Ab-initio calculation of $\pi^0/\eta/\eta'$ photoproduction off light nuclei

Photoproduction studies on the deuteron and Helium-3 in Hall D
JLab Workshop, April 18, 2026

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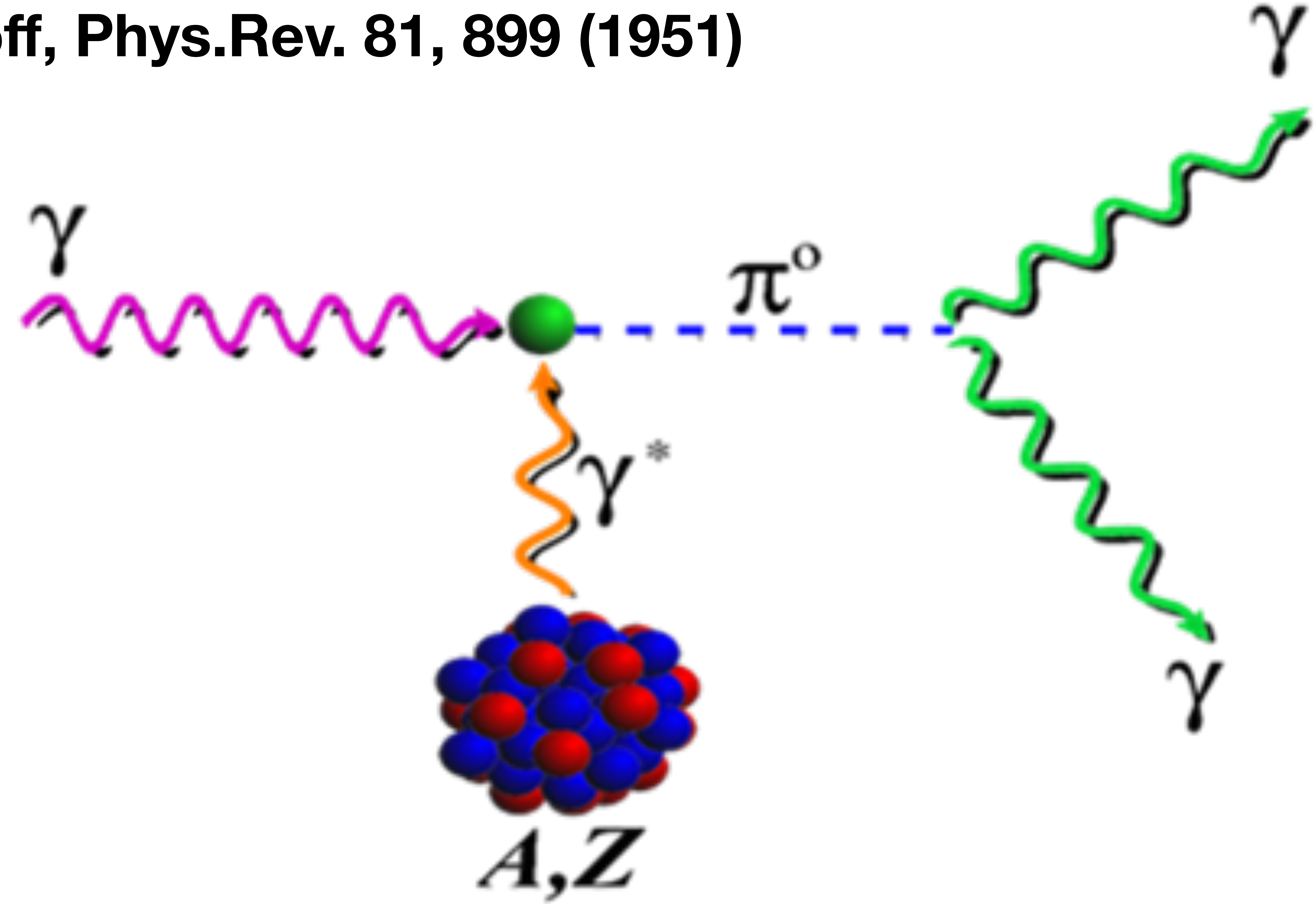
feat. Lorenzo Andreoli

Old Dominion University

Jefferson Lab

The Primakoff effect

H. Primakoff, Phys.Rev. 81, 899 (1951)



How to model the nuclear background?

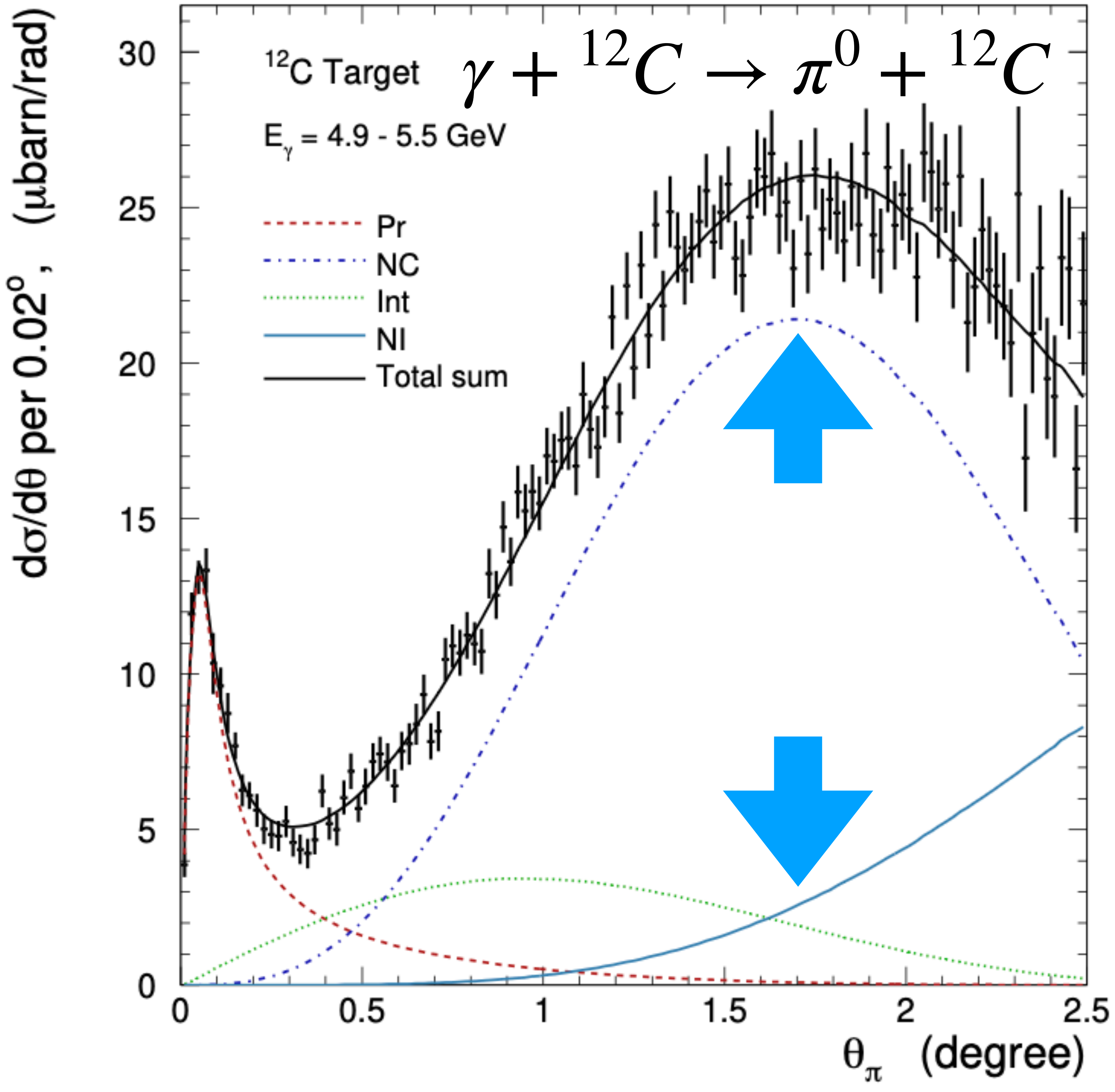
$$\frac{d\sigma}{d\Omega} = |T_C + e^{i\phi}T_S|^2 + \left(\frac{d\sigma}{d\Omega}\right)_{inc}$$

- Competitive processes

$$\gamma + A \rightarrow P + X$$

- Background in the Primakoff peak

- **Extraction of the P decay width relies on theoretical modeling of the nuclear background**



[PrimEx experiment PRL 106, 162303 (2011)]

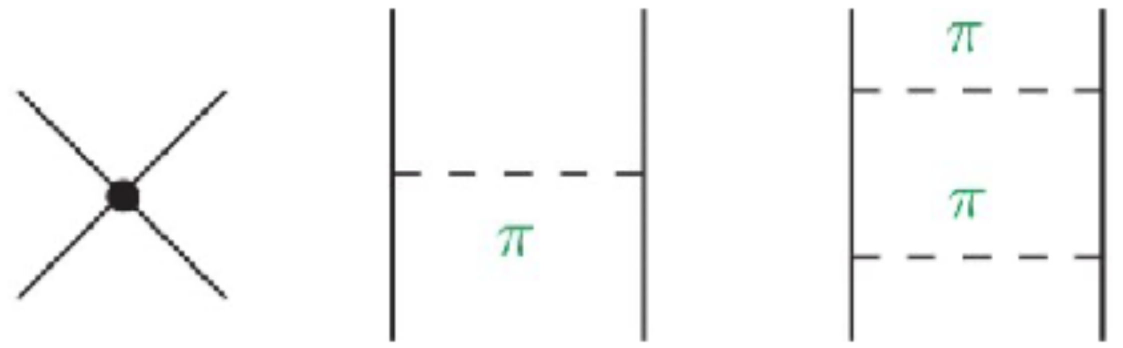
Outline

- Overview of ab-initio theory
- The Primakoff process in the ab-initio framework
- Coherent and Incoherent production
- Some preliminary results for $\gamma + {}^4He \rightarrow \eta + {}^4He$
- Beyond Primakoff: what can ab-initio do?

Ab-initio nuclear theory

- Nuclear Hamiltonian that describes the correlations among nucleons

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$



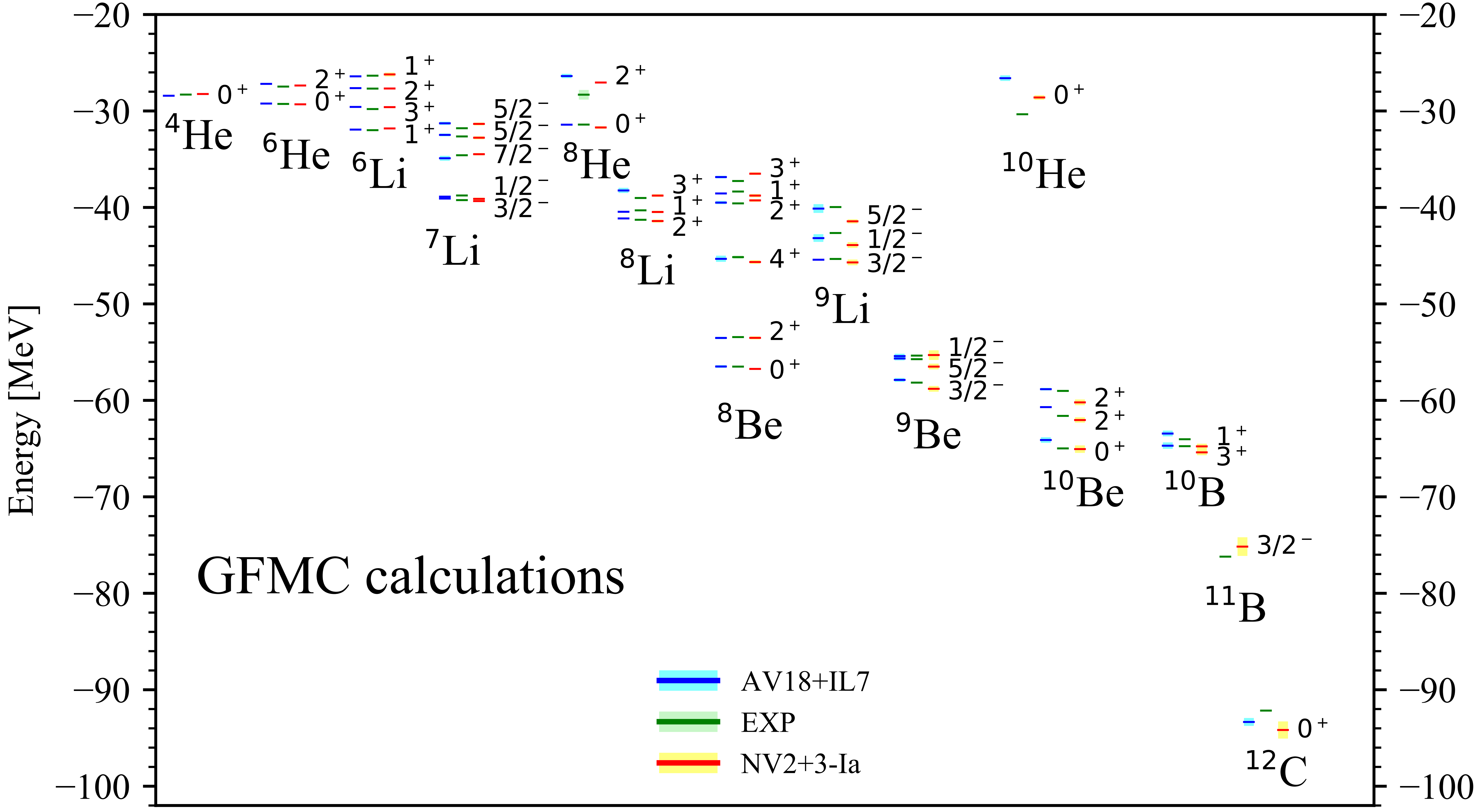
Contact term: short-range
Two-pion range: intermediate-range $r \propto (2m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$

- A numerical approach to solve the many-body nuclear problem

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

Quantum Monte Carlo: a Computational Methods to solve (numerically) exactly or with control approximations

Ab-initio nuclear theory



Spectra of light nuclei
Piarulli et al. PRL 120, 052503 (2018)

Electro scattering on nuclei

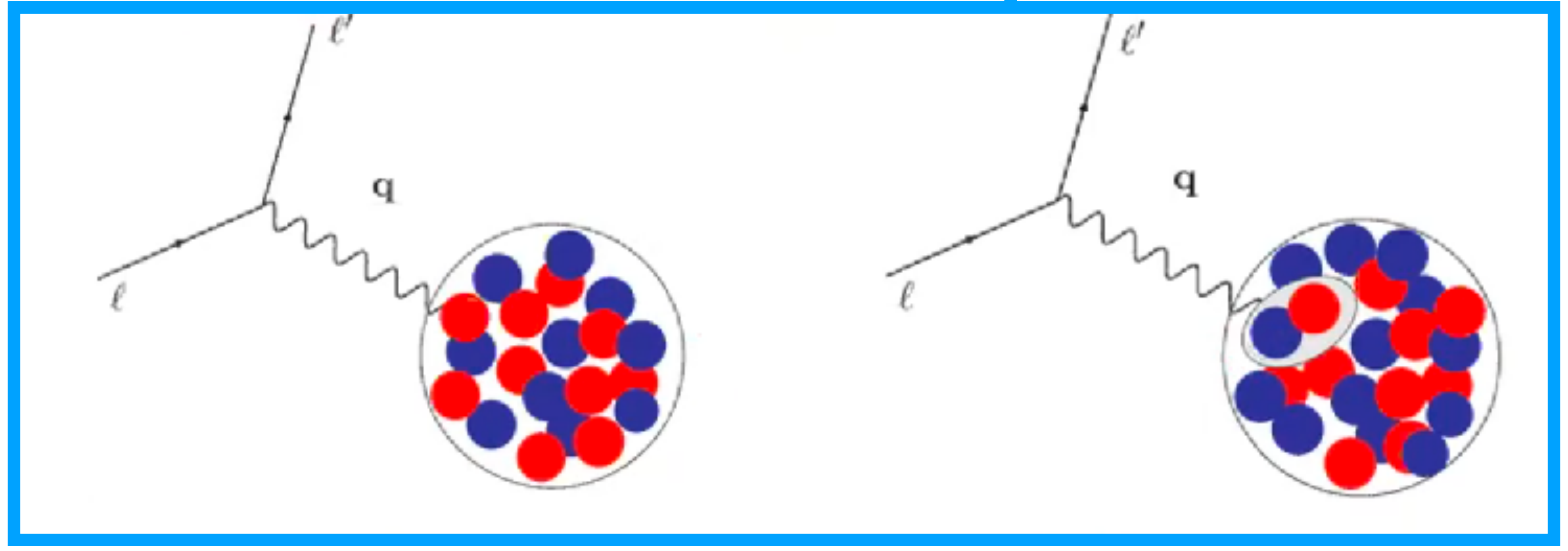
$$\frac{d^2\sigma}{d\Omega d\omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \left[\frac{Q^4}{q^4} R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\theta_e/2\right) R_T(q,\omega)\right]$$

$$R_L(q,\omega) \propto \langle \psi_i | \rho(q,\omega) | \psi_f \rangle \qquad R_T(q,\omega) \propto \langle \psi_i | j_y(q,\omega) | \psi_f \rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

+ Quantum Monte Carlo method (VMC)



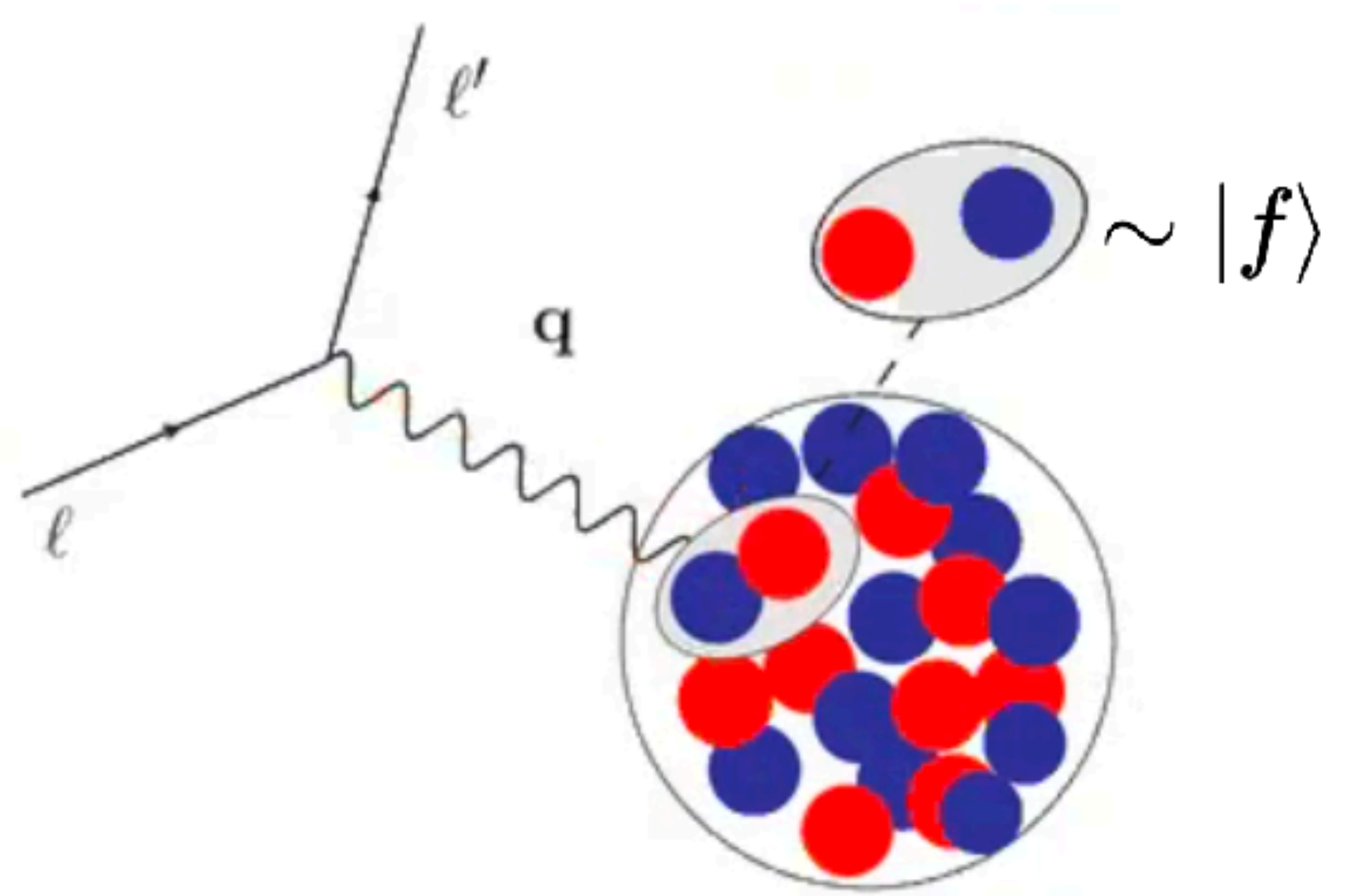
S. Pastore et al., PRC 101, 044612 (2020)

Short time approximation

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)]$$

We can't really compute it

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f)|\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$



$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

The sum over all final states is replaced by a two nucleon propagator:

- Inclusion of full two-body dynamics
- High momentum transfered (q>300 MeV/c)

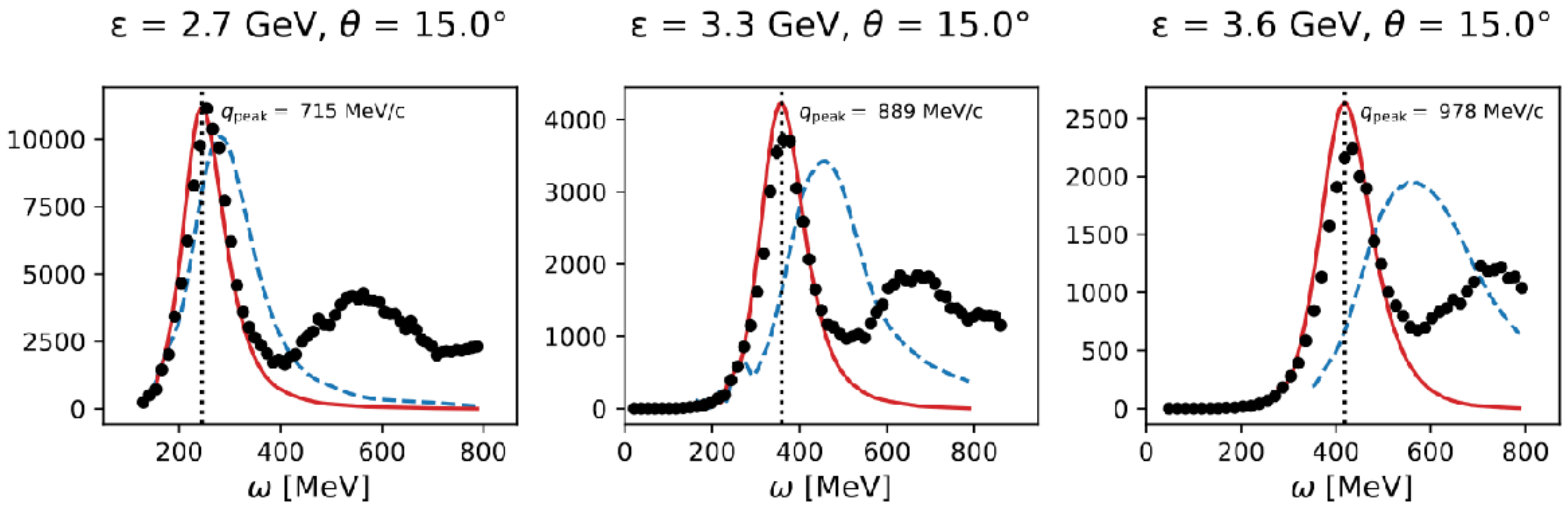
Figures and formulas courtesy of L. Andreoli and S. Pastore

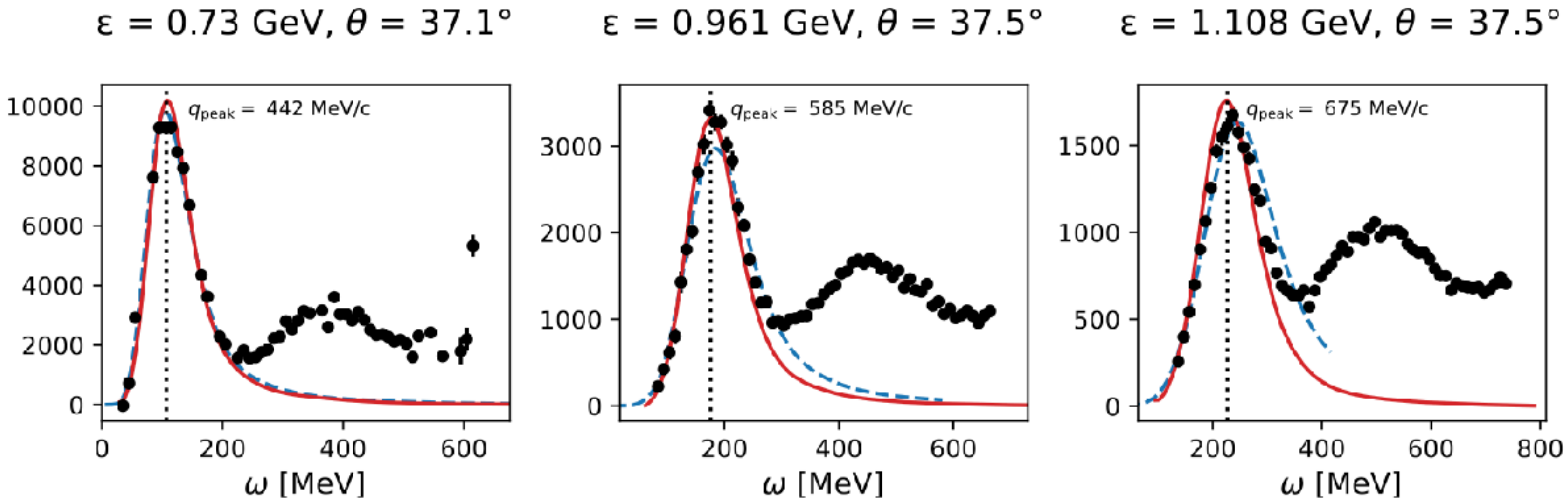
L. Andreoli, R. Weiss et al., PRC 113, 045501 (2026)

Validation electron-scattering cross sections

AV18+UIX with phenomenological 1-body current

Helium-3





Helium-4

S. Bacca et al., PRL 102, 162501 (2009)

Lorentz Integral Transform

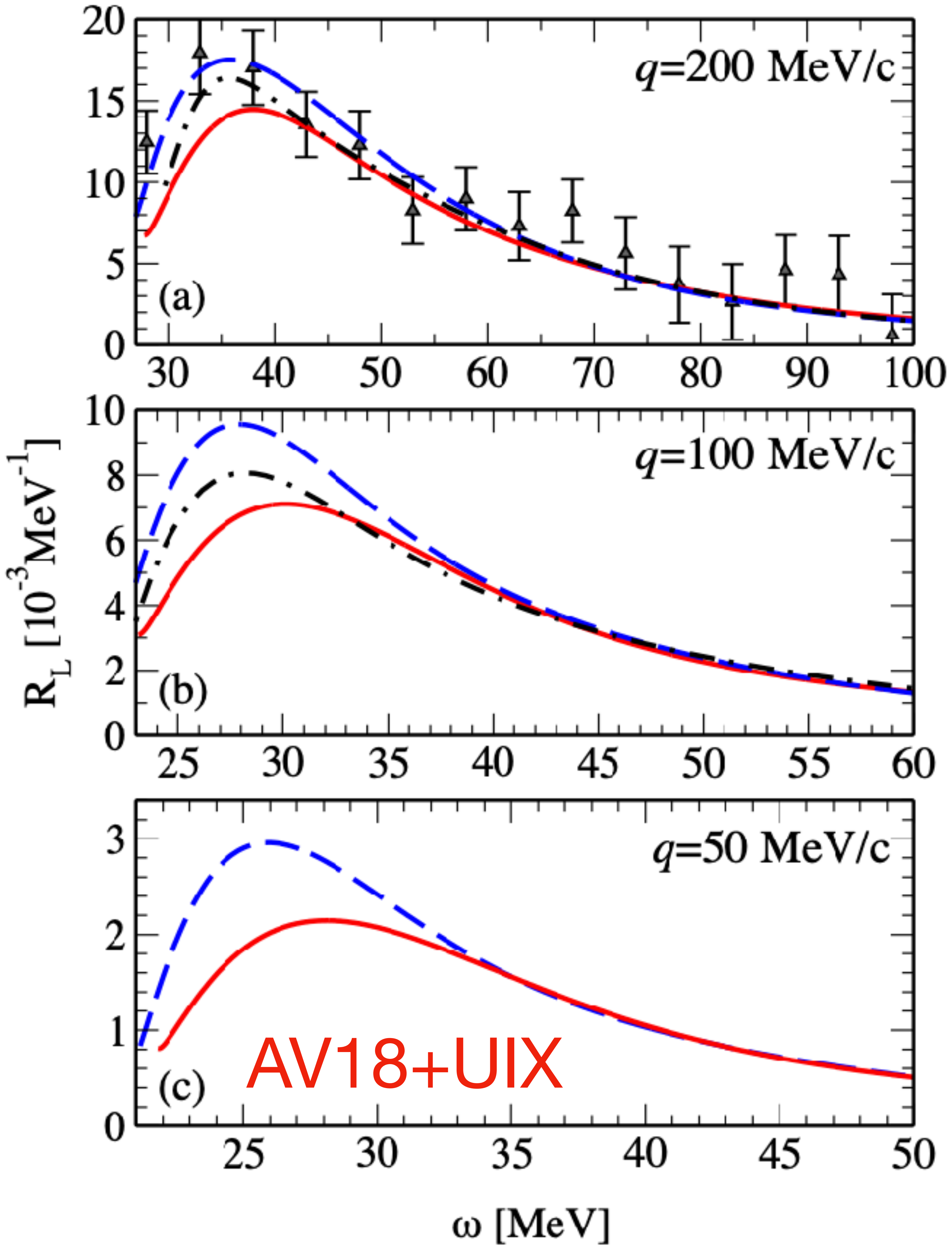
feat. Sonia Bacca (Mainz)

- Response functions at low momentum transfered
- Contain the full many-body dynamics
- Inversion of the transform is complicated

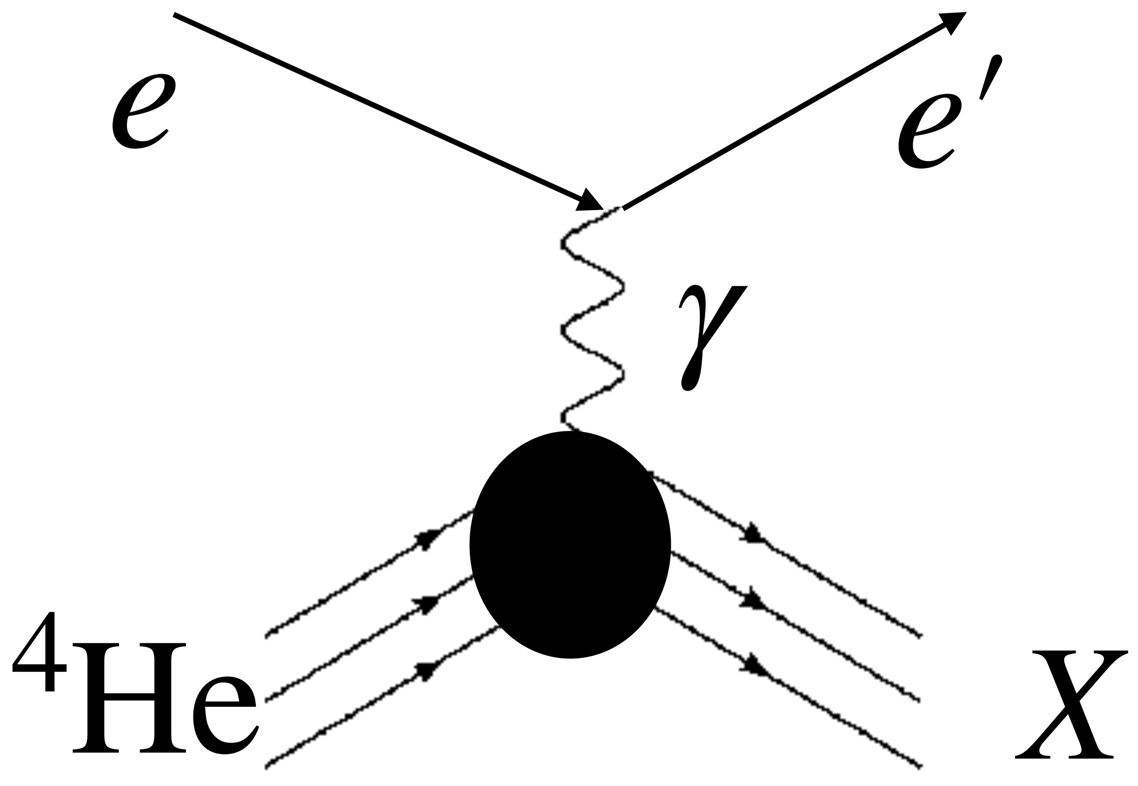
$$\mathscr{L}_L(\sigma, q) = \int d\omega \frac{R_L(\omega, q)}{(\omega - \sigma_R)^2 + \sigma_I^2}$$

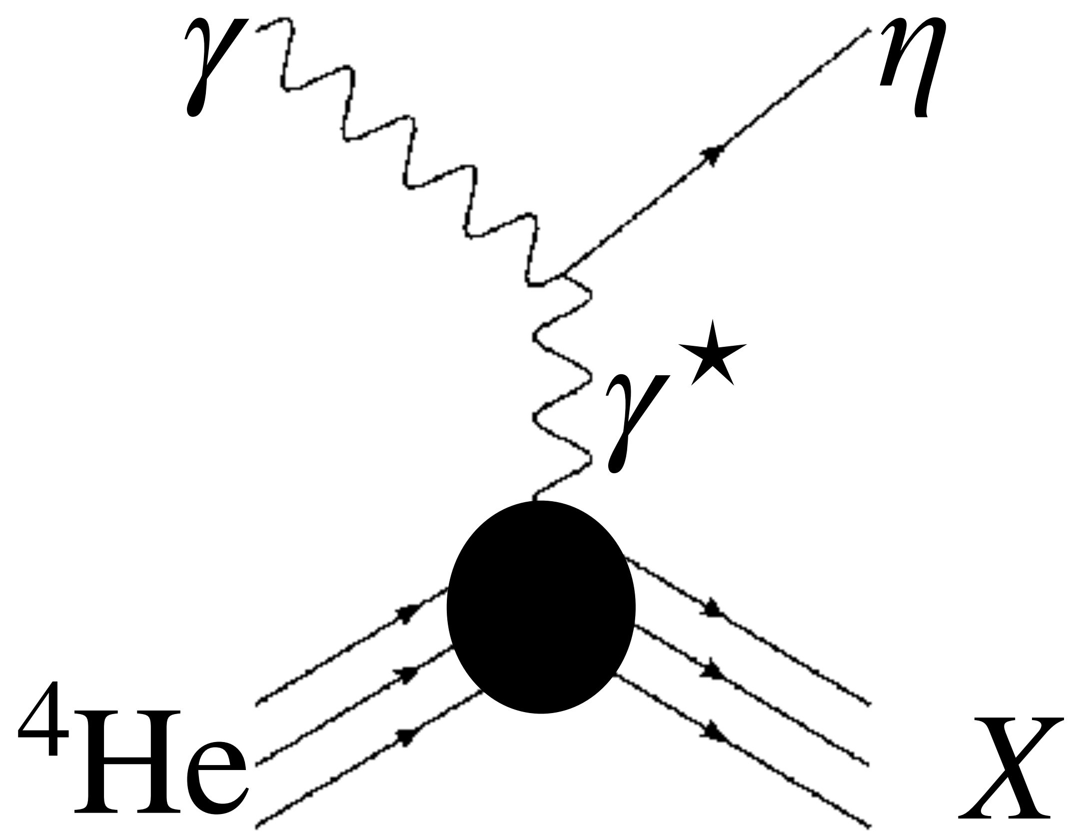
What is computed ($\mathscr{L}_L(\sigma, q)$)

Extracted through inversion ($R_L(\omega, q)$)



From electron-scattering to Primakoff





$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^3}{8\pi^3 f_\pi^2}\frac{|\boldsymbol{k}'|^3 k}{q^4}\left(\sin^2\theta \frac{q^4}{|\boldsymbol{q}|^4} R_L(\omega, |\boldsymbol{q}|) - \left(\frac{q^2}{2|\boldsymbol{q}|^2}\sin^2\theta + \left(\frac{E'}{|\boldsymbol{k}'|} - \cos\theta\right)^2\right) R_T(\omega, |\boldsymbol{q}|)\right)$$

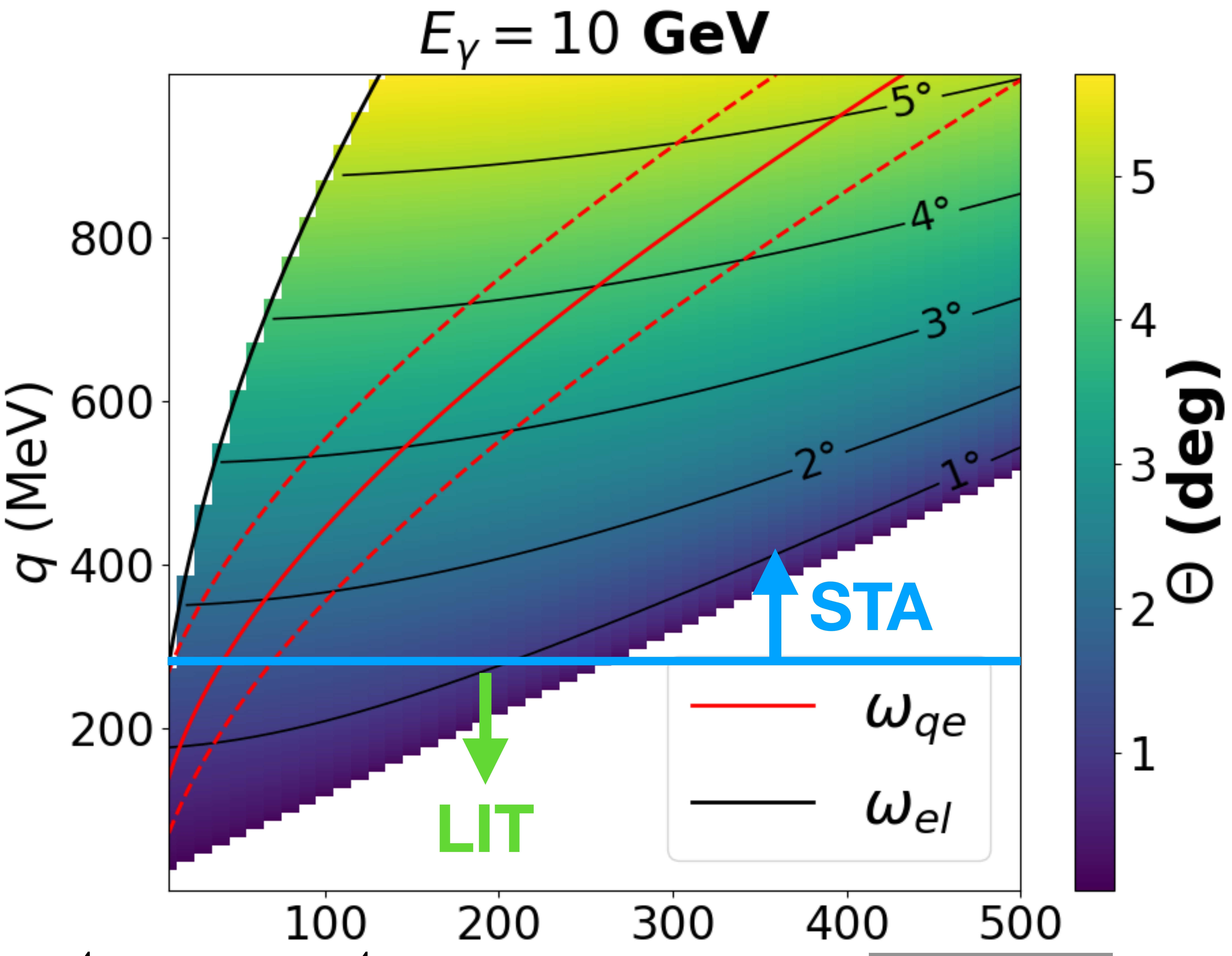
- Used only covariance and conservation of currents.
- General for any nucleus
- Contain coherent and incoherent parts

Nuclear Response Functions

They are exactly the same appearing in the electron-scattering

A. G., L. Andreoli, et al., in preparation

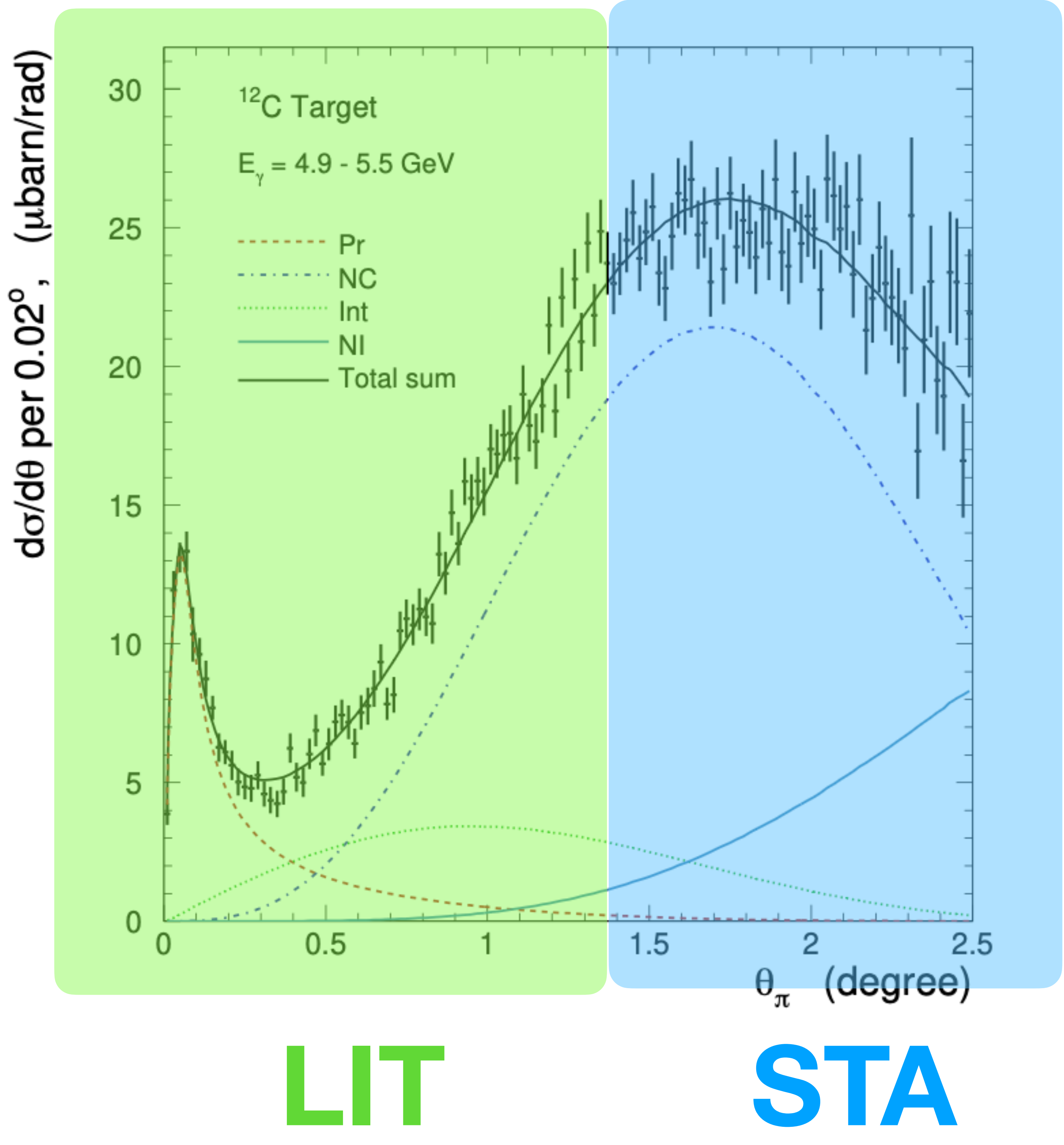
Incoherent cross-section (kinematics)



$\gamma + {}^4He \rightarrow \eta + {}^4He$

Preliminary

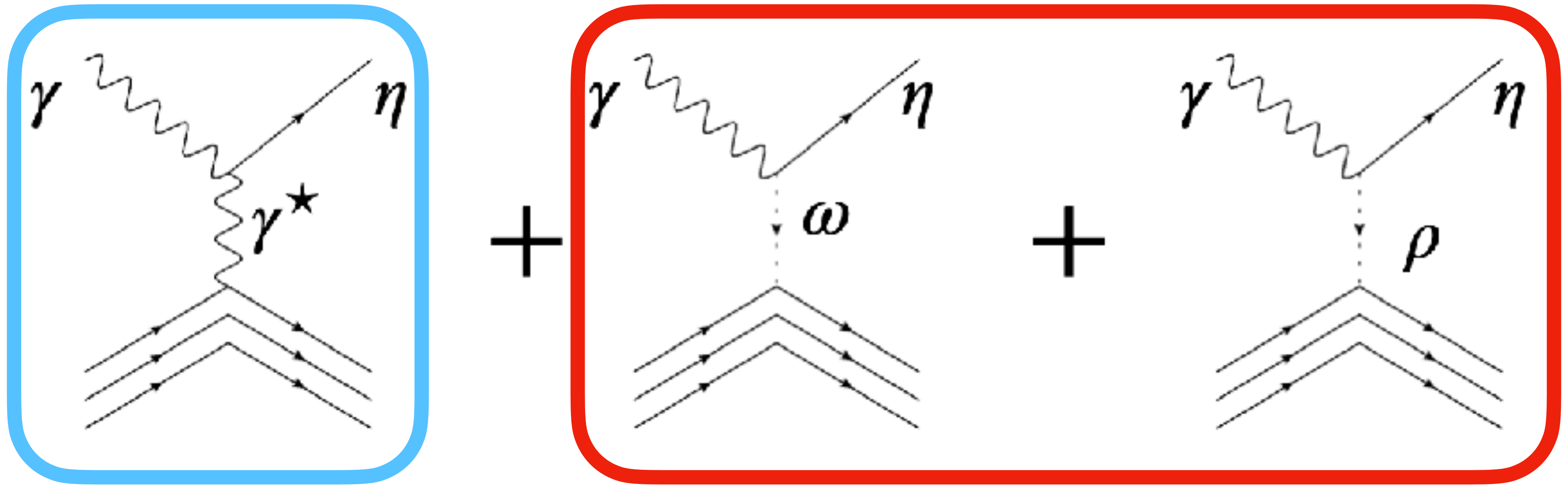
$$\frac{d\sigma}{d\Omega} = |T_C + e^{i\phi} T_S|^2 + \left(\frac{d\sigma}{d\Omega}\right)_{inc}$$

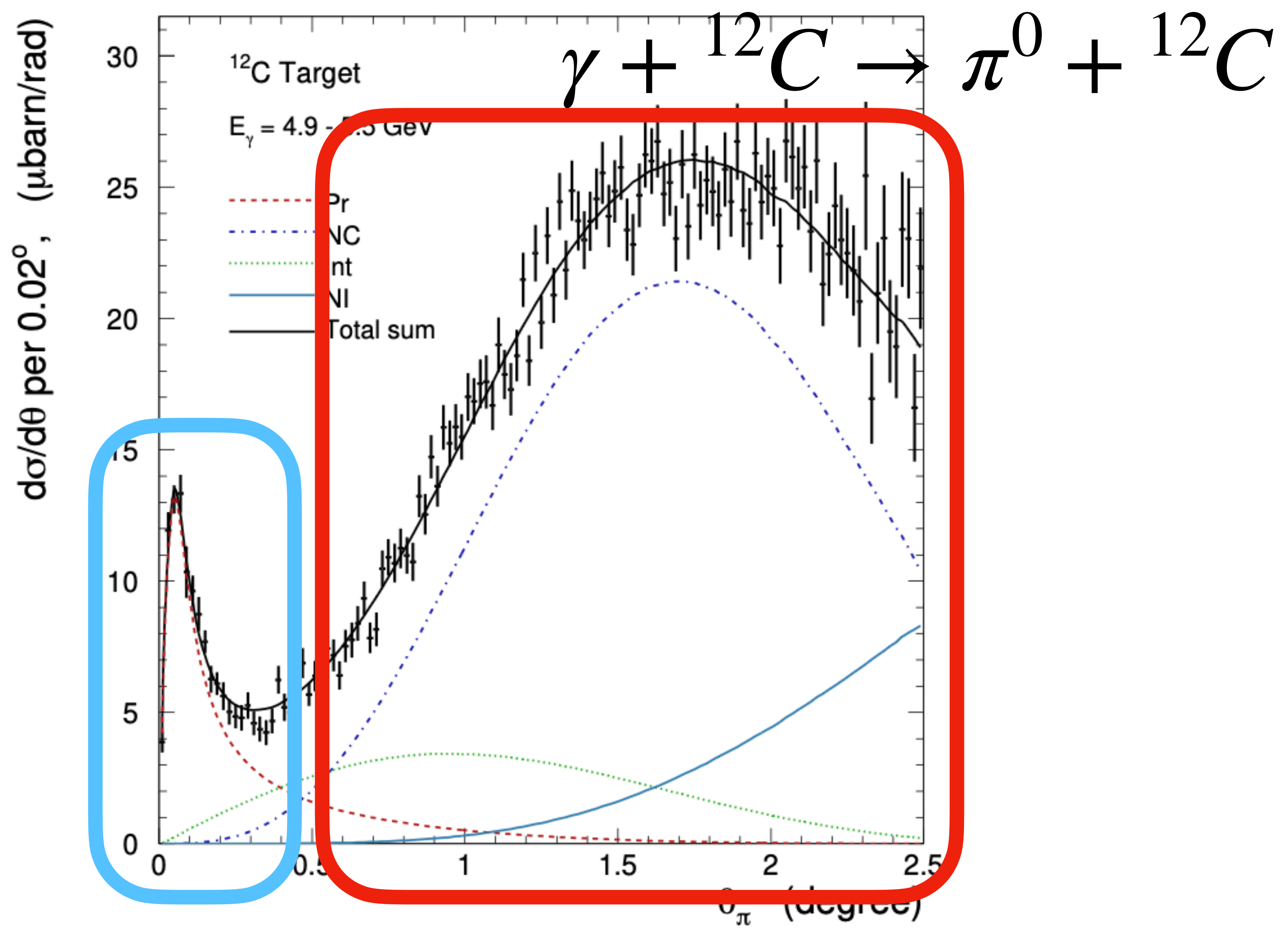


We thank S. Bacca for the LIT results

Process description

- Vector meson dominance model



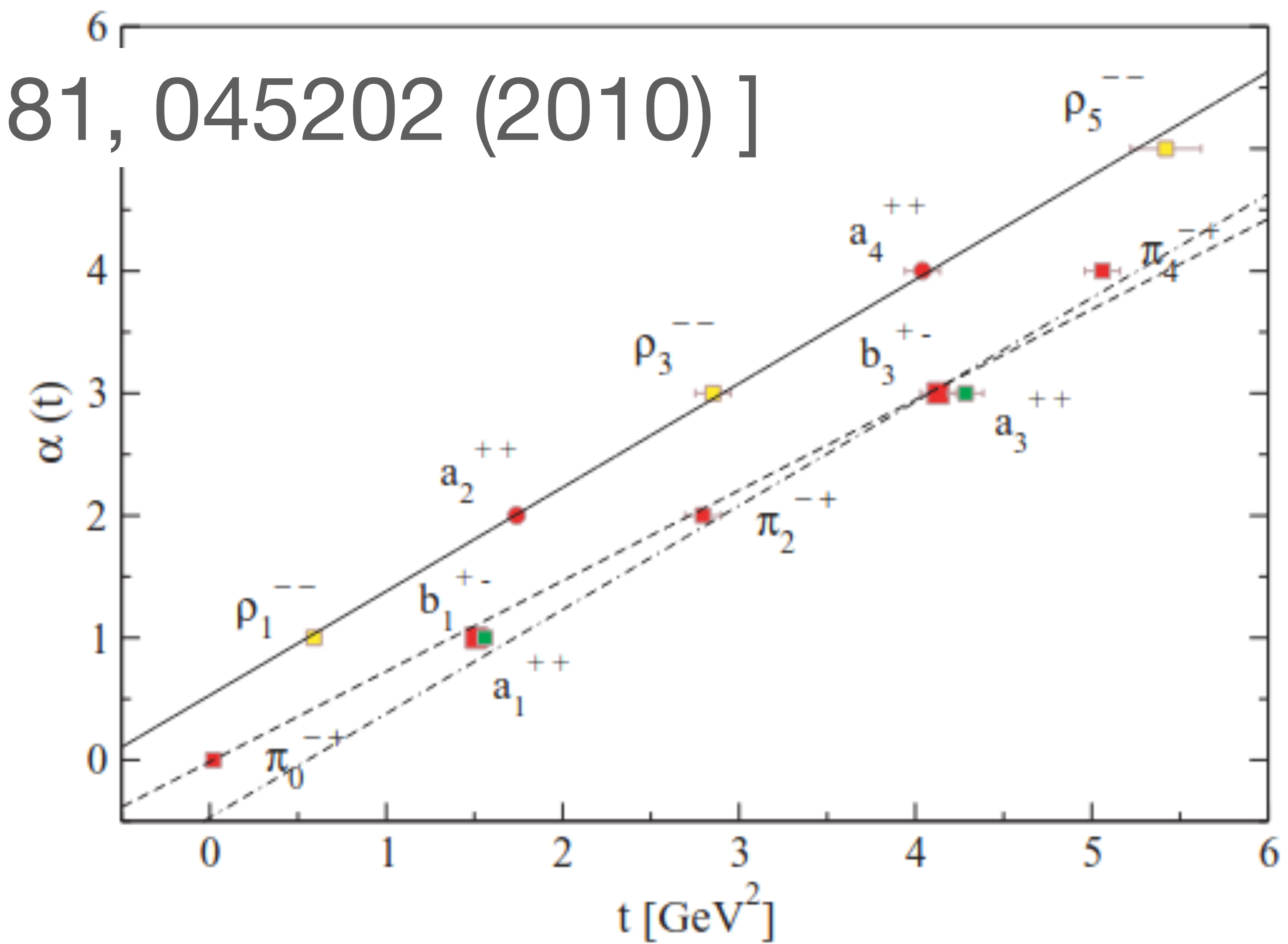


- **Regge theory:** matching between low-energy and high-energy domains of the amplitudes.

$$\frac{1}{q^2 - m_V^2 + i\epsilon^+} \rightarrow R_V(q^2, s)$$

$$\alpha_V(q^2) = \alpha_V^0 + \alpha_V' q^2$$

[PRC 81, 045202 (2010)]



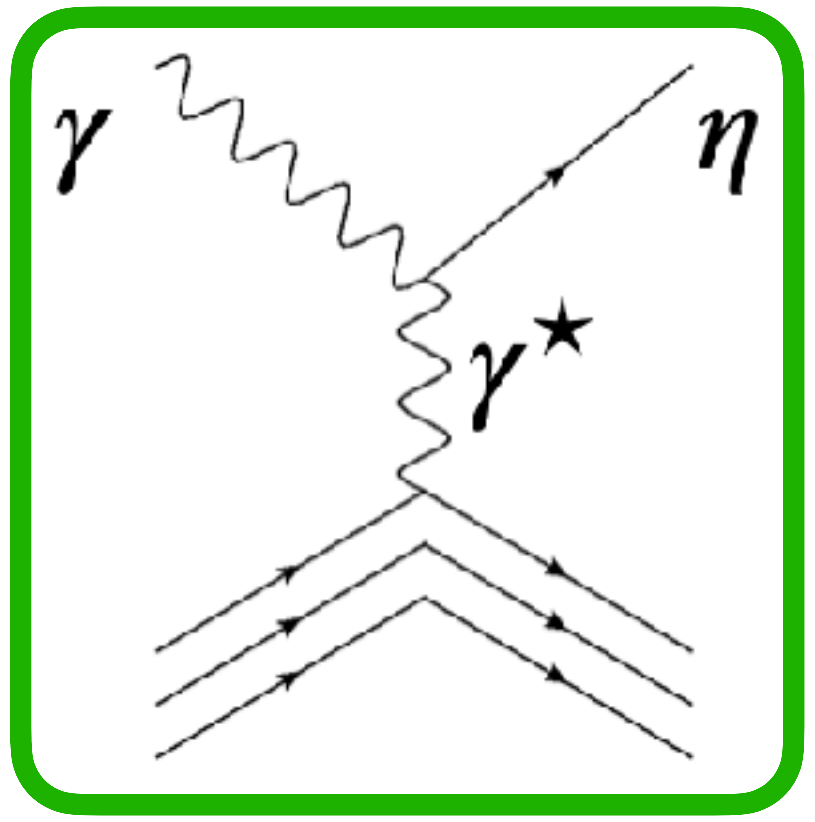
Include the Regge theory in our calculation

- We limit our-selves to impulse approximation (one-body current)
- The VMD and the Reggerized theory does not change the operator structure of the currents.

$$j_0 = \sum_{i=1,A} \frac{G_E^S(q^2) + G_E^V(q^2)\tau_z^i}{2} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} \rightarrow j_0^{VMD} = \sum_{i=1,A} \frac{A_E^S(q^2,s) + A_E^V(q^2,s)\tau_z^i}{2} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

Isoscalar term

$$A_E^S(q^2,s) = \frac{c_\eta}{2}(G_E^p(q^2) + G_E^n(q^2)) + \tilde{G}_\omega q^2 R^\omega(q^2,s)$$



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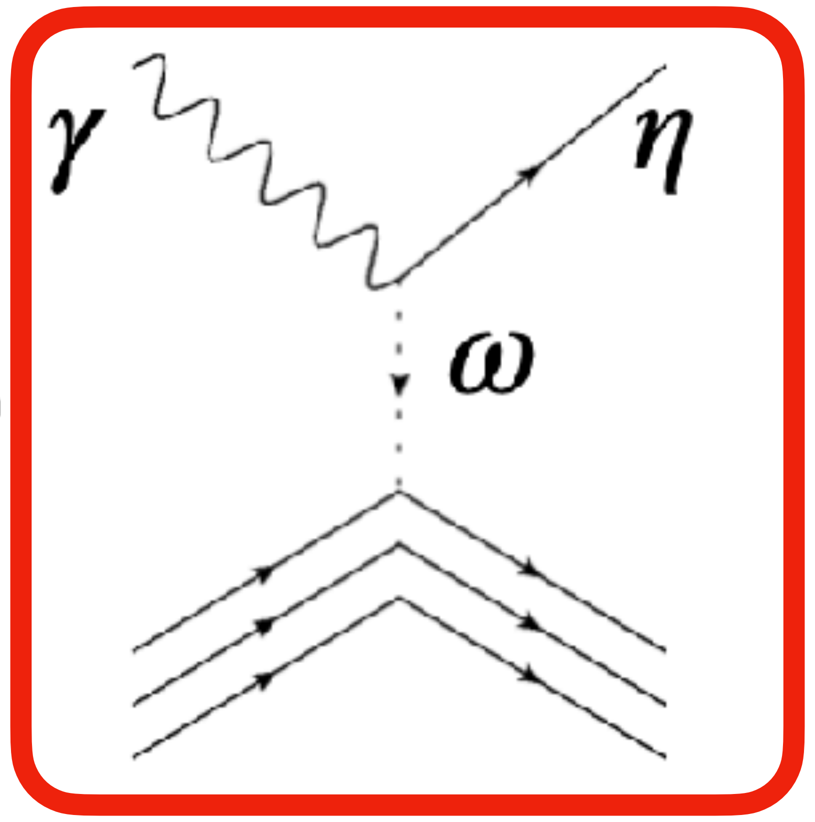


Coherent (elastic) cross section

$$\frac{d\sigma}{d\Omega} = |T_C + e^{i\phi}T_S|^2 + \left(\frac{d\sigma}{d\Omega}\right)_{inc}$$

$$\gamma + A \to \gamma + A$$

$$\left(\frac{d\sigma}{d\Omega}\right)_C = \frac{\alpha^3}{8\pi^3 f_\pi^2}\left(F^{\eta}_{\gamma\gamma^\star}(\overline{q}^2)\right)^2 \frac{1}{f_{rec}}\frac{\overline{k}'^4}{\overline{q}^4}\frac{k}{\overline{E}'}\left(\sin^2\theta\frac{\overline{q}^4}{|\overline{\boldsymbol{q}}|^4}F^2_{VMD-L}(\overline{q}^2) - \left(\frac{\overline{q}^2}{2|\overline{\boldsymbol{q}}|^2}\sin^2\theta + \left(\frac{\overline{E}'}{|\overline{\boldsymbol{k}}'|} - \cos\theta\right)^2\right)F^2_{VMD-T}(\overline{q}^2)\right)$$

For $J = 0$ $M_A \to \infty$ nuclei

$$F_L^2 \to Z^2 F_A^2(q^2)$$

$$F_T^2 \to 0$$

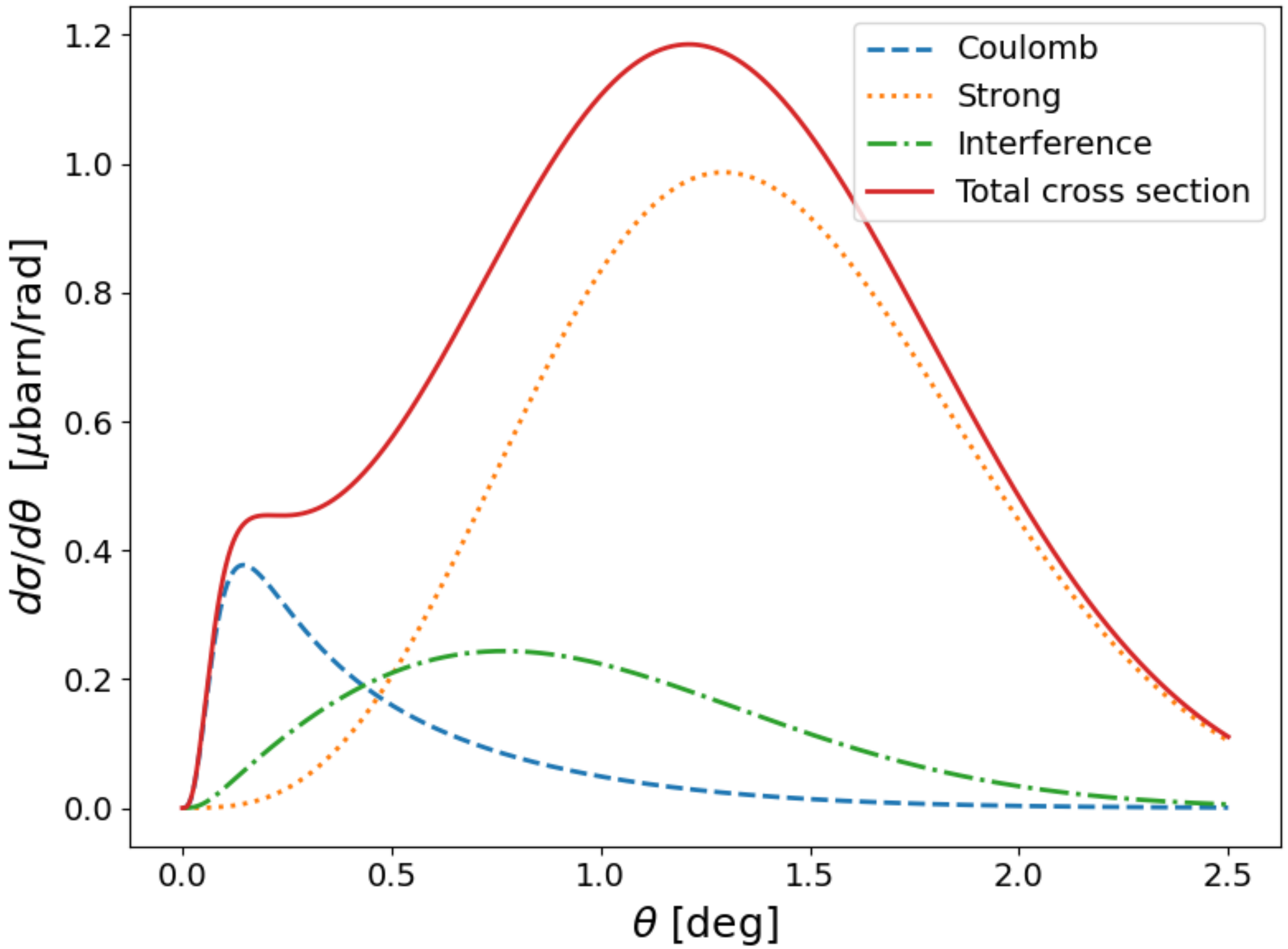
$$\left(\frac{d\sigma}{d\Omega}\right)_C = \Gamma_{P\to\gamma\gamma}\frac{8Z^2\alpha}{m_P^3}\frac{|\boldsymbol{k}'|^3 k}{q^4}\sin^2\theta F_A^2(q^2)$$

Coherent production of η on ^{4}He

$\gamma + {}^4\text{He} \to \eta + {}^4\text{He}$

$$\left(\frac{d\sigma}{d\Omega}\right)_C = \frac{Z^2\alpha^3}{8\pi^3 f_\pi^2}\left(F^{\eta}_{\gamma\gamma^\star}\left(\bar{q}^2\right)\right)^2 \frac{1}{frec}\frac{\bar{k}'^4}{|\bar{\boldsymbol{q}}|^4}\frac{k}{\overline{E'}}\sin^2\theta F^2_{VMD}\left(\bar{q}^2\right)$$

Preliminary



$$F^2_{VMD}(\bar{q}^2) = \left|\langle\psi_{^4\text{He}}(0)\,|\,j_0^{VMD}(\omega,\boldsymbol{q})\,|\,\psi_{^4\text{He}}(0)\rangle\right|^2,$$

- AV18+UIX interaction
- $E_\gamma = 10$ GeV
- $\Gamma_{\eta\to\gamma\gamma} = 0.52$ keV [PDG]

Kinematic effects

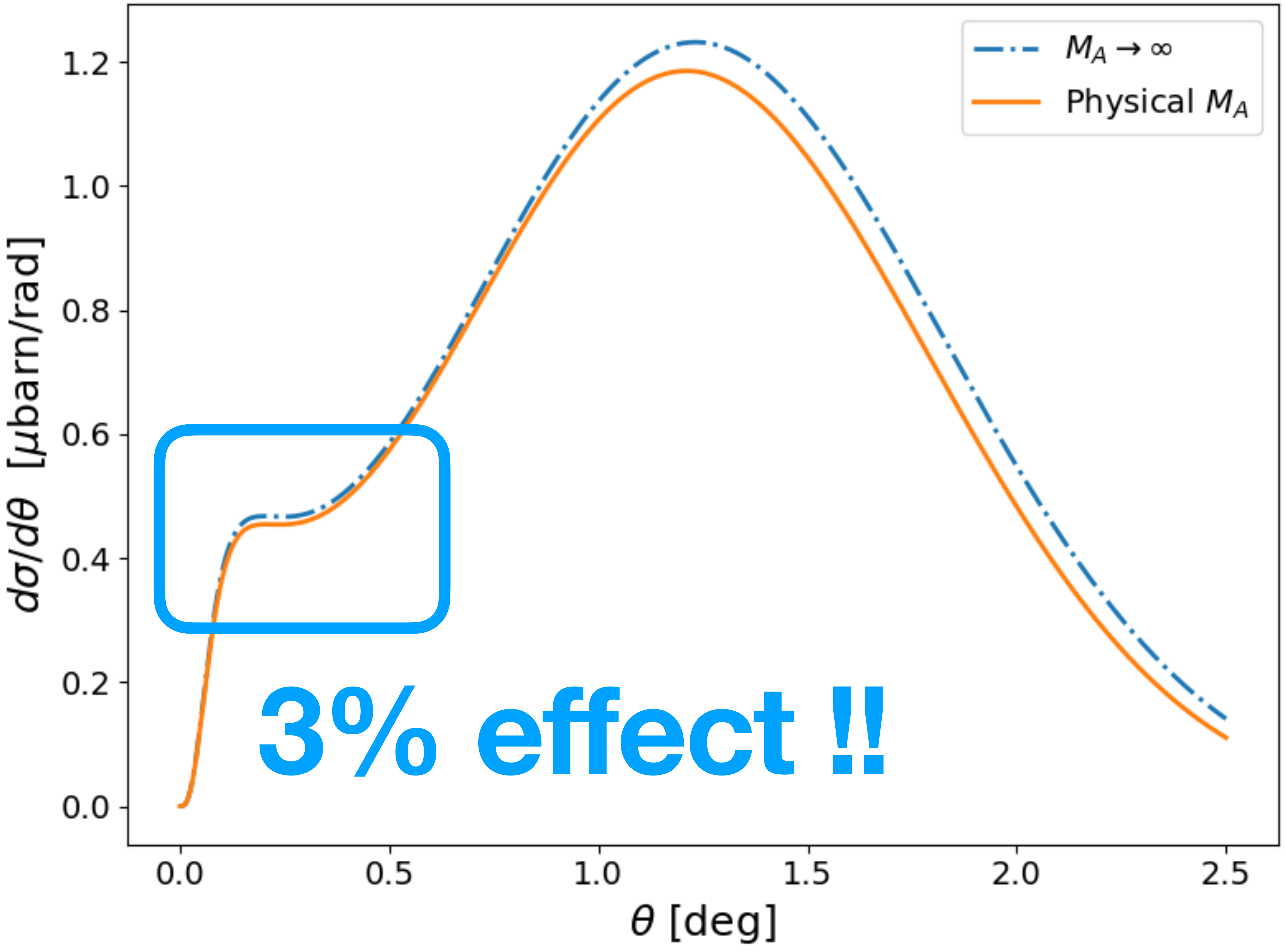
An important note on the calculation

- In literature the condition $M_A \to \infty$ is used giving rise to the cross-section

$$\left(\frac{d\sigma}{d\Omega}\right)_C = \Gamma_{P\to\gamma\gamma}\frac{8Z^2\alpha}{m_P^3}\frac{|\boldsymbol{k}'|^3 k}{q^4}\sin^2\theta F_A^2(q^2)$$

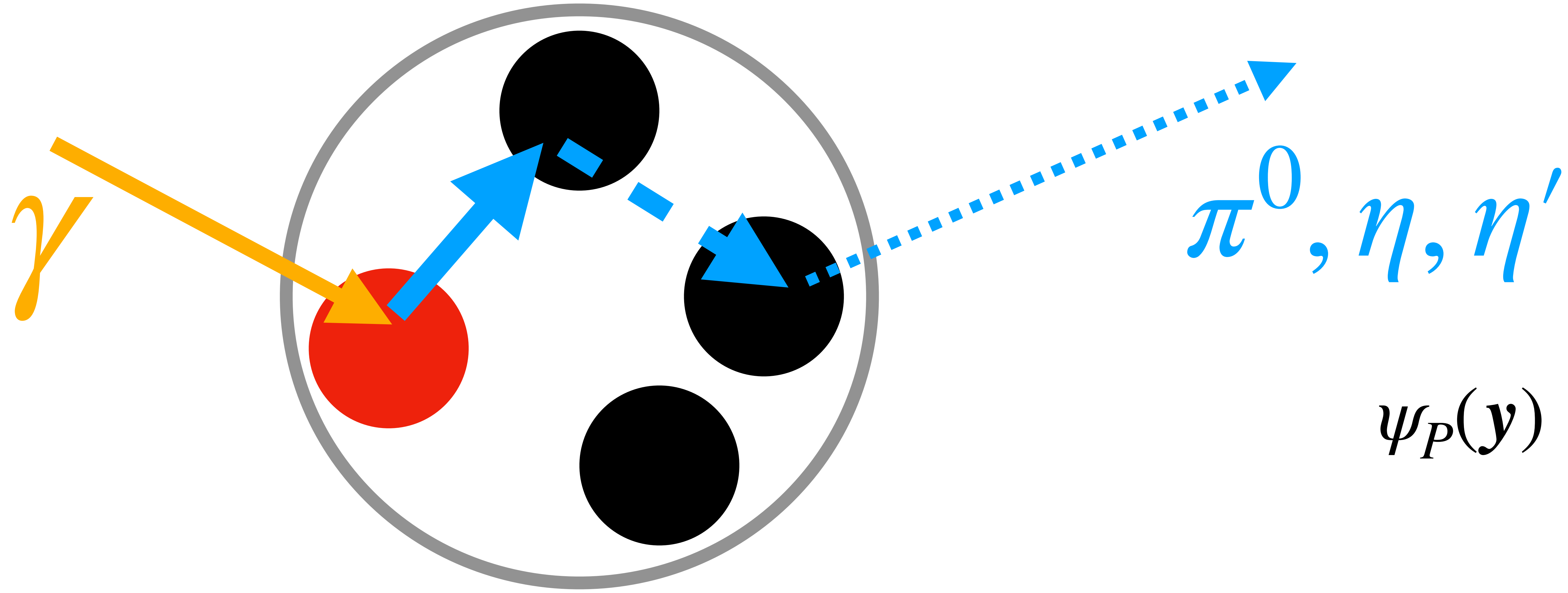
- **Kinematic effects are important and MUST be included**

Preliminary



Final state interactions

- The produced pseudoscalar meson interacts with the nuclear matter
- Used an optical model [1-2]



Main effect: absorption of P in the Nucleus

WKB approximation

$$\psi_P(\boldsymbol{y}) = \exp\left(i\boldsymbol{k}' \cdot \boldsymbol{y} - \frac{\sigma_{PN} A}{2} \int_{-\infty}^{z} \rho(\boldsymbol{b}, z') dz' \right)$$

$\frac{\sigma_{PN} A}{2}$: Total Cross Section of PN scattering

$\rho(\boldsymbol{b}, z')$: Nuclear density

[1- Morpurgo, Il Nuovo Cimento 31, 569 (1964)]
[2-Engelbrecht, Phys. Rev. 133, B988 (1964)]

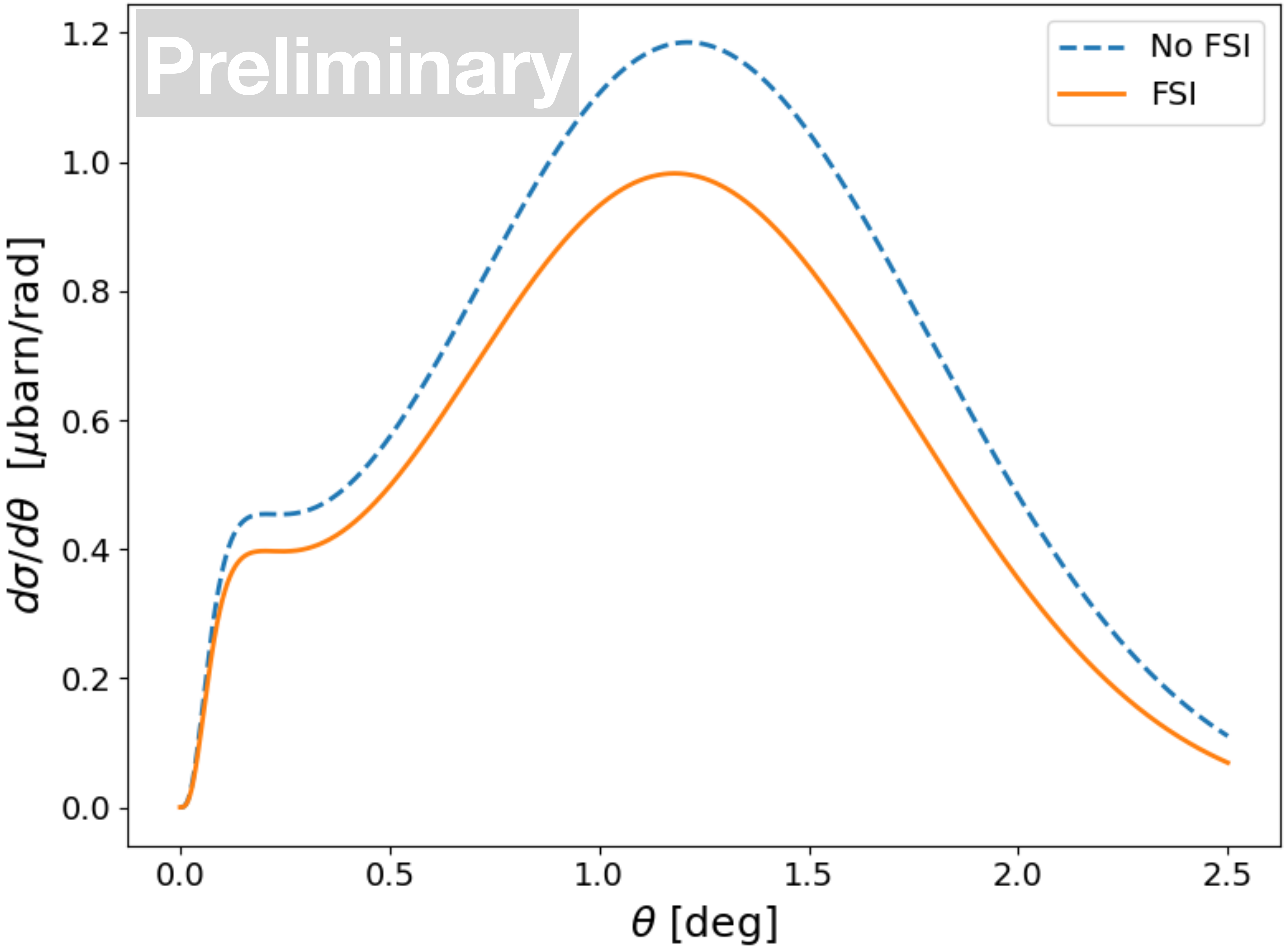
Final state interaction

- For our first attempt we follow the strategy of Engelbercht [1] and Fix [2]

$$\langle \psi | \sum_i \hat{O}_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} | \psi \rangle \rightarrow \langle \psi | \sum_i \hat{O}_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} \chi^\dagger(\boldsymbol{r}_i) | \psi \rangle$$

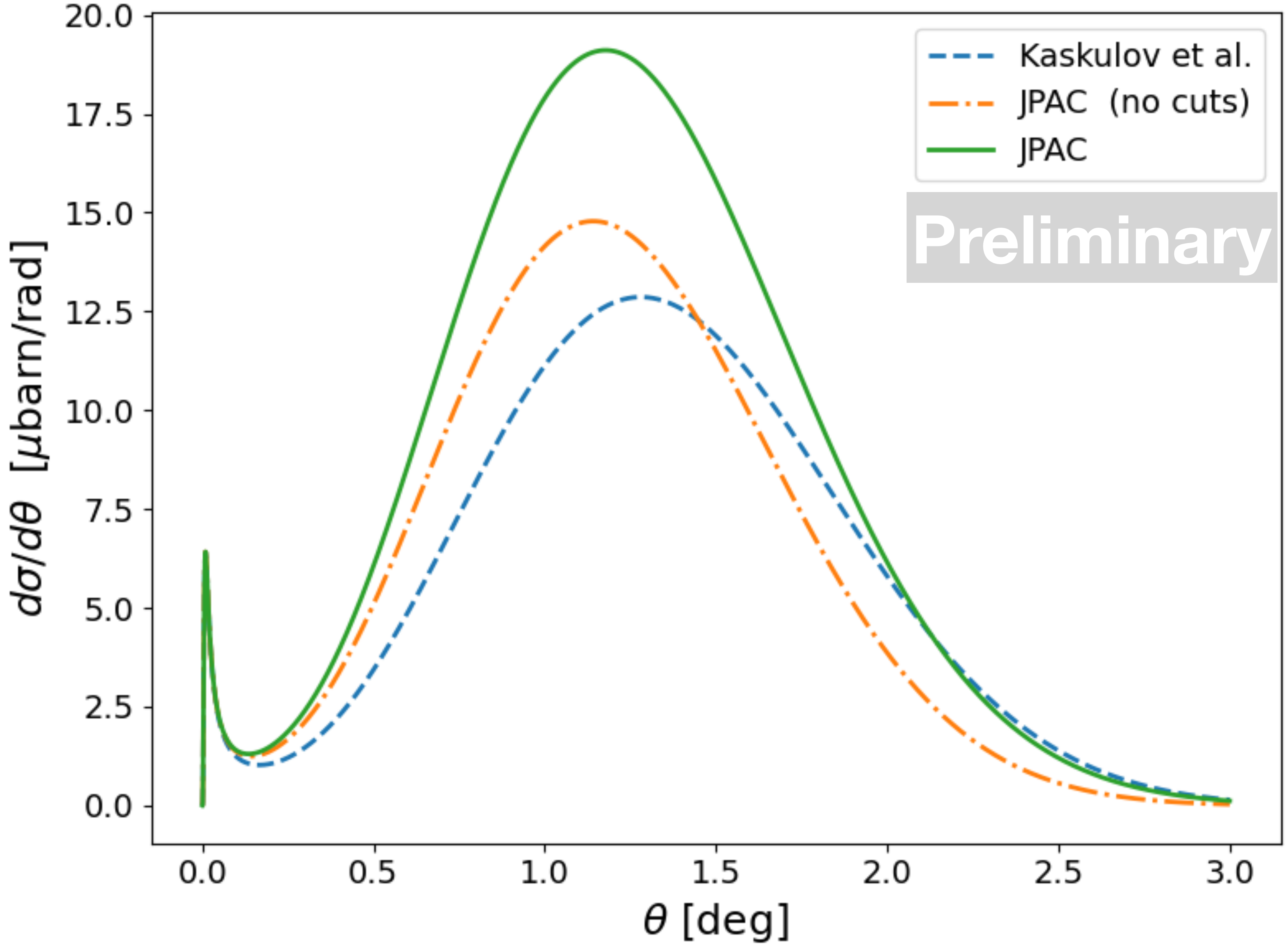
- Does not consider that the meson is produced in a different vertex!!

Implementation similar to Kaskulov [3] or Gevorkyan [4] adapted to ab-initio in progess



[1- Engelbrecht, Phys. Rev. 133, B988 (1964)]
[2- Fix, PRC 108(4), 044607 (2023)]
[2- Gevorkyan et al., PRC 80, 055201,(2009)]
[3- Kaskulov & Mosel, PRC 84, 065206, (2011)]

Flexible implementation



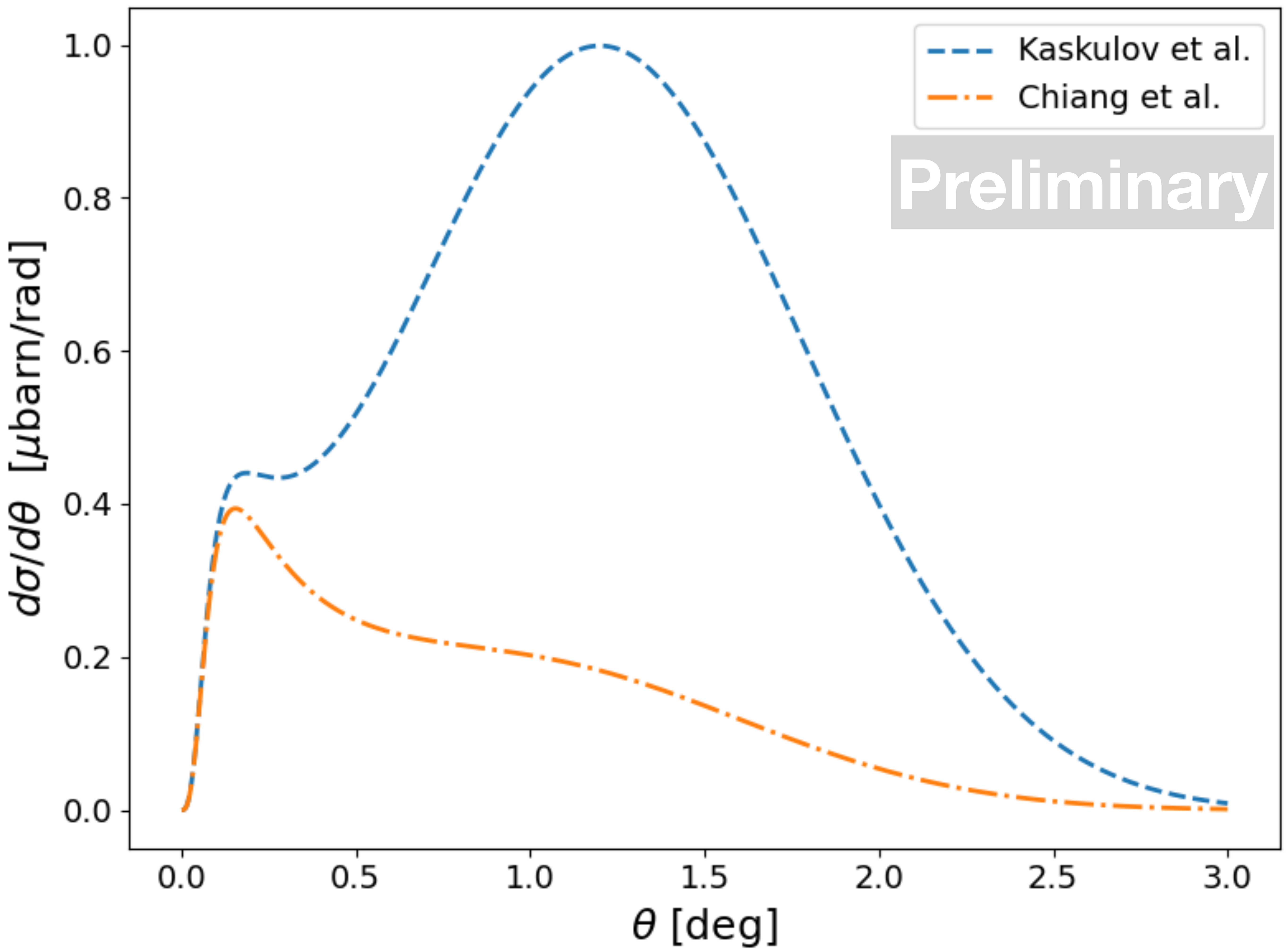
$$\gamma + {}^4\text{He} \rightarrow \pi^0 + {}^4\text{He}$$

Different strong production models can be implemented:

- Lagrangian implementation (Kaskulov et al. PRC84, 065206)
- Amplitude formalism (Mathieu et al. PRD92, 074013 - JPAC)

Strong production models give very different results

Strong production comparison



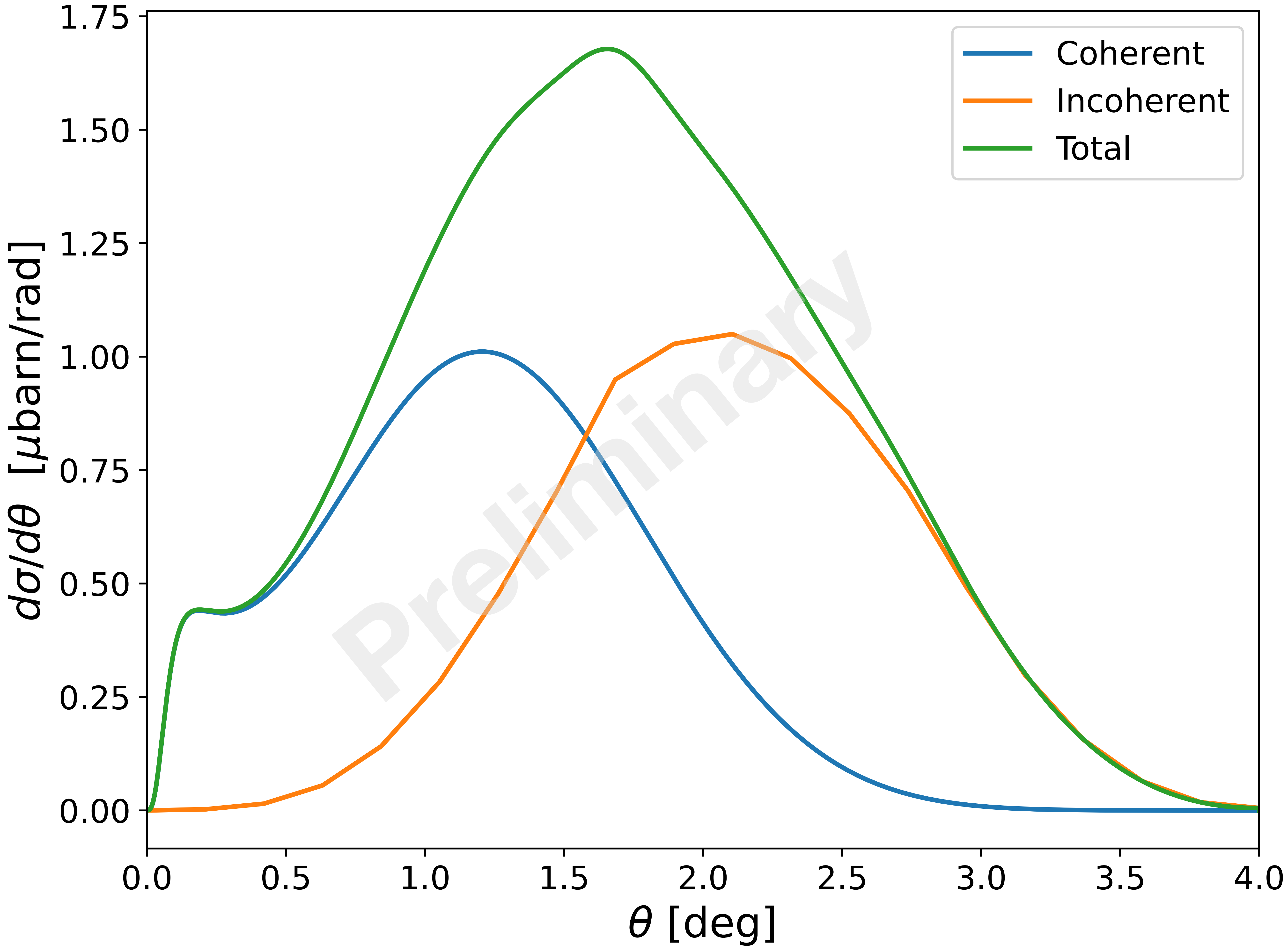
$$\gamma + {}^4\mathrm{He} \rightarrow \eta + {}^4\mathrm{He}$$

Different strong production models can be implemented:

- Lagrangian implementation (Kaskulov et al. PRC84, 065206)
- Lagrangian implementation (Chiang et al. PRC68, 045202)

Strong production models give very different results

Preliminary result for $\gamma + {}^4\mathrm{He} \rightarrow \eta + {}^4\mathrm{He}$



First consistent calculation of coherent and incoherent part!

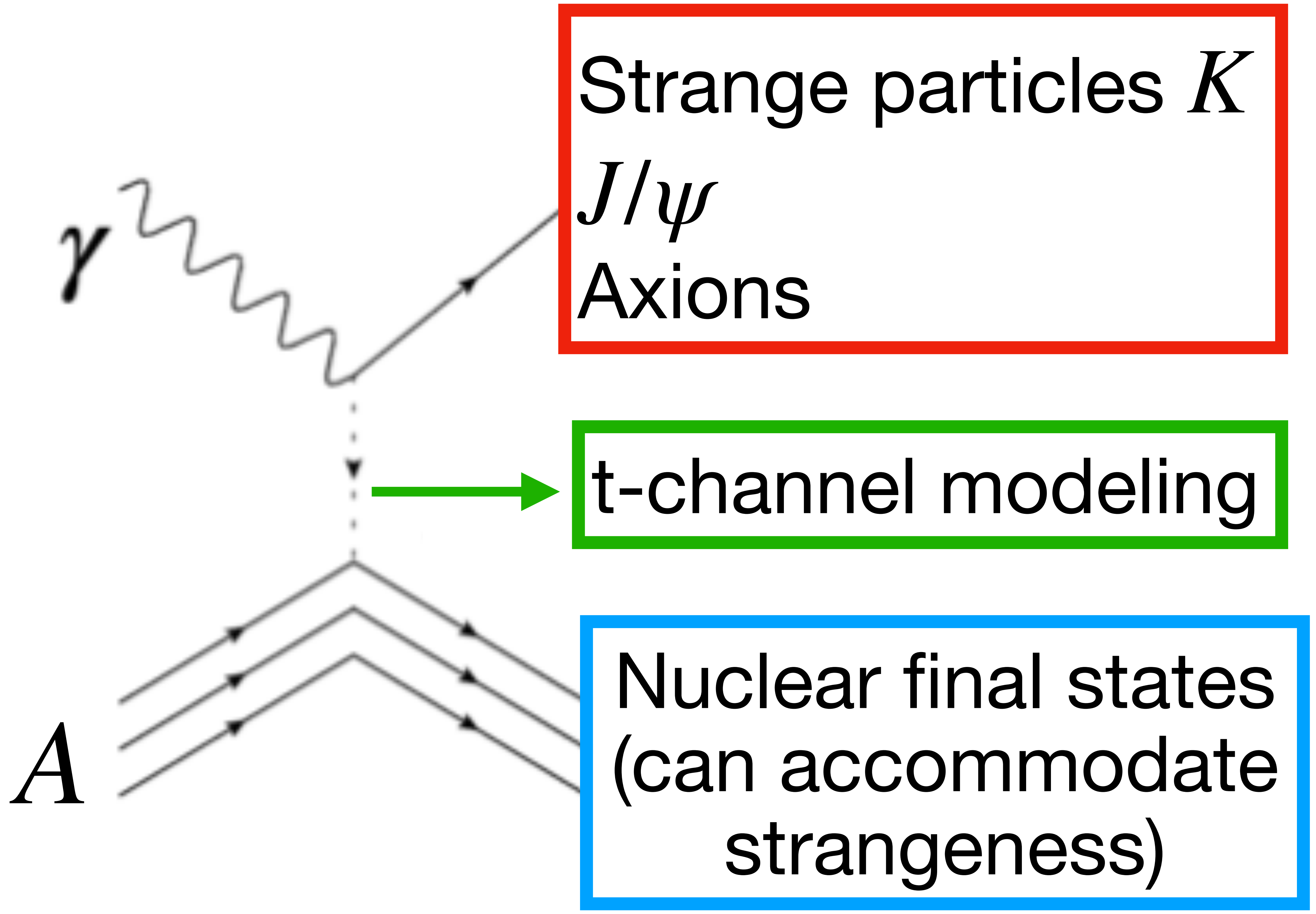
- AV18+UIX interaction
- $E_\gamma = 10$ GeV
- LIT + STA result
- Regge model from Kaskulov & Mosel (PRC 84, 065206)

Beyond Primakoff

Any process that can be parametrized as a t-channel exchange can, in principle, be included in our calculations

Limitations!

- Low energy and momentum exchanged
- Inclusive processes (we are working for exclusive cases in $A \geq 3$)
- Light nuclei (up to ^{12}C)



Conclusions

- Ab-initio calculation includes the full nuclear dynamics
- Coherent and incoherent processes are treated consistently (same physics)
- Preliminary result for the Primakoff are available for ^4He. Easily extensible to Helium-3 and deuteron
- This approach can be extended to other process that are within the kinematics we can cover
- Extension to other kinematics is in progress for electro-scattering

Collaborators

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GlueX collaboration:
Malte Albrecht, Igal Jaegle,
Drew Smith

+ Arkaitz B. Rodas (ODU)

Acknowledgments

NTNP
DOE Topical Collaboration

U.S. DEPARTMENT OF ENERGY

NERSC
National Energy Research Scientific Computing Center

Sparse

L. Andreoli, R. Weiss et al., arXiv:2510.21576

Response densities

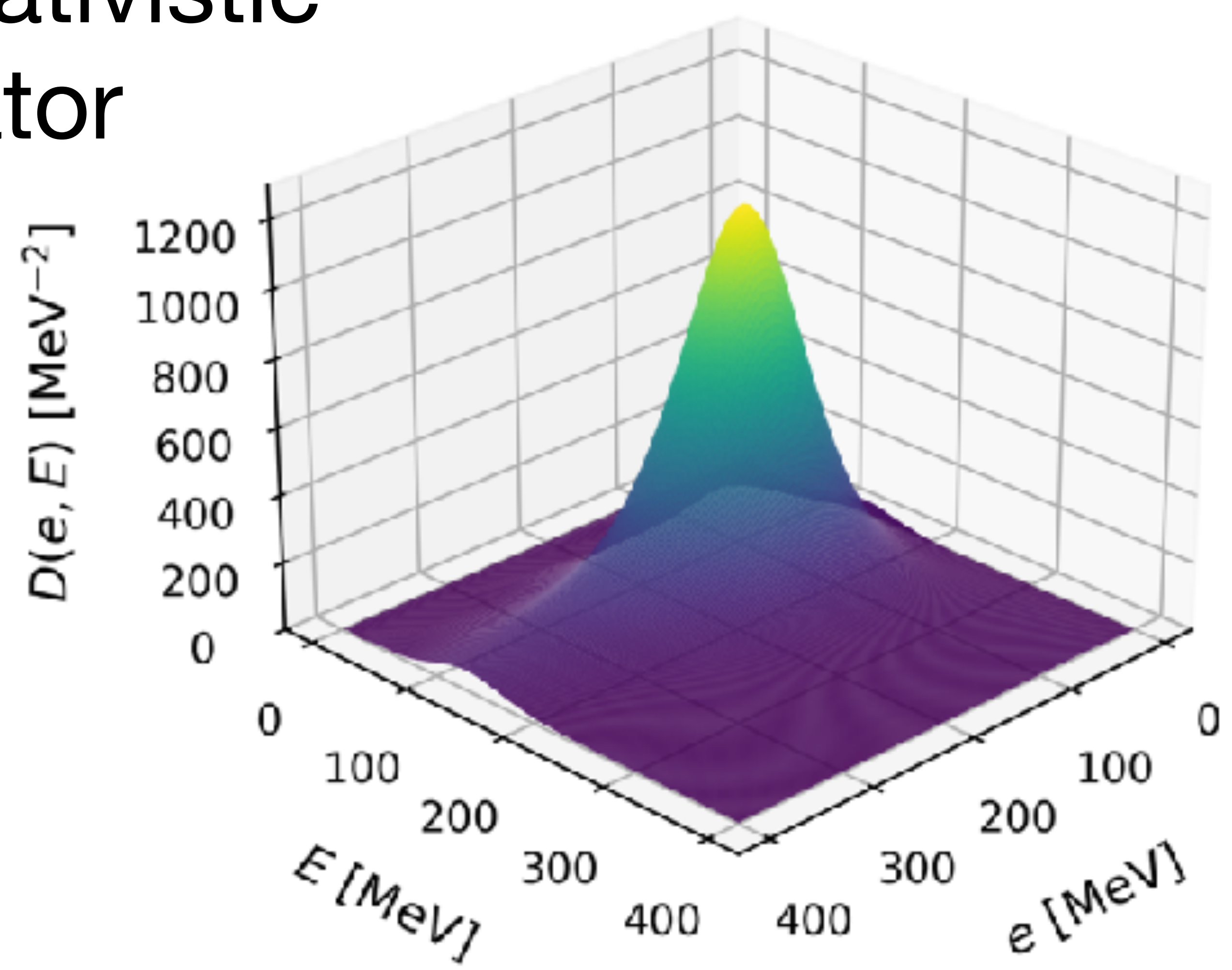
$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

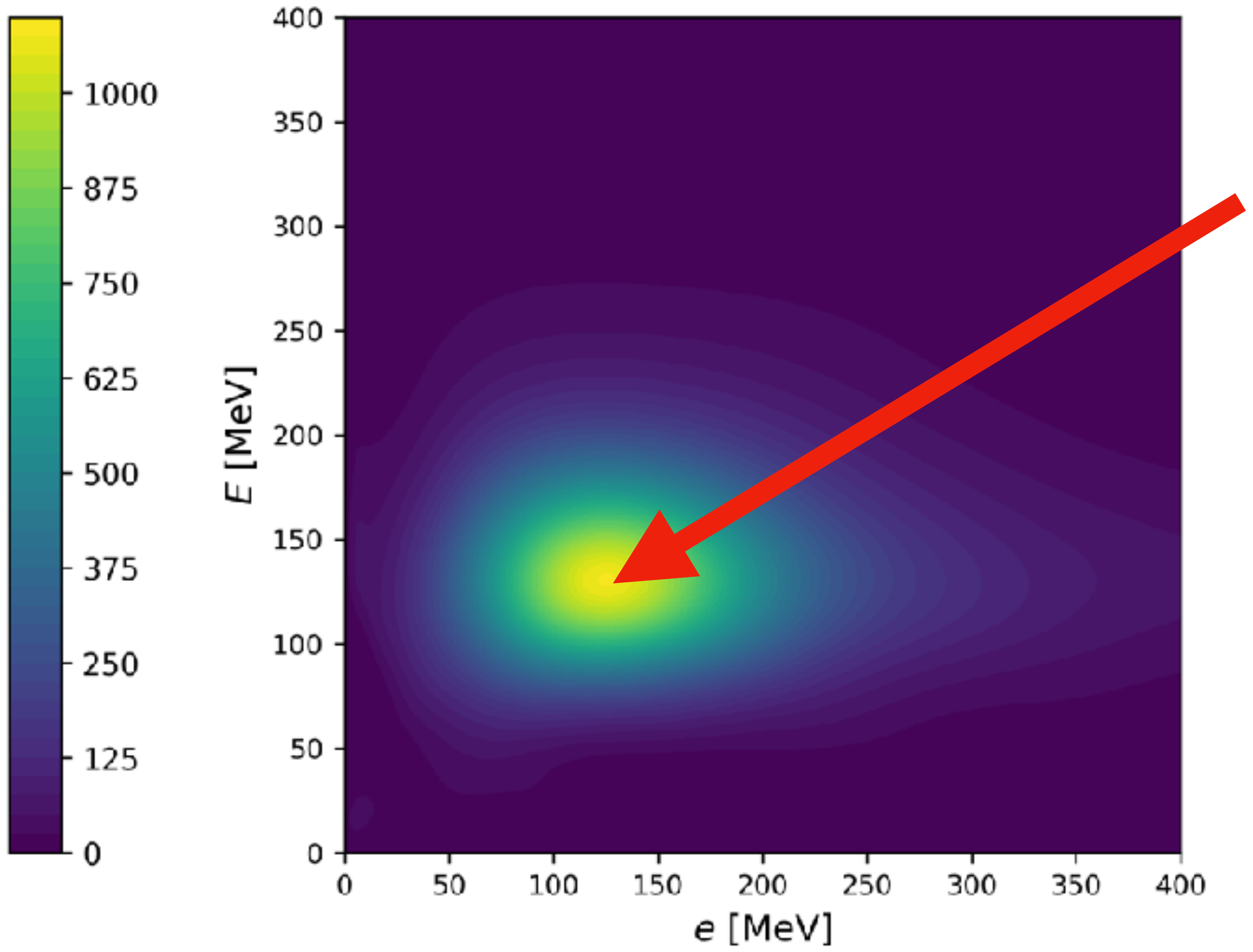
STA

$$R_\alpha(\boldsymbol{q},\omega) = \int de dE_{cm} D(e, E_{cm}) \delta(\omega - E_{cm} - e)$$

q=700 MeV/c
Transverse response
Full non—relativistic
propagator

^{4}He





$e = E = q^2/4m$

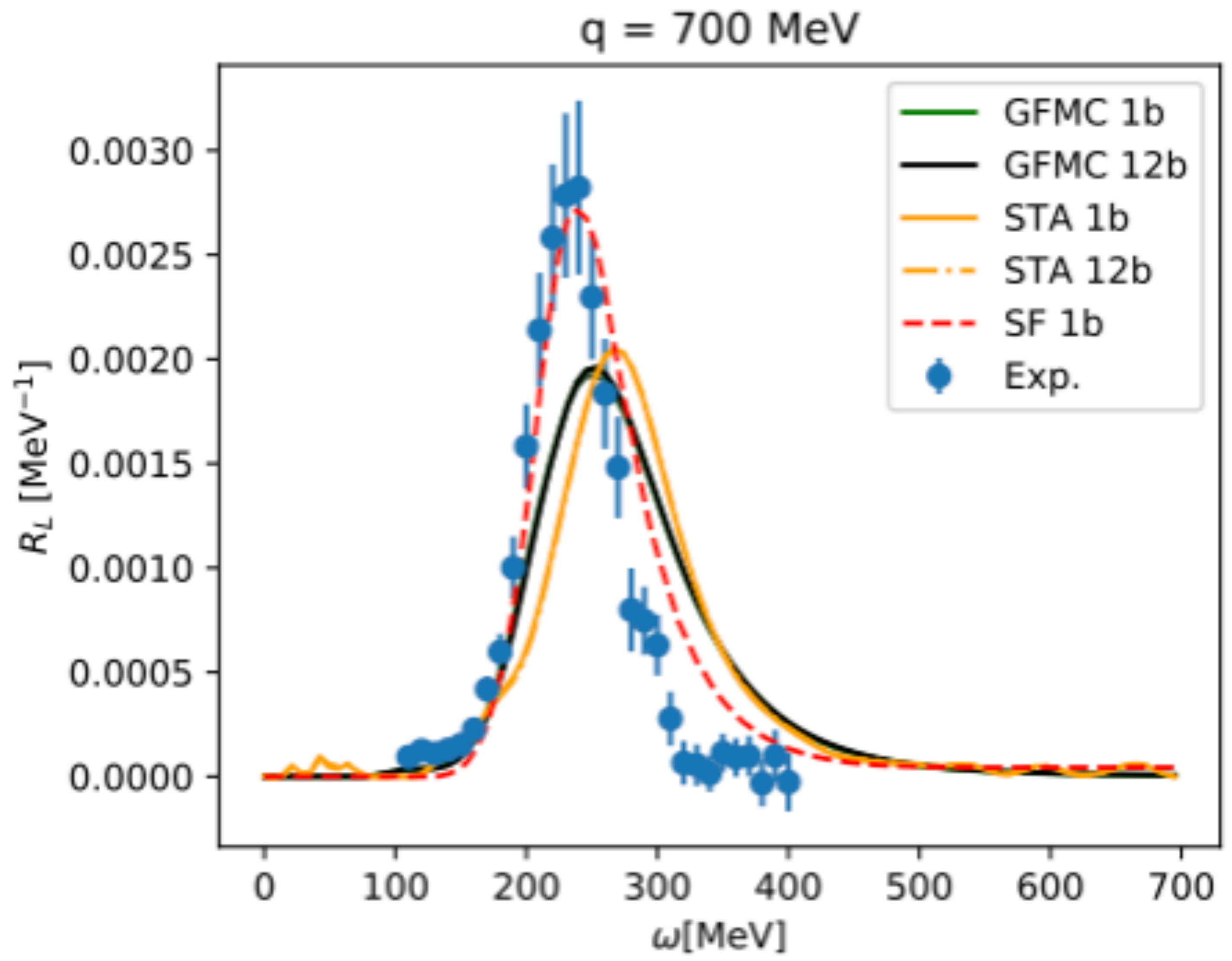
L. Andreoli et al., PRC 105, 014002 (2022)

Previous results

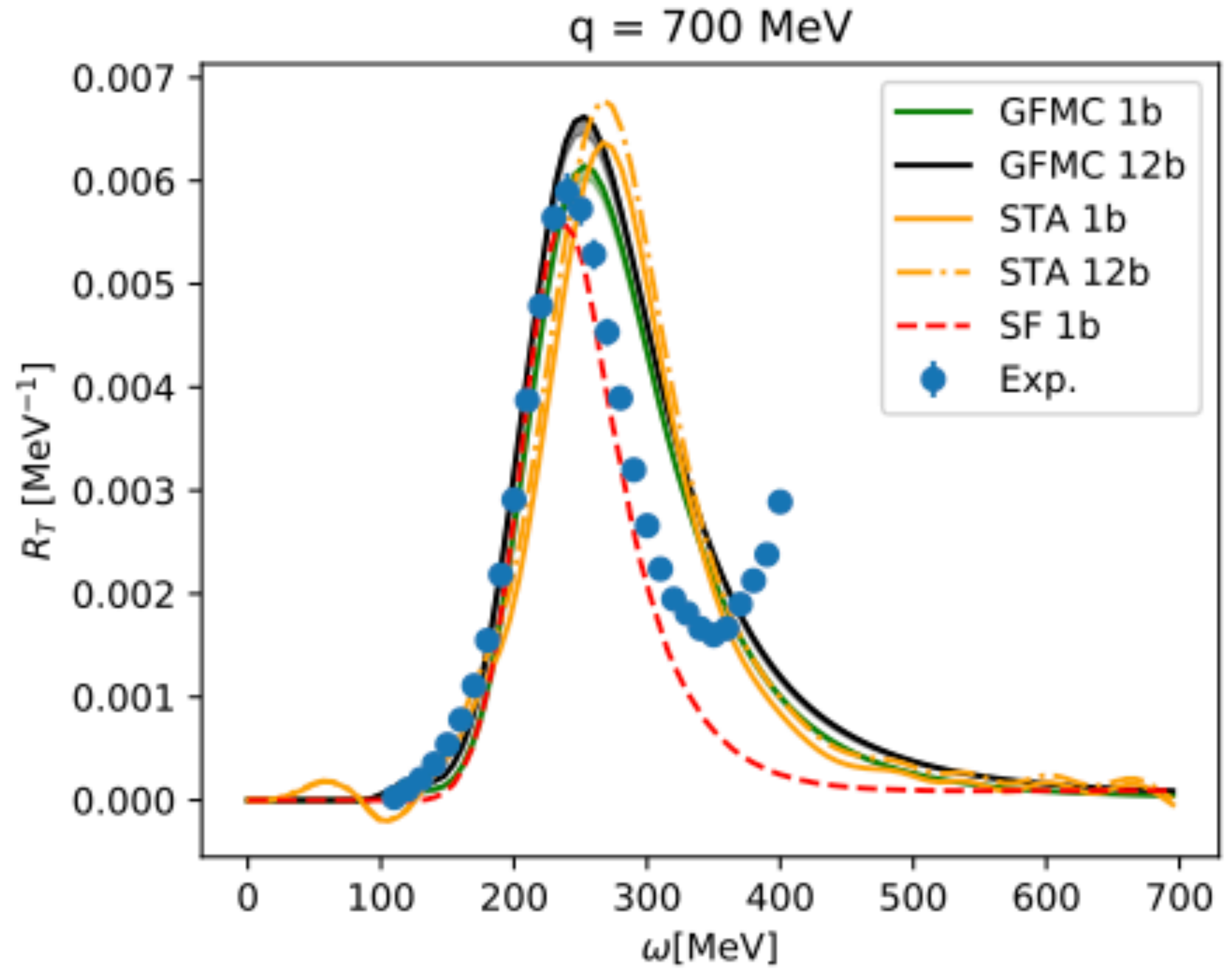
- The STA contain the full 2-body dynamics.
- The spectral function can use the full relativistic dynamic exactly.

↓

Correct the STA including (at least partially) the relativistic dynamic and kinematic



Helium-3



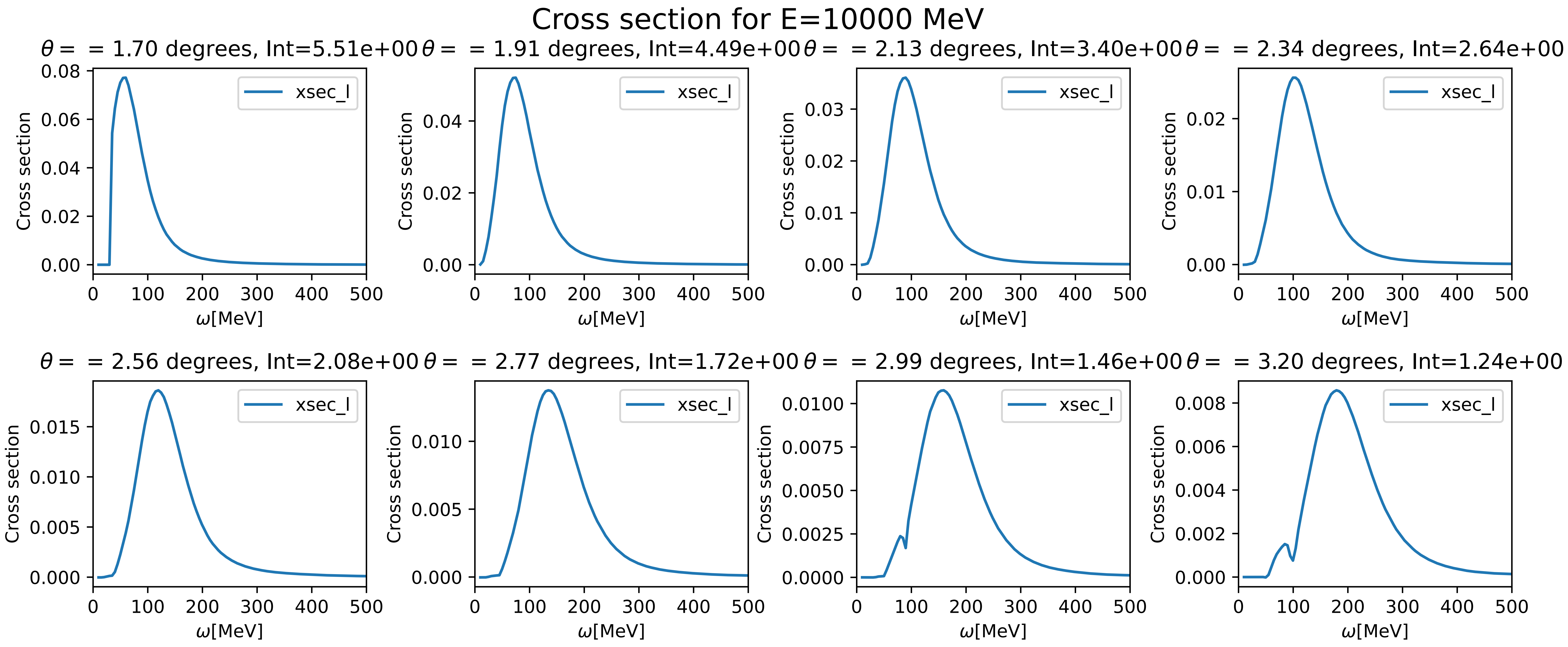
Preliminary

To Primakoff effect

All credits go to L. Andreoli

Bare cross-section (not included form factors and VMD)

- The response functions can be factorized dividing the nuclear operator from the VMD when computed in impulse approximation.
- Here we report them without including the VMD structure and the nucleon form factors.

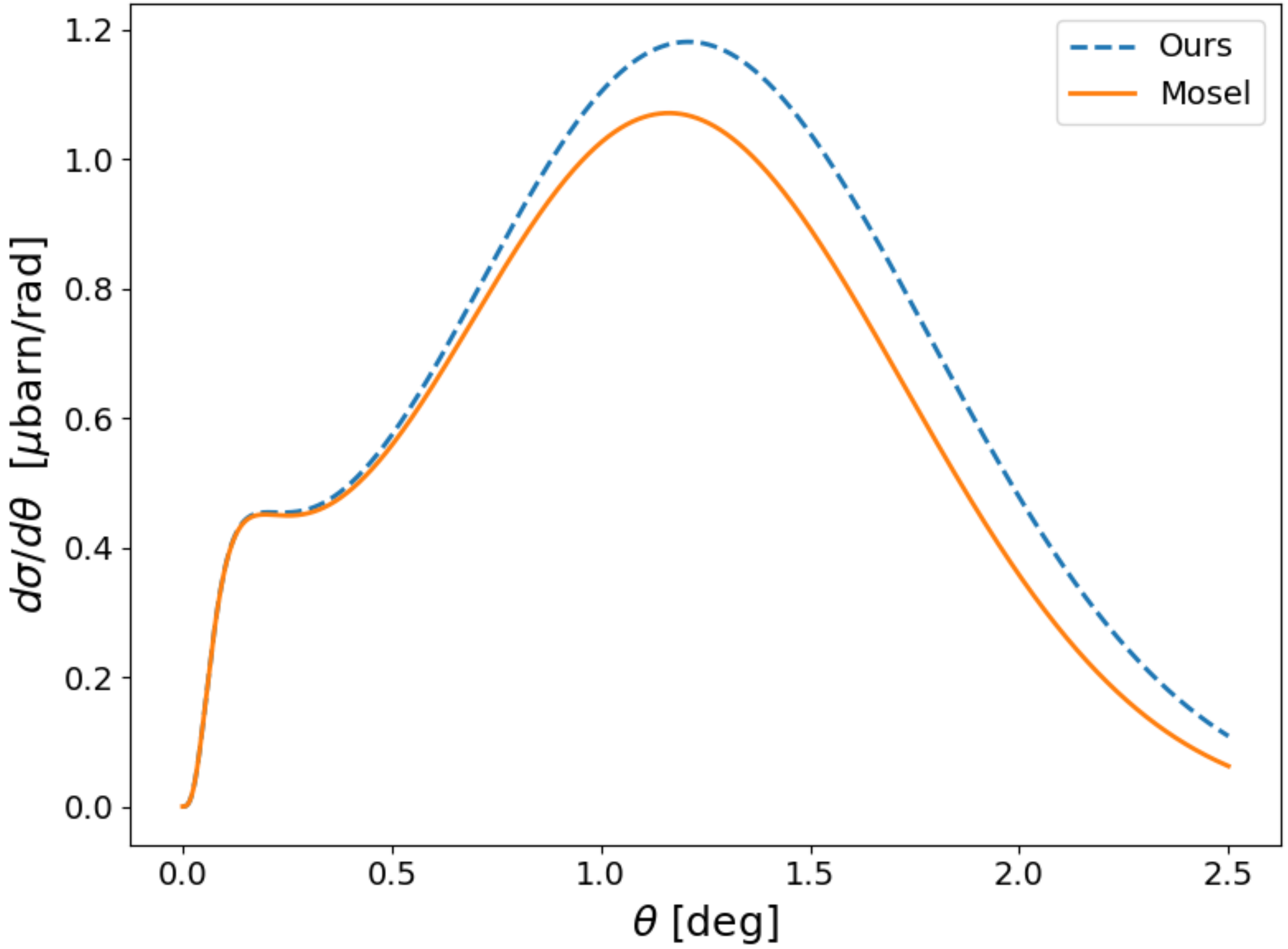


Longitudinal response AV18+UIX with STA

Comparison with Kaskulov model

Kaskulov & Mosel, PRC 84, 065206, (2011)

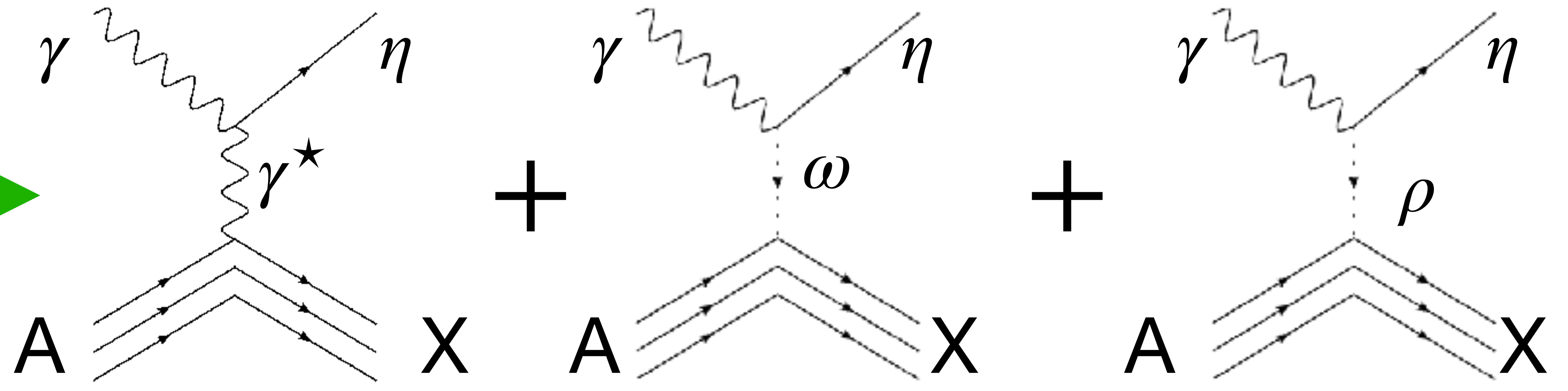
- Same initial Lagrangian.
- Phenomenological treatment of the nucleus through one-body density.
- No discussion regarding the incoherent part.
- Corrected for double counting of nucleon form factors and using correct kinematic.

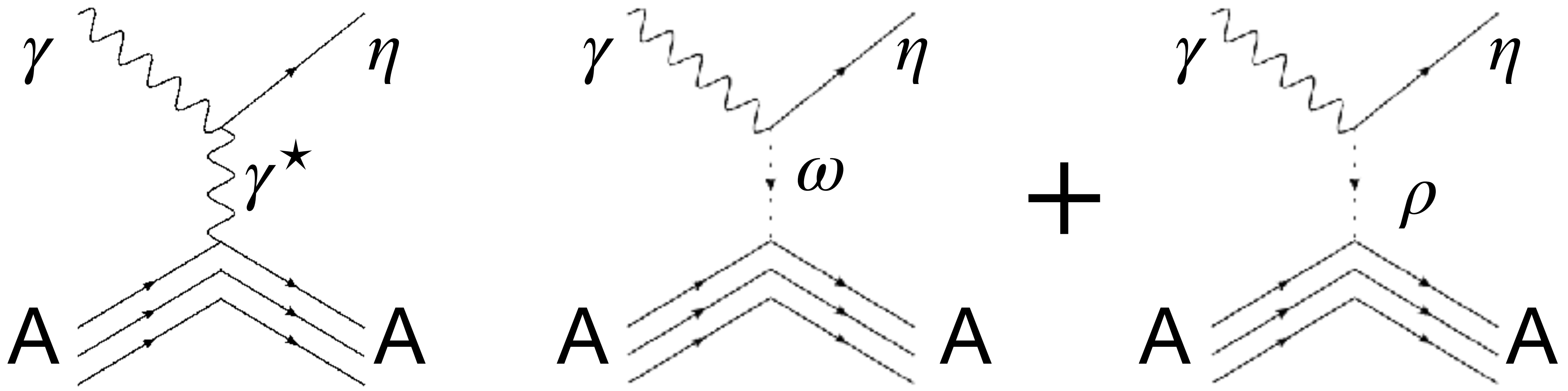


The nuclear background

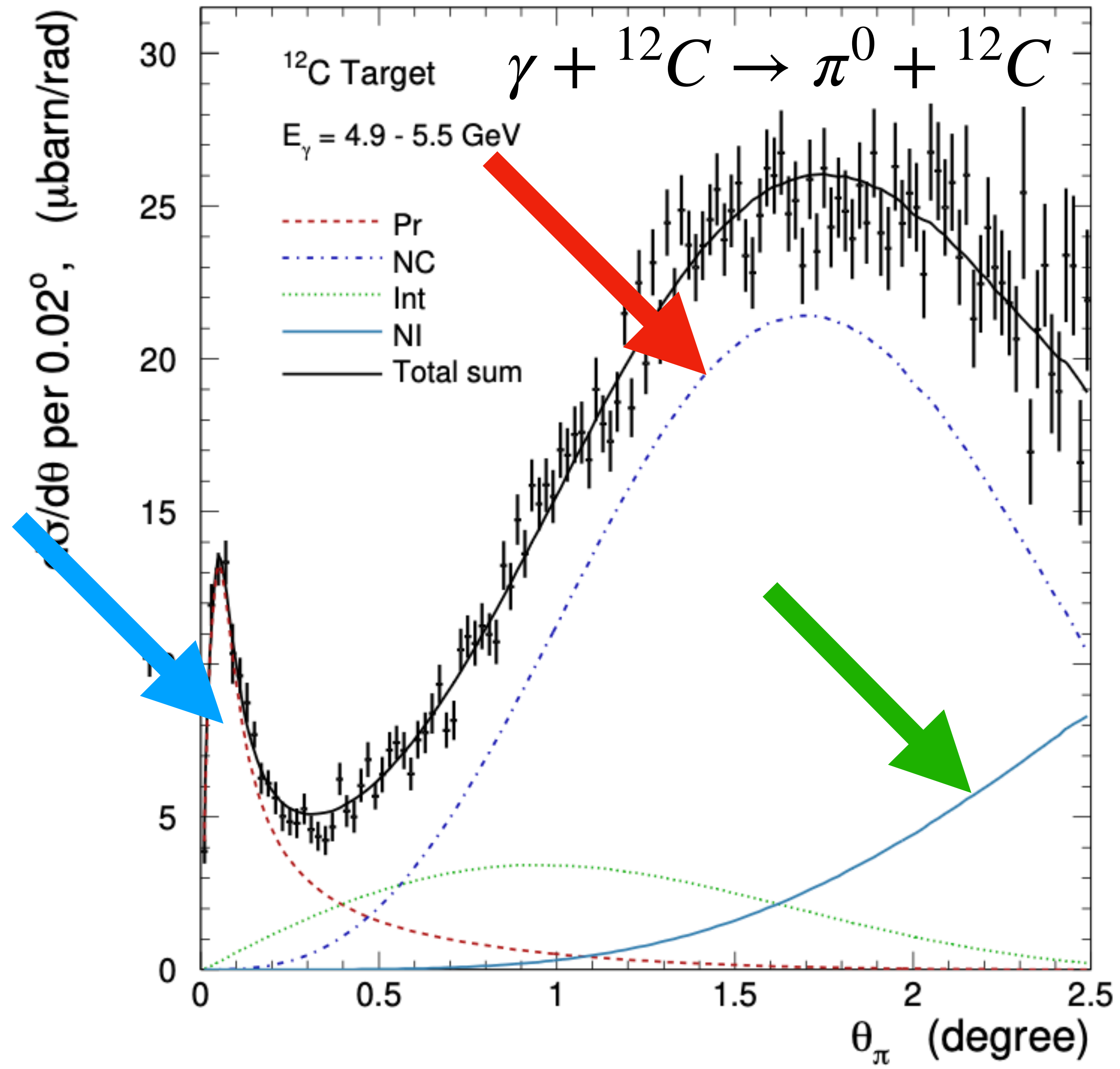
$$\gamma + A \to \gamma + X$$

$$\frac{d\sigma}{d\Omega} = |T_C + e^{i\phi}T_S|^2 + \left(\frac{d\sigma}{d\Omega}\right)_{inc}$$





$$\gamma + A \xrightarrow{C} \gamma + A \qquad \gamma + A \xrightarrow{S} \gamma + A$$



Usually computed using Glauber models

[Phys. Rev. C 80, 055201 (2009)]