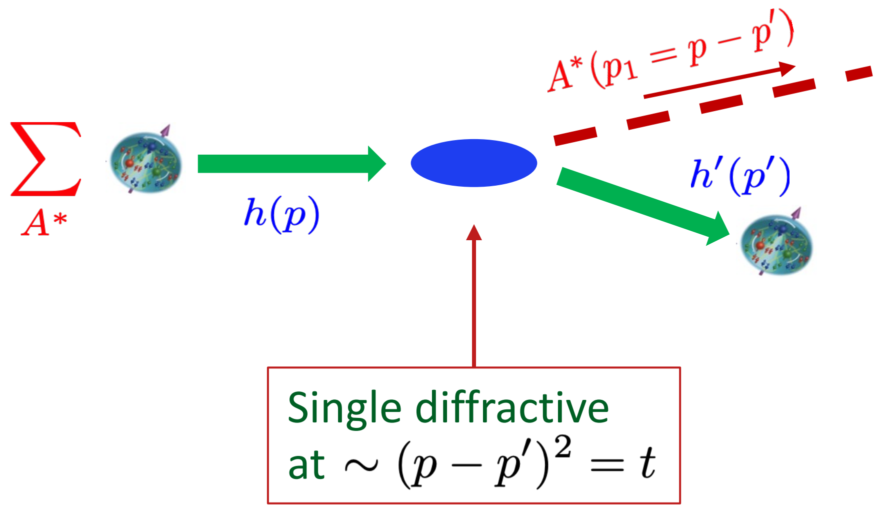
Need to know the x-dependence of GPDs to construct the proper radii !

How to Find Physical Processes to be Sensitive to GPDs?

- **Need an exclusive process – Not to break the hadron:**

Single diffractive: $h(p) \to h'(p') + A^*(p_1 = p - p')$ ← Momentum, spin, composition of particles (or partons)

A "soft" scale: $t = (p - p')^2 \equiv \Delta^2$



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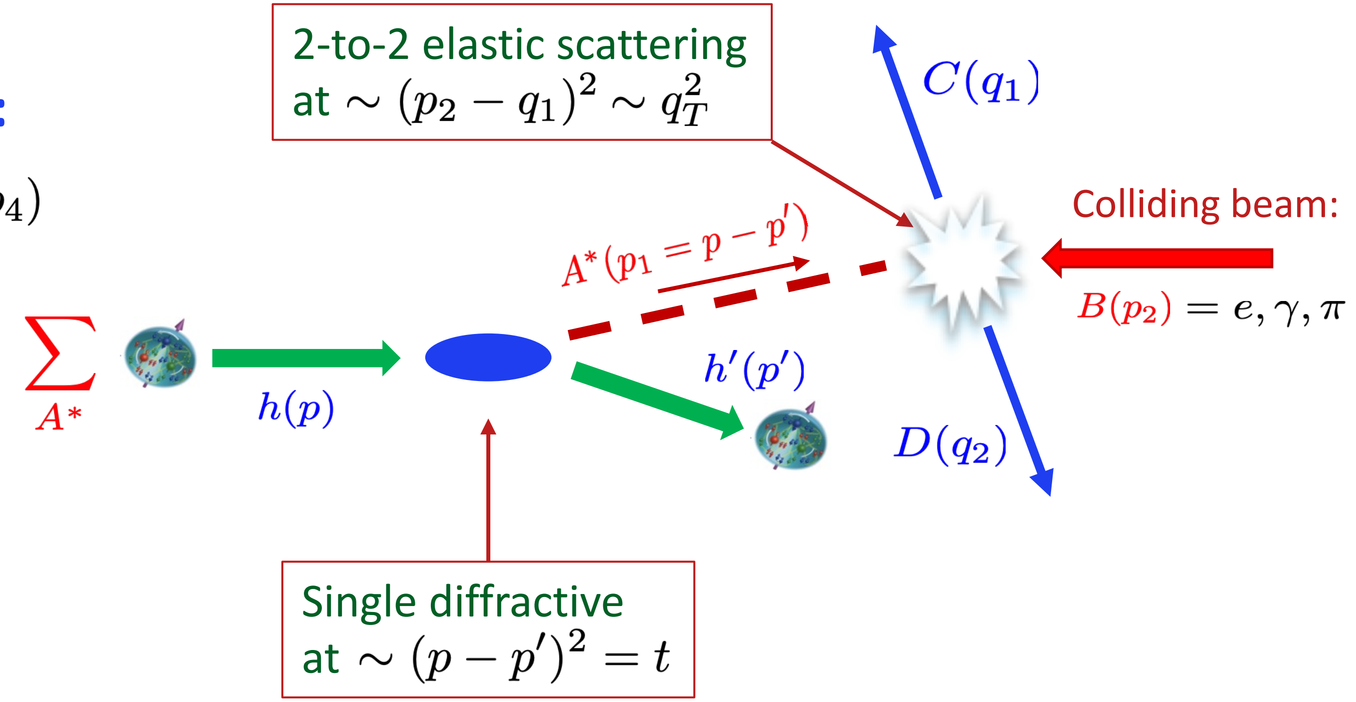
❑ Need a hard collision – to see the parton(s):

Hard exclusive: $A^*(p_1) + B(p_2) \to C(p_3) + D(p_4)$
($2 \to 2$ minimum)

A "hard" scale: $\hat{s} = (p_1 + p_2)^2 \gg \Lambda^2_{\rm QCD}$

$$\hat{t} = (p_1 - q_1)^2 \sim (p_1 - q_2)^2 = \hat{u}$$



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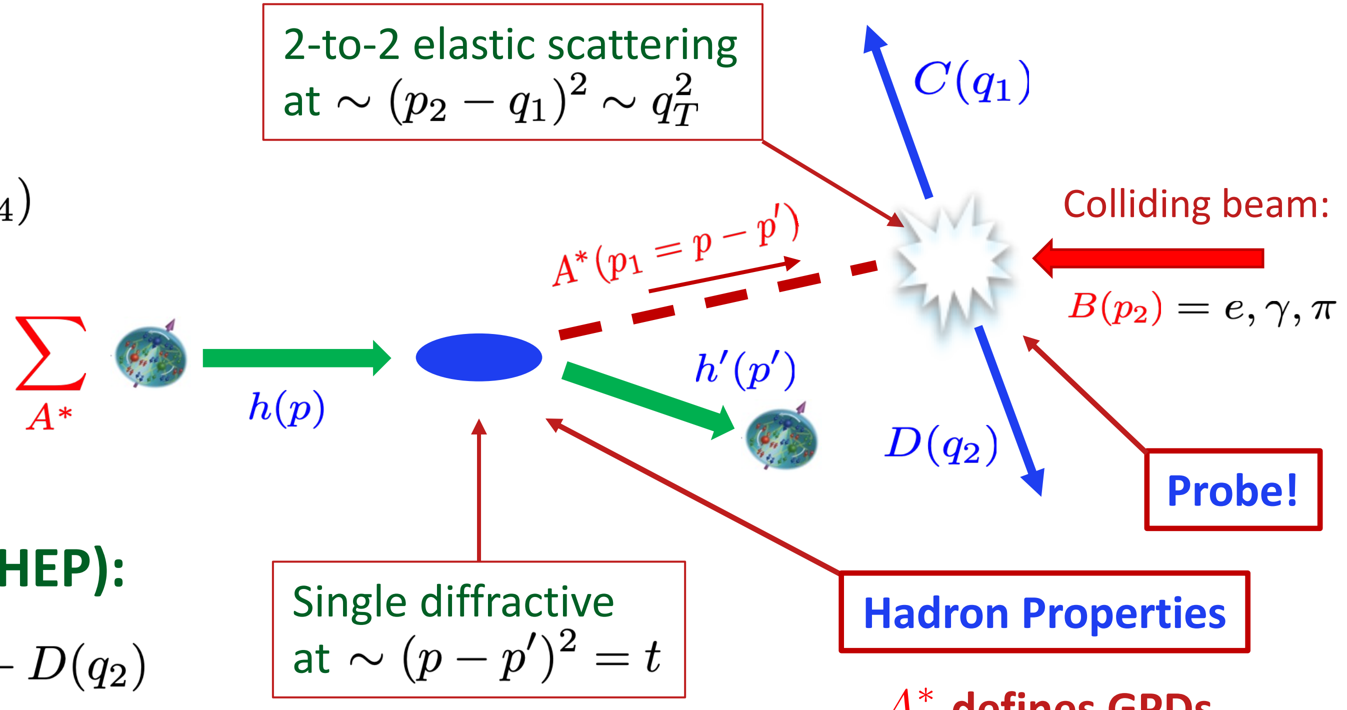

A^* defines GPDs & what we are probing!

❑ Single diffractive hard exclusive process (SDHEP):

SDHEP Process: $h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

(Need a minimum $2 \to 3$ SDHEPs)

Factorization: $Q^2 \sim \hat{s} \sim -\hat{t} \sim -\hat{u} \sim q_T^2 \gg -t$

A necessary condition!

How to Choose the Light-cone Vector "n" to Define GPDs?

❑ Choose the z-axis of the hard collisions – a choice of frame:

Recall – Inclusive DIS: $h(p) + l(\ell) \rightarrow l'(\ell') + X$

Let p and ℓ define the **z**-axis of the collision & p along the **+z** direction and ℓ long the **–z** direction,

$p^\mu = (E, \mathbf{0}_\perp, p_z) \rightarrow (p^+, p^\perp, p^-)$ with

$p^+ = (E + p_z)/\sqrt{2}$ and $p^- = (E - p_z)/\sqrt{2} = m^2/2p^+$

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$

along the direction of $l(\ell)$

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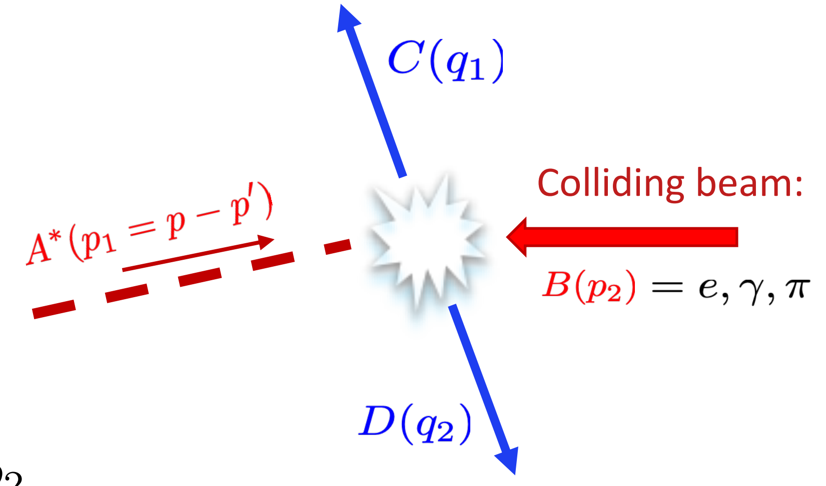

❑ Choose the z-axis of the hard exclusive collision – SDHEP frame:

Let momentum of *A**, $p_1 = p - p'$ and the momentum of colliding beam, p_2

define the **z**-axis of the hard exclusive collision with p_1 along with **+z** direction

➡ Light-cone vector: $n^\mu = (n^+, \mathbf{n}^\perp, n^-) = (0, \mathbf{0}^\perp, 1)$

along the direction of $B(p_2)$

$p_1^+ = (p - p')^+ = 2\,\xi P^+ = \xi\,(p + p')^+$ and $p_1^- = t\,/\,2p_1^+$

A^* defines GPDs & what we are probing!

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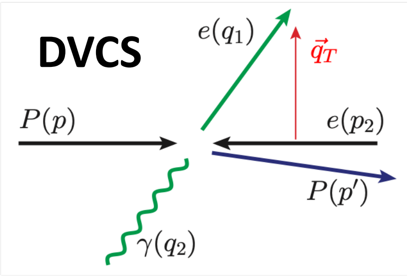
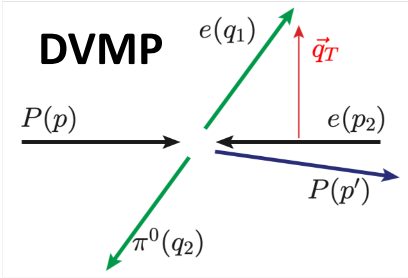
All Known Processes for Extracting GPDs – Fit the Kinematics of SDHEPs

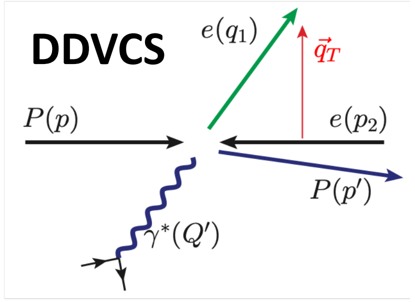
Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

- **Electro-production (JLab, EIC, …)**



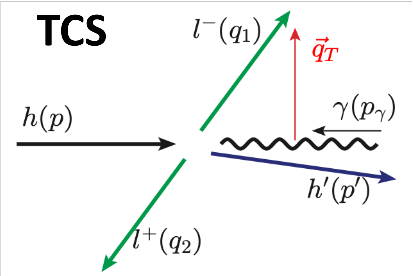


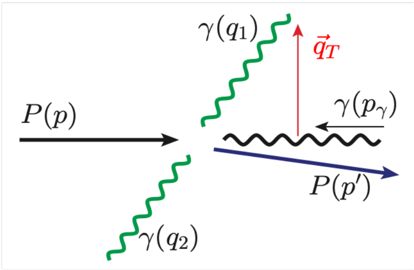
add virtuality

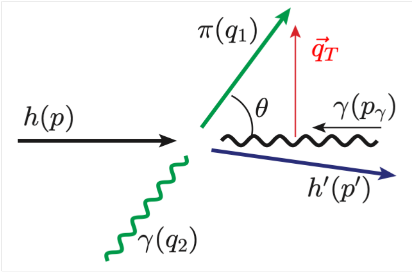


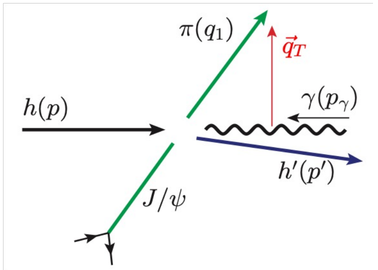
DHQP…

- **Photo-production (JLab, EIC, …)**

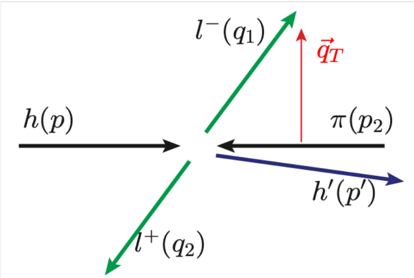


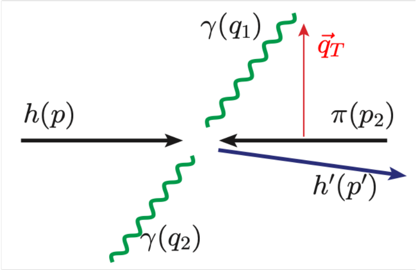






- **Meso-production (AMBER, J-PARC, …)**





…

Unified frame for all these observables

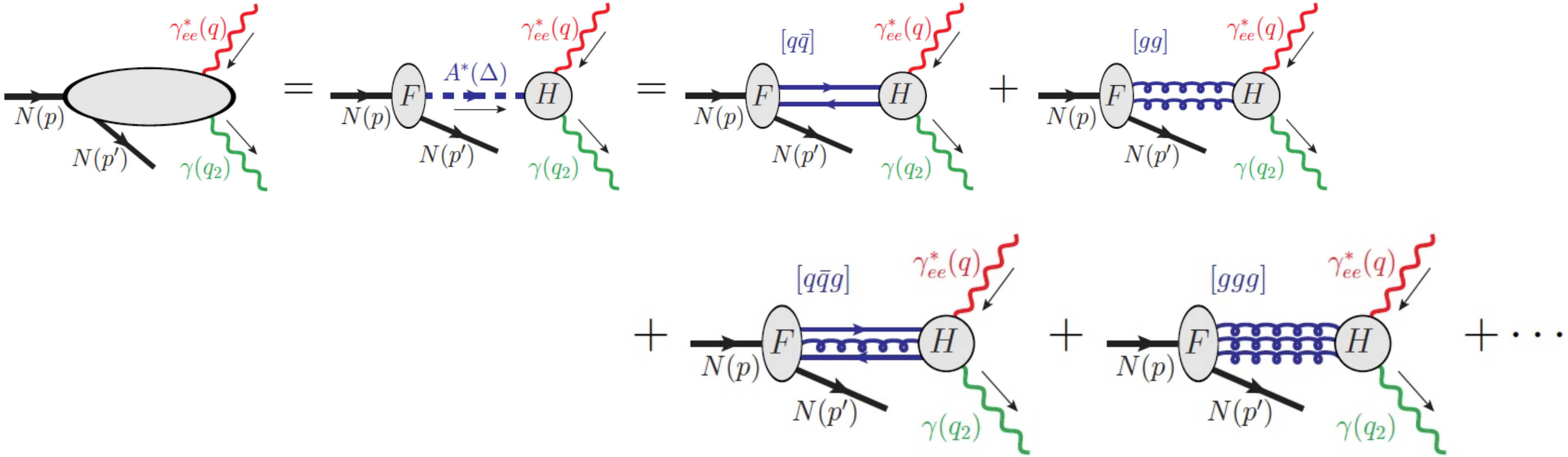
Same "n" along p_2

- Unified definition for all GPDs
- Ready for Global analysis of GPDs
- Easy to implement experimentally

DVCS Compton Form Factors (CFFs)

- **Like structure functions, CFFs include all power contributions:**

Qiu, Sato & Yu
Phys.Rev.D 113 (2026) 054037



In terms of EM gauge invariance, this virtual photon Compton scattering amplitude can be decomposed into 18 scalar Compton Form Factors.

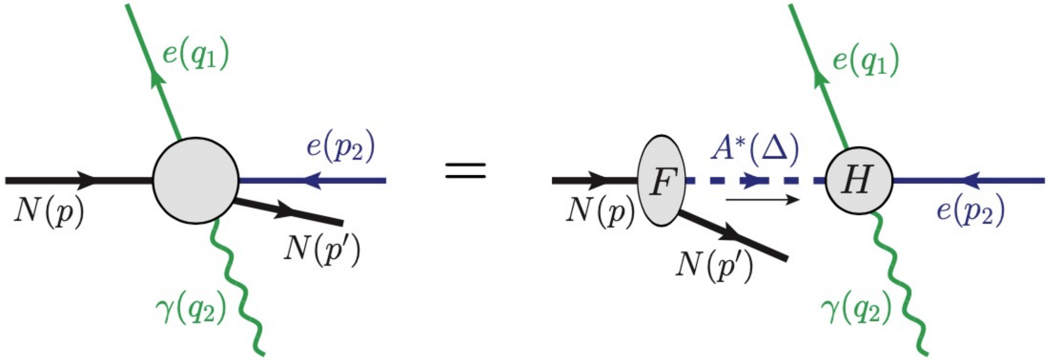
- **Extraction of GPDs from QCD global analyses:**

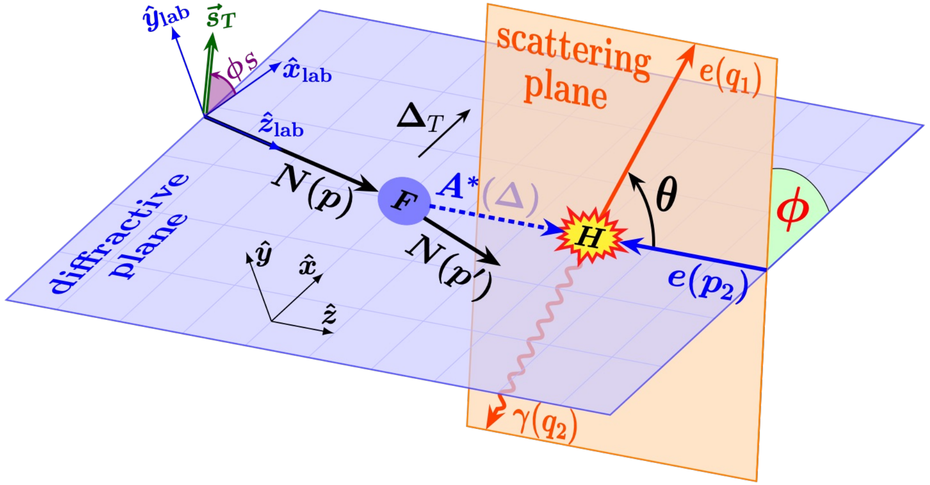
1) **Universality of GPDs – unlike PDFs, the choice of the light-cone vector to define "+" component is not unique! When considering the power corrections, it is critical to carefully define the GPDs!**

2) **Angular modulation is critically important for separating different GPDs, but, consistent choice of the "angle" to define the modulation is crucial!**

Angular Modulations – Separation of Different GPDs & Global Analyses

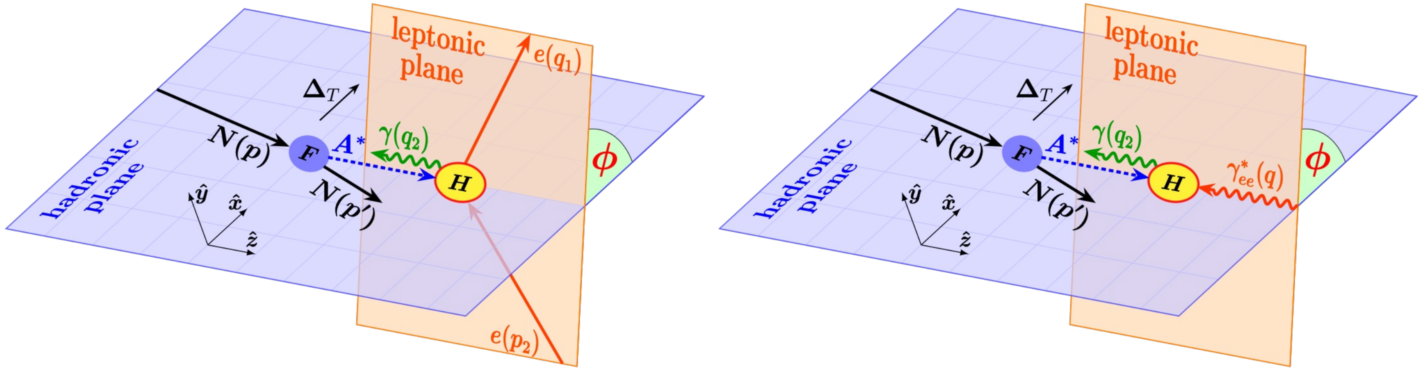
- **GPDs depend on the choice of Frame (light-cone n-vector to define the "+" component):**



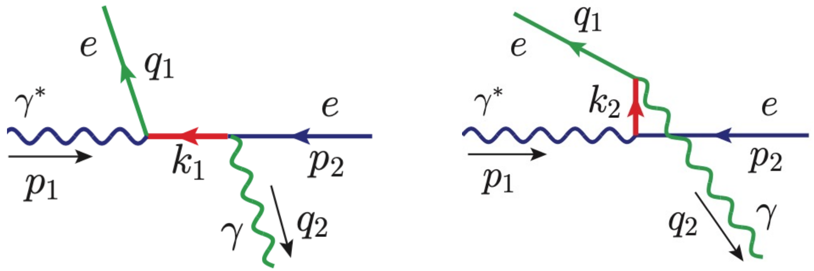


Angular modulation between "diffractive" and "scattering" planes to select the spin-state of A* - different GPDs

- **Experimental Breit frame is not ideal:**



BH is not a "t"-channel process:



Propagators of k_1 & k_2 have ϕ-dependence!

Angular modulation between "leptonic" and "hadronic" planes do not necessarily select the definite spin-state of A* - different GPDs!

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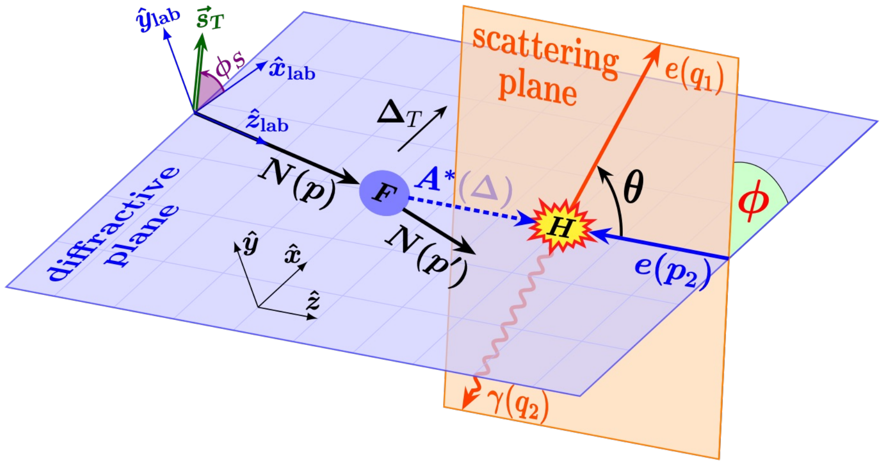
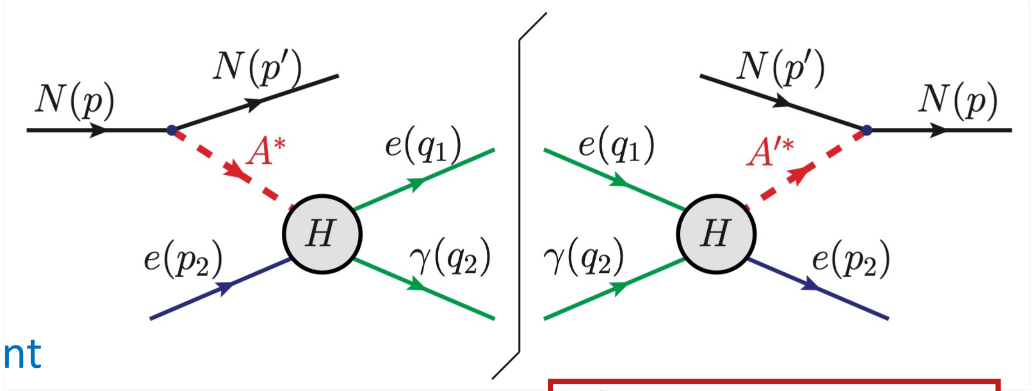
SDHEP Frame vs. Breit Frame – Use DVCS as an Example

❑ SDHEP Frame = A^* - Beam Frame:



Hadron plays the role of the lepton in Breit frame



- Easy to implement
- A^* defines the GPDs
- Angular modulation to separate GPDs
- Same for all GPD finding processes!

$\phi-$dependence
⇔ quantum # of A^*

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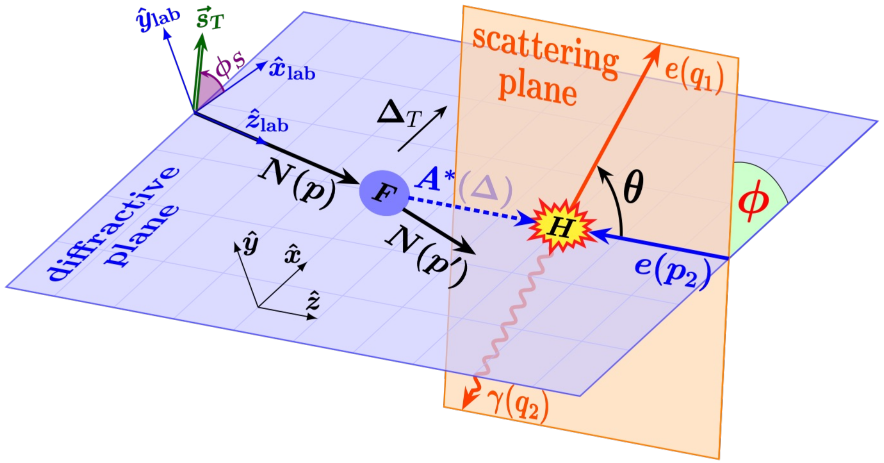

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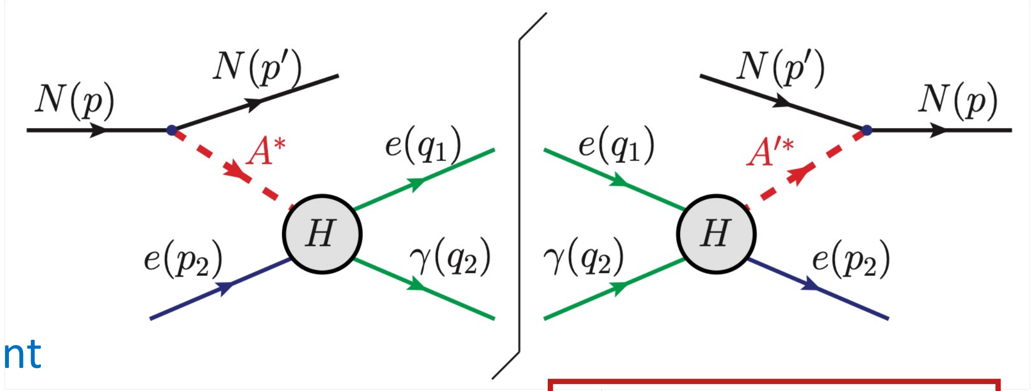

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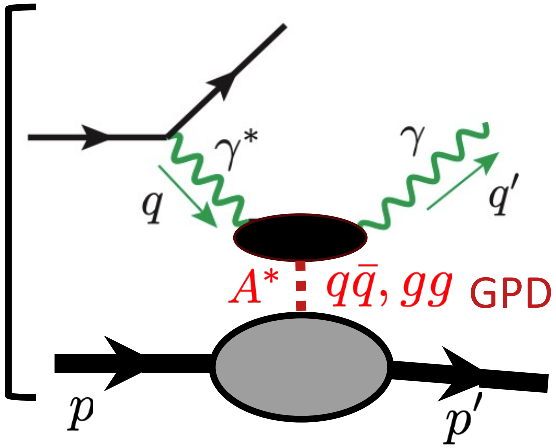
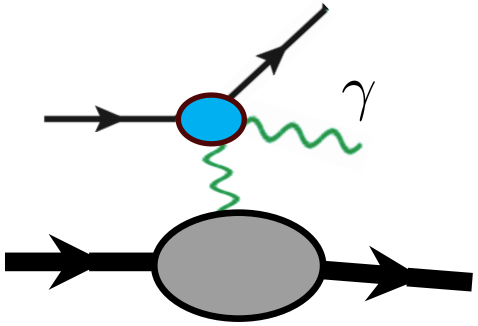
$\phi-$**dependence**
⇔ quantum # of A^*

❑ Breit Frome = γ^* - Beam Frame: $e(\ell)+h(p)\to e(\ell')+h(p')+\gamma(q')$



DVCS: in terms of factorized GPDs

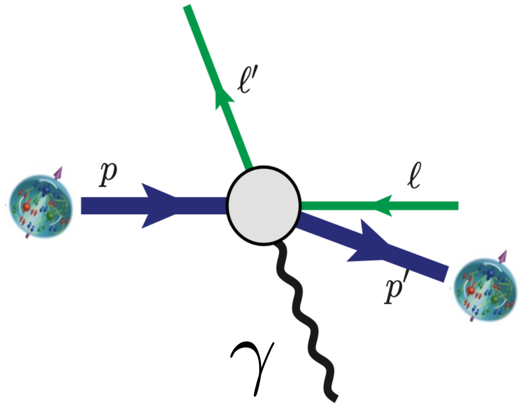
\+



2 EM form factors

Bethe-Heitler subprocess in terms of EM form factors

$]^2$



Breit frame is a natural extension from the treatment of DIS, SIDIS

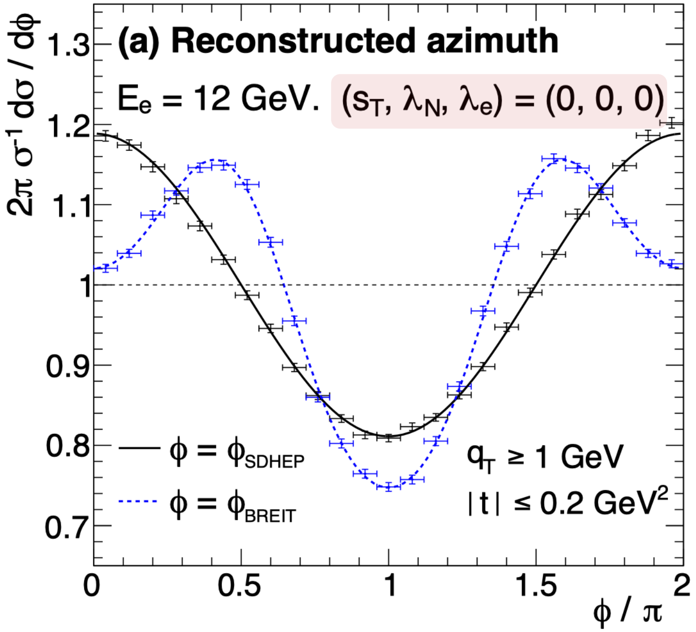
Angular distribution between lepton plane ($\ell\to\ell'$) and hadron plane ($p\to p'$) DOES not have a simple relation to GPDs to help their extraction, due to mixing of BH!

Simple Numerical Examples for Angular Distribution/Modulation

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta}\cdot\Big[1 + \lambda_e\lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$
$$+ \left(\boxed{A_{UU}^{\rm NLP}} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct

unpolarized



(normalized)

SDHEP fit to $1.00 + 0.190\cos\phi$ $\Rightarrow$ $\langle A_{UU}^{\rm NLP}\rangle = 0.190$

Breit fit to $1.00 + 0.15\cos\phi - 0.12\cos 2\phi - 0.01\cos 3\phi + 0.01\cos 4\phi$

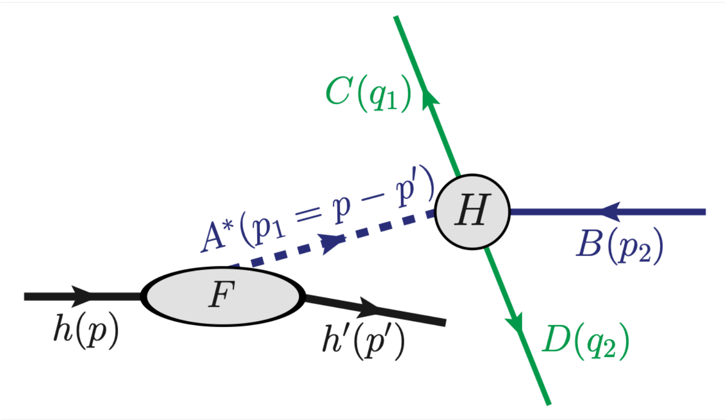
- **No straightforward interpretation of the coefficients.**
- **Need to introduce more gears in GPD extraction.**

[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
…

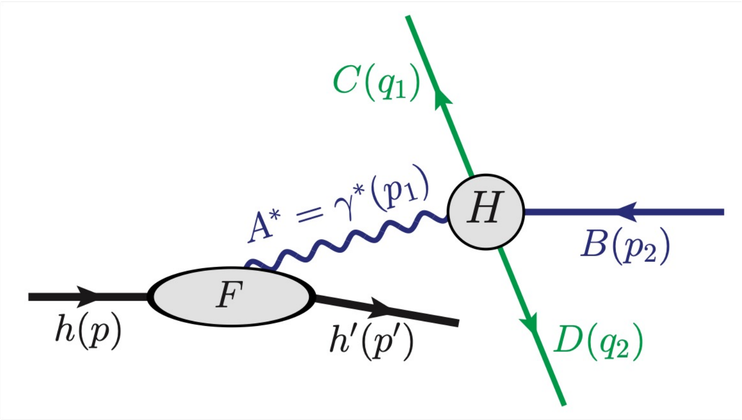
Jefferson Lab

Nature of the Exchange State A* - An Expansion in Powers of $\sqrt{-t}/q_T$

$\sum_{A^*}$

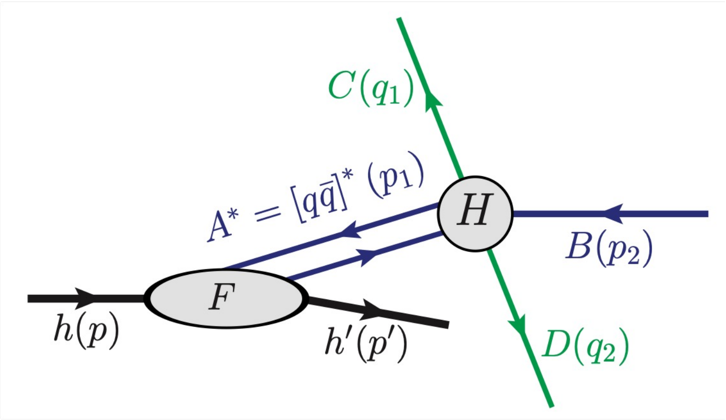


$=$

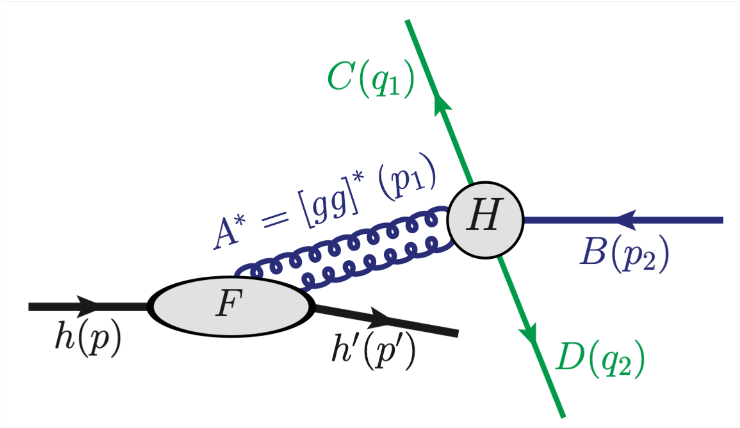


Leading Power in $\frac{\sqrt{-t}}{Q}$

$+$



$+$



Next Leading Power in $\frac{\sqrt{-t}}{Q}$

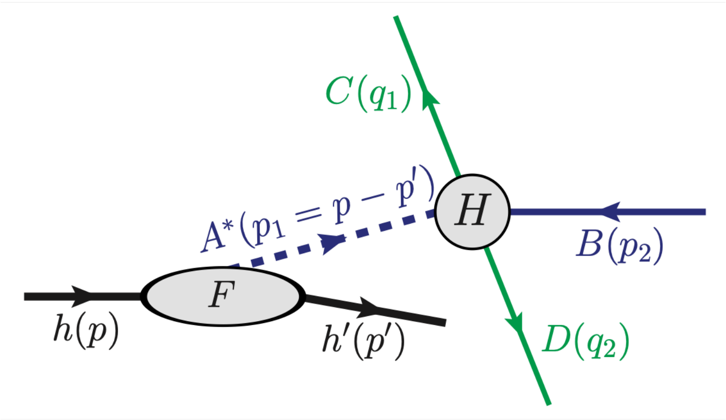
$+ \cdots$

$\geq$ 3 parton connection: further Power suppressed

If A* cannot be factorized in terms of partons, one might interpretate them as quasi-states, like pomeron, odderon, …, from the diffractive scattering

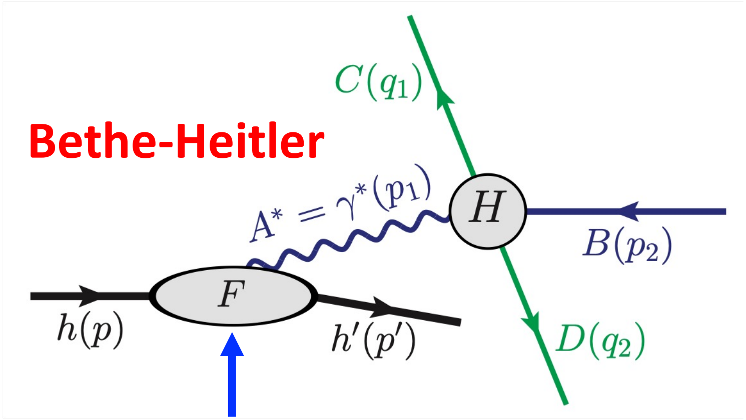
Nature of the Exchange State A* - An Expansion in Powers of $\sqrt{-t}/q_T$

$\sum_{A^*}$



$=$

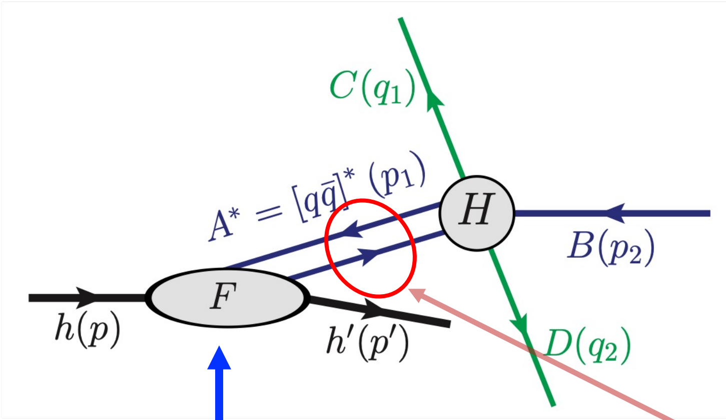
Bethe-Heitler



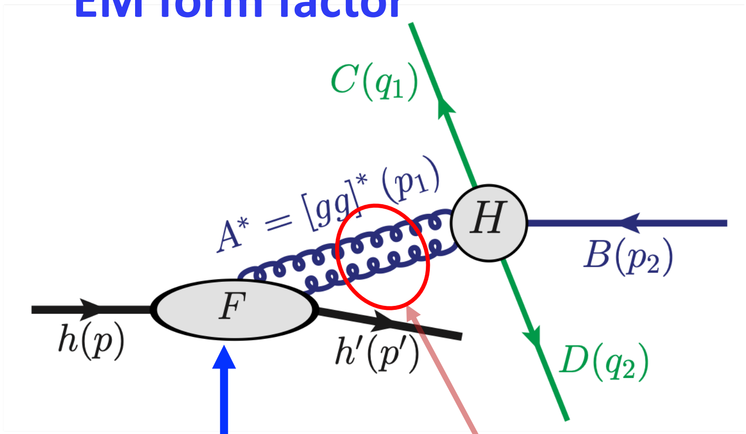
EM form factor

Leading Power in $\frac{\sqrt{-t}}{Q}$

$+$



$+$



Next Leading Power in $\frac{\sqrt{-t}}{Q}$

Quark GPD

Gluon GPD

$+ \cdots$

$\geq$ 3 parton connection: further Power suppressed

Ensured by $\sqrt{-t}/q_T \ll 1$

Energy $\gg$ virtuality

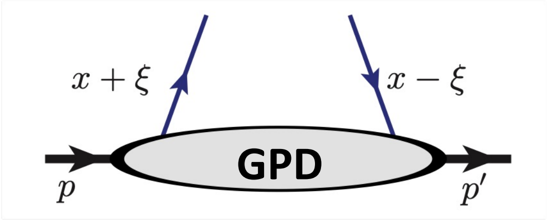
Factorization into GPDs

GPDs are defined by the quantum states of *A**

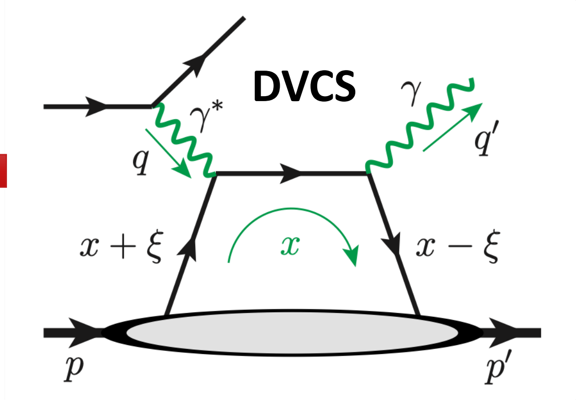
Jefferson Lab

Why the GPD's x-Dependence is so *Difficult* to Measure?

- **Amplitude nature: exclusive processes**



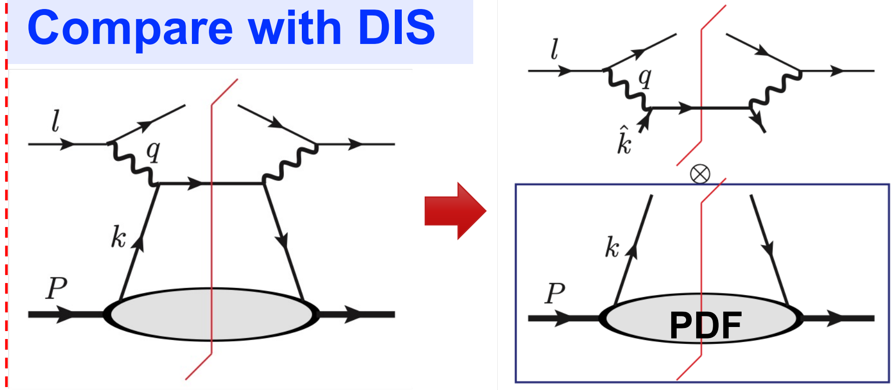
x ~ loop momentum



$$iM \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x=0;\quad x=\pm\xi$

Compare with DIS



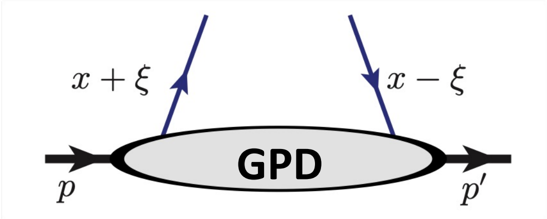
cross section: cut diagram

$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

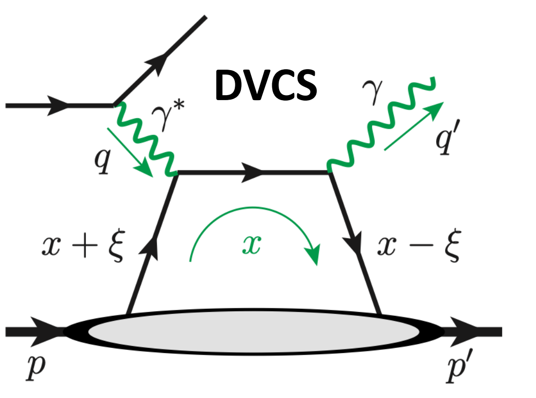
Jefferson Lab

Why the GPD's *x*-Dependence is so *Difficult* to Measure?

- **Amplitude nature: exclusive processes**



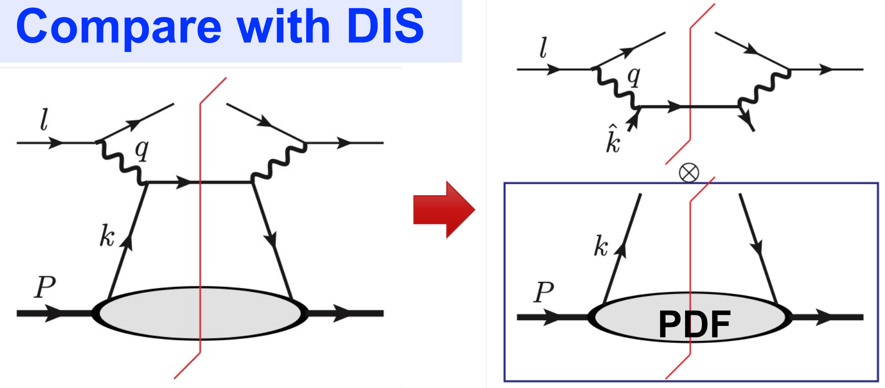
x ~ loop momentum



$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t)\cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x=0;\quad x=\pm\xi$

Compare with DIS



cross section: **cut diagram**

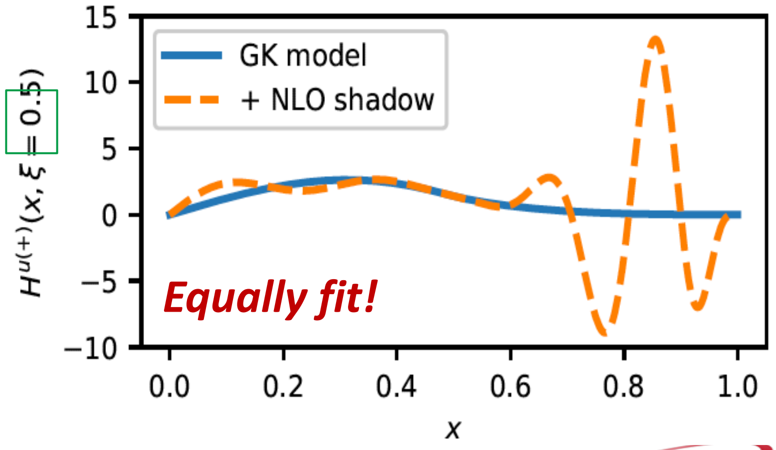
$$\sigma_{\text{DIS}} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\,\hat{\sigma}(x/x_B)$$

- **Sensitivity to x: comes from $C(x,\xi;Q/\mu)$**

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1}\mathrm{d}x\,\frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

“x-moment”

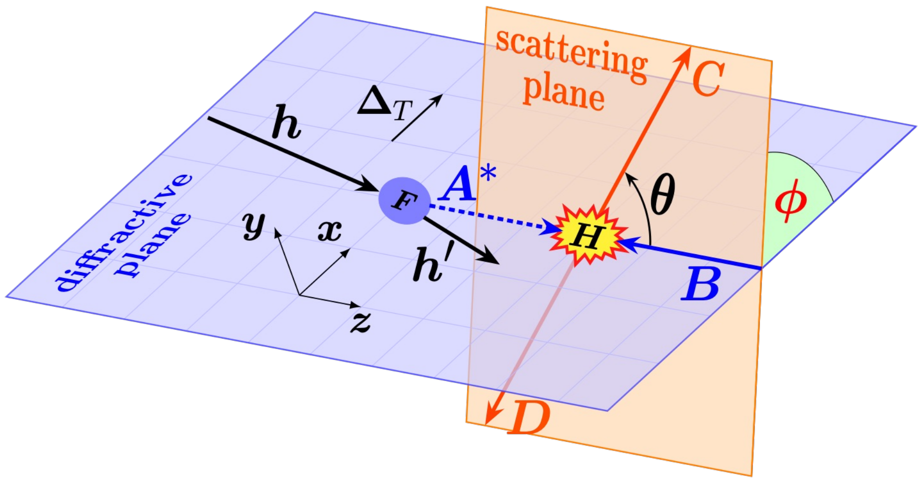


[Bertone et al. PRD `21]

20

Jefferson Lab

Where Does the x-sensitivity Come From?



❑ x-sensitivity ⇔ 2 → 2 hard scattering:

Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⟵ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\,\sin\theta$ ⟷ x
3. ϕ ⟵ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi}\cdot\int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q=\theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

- **Moment-type sensitivity:** $C(x;Q) = G(x)\cdot T(Q)$ ⟶ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ Independent of Q, Scaling for F_G

➡ Inversion problem: *shadow GPD* $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

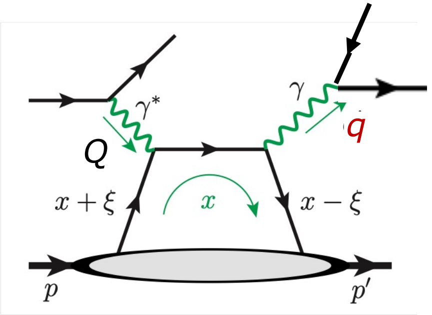
- **Enhanced sensitivity:** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟶ $d\sigma/dQ \sim \left|C(x;Q)\otimes_x F(x,\xi,t)\right|^2$

What Kind of Process Could be Sensitive to the *x*-Dependence?

- **Create an entanglement between the internal *x* and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external *q* to sample different part of x.



- **Double DVCS (two scales):**

$$x_p(\xi,q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

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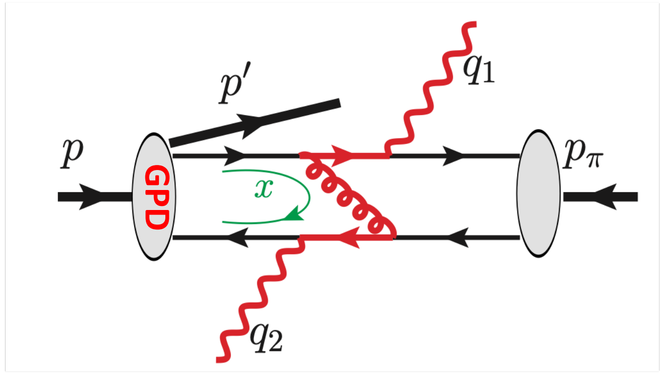
Hard scale: $q_T \gg \Lambda_{\rm QCD}$ **Soft scale:** $t \sim \Lambda^2_{\rm QCD}$



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu)\cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$

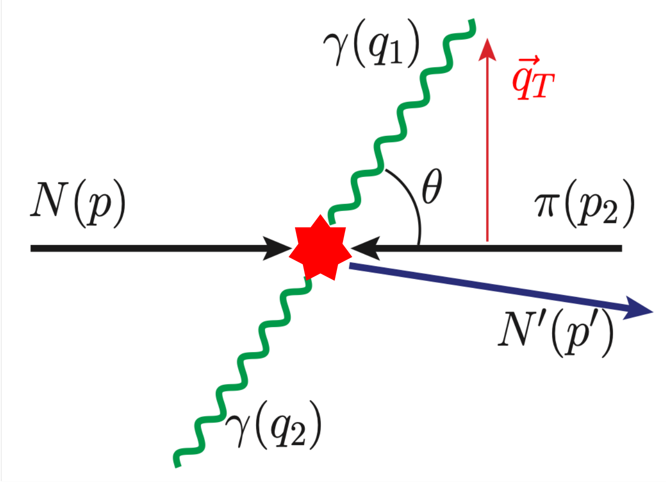
$$\Longrightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}t\;\mathrm{d}\xi\;\mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

q_T distribution is "conjugate" to x distribution

$$x \leftrightarrow q_T$$

New Physical Processes to have Enhanced x-Sensitivity

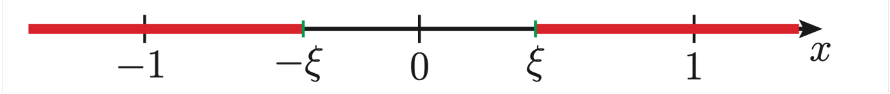
- **Diphoton process** $N\pi \to N'\gamma\gamma:\ (1)\ p\pi^- \to n\gamma\gamma;\ (2)\ n\pi^+ \to p\gamma\gamma$



$i\mathcal{M}$ **also contains**

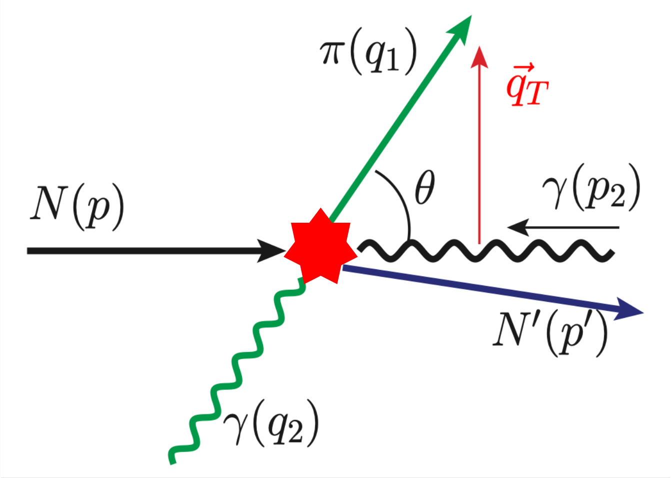
$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho(z;\theta) + i\epsilon\, \mathrm{sgn}\left[\cos^2(\theta/2) - z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2)\, z}{1 - z - \tan^2(\theta/2)\, z}\right] \in (-\infty, -\xi] \cup [\xi, \infty)$$



Covers the DGLAP region

- **γ-π Pair Photoproduction:**



$i\mathcal{M}$ **also contains:**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

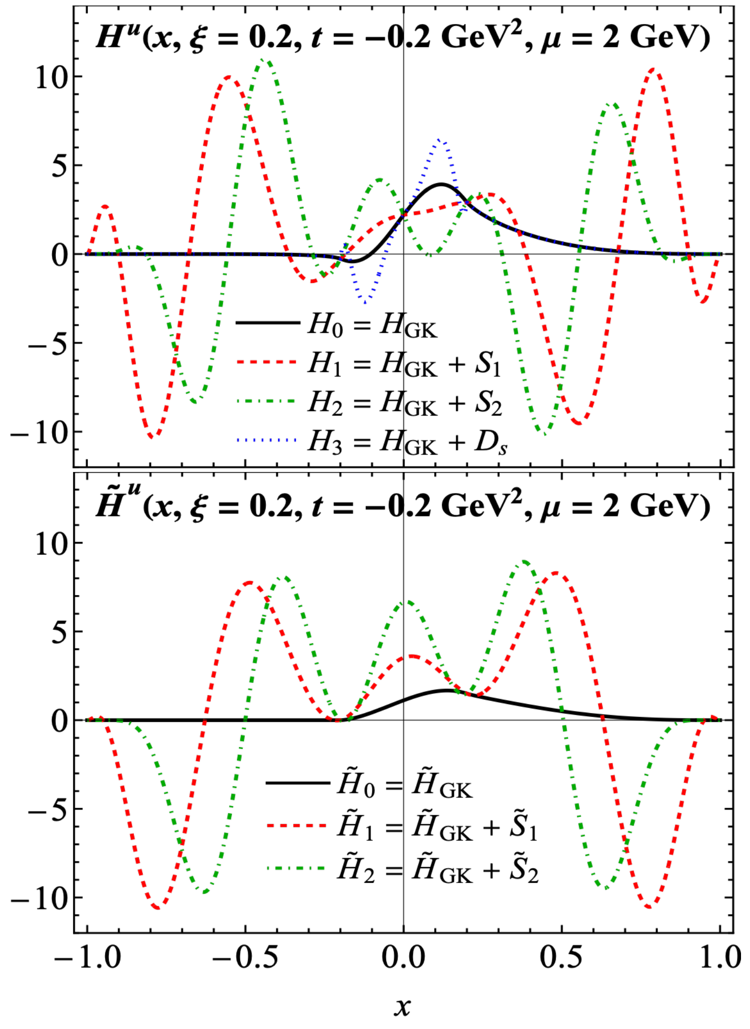
$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$

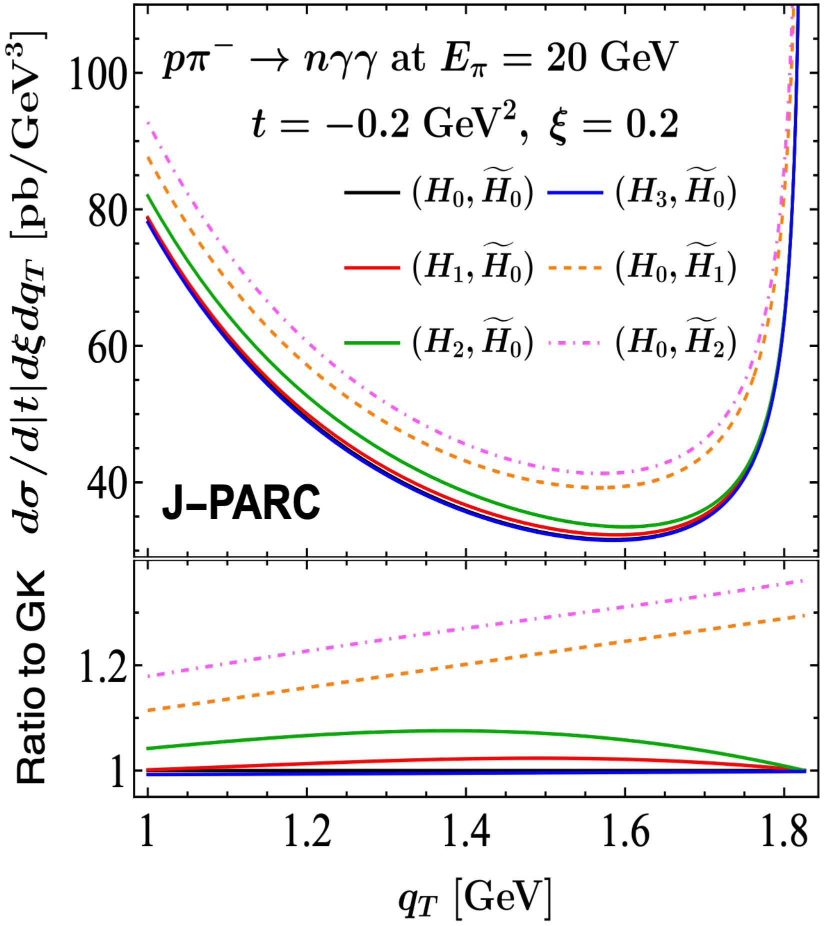
Covers the ERBL region

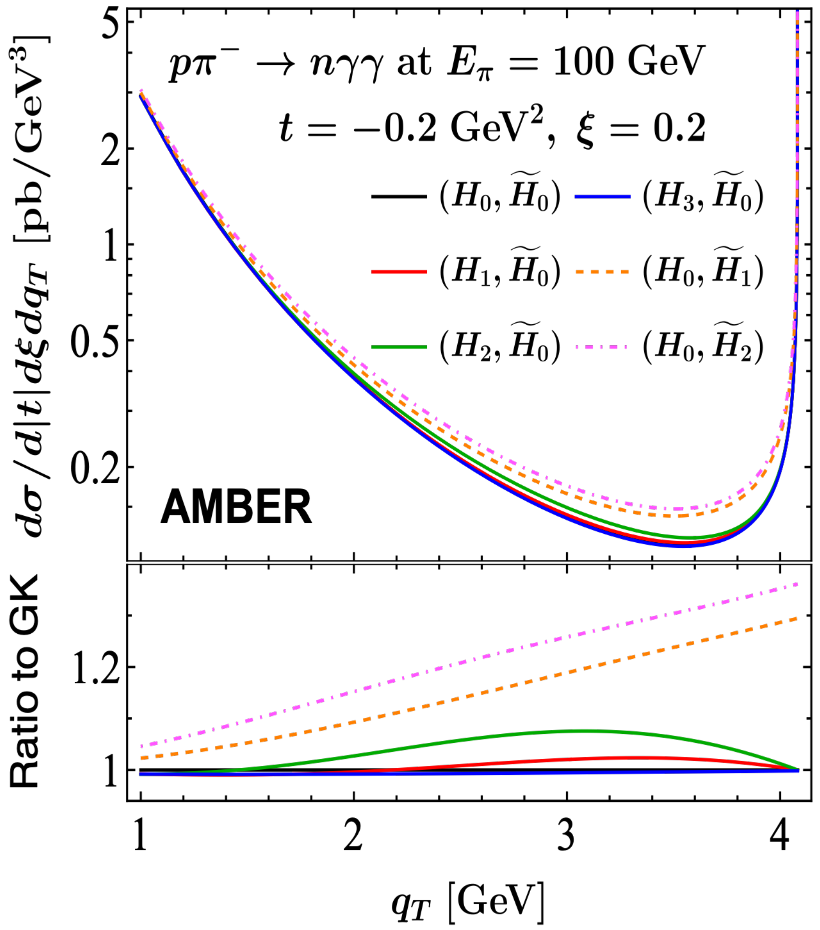
Test the Sensitivity on the GPDs' x-Dependence

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23

GPD models = GK model + shadow GPDs ⟸ $$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$







Test the Sensitivity on the GPDs' x-Dependence

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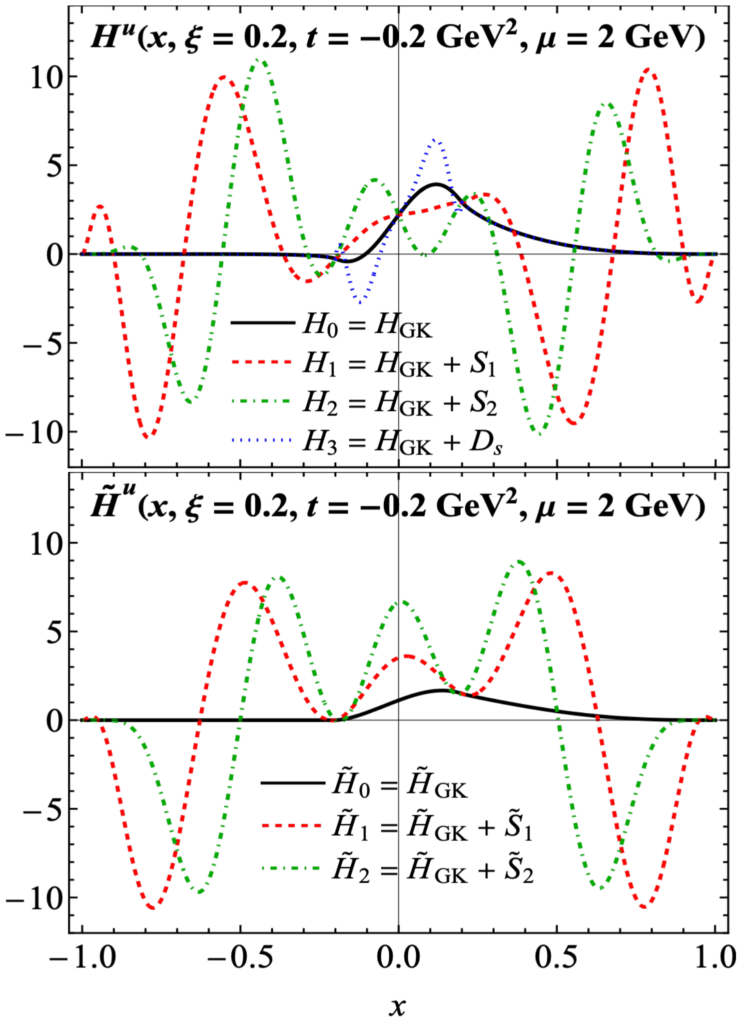
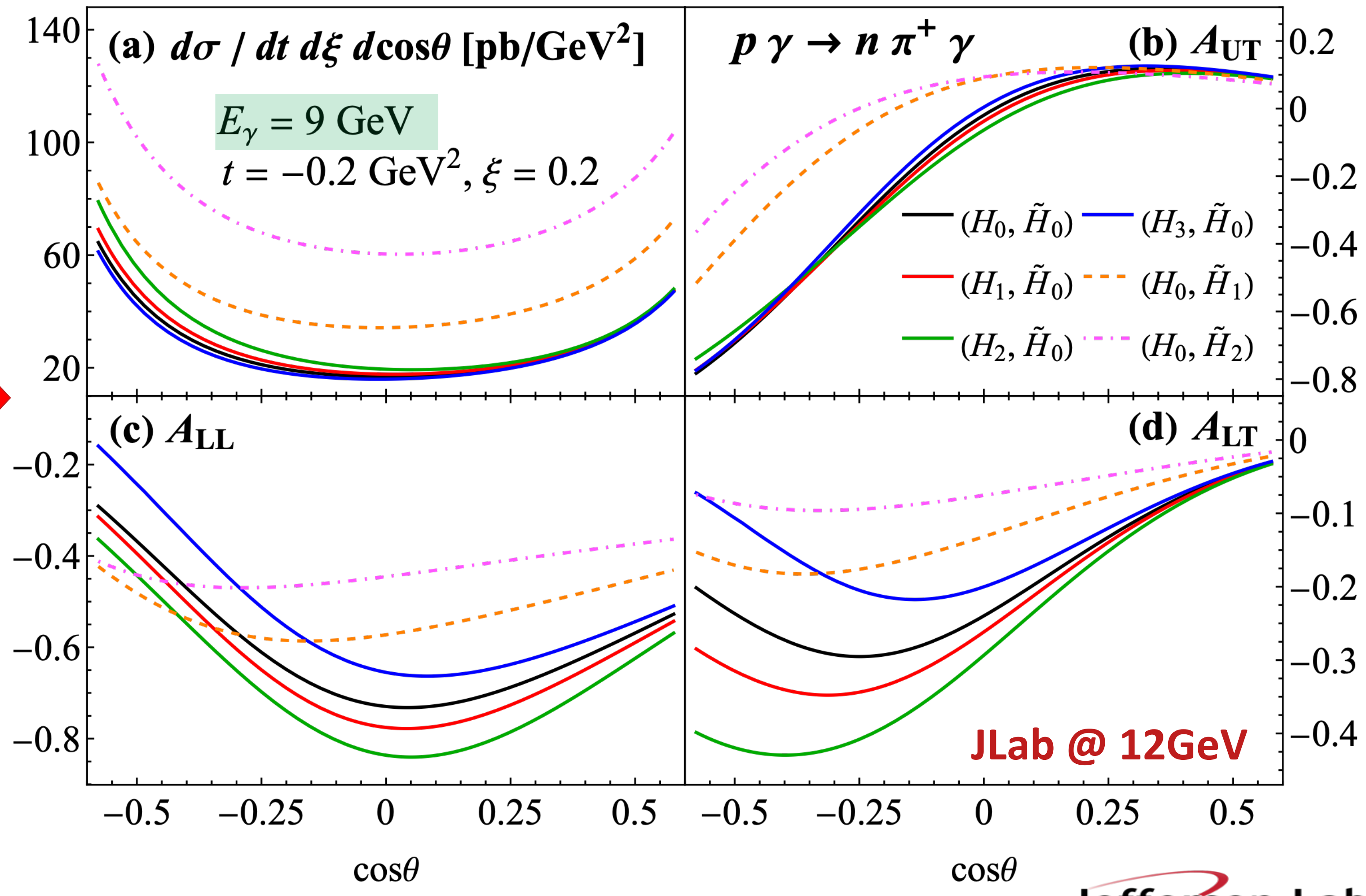


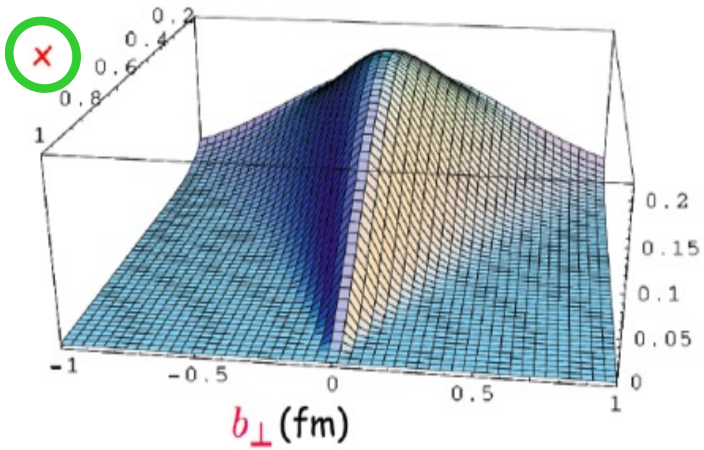


Summary and Outlook

- **GPDs are fundamental, carrying rich information on:**
  - Tomographic images of confined quarks and gluons
  - Underline dynamics of hadronic properties
- **The $2 \to 3$ SDHEPs are excellent physical processes for extracting of GPDs**
  - SDHEP frame is the right one for evaluating angular modulations
  - Need SDHEPs with ***x*** of GPDs entangled with measured hard scales!
- **QCD Global analyses to extract GPDs:**
  - With $p \neq p'$, the choice of "+" component is not unique
  - SDHEP frame for all known SDHEPs provides a unique way to define the GPDs, necessary for Global analyses
  - Need to identify more factorizable SDHEPs for extracting GPDs through Global analyses
  - …

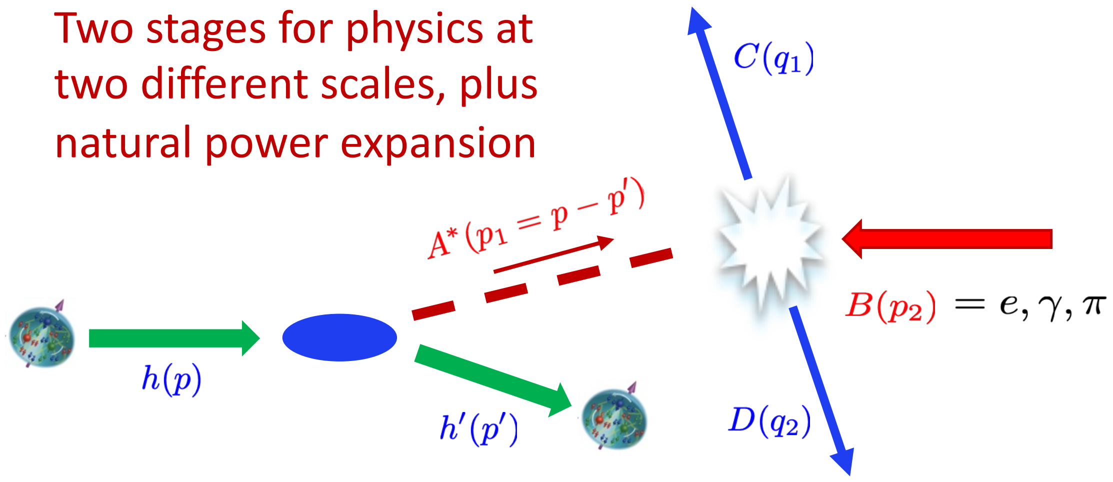
A long but challenging & exciting way to go!

3D image



SDHEPs:

Two stages for physics at two different scales, plus natural power expansion

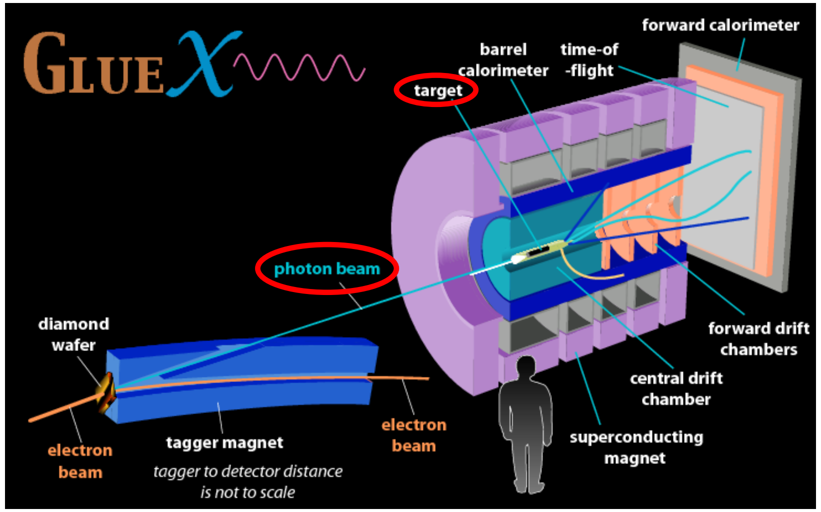


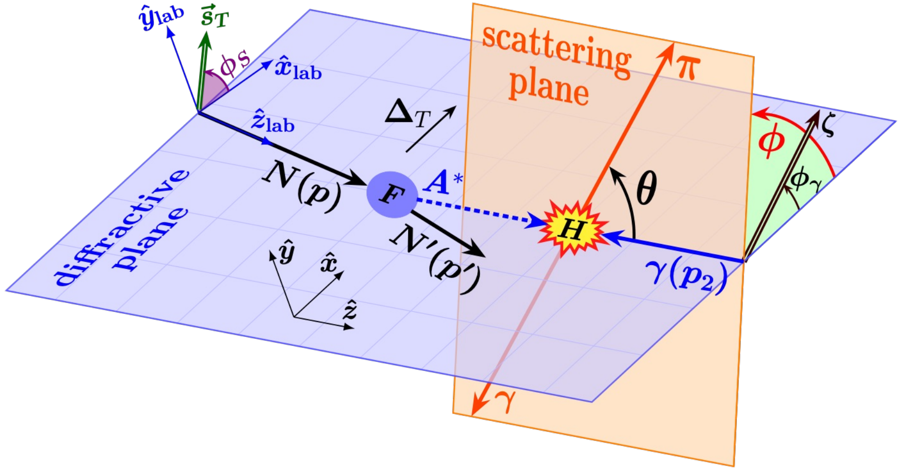
Thanks!

Backup Slides

Enhanced x-Sensitivity: (2) γ-π Pair Photoproduction (at JLab Hall D)

Qiu & Yu, PRL 131 (2023), 161902





❑ Polarization asymmetries:

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma A_{LL} + \zeta A_{UT}\cos 2(\phi-\phi_\gamma) + \lambda_N\zeta A_{LT}\sin 2(\phi-\phi_\gamma)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\,(\alpha_e\alpha_s)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2 + |\mathcal{M}_-^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel