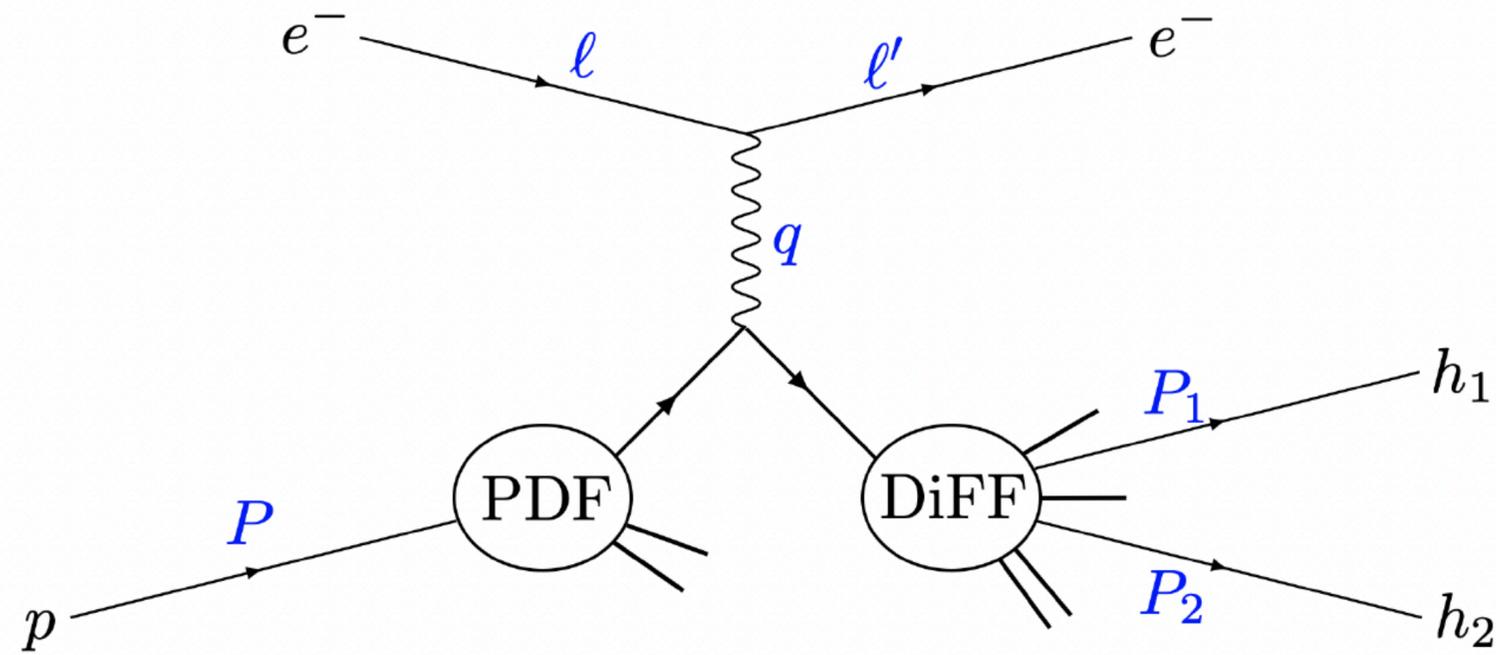


$\pi^+ \pi^-$ dihadron asymmetries from RG-C

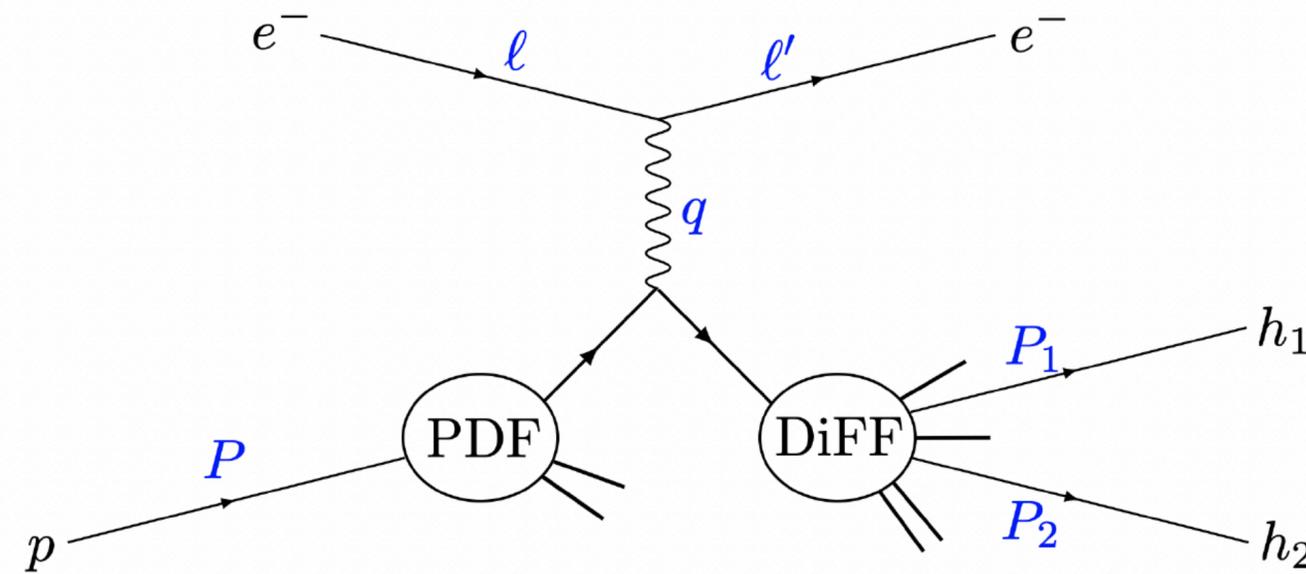


Nilanga Wickramaarachchi

Duke



Dihadron SIDIS



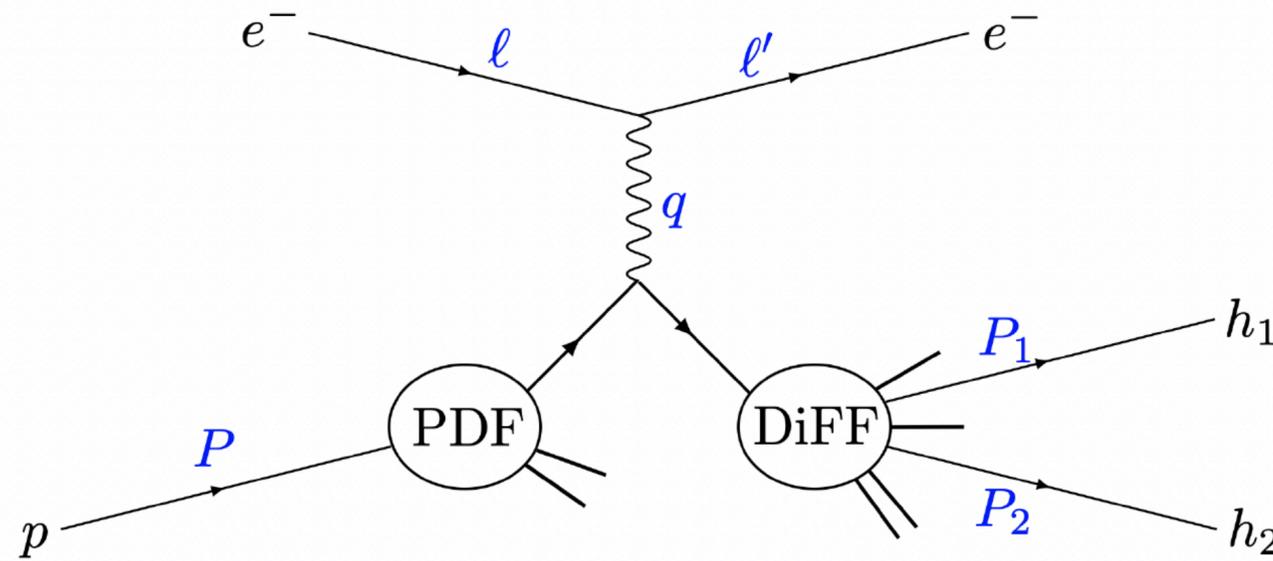
[1]

- In the dihadron SIDIS we detect two hadrons in the final state

$$e^- p \rightarrow e^- h_1 h_2 X$$

- Dihadron fragmentation function (DiFF) provide information on how the struck quark produces two hadrons
- Relative angular momentum between two hadrons provide more information on hadronization compared to single hadron case
 - Quark transverse momentum - angular momentum correlations
 - Access more structure functions

Dihadron kinematics



- Dihadron momentum is defined as sum of momenta of two hadrons

$$P_h = P_1 + P_2 \quad M_h - \text{dihadron mass}$$

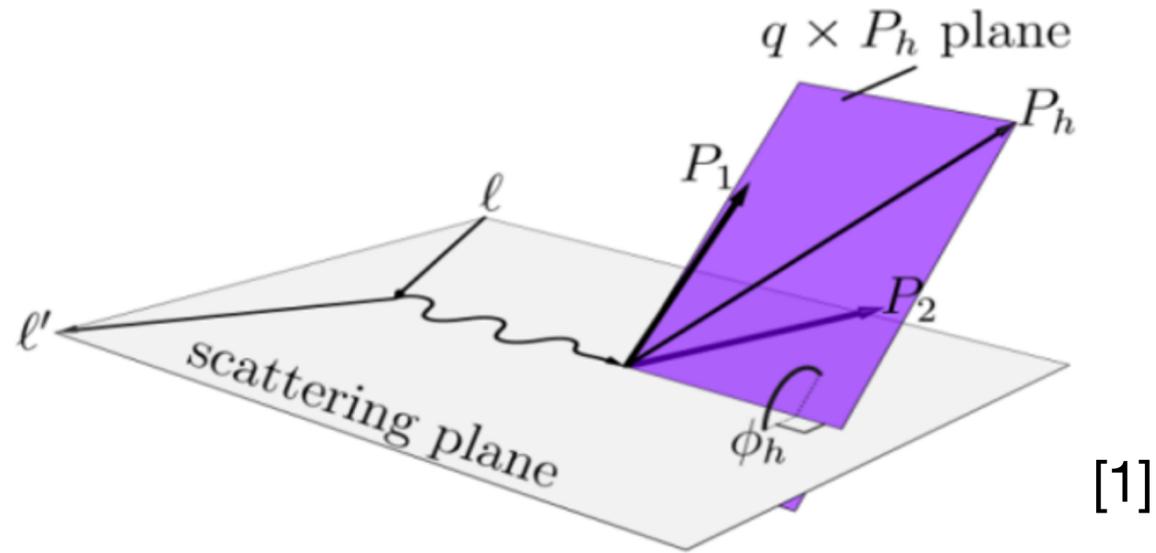
- Fractional energy of dihadron

$$z = \frac{P \cdot P_h}{P \cdot q}$$

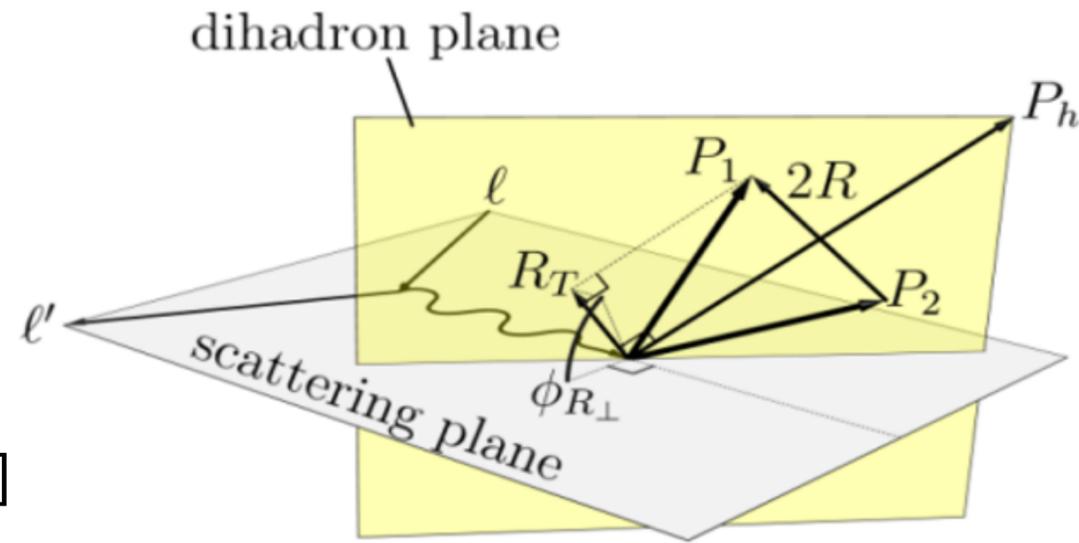
- Feynman-x variable used to consider current fragmentation region

$$x_F = \frac{2 P_h \cdot q}{|q| W}$$

Dihadron kinematics



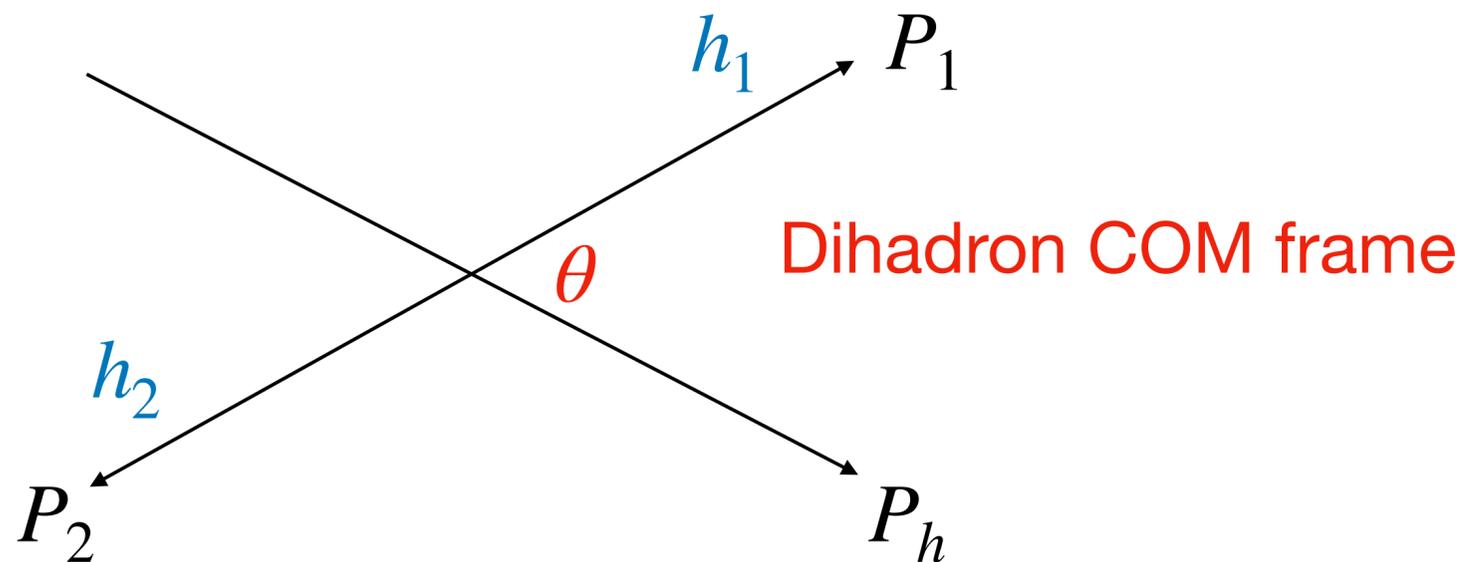
[1]



$$\phi_h = \frac{(\vec{q} \times \vec{l}) \cdot \vec{P}_h}{|(\vec{q} \times \vec{l}) \cdot \vec{P}_h|} \arccos \frac{(\vec{q} \times \vec{l}) \cdot (\vec{q} \times \vec{P}_h)}{|\vec{q} \times \vec{l}| |\vec{q} \times \vec{P}_h|}$$

$$\phi_R = \frac{(\vec{q} \times \vec{l}) \cdot \vec{R}_T}{|(\vec{q} \times \vec{l}) \cdot \vec{R}_T|} \arccos \frac{(\vec{q} \times \vec{l}) \cdot (\vec{q} \times \vec{R}_T)}{|\vec{q} \times \vec{l}| |\vec{q} \times \vec{R}_T|}$$

$$\vec{R}_T = \frac{z_2 \vec{P}_1^\perp - z_1 \vec{P}_2^\perp}{z}$$



Provides additional degrees of freedom
 ϕ_R , θ and M_h compared to single hadron SIDIS

Dihadron SIDIS cross section

- Cross section components for certain polarization states of beam and target

$$d\sigma_{LU} = \frac{\alpha^2}{4\pi xy Q^2} \left(1 + \frac{\gamma^2}{2x}\right) \lambda_e \sum_{l=0}^{l_{\max}} \left\{ C(x, y) \sum_{m=1}^l \left[P_{l,m} \sin(m(\phi_h - \phi_{R_\perp})) 2 \left(F_{LU,T}^{P_{l,m} \sin(m(\phi_h - \phi_{R_\perp}))} + \epsilon F_{LU,L}^{P_{l,m} \sin(m(\phi_h - \phi_{R_\perp}))} \right) \right. \right. \\ \left. \left. + W(x, y) \sum_{m=-l}^l P_{l,m} \sin((1-m)\phi_h + m\phi_{R_\perp}) F_{LU}^{P_{l,m} \sin((1-m)\phi_h + m\phi_{R_\perp})} \right] \right\}$$

- $F_{XY}^{m(\phi_h, \phi_R)}$'s are structure functions which can be written as convolutions of **TMD PDF** and **DIFF**
- $P_{l,m}$'s are Legendre polynomials depend on **cos θ**
- Limit $l_{\max} = 2$ (dihadron invariant mass in CLAS12 limited to around 1 GeV)
- Consider integration over **θ**
- Cross section is related to azimuthal modulations of angles ϕ_h and ϕ_R

Dihadron SIDIS cross section

$$\begin{aligned}
 d\sigma_{UL} = & \frac{\alpha^2}{4\pi xy Q^2} \left(1 + \frac{\gamma^2}{2x}\right) S_L \left\{ A(x, y) \sum_{\ell=1}^{\ell_{\max}} \sum_{m=1}^{\ell} P_{\ell, m} \sin(-m\phi_h + m\phi_{R_{\perp}}) F_{UL}^{P_{\ell, m} \sin(-m\phi_h + m\phi_{R_{\perp}})} \right. \\
 & + B(x, y) \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell, m} \sin((2-m)\phi_h + m\phi_{R_{\perp}}) F_{UL}^{P_{\ell, m} \sin((2-m)\phi_h + m\phi_{R_{\perp}})} \\
 & \left. + V(x, y) \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell, m} \sin((1-m)\phi_h + m\phi_{R_{\perp}}) F_{UL}^{P_{\ell, m} \sin((1-m)\phi_h + m\phi_{R_{\perp}})} \right\}. \quad [2]
 \end{aligned}$$

$$\begin{aligned}
 d\sigma_{LL} = & \frac{\alpha^2}{4\pi xy Q^2} \left(1 + \frac{\gamma^2}{2x}\right) \lambda_e S_L \\
 & \times \sum_{\ell=0}^{\ell_{\max}} \left\{ C(x, y) \sum_{m=0}^{\ell} 2^{2-\delta_{m0}} P_{\ell, m} \cos(m(\phi_h - \phi_{R_{\perp}})) F_{LL}^{P_{\ell, m} \cos(m(\phi_h - \phi_{R_{\perp}}))} \right. \\
 & \left. + W(x, y) \sum_{m=-\ell}^{\ell} P_{\ell, m} \cos((1-m)\phi_h + m\phi_{R_{\perp}}) F_{LL}^{P_{\ell, m} \cos((1-m)\phi_h + m\phi_{R_{\perp}})} \right\}. \quad [2]
 \end{aligned}$$

- Longitudinally polarized target data from RG-C sensitive to $d\sigma_{LU}$, $d\sigma_{UL}$ and $d\sigma_{LL}$

Structure functions, PDFs and DIFFs

Structure function and modulation	PDF and DIFF	Depolarization factor
$F_{LU}^{\sin(\phi_R)}$	eH_1^\perp	W
$F_{LU}^{\sin(\phi_h - \phi_R)}$	$f_1 G_1$	C
$F_{UL}^{\sin(\phi_R)}$	$h_L H_1^\perp$	V
$F_{UL}^{\sin(2\phi_R)}$	$h_{1L}^\perp H_1^\perp$	B
$F_{UL}^{\sin(-\phi_h + \phi_R)}$	$g_{1L} G_1$	A
F_{LL}^{const}	$g_{1L} D_1$	C
$F_{LL}^{\cos(\phi_R)}$	$g_{1L} \tilde{D}$	W
F_{UU}	$f_1 D_1$	A

$$A(\epsilon, y) = \frac{y^2}{2(1 - \epsilon)}$$

$$V(\epsilon, y) = \frac{y^2}{2(1 - \epsilon)} \sqrt{2\epsilon(1 + \epsilon)}$$

$$B(\epsilon, y) = \frac{y^2}{2(1 - \epsilon)} \epsilon$$

$$W(\epsilon, y) = \frac{y^2}{2(1 - \epsilon)} \sqrt{2\epsilon(1 - \epsilon)}$$

$$C(\epsilon, y) = \frac{y^2}{2(1 - \epsilon)} \sqrt{1 - \epsilon^2}$$

[3]

$$\epsilon = \frac{1 - y - \gamma^2 y^2 / 4}{1 - y + y^2 / 2 + \gamma^2 y^2 / 4}$$

$$\gamma = \frac{2Mx}{Q}$$

Event selection

$$e^-p \rightarrow e^- \pi^+ \pi^- X$$

Cuts for electron

- $Q^2 > 1 \text{ GeV}^2$
- $W > 2 \text{ GeV}$
- $y < 0.75$
- $M_X > 1.3 \text{ GeV}$
- $x_F > 0$ for π^+ , π^-
- $z < 0.85$ for dihadron

- **QADB cuts**

- **Only inbending data**

- **Vertex z cut**
 - **Summer22**: $-9.0 \leq V_z \leq 1.0 \text{ cm}$
 - **Fall22** & **Spring23**: $-7.5 \leq V_z \leq 2.5 \text{ cm}$
- $|V_x| < 2 \text{ cm}$, $|V_y| < 2 \text{ cm}$
- **DC fiducial cuts**
 - **EdgeR1** $> 4 \text{ cm}$, **EdgeR2** $> 5 \text{ cm}$, **EdgeR3** $> 8 \text{ cm}$
- **PCAL fiducial cuts**
 - $9 \text{ cm} < L_v, L_w < 400 \text{ cm}$
- **Sampling fraction cuts**
- **Diagonal cut for E/p of EC_{in} vs PCAL**

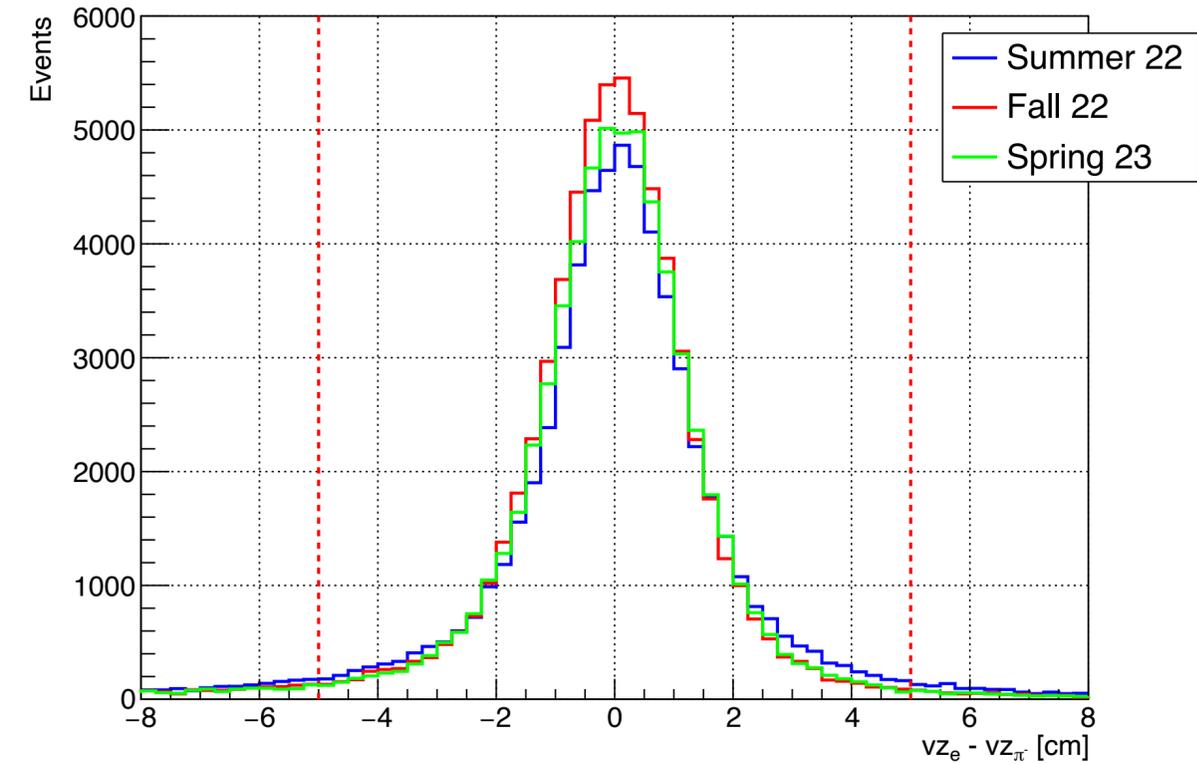
Event selection

$$e^- p \rightarrow e^- \pi^+ \pi^- X$$

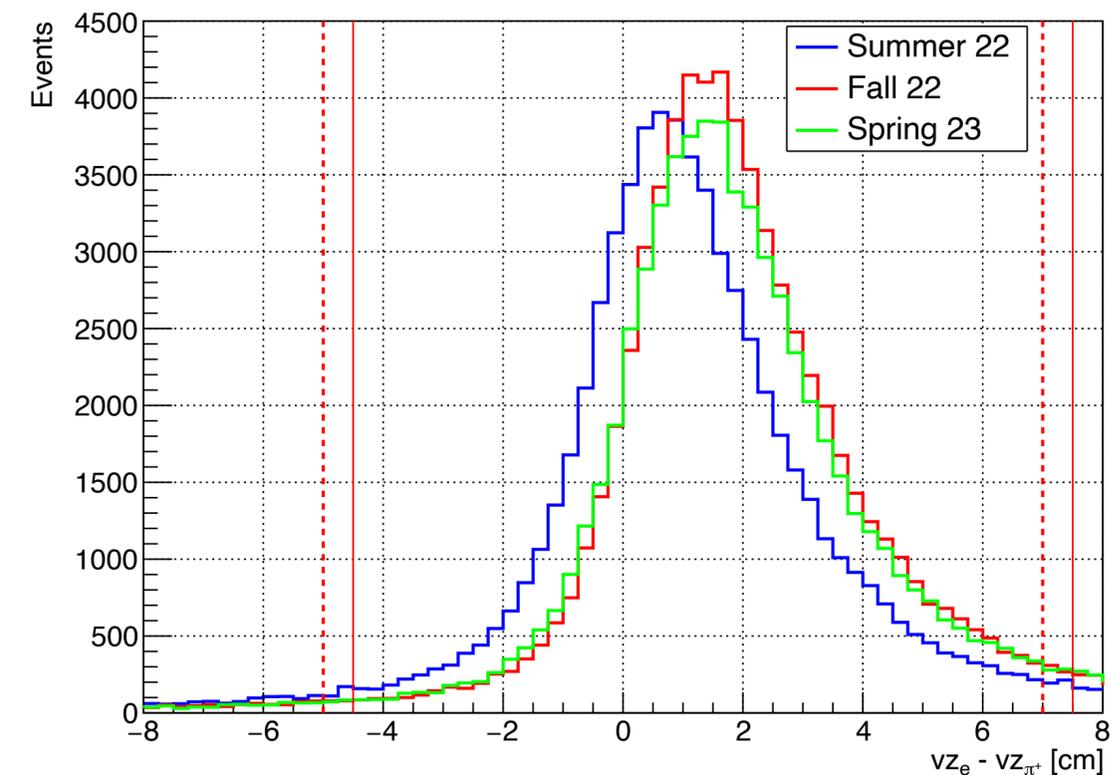
Cuts for π^+ , π^-

- Vertex z difference between e- and pion
- $\pi^- \rightarrow |V_z(e^-) - V_z(\pi^-)| < 5 \text{ cm}$
- $\pi^+ \rightarrow |V_z(e^-) - V_z(\pi^+) - 1| < 6 \text{ cm}$
Summer22
- $\pi^+ \rightarrow |V_z(e^-) - V_z(\pi^+) - 1.5| < 6 \text{ cm}$
Fall22, Spring23
- χ_{PID}^2 cuts
- $5^\circ < \theta < 35^\circ$

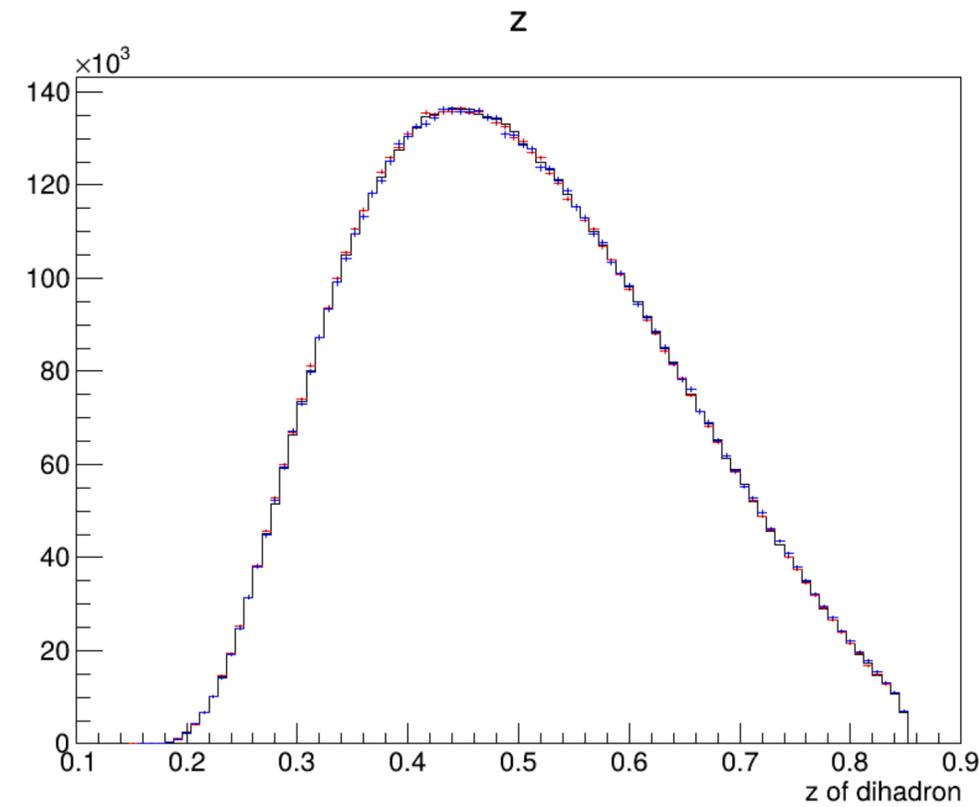
Vz difference between electron and pi-



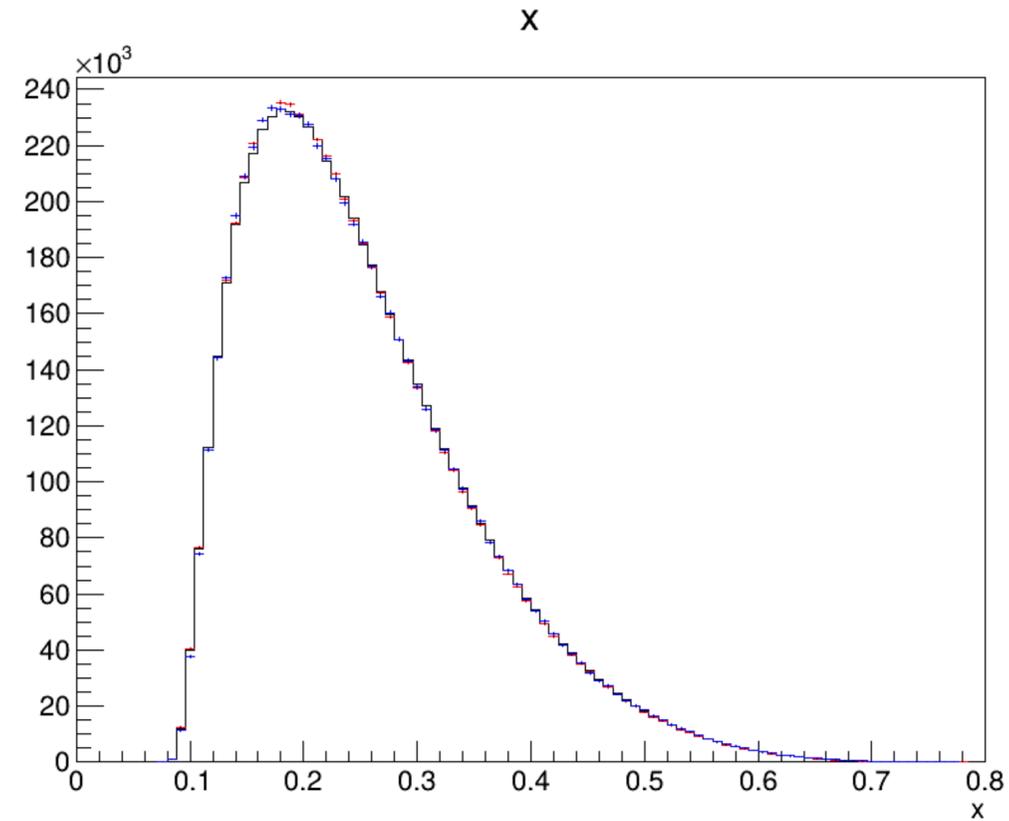
Vz difference between electron and pi+



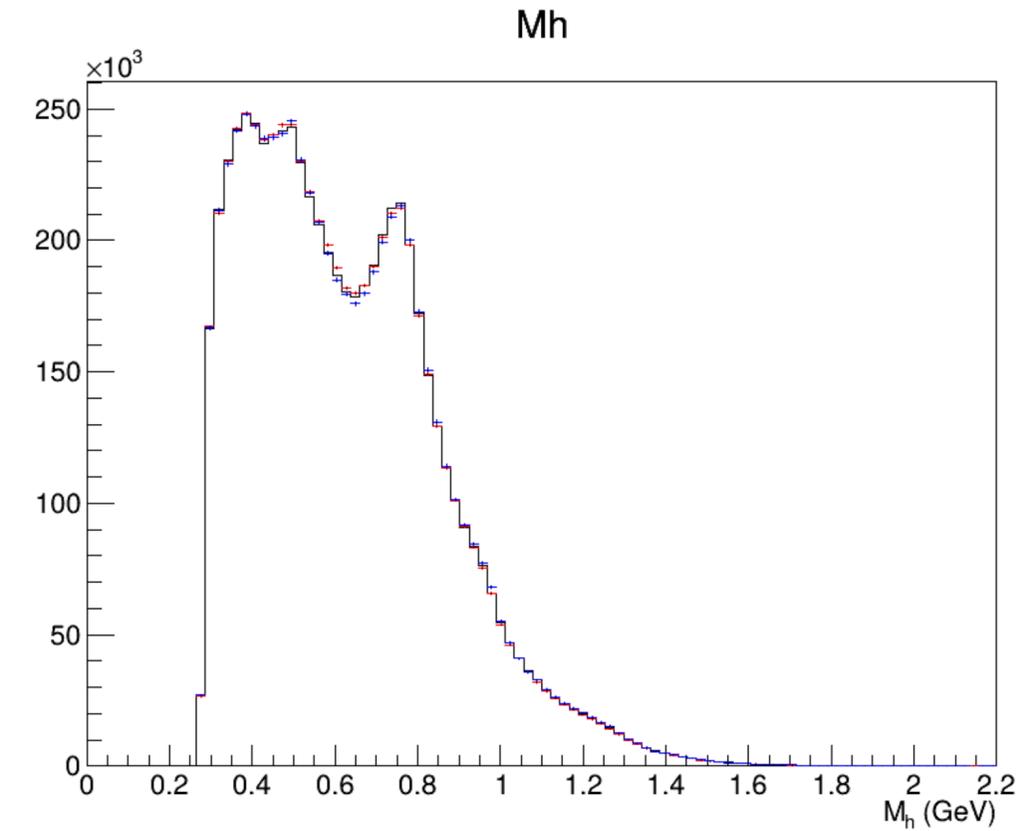
Kinematic variables



z



x



$M_{\pi^+\pi^-}$ (GeV)

- Asymmetries extracted for 10 bins in each kinematic variable
- Dilution factor calculated for each kinematic bin

Calculating packing fraction and dilution factor

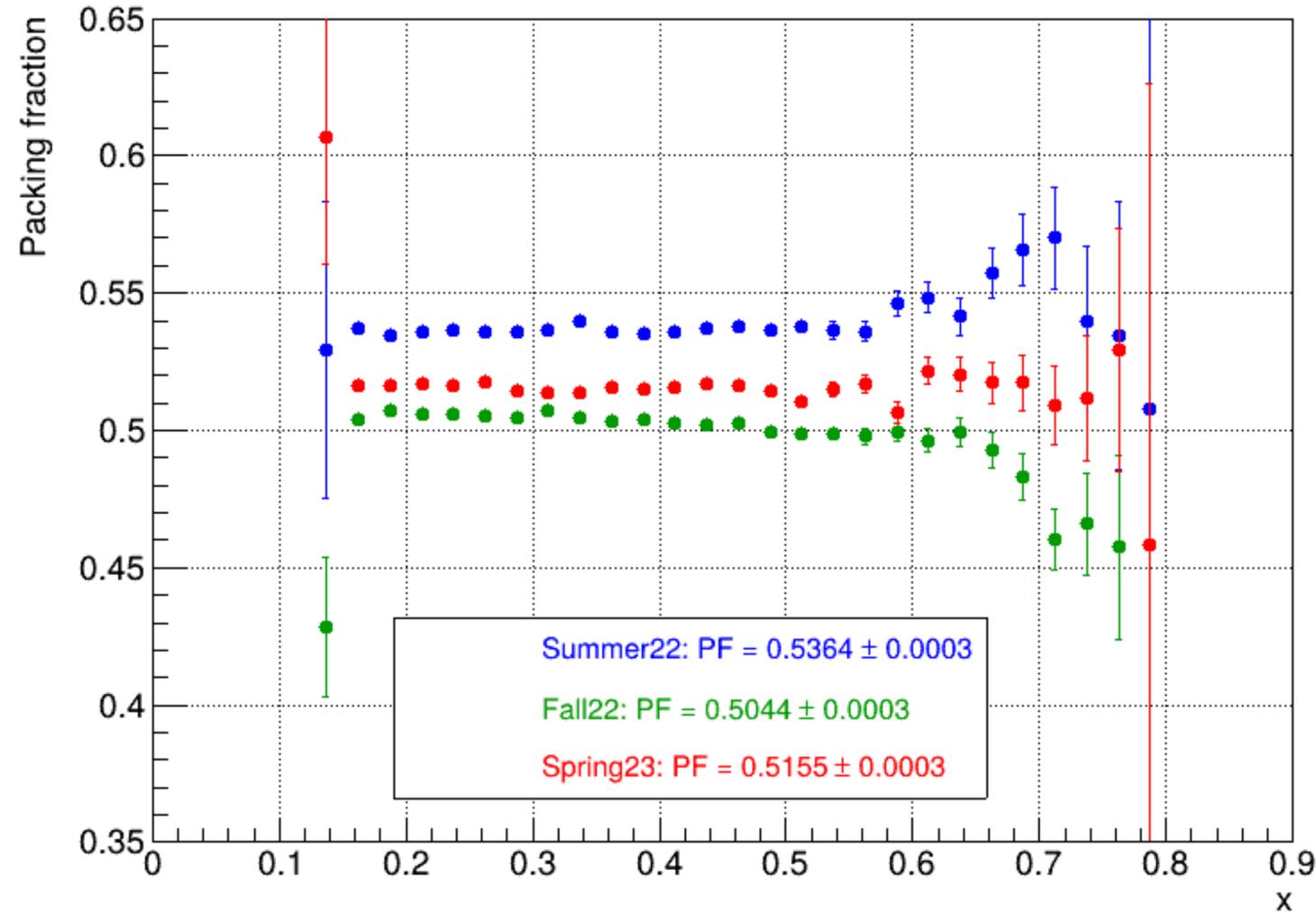
- **Packing fraction** - target volume fraction occupied by the ammonia crystals
- **Dilution factor** - fraction of events coming from polarized H in the target
- Compare two channels - inclusive and dihadron
- **New** equations to extract packing fraction and dilution factor !
(https://clasweb.jlab.org/wiki/images/4/46/RG-C_Dilution_v3.1.pdf)

$$PF = 0.50734 \frac{n_A - n_{MT}}{n_{CH} - 0.195667n_C - 0.82744n_{MT} + 0.023106n_F}. \quad (11)$$

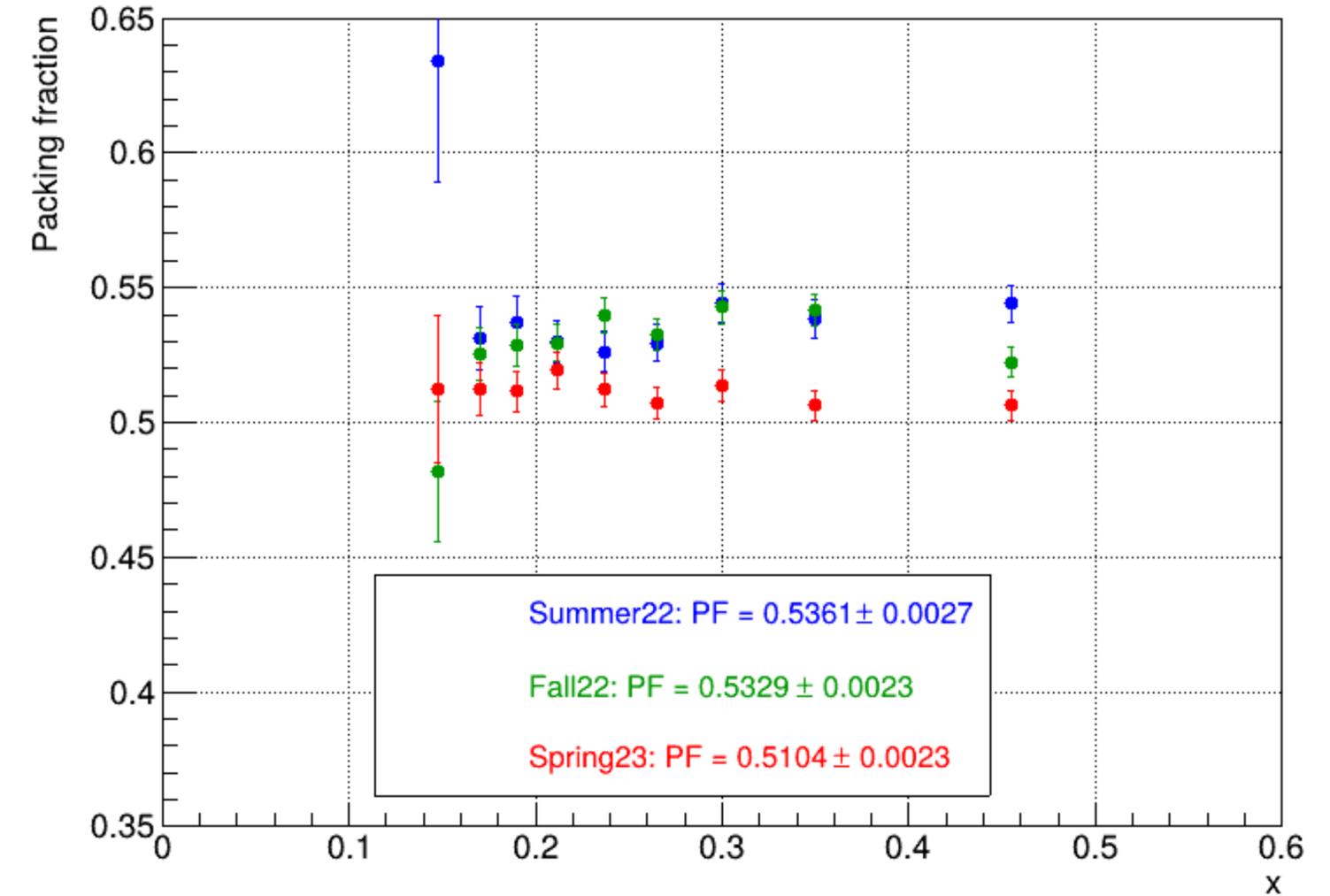
$$DF = \frac{n_{CH} - 0.880502n_C + 0.16604n_{MT} - 0.285536n_F}{n_{CH} - 0.195667n_C - 0.82744n_{MT} + 0.023106n_F) + 0.50734n_{MT}/PF}. \quad (14)$$

Packing fraction comparison from two channels

Packing fraction vs x from e^-X , $Q^2 > 2.2277 \text{ GeV}^2$



Packing fraction vs x from $e^-\pi^+\pi^-X$, $Q^2 > 2.2277 \text{ GeV}^2$

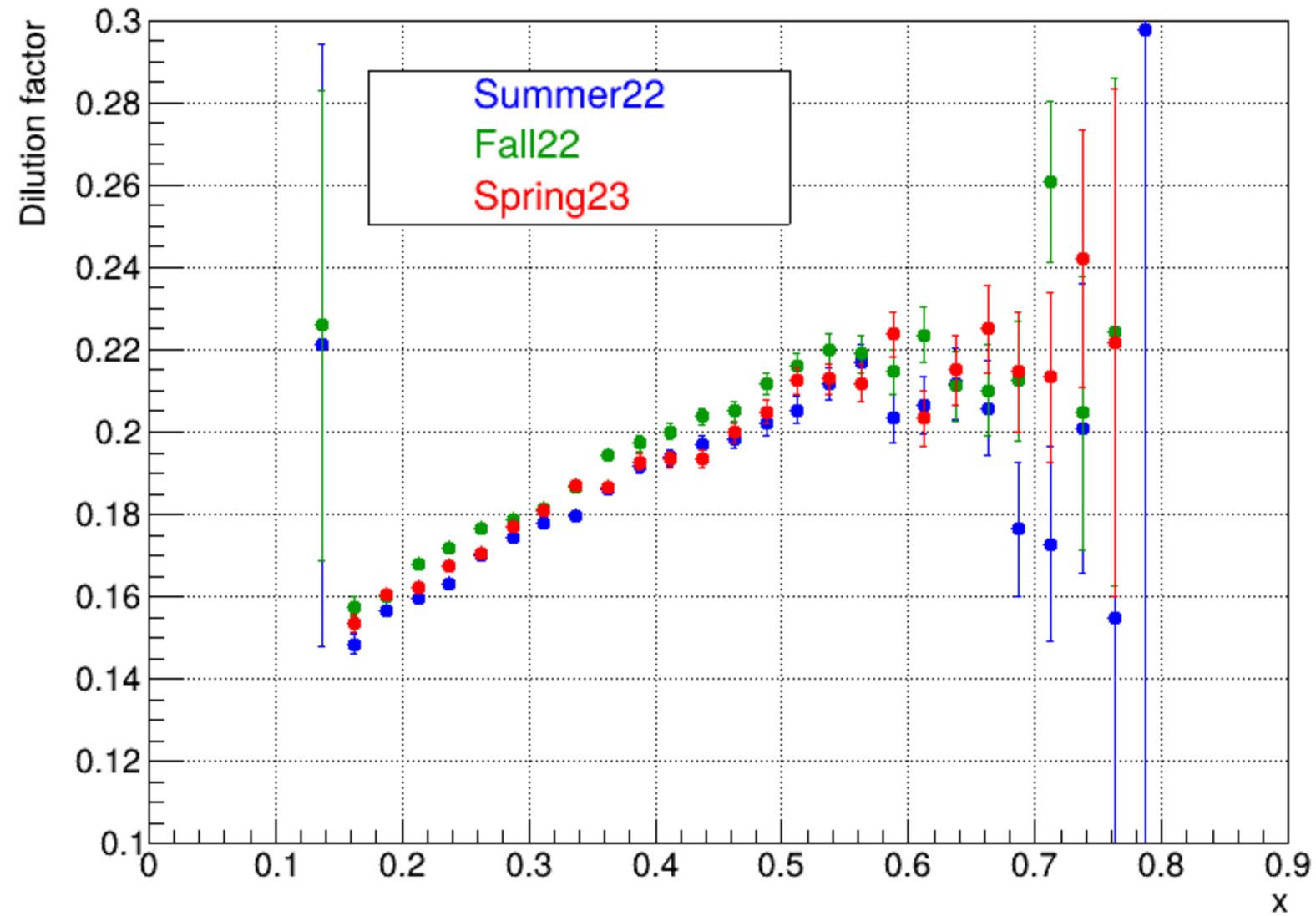


Run Period	e^-X	$e^-\pi^+\pi^-X$
Summer 2022	0.5364 ± 0.0003	0.5361 ± 0.0027
Fall 2022	0.5044 ± 0.0003	0.5329 ± 0.0023
Spring 2023	0.5155 ± 0.0003	0.5104 ± 0.0023

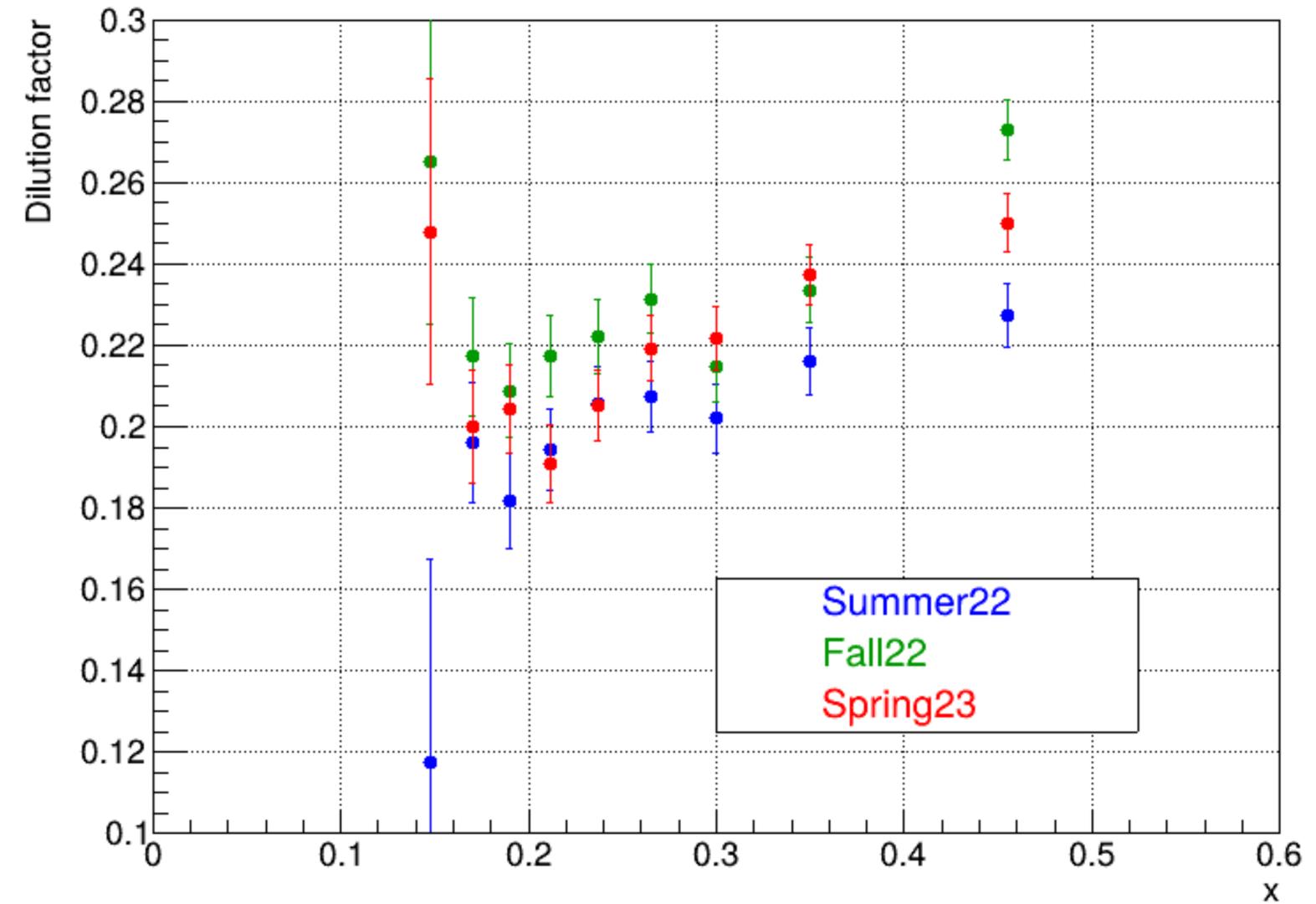
- Summer 2022 and Spring 2023 results agree between two channels
- Discrepancy for Fall 2022

Dilution factor comparison from two channels

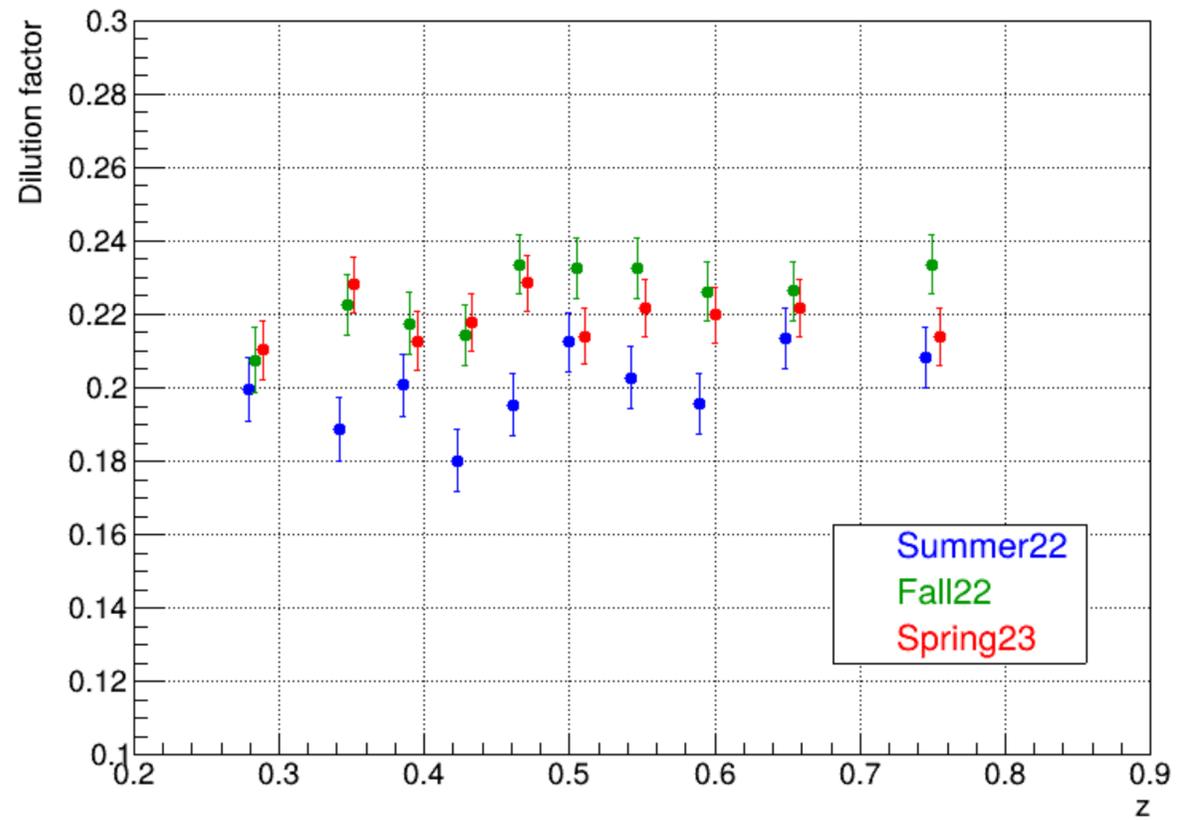
Dilution factor vs x from e^-X , $Q^2 > 2.2277 \text{ GeV}^2$



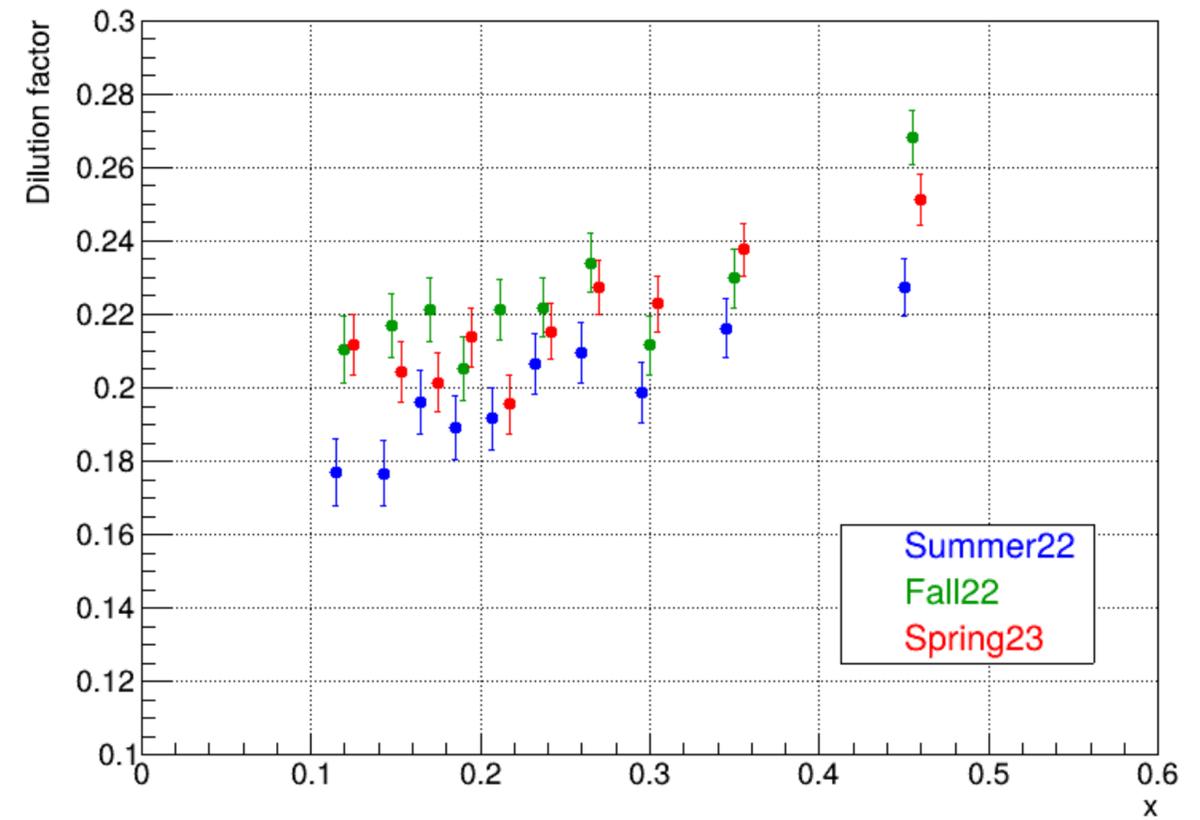
Dilution factor vs x from $e^-\pi^+\pi^-X$, $Q^2 > 2.2277 \text{ GeV}^2$



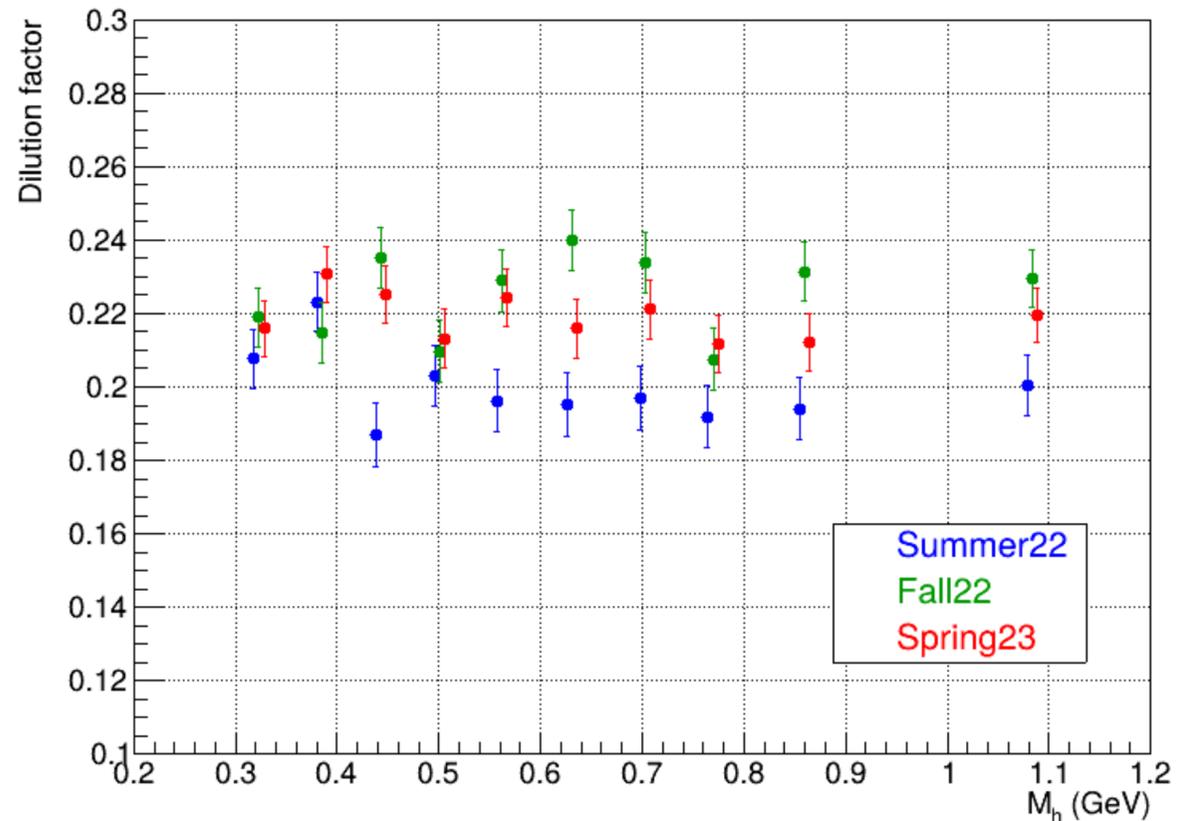
Dilution factor vs z from $e^- \pi^+ \pi^- X$



Dilution factor vs x from $e^- \pi^+ \pi^- X$



Dilution factor vs M_h from $e^- \pi^+ \pi^- X$



- Used packing fraction from inclusive channel ($Q^2 > 2.2277 \text{ GeV}^2$)
- $Q^2 > 1 \text{ GeV}^2$ for dihadron analysis

Extracting asymmetries - likelihood PDF

- Use a “combined fit” to extract $A_{LU}^{\psi_i}$, $A_{UL}^{\psi_i}$ and $A_{LL}^{\psi_i}$ simultaneously

- Probability distribution has the form

$$p = 1 + P_b h_b \frac{W(\epsilon, y) F_{LU}^{\sin \phi_R}}{A(\epsilon, y) F_{UU}^{\text{const}}} \sin \phi_R + \dots$$

$$+ P_t h_t f \frac{V(\epsilon, y) F_{UL}^{\sin \phi_R}}{A(\epsilon, y) F_{UU}^{\text{const}}} \sin \phi_R + \dots$$

$$+ P_b h_b P_t h_t f \frac{C(\epsilon, y) F_{LL}^{\text{const}}}{A(\epsilon, y) F_{UU}^{\text{const}}} + P_b h_b P_t h_t f \frac{W(\epsilon, y) F_{LL}^{\cos \phi_h}}{A(\epsilon, y) F_{UU}^{\text{const}}} \cos \phi_h + P_b h_b P_t h_t f \frac{W(\epsilon, y) F_{LL}^{\cos \phi_R}}{A(\epsilon, y) F_{UU}^{\text{const}}} \cos \phi_R + \dots$$

- Total of 27 modulation amplitudes

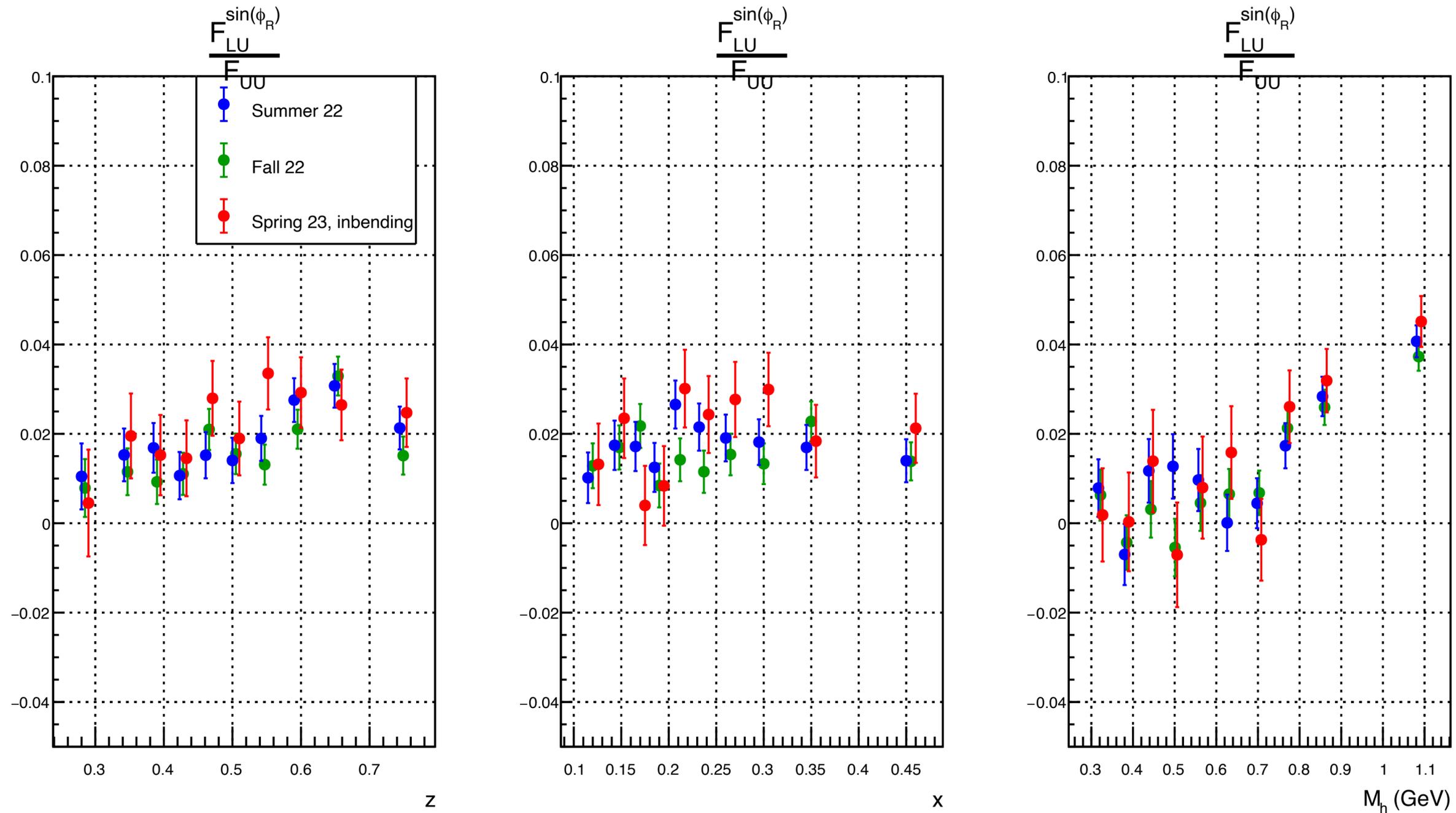
(fit parameters give structure functions ratio $\frac{F_{XY}^{\psi_i}}{F_{UU}^{\text{const}}}$)

A, C, V, W - depolarization factors

weight was applied (p^w) to account for imbalance between accumulated charges for +, - helicity states of beam, target

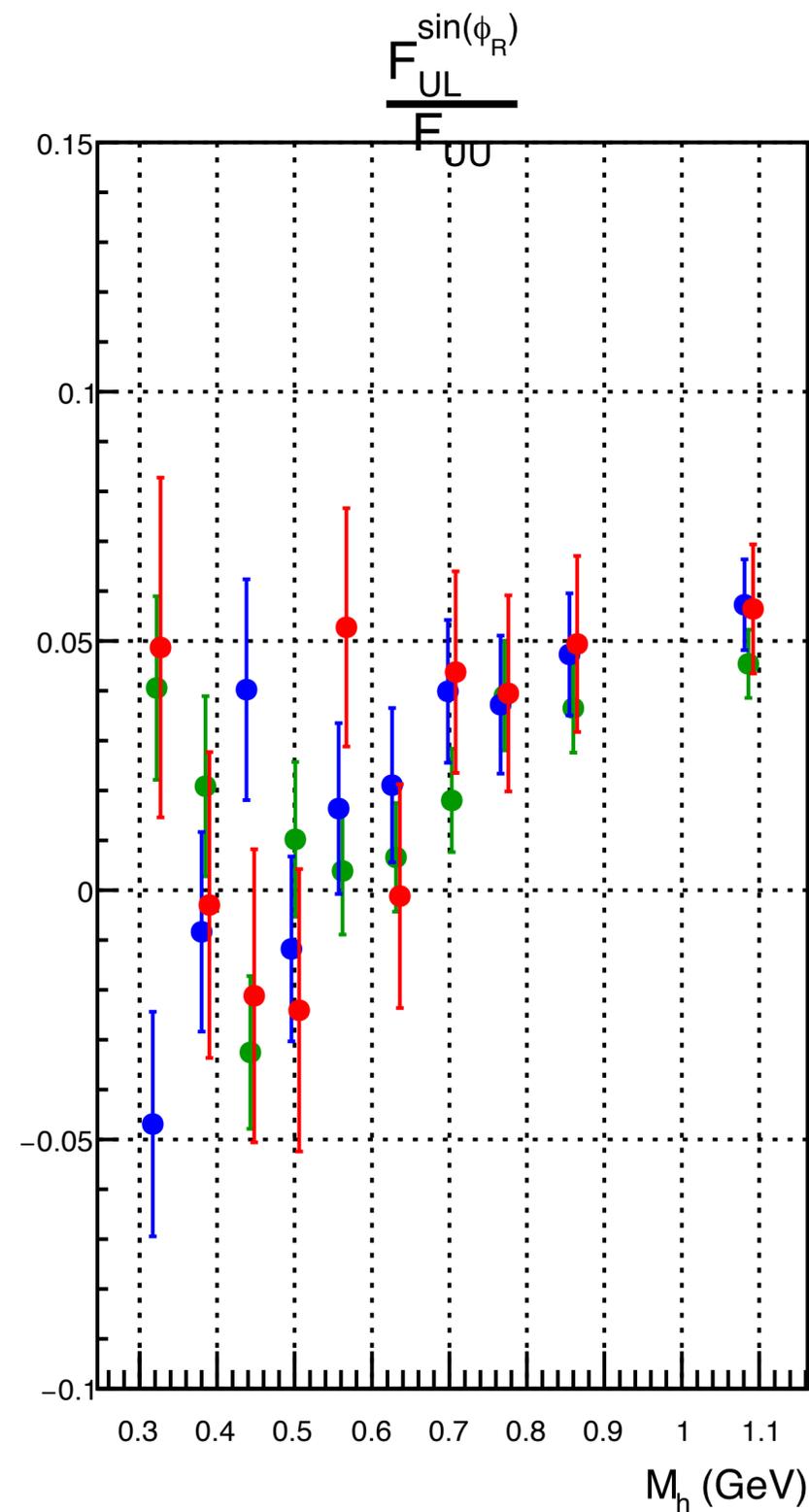
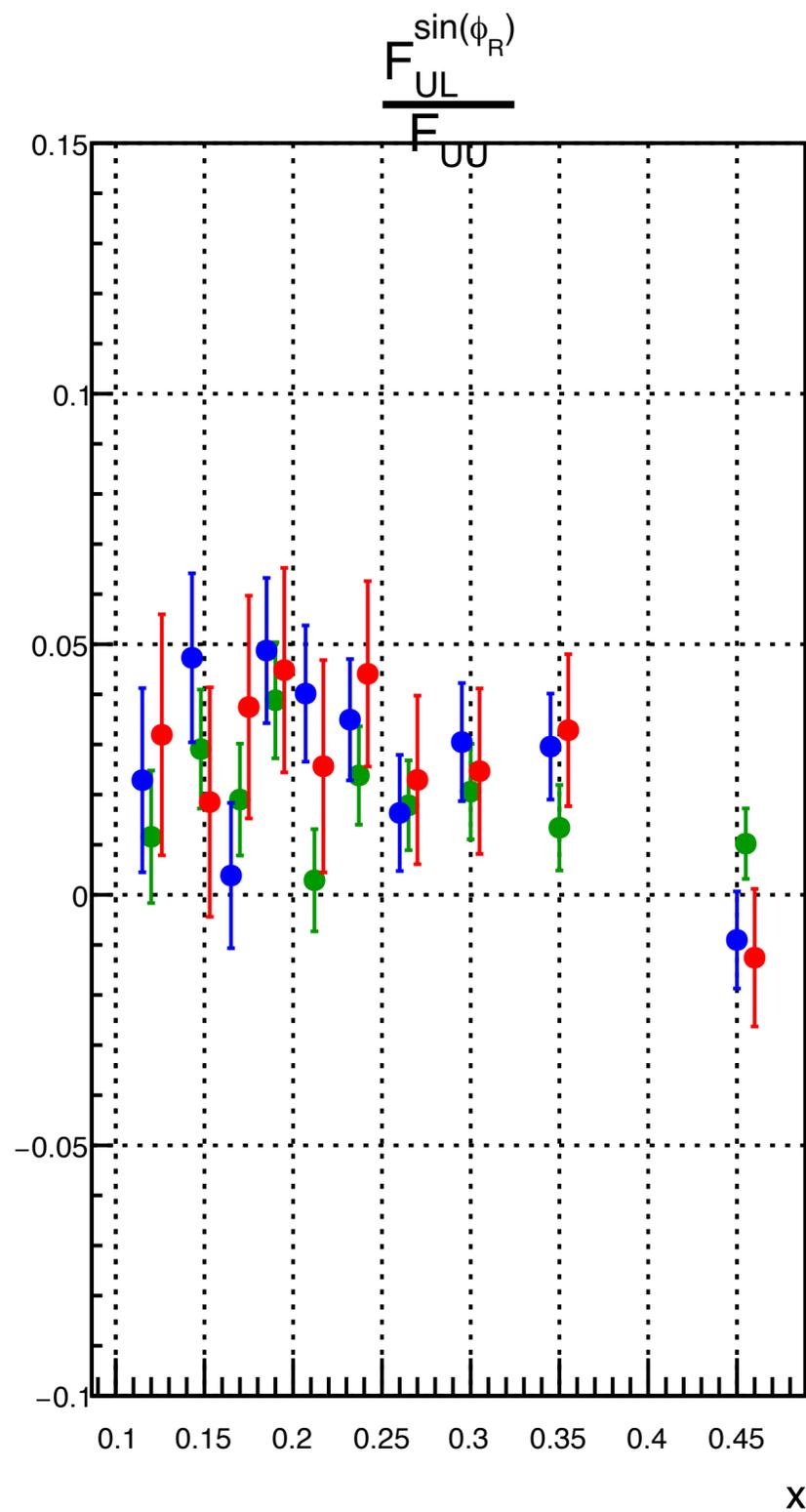
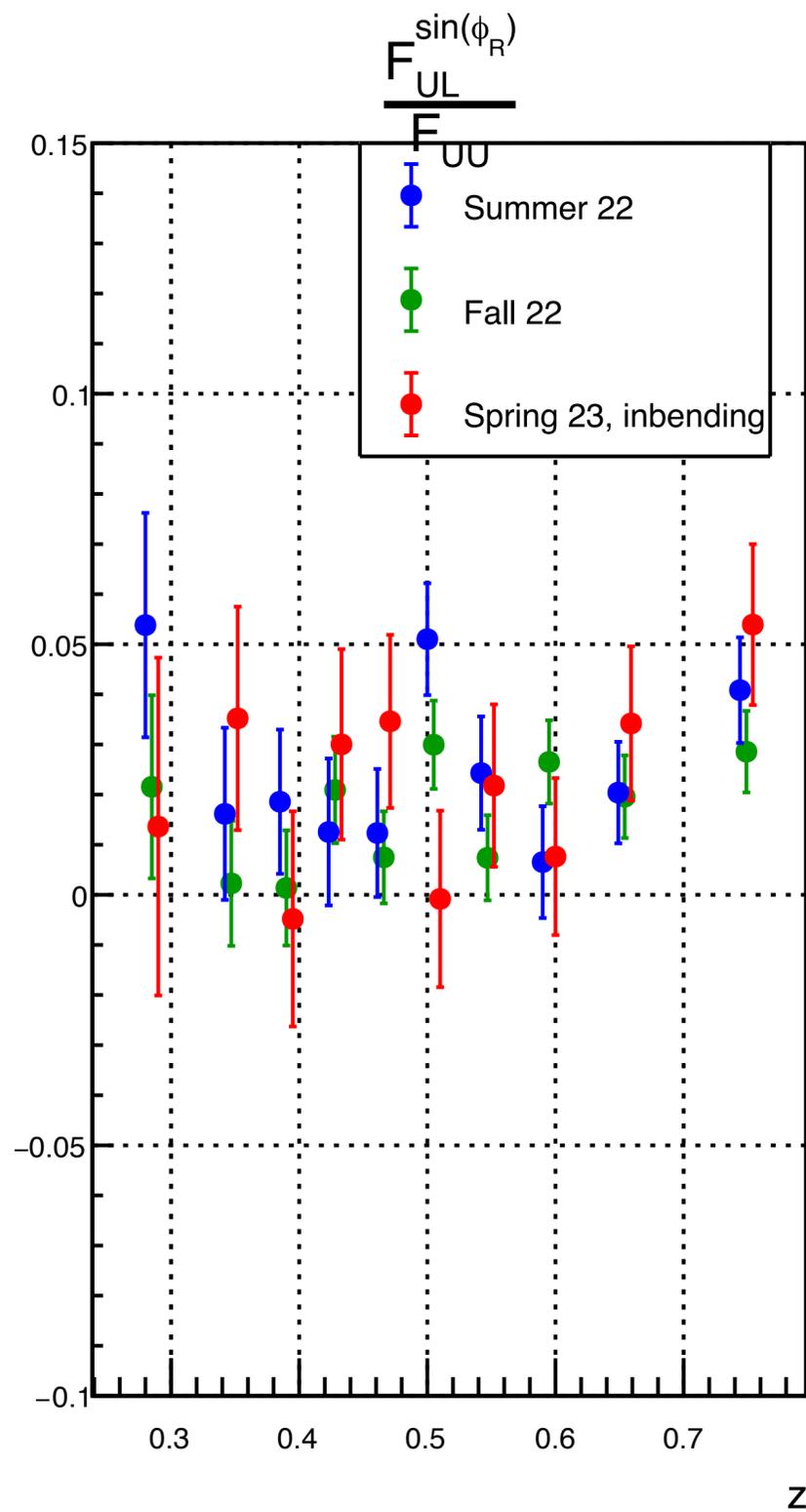
(https://clas12-docdb.jlab.org/DocDB/00111/0011176/001/CLAS_MLM_Analysis_Note.pdf)

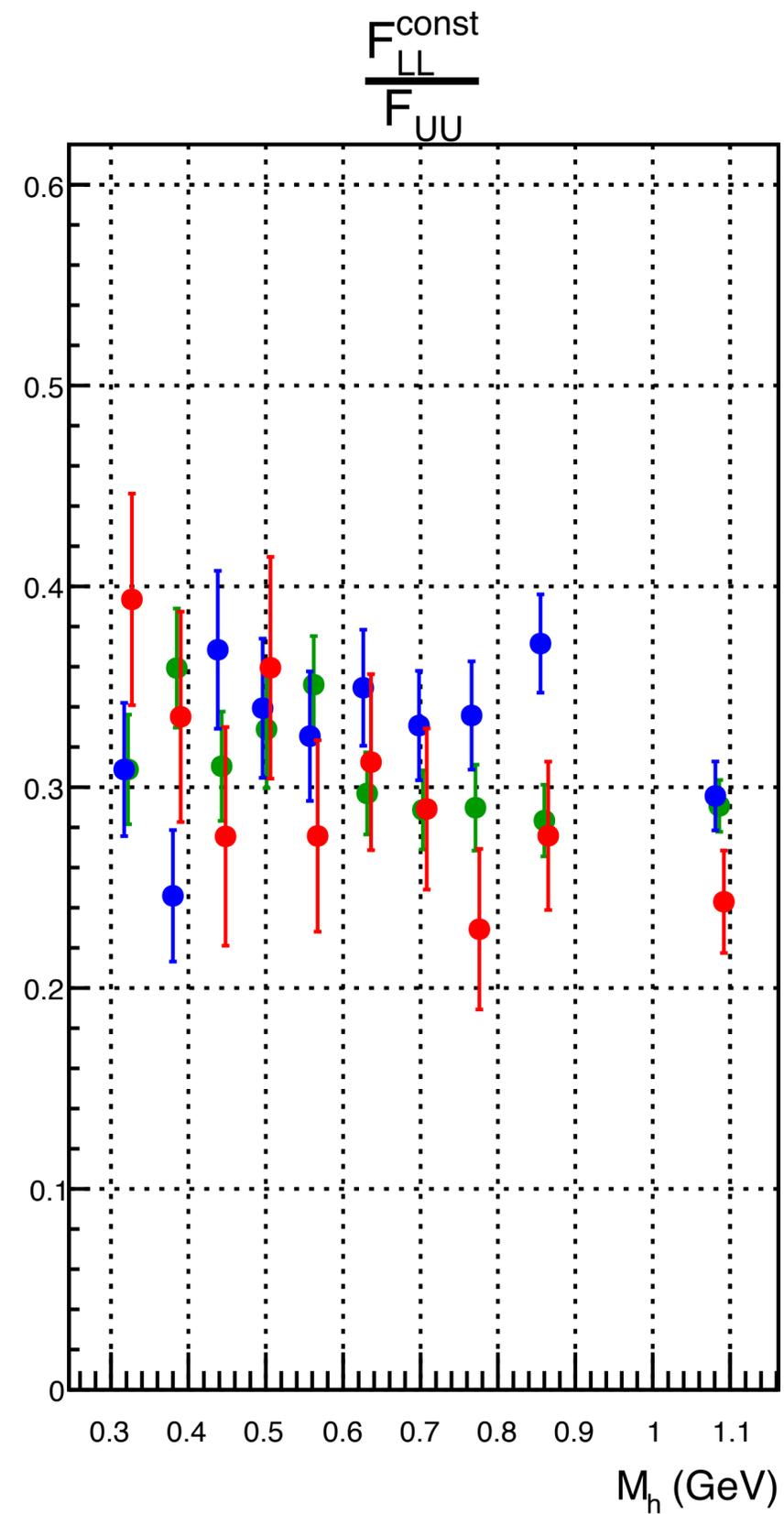
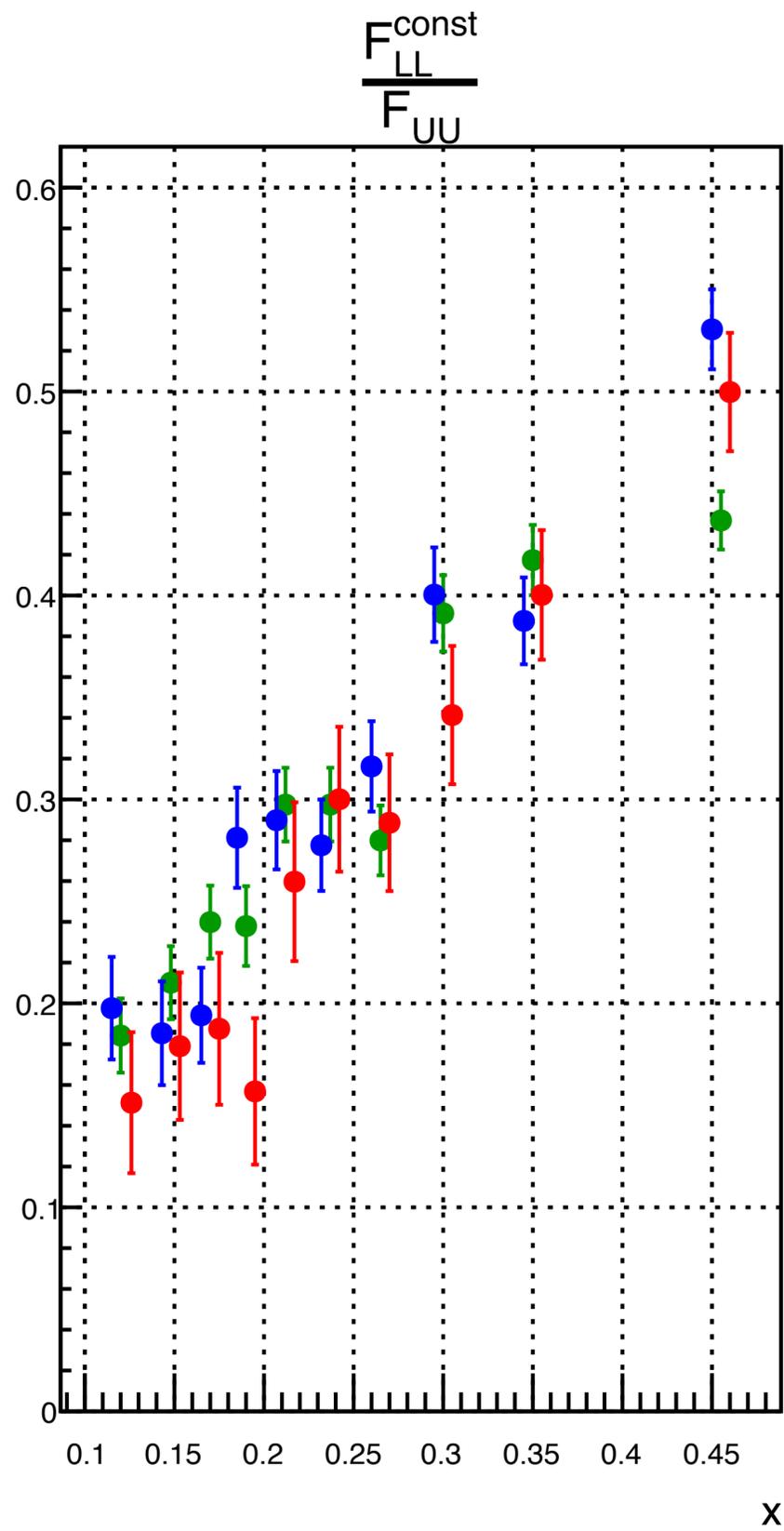
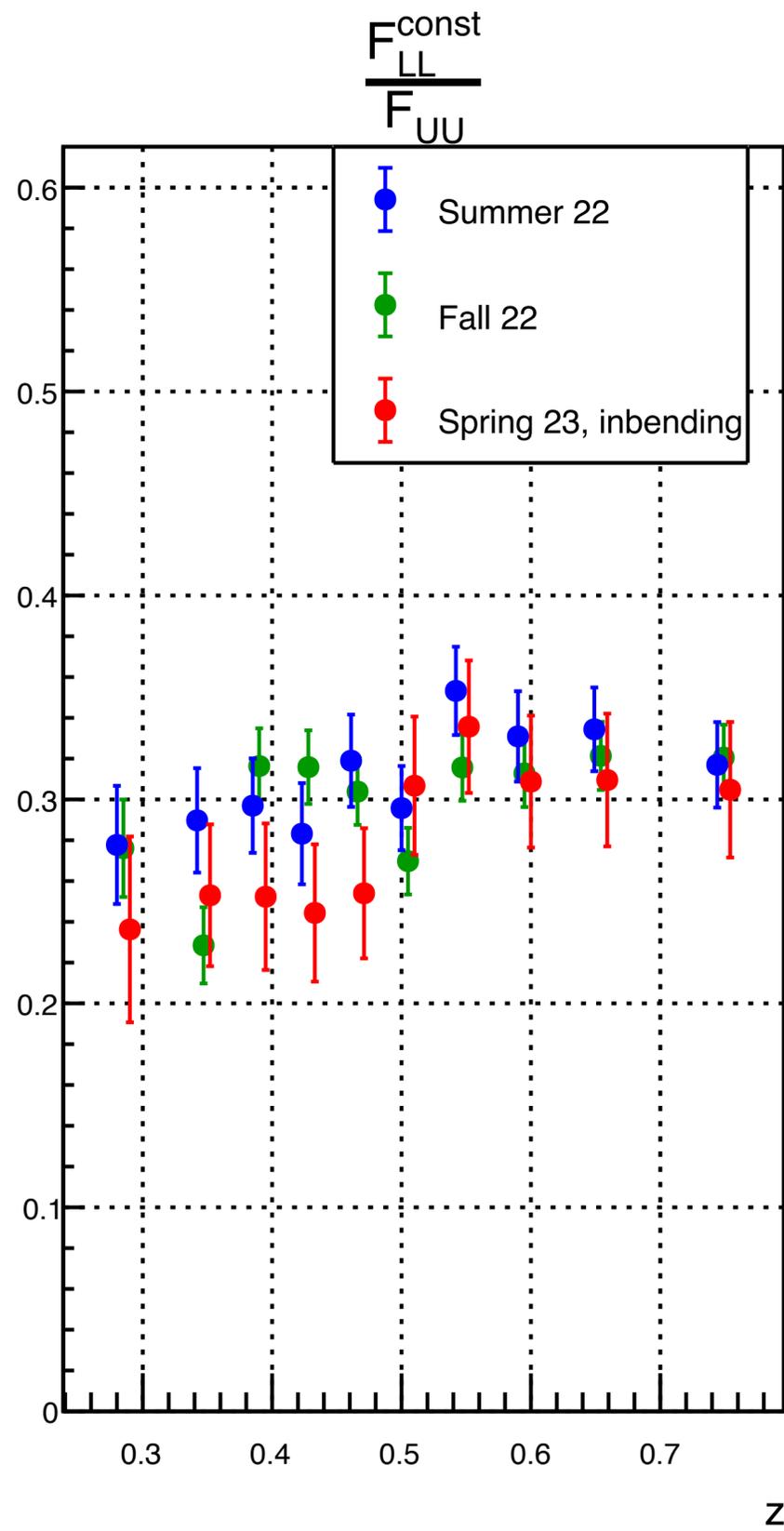
Asymmetry results for different run periods

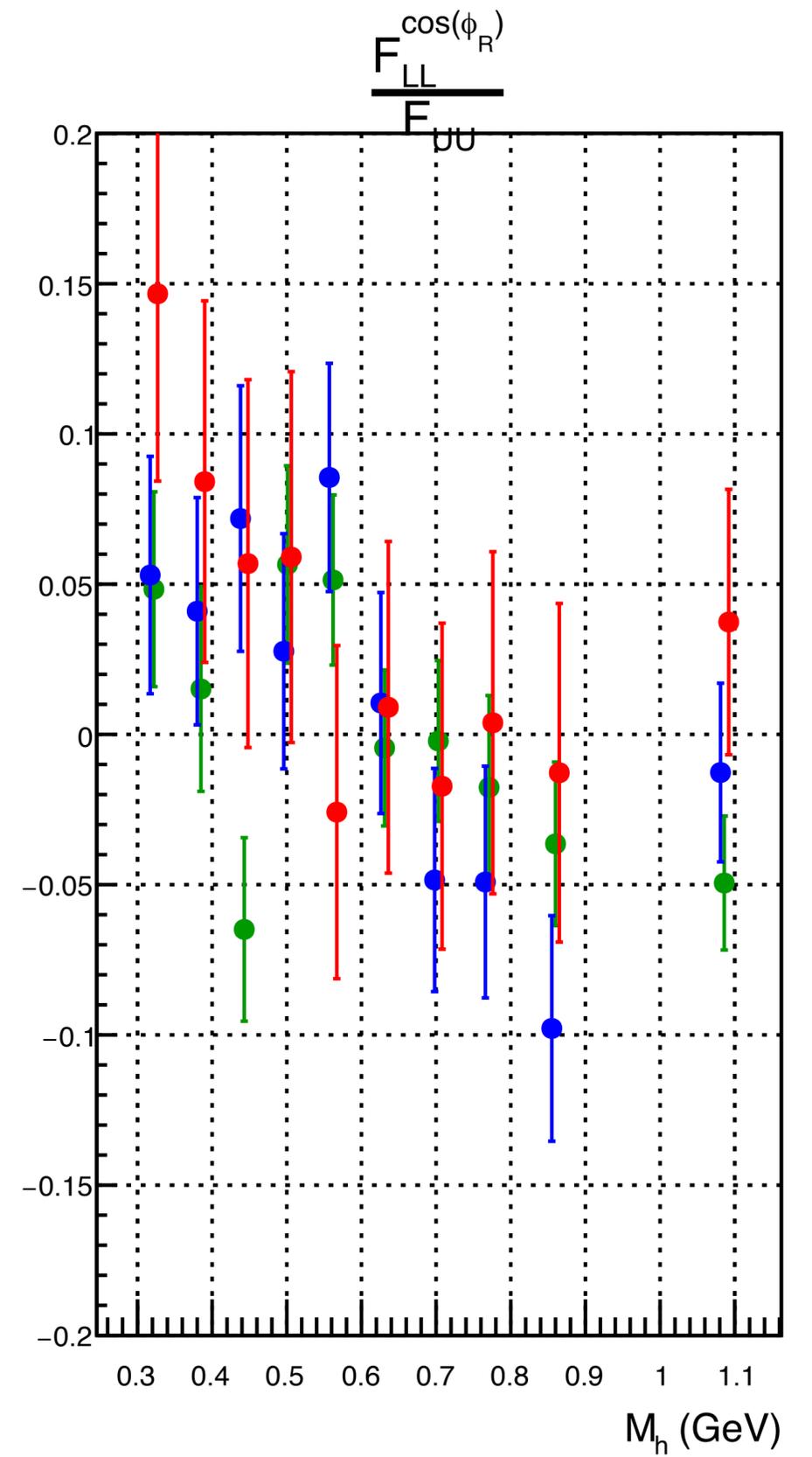
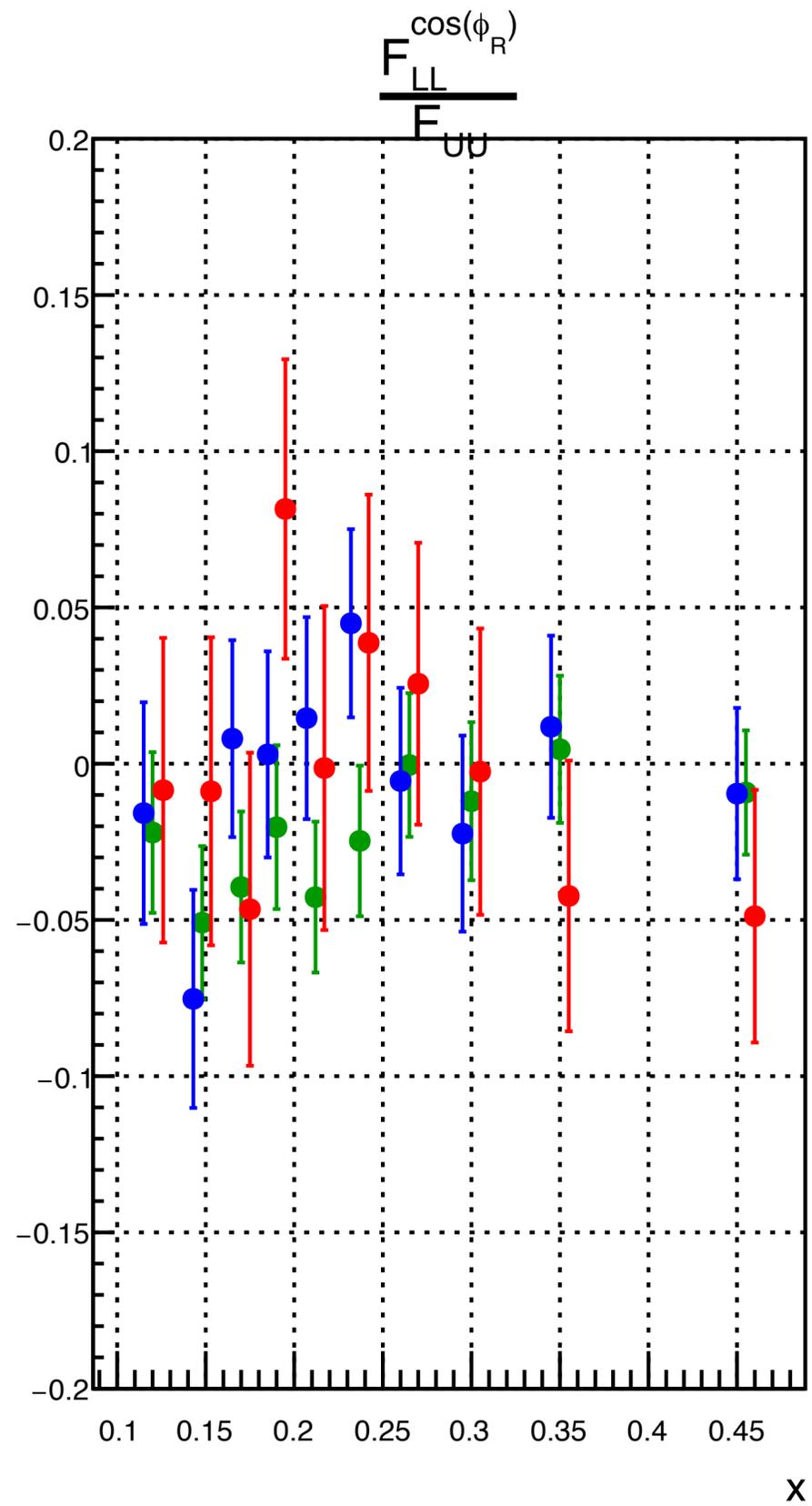
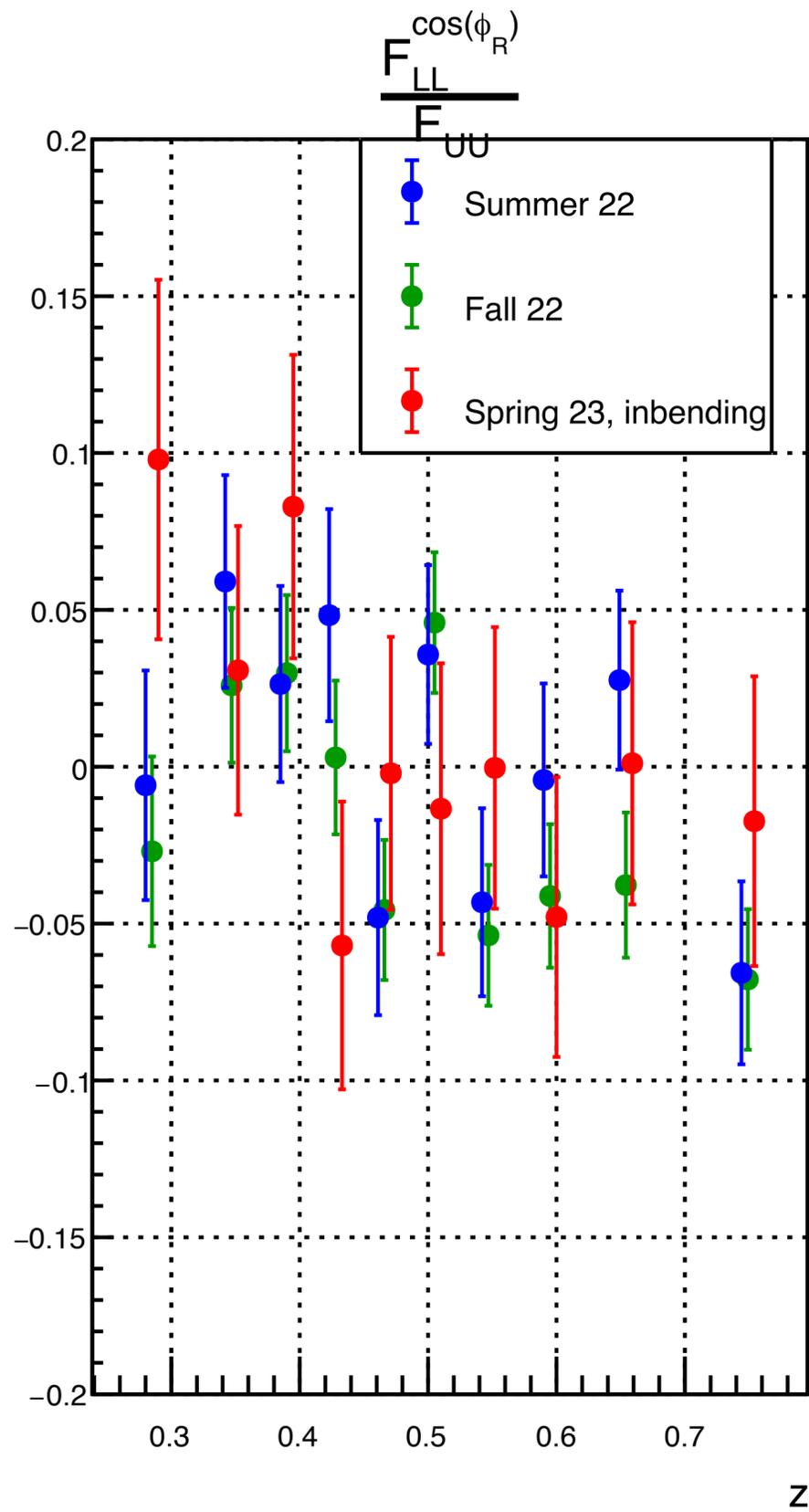


- Use weighted average method to combine results from 3 run periods

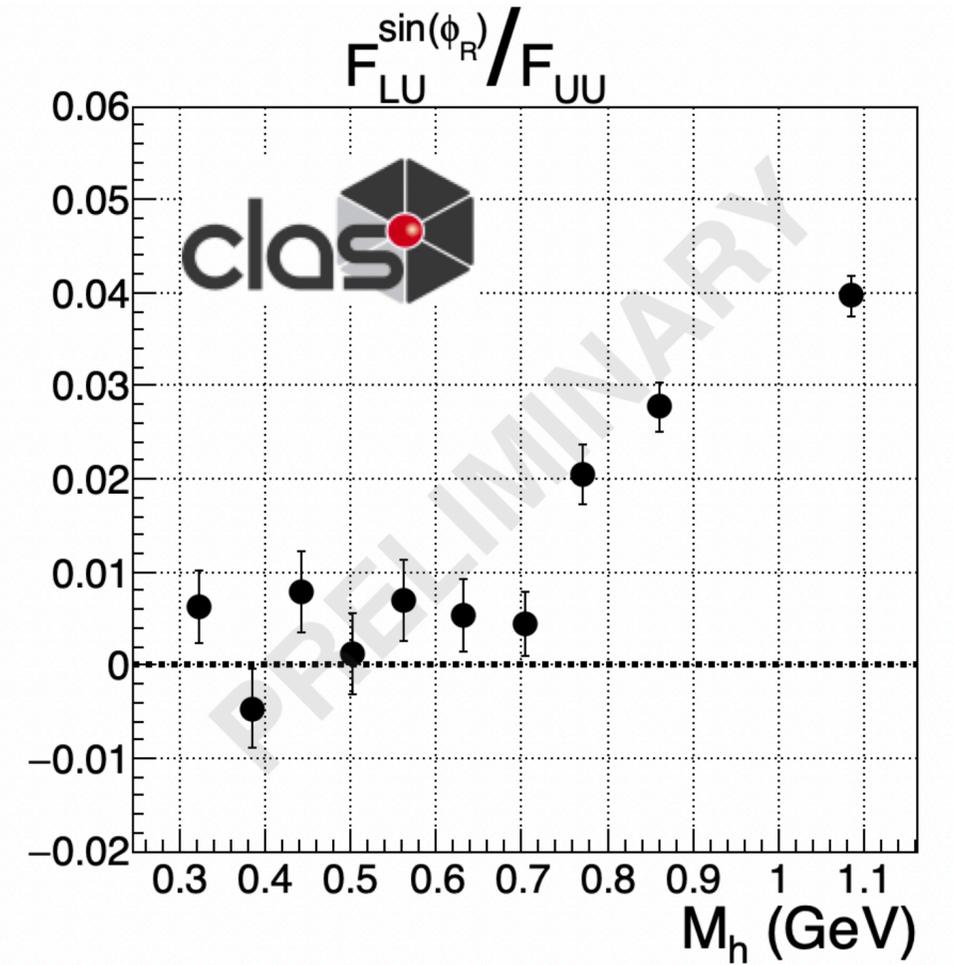
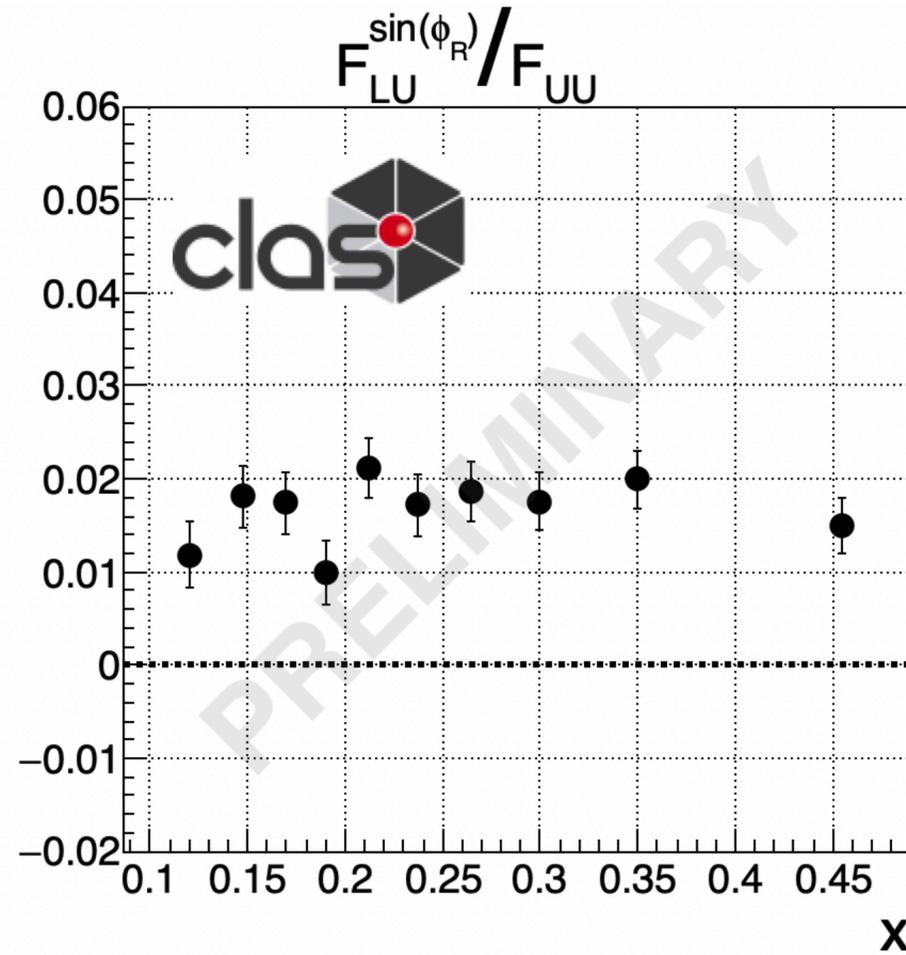
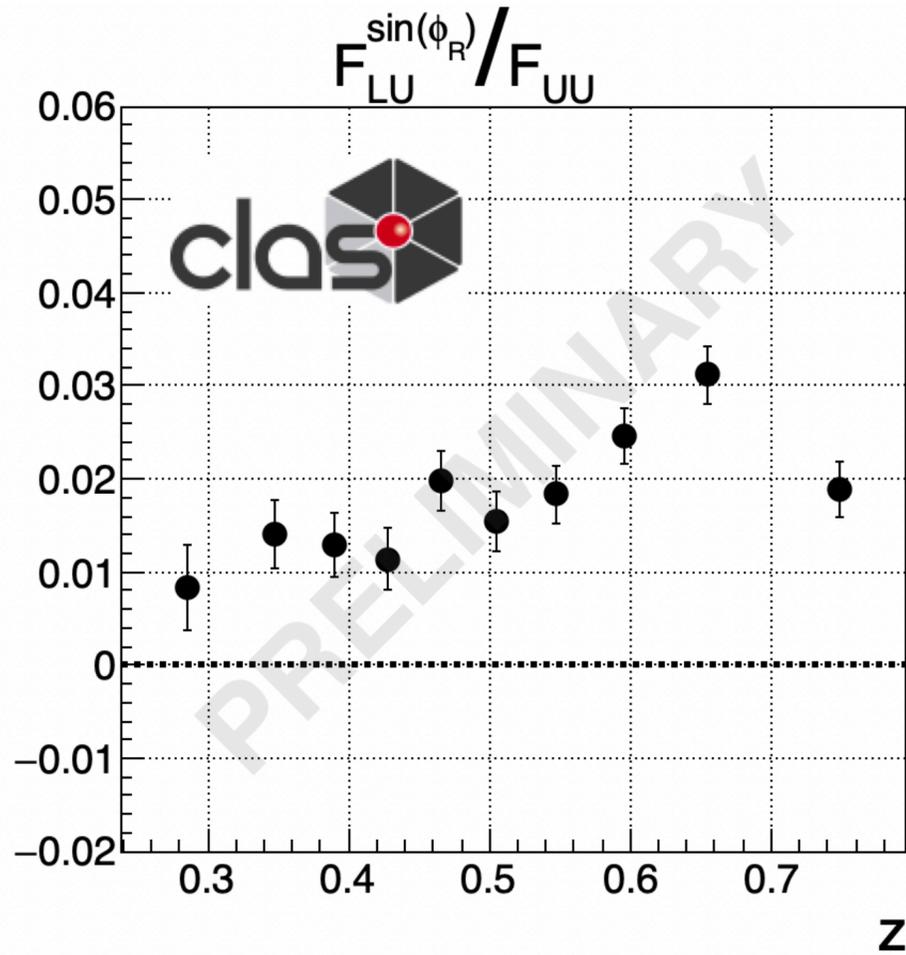
$$\bar{y} = \frac{\sum (y_i/\sigma_i^2)}{\sum (1/\sigma_i^2)} \quad \sigma_{\bar{y}} = \sqrt{\frac{1}{\sum (1/\sigma_i^2)}}$$





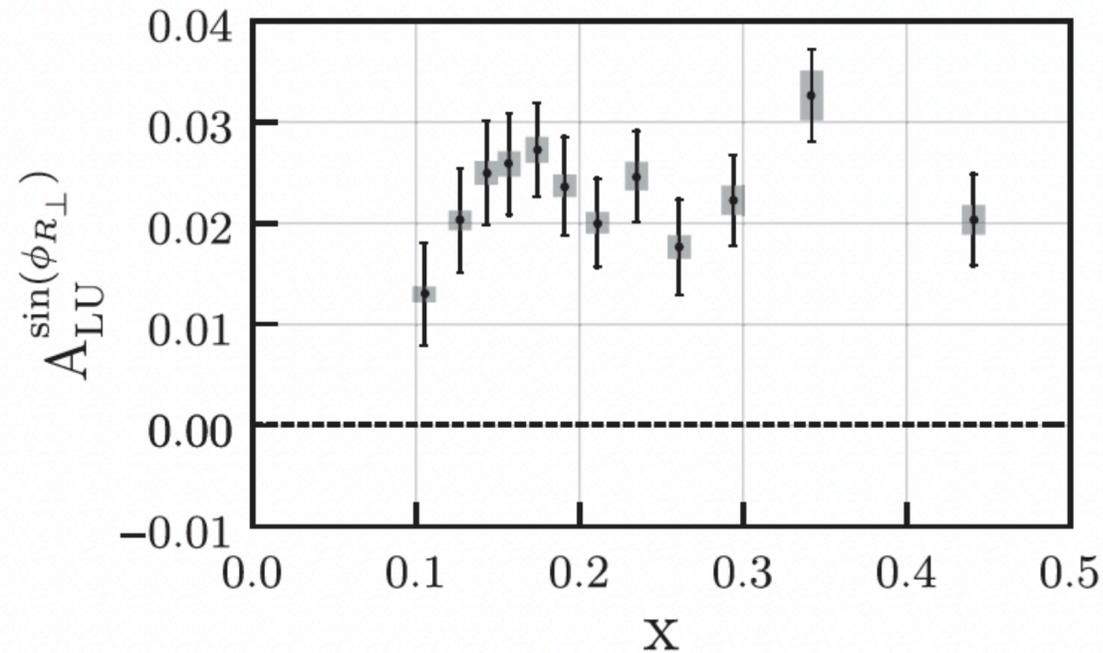


Weighted average results - $F_{LU}^{\sin(\phi_R)} / F_{UU}$



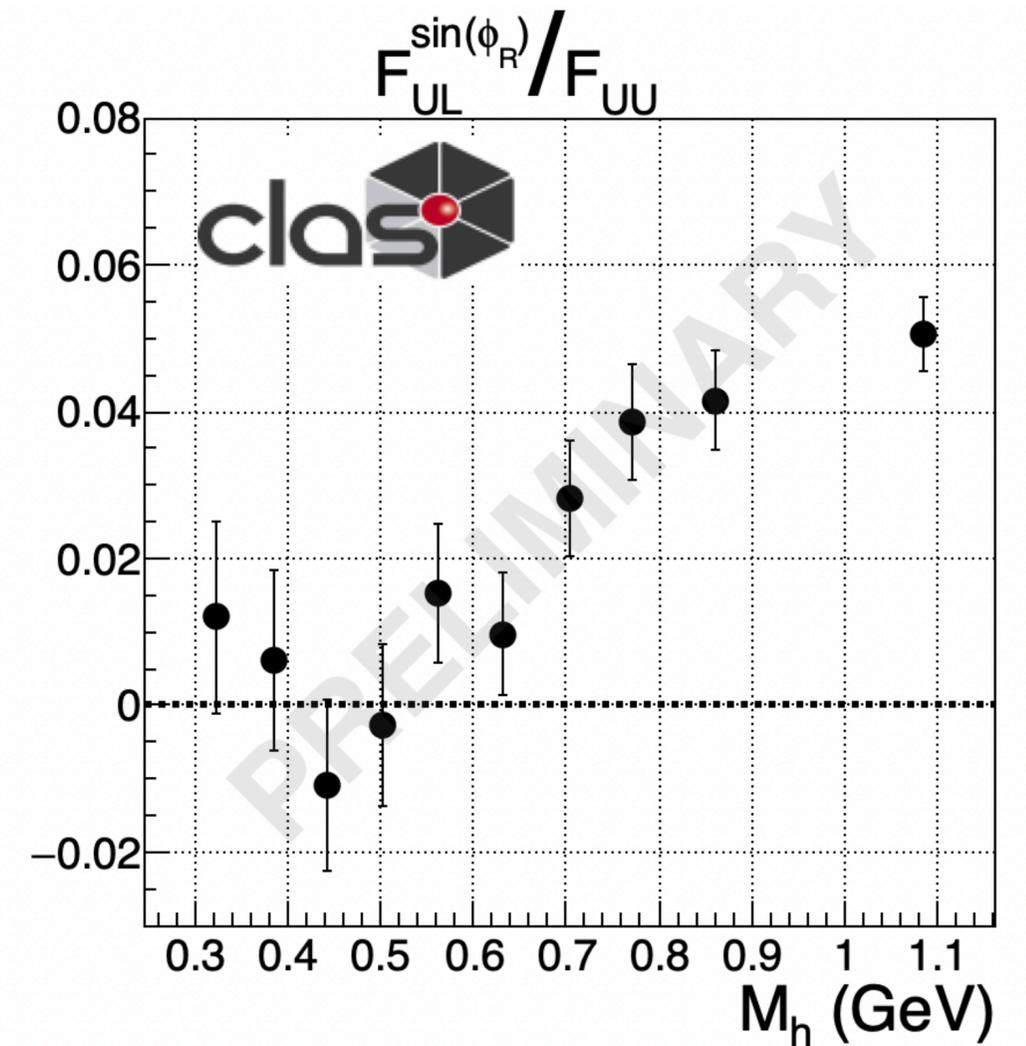
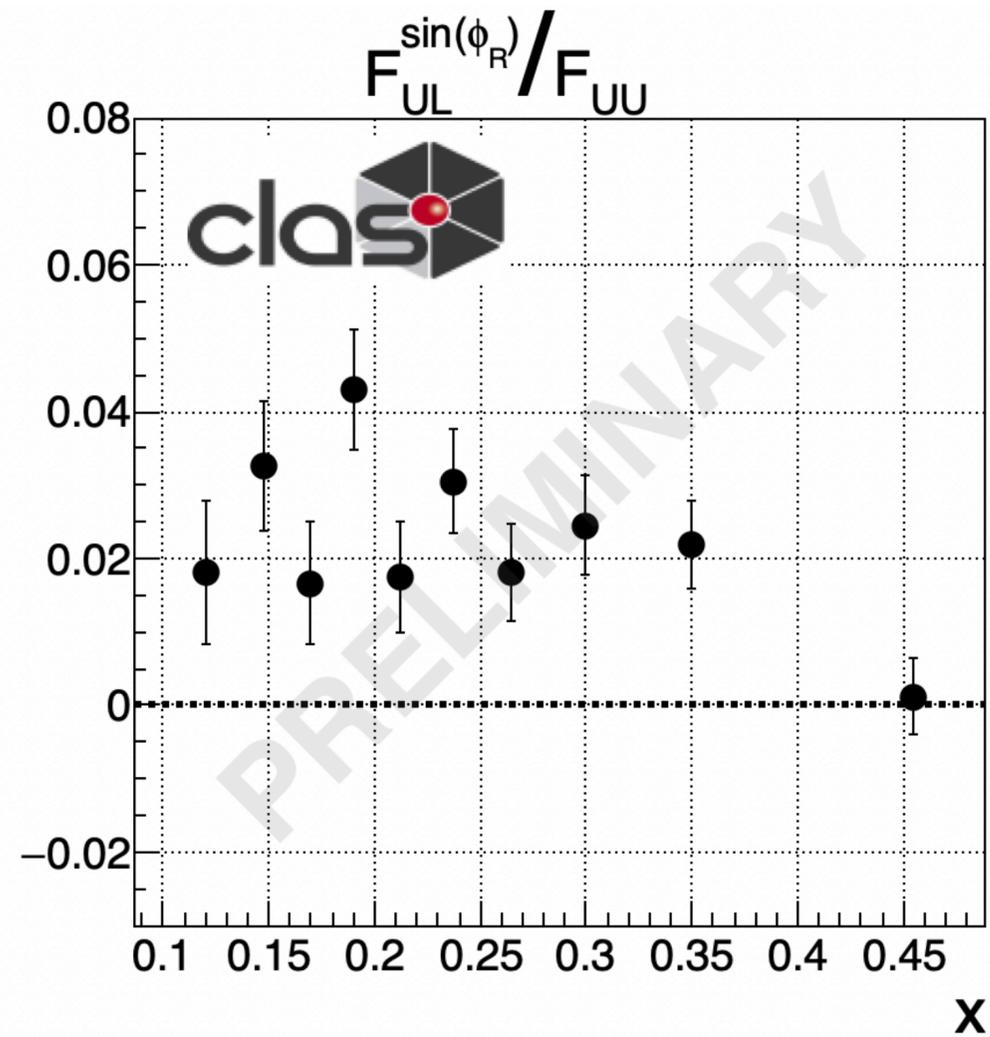
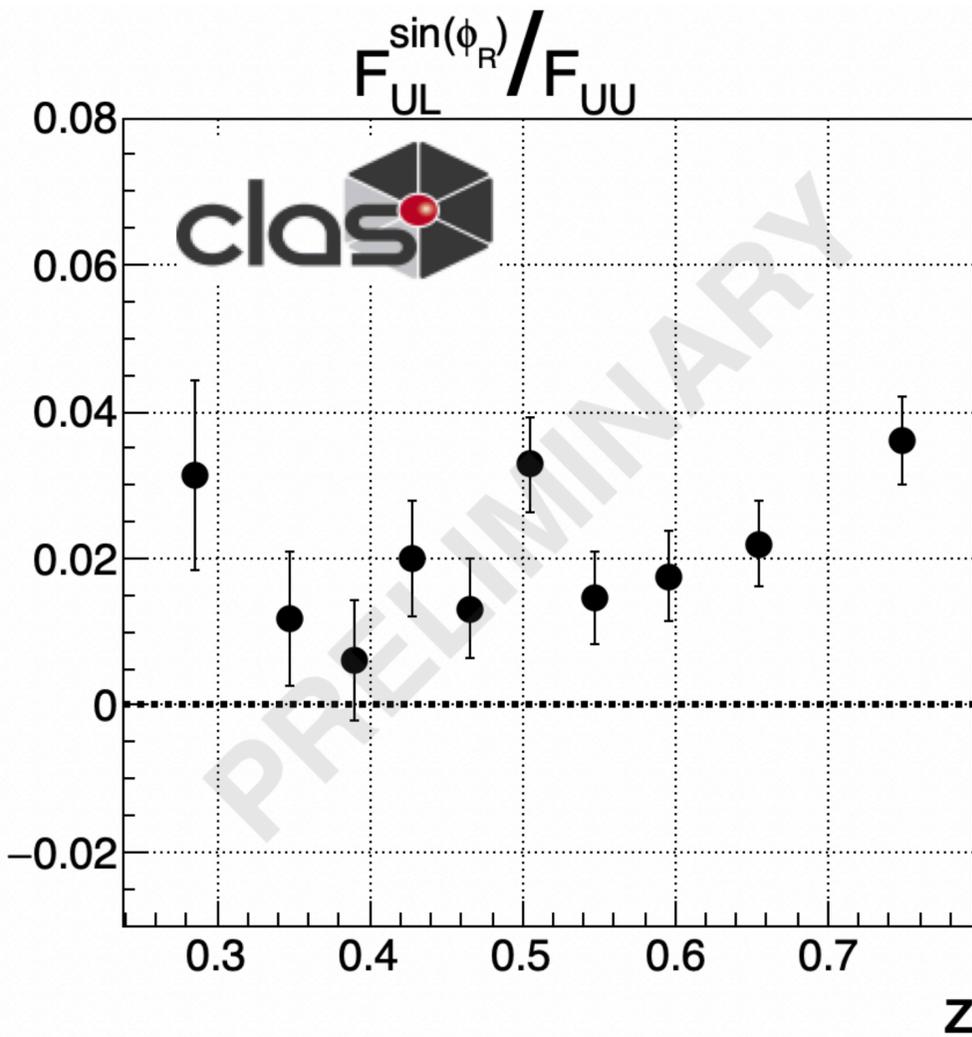
- Flat wrt x
(Consistent with published results from RG-A data)

Sensitive to eH_1^\perp



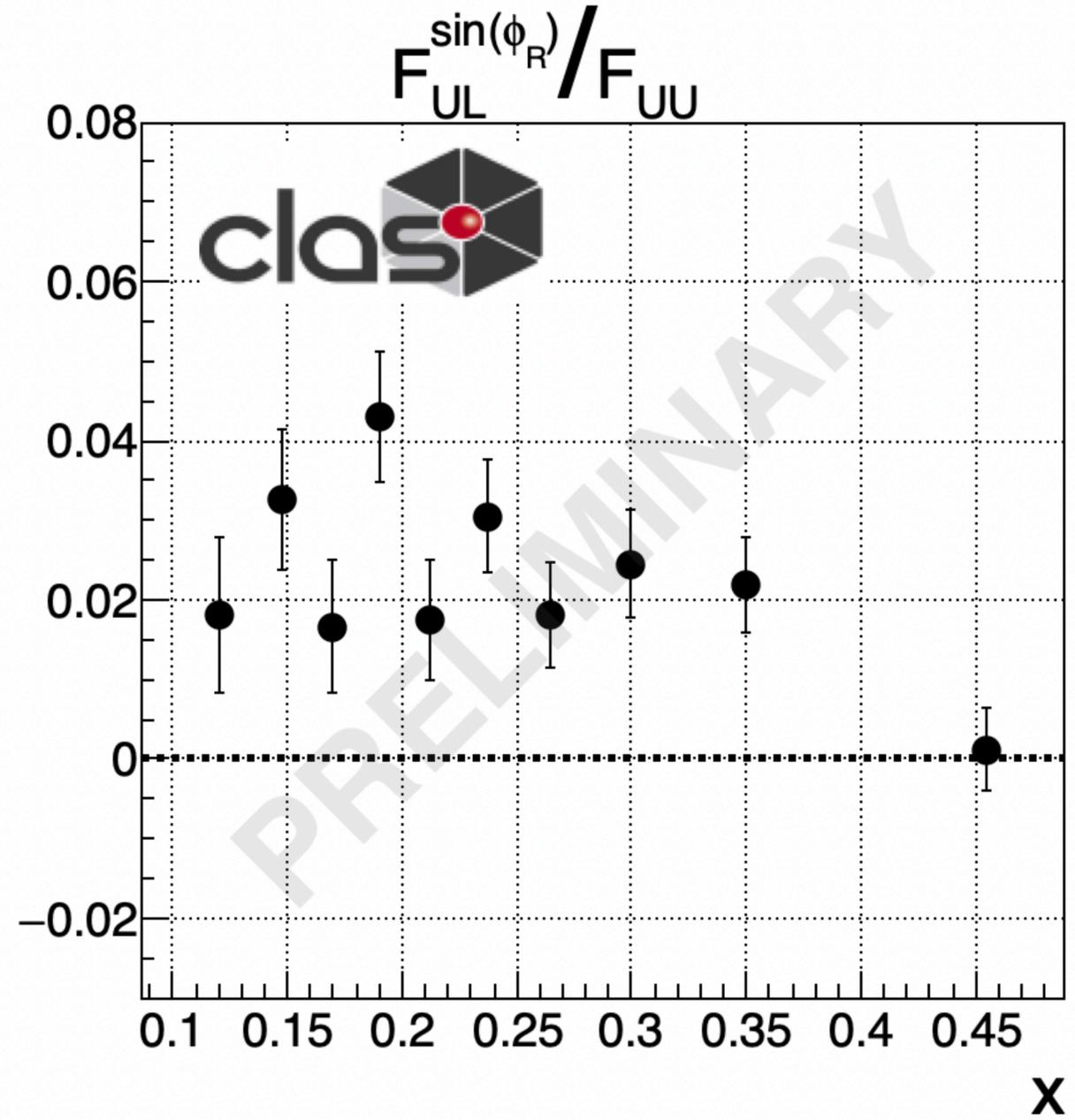
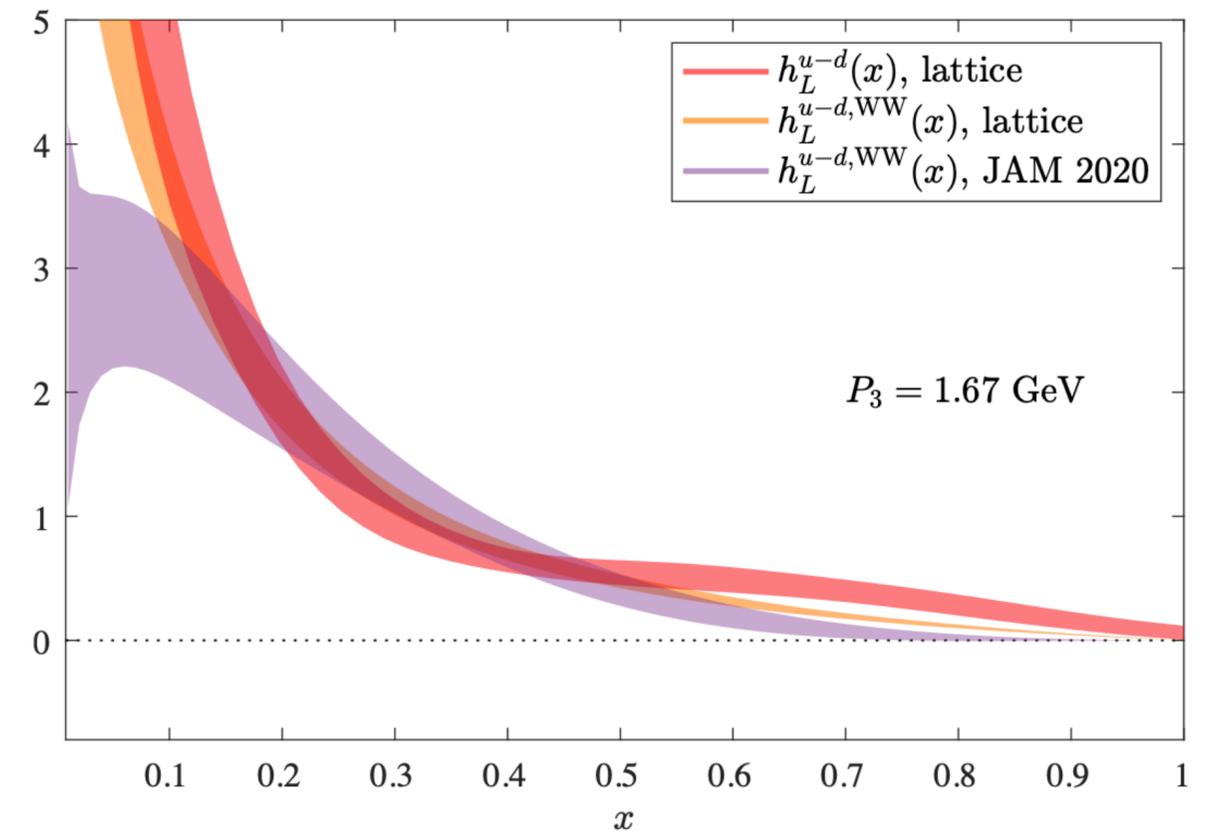
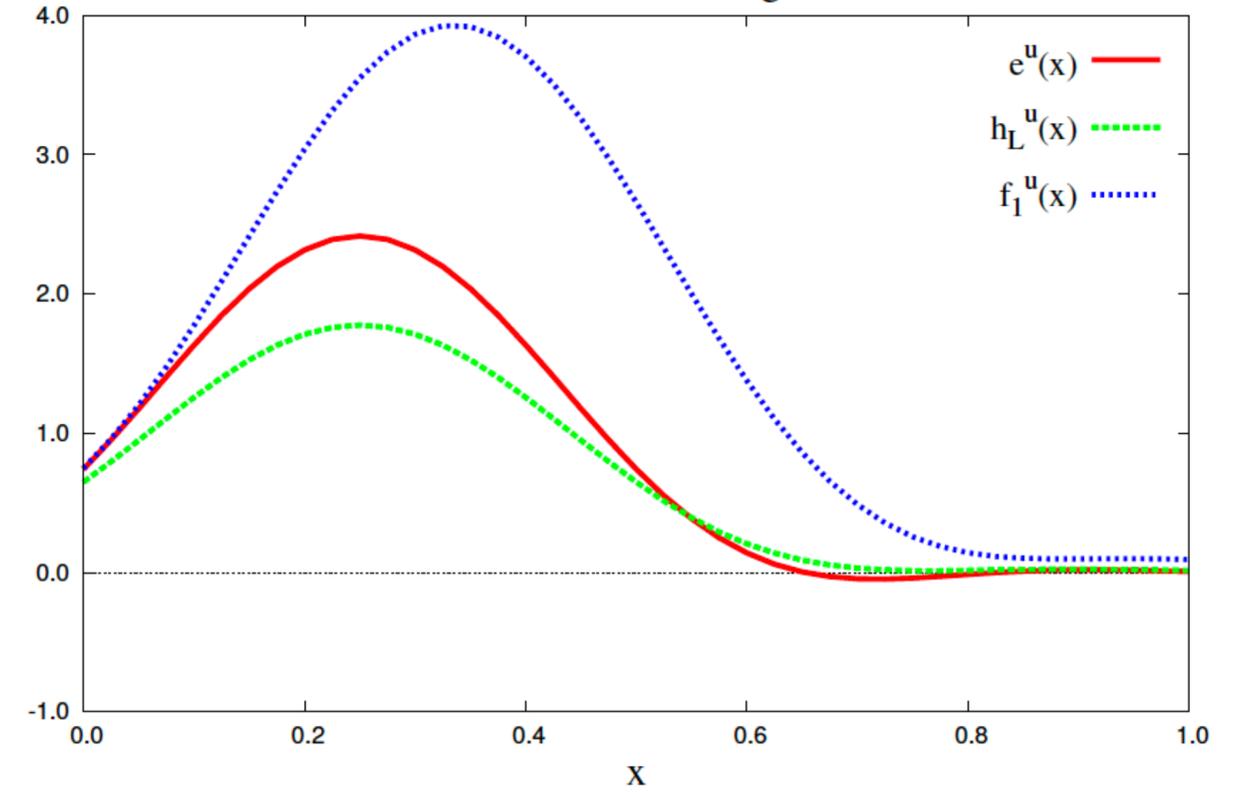
CLAS12
PRL 126, 152501 (2021)

$$F_{UL}^{\sin(\phi_R)} / F_{UU}$$



- Non-zero and positive amplitude
- Sensitive to $h_L H_1^\perp$
- No published results on twist-3 PDF h_L

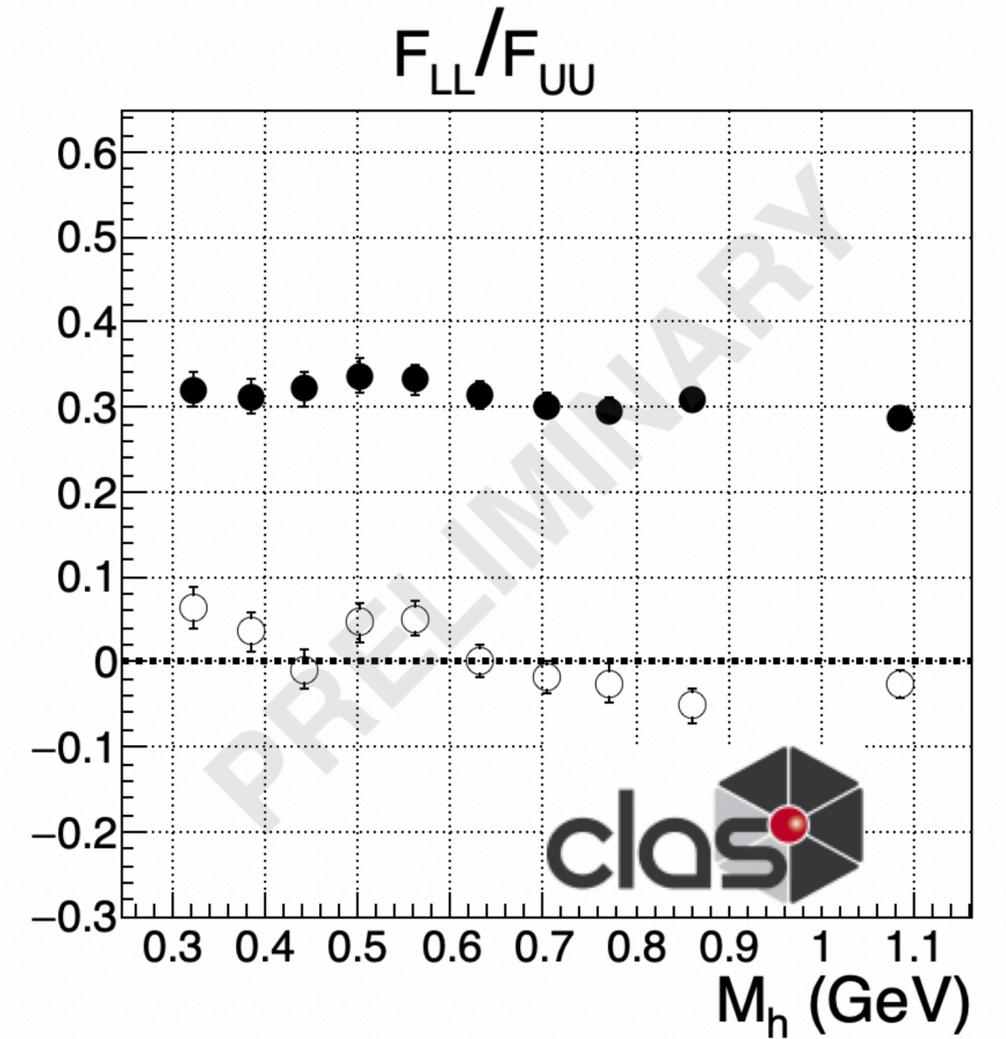
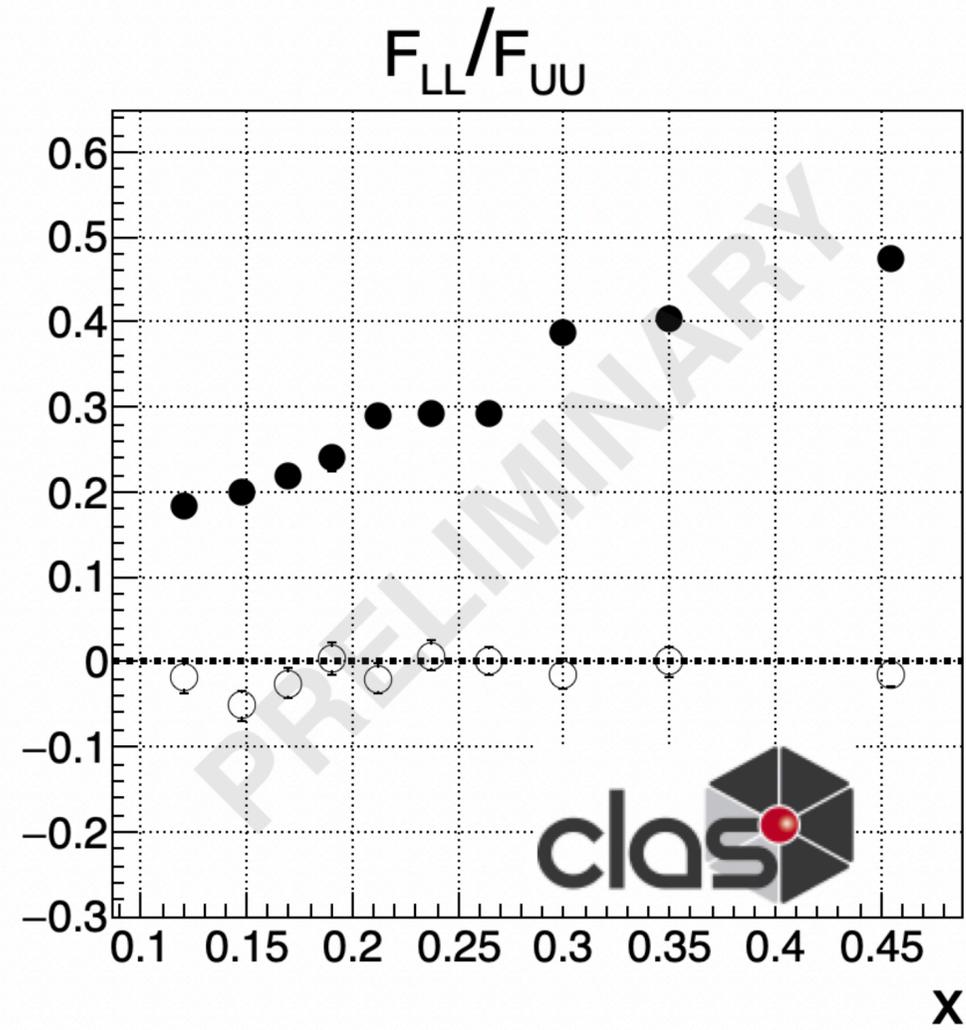
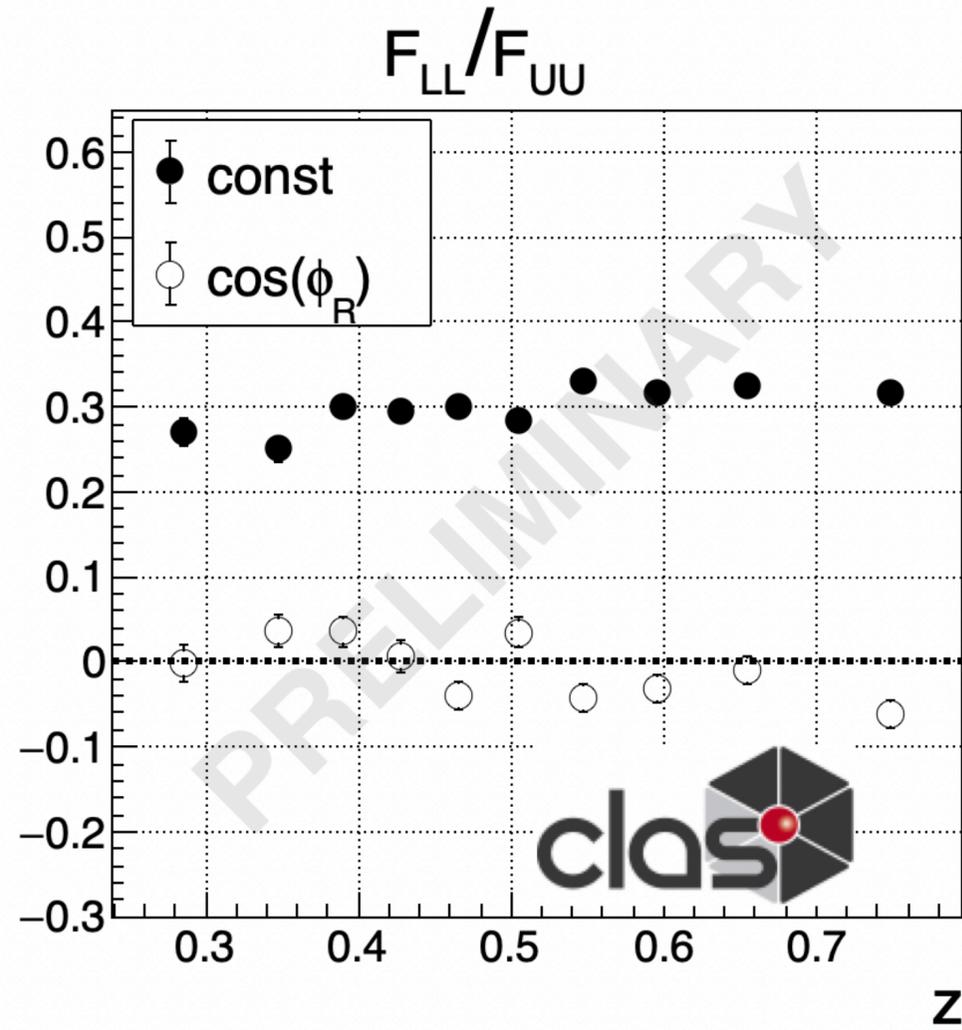
Twist-3 PDFs in the Bag Model



CLAS12 results will allow extraction of h_L from data

arXiv:2111.01056

$$F_{LL}/F_{UU}$$



$$\frac{F_{LL}^{const}}{F_{LL}^{\cos(\phi_R)}} \text{ Sensitive to } \frac{g_{1L}D_1}{g_{1L}\tilde{D}}$$

- Suggests \tilde{D} is roughly one order of magnitude smaller than D_1

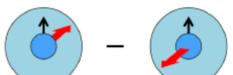
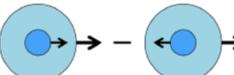
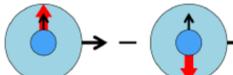
Outlook

- Dilution factor values calculated for $e^-p \rightarrow e^-\pi^+\pi^-X$ channel
- Results of beam, target and double spin asymmetries obtained from longitudinally polarized NH_3 target data
- Analysis note mostly complete (except for systematics)
- Systematic studies will be done with new MC (correct vertex positions for RG-C)

Backup - transverse momentum dependent PDFs

- Depends on x and transverse momentum p_T^2 of struck quark

- **Twist-t** means contribution to cross section suppressed by $\left(\frac{M}{Q}\right)^{t-2}$

		Quark		
		U	L	T
Nucleon	U	Unpolarized $f_1 =$ 		Boer-Mulders $h_1^\perp =$ 
	L		Helicity $g_{1L} =$ 	Worm-gear 1 $h_{1L}^\perp =$ 
	T	Sivers $f_{1T}^\perp =$ 	Worm-gear 2 $g_{1T}^\perp =$ 	Transversity $h_1 =$  Pretzelosity $h_{1T}^\perp =$ 

Twist-2

[4]

U - unpolarized
L - longitudinally polarized
T - transversely polarized

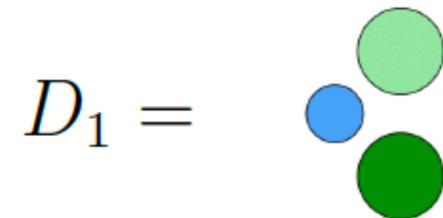
		Quark		
		U	L	T
Nucleon	U	f^\perp	g^\perp	h, e
	L	f_L^\perp	g_L^\perp	h_L, e_L
	T	f_T, f_T^\perp	g_T, g_T^\perp	$h_T, e_T, h_T^\perp, e_T^\perp$

Twist-3
sensitive to quark-gluon correlations

Dihadron fragmentation functions

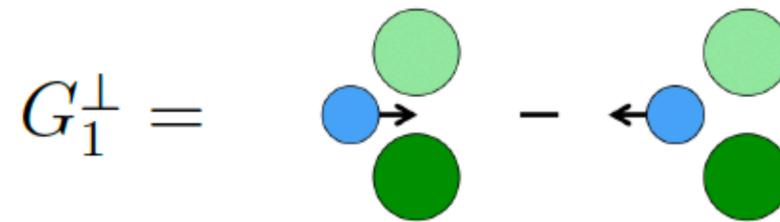
- Depends on z , M_h and transverse momentum k_T^2 of fragmenting quark
($k = p + q$)
- Probability difference of quark with a certain polarization to fragment into unpolarized hadrons

Unpolarized



\tilde{D}^\perp

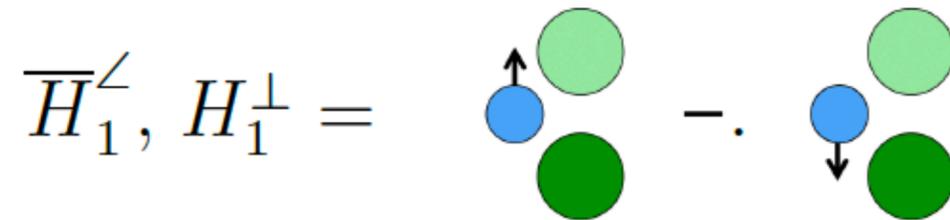
Longitudinally polarized



\tilde{E}

\tilde{G}^\perp

Transversely polarized



[4]

\tilde{H}

Twist-2

Twist-3

References

- [1] T. Hayward, Ph.D. thesis - “Dihadron beam spin asymmetries on an unpolarized hydrogen target with CLAS12” (2021)
- [2] S. Gliske, A. Bacchetta, and M. Radici, Phys. Rev. D90, 114027 (2014)
- [3] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, JHEP 02, 093 (2007)
- [4] S. Sirtl, Ph.D. thesis - “Azimuthal asymmetries in semi-inclusive deep-inelastic hadron muoproduction on longitudinally polarized protons” (2016)