UNIVERSITÀ
DI TORINO

INFN
Istituto Nazionale di Fisica Nucleare

Andrea Signori

University of Turin and INFN

The Sivers effect in atomic Compton scattering

JLab Positron Working Group Meeting

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Motivation

Can we devise analogues of hadron structure observables that can be studied in a low-energy framework?

YES!

protons → atoms

QCD → QED bound states

GeV → MeV physics

Atomic Compton scattering

$\gamma A \to \gamma e^-$

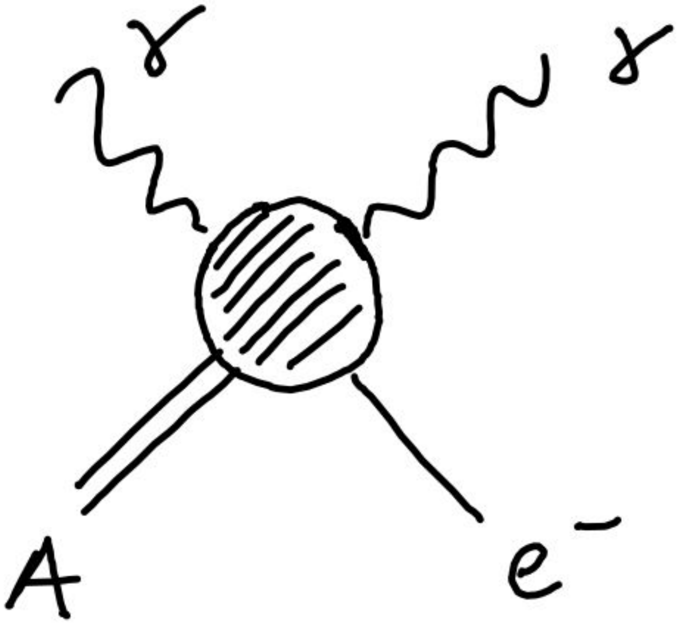
Unpolarized TMD PDF e/A

Sivers TMD PDF e/A

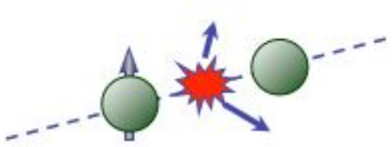
$$\boxed{f_{1T}^{[U]\perp}(x, k_T^2)}\,\epsilon_T^{\alpha\beta} S_{T\,\alpha} k_{T\,\beta}$$

Transverse spin of target

Transverse momentum of “parton”

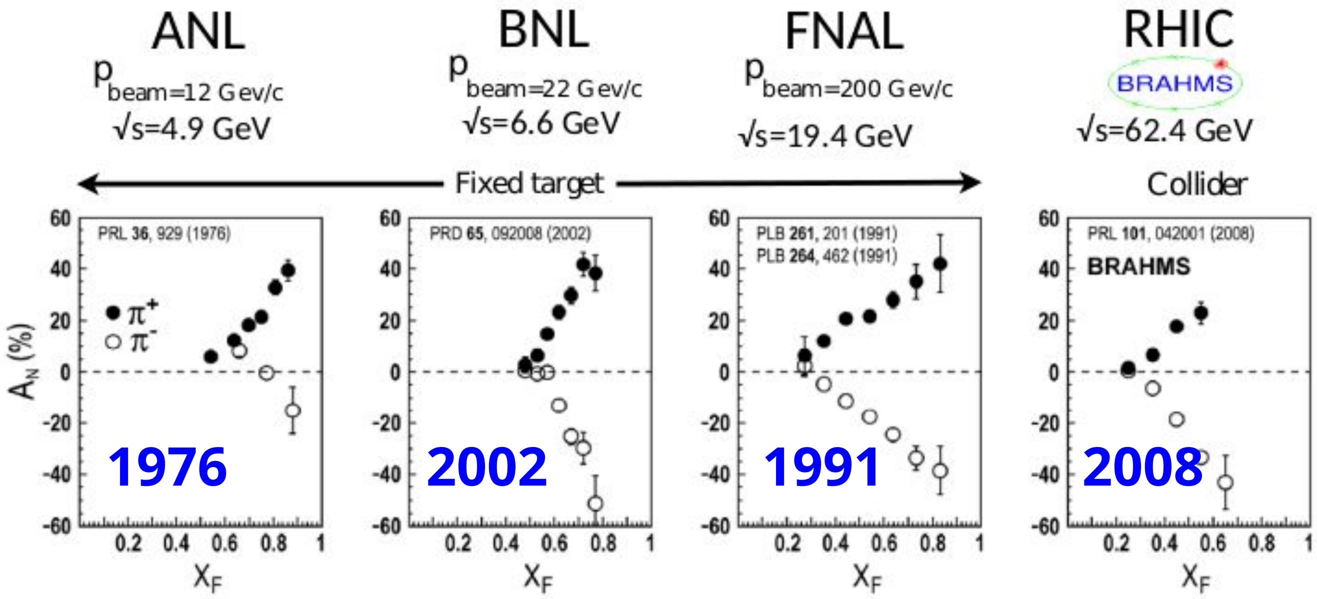


Single-spin asymmetries



$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

$$x_F = 2p_l/\sqrt{s}$$



$$A_N^{\text{pQCD}} \sim \alpha_s\, m_q/\sqrt{s} \ll A_N^{\text{exp}}$$

Need different mechanism: non-perturbative → Sivers effect

See also https://inspirehep.net/literature/1410100 (review for asymmetries in pp collisions)

Outline of the content

1. **The Sivers effect and Wilson lines**
2. **Atomic Compton scattering**
3. **Atomic bound states**
4. **Results**

Contributors


Christine Aidala


Dylan Manna


Me

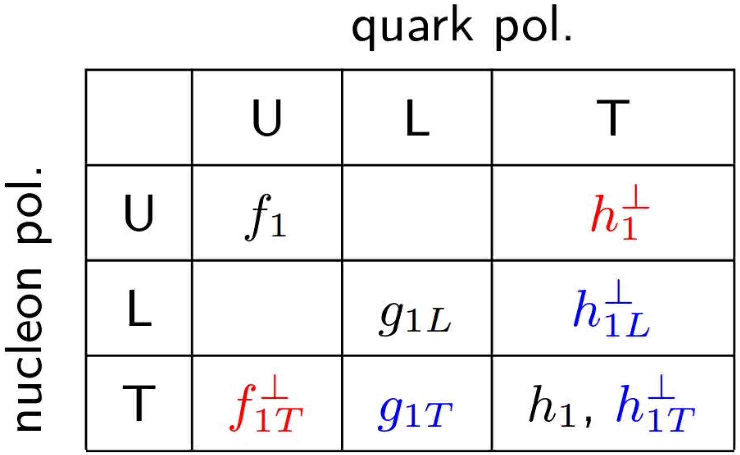

Alessandro Bacchetta

Andrea Minotti
(BSc Turin)

Andrea Perron Cabus
(BSc, Turin)

1. The Sivers effect and Wilson lines

Quark TMD PDFs in a proton



quark pol.

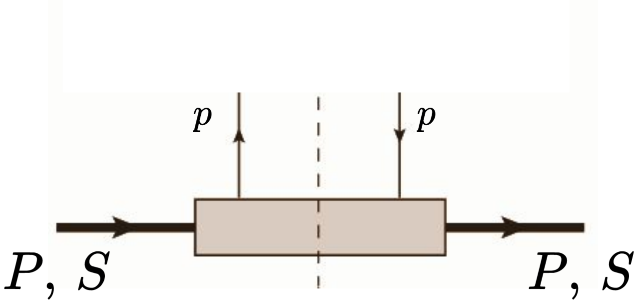
nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

The **symmetries of QCD** play a **crucial role** in this classification

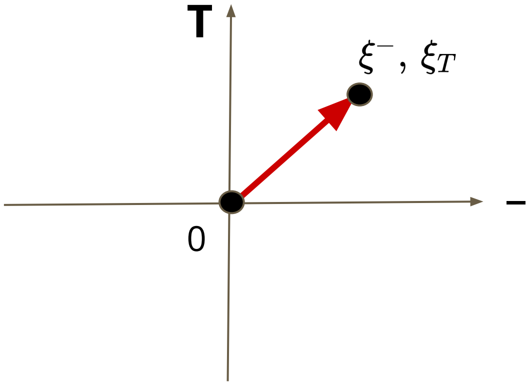
Quark TMD correlator

$$\Phi_{ij}(p,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p\,\cdot\,\xi}\, \langle PS \big| \overline{\psi}_j(0)\,\psi_i(\xi) \big| PS \rangle$$



$$\Phi_{ij}(x,\,\mathbf{p}_T,S) = \int dp^+\,dp^-\,\delta\big(p^+ - xP^+\big)\Phi(p,P,S) =$$

$$= \int \frac{d\xi^-\,d^2\xi_T}{2\pi}\, e^{i\,p\,\cdot\xi}\, \langle PS \big| \overline{\psi}_j(0)\,\psi_i(\xi)\, \big| PS \rangle_{\,\xi^+=0}$$



Implications of time reversal symmetry

$$\Phi^*_{ij}(k,P,S) = (-i\gamma_5 C)_{ik}\,\Phi_{kl}(\tilde{k},\tilde{P},\tilde{S})\,(-i\gamma_5 C)_{lj}$$

$$f_1(x,k_T^2) = f_1(x,k_T^2)$$

Trivial …

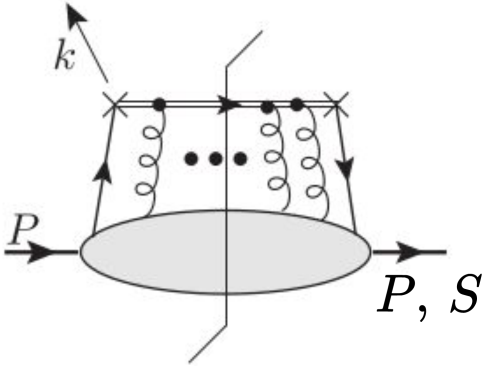
$$f_{1T}^{\perp}(x,k_T^2) = -f_{1T}^{\perp}(x,k_T^2)$$

There is no Sivers function!

Where is the problem ?

Gauge invariant quark correlator

See Collins book



$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,k\cdot\xi}\, \langle PS| \boxed{\overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)} |PS\rangle$$

GAUGE INVARIANT!

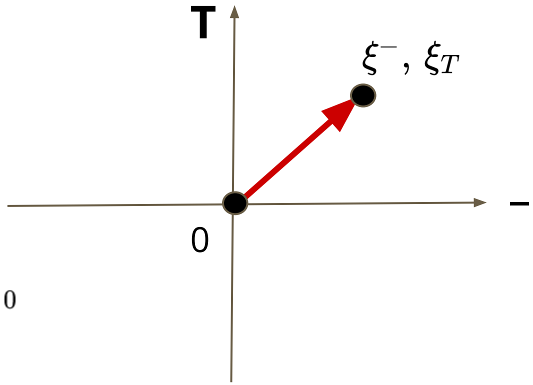
$$U(0,\xi) = \mathcal{P}\, e^{-ig\int_0^\xi ds^\mu A^a_\mu(s) T^a}$$

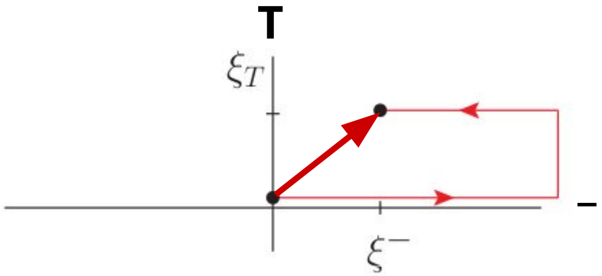
→ Eventually the correlator and the (TMD) PDFs **depend on the gauge link and its path** in spacetime

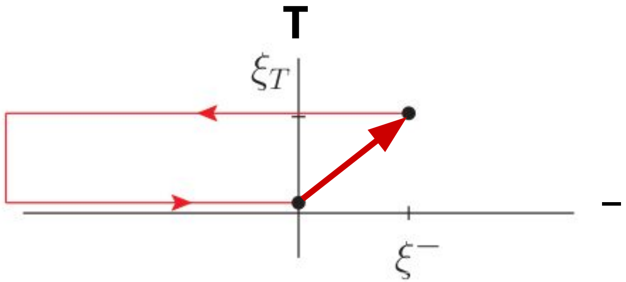
Gauge links for TMD PDFs

$$\Phi_{ij}^{[U]}(x,\, \mathbf{p}_T, S) \;=\; \int dp^+\, dp^-\, \delta\big(p^+ \;-xP^+\big)\,\Phi^{[U]}(p, P, S) =$$

$$= \int \frac{d\xi^-\, d^2\xi_T}{2\pi}\; e^{i\,p\cdot\xi}\, \langle PS \big|\, \overline{\psi}_j(0)\, U(0,\xi)\, \psi_i(\xi)\,\big| PS\rangle_{\,\xi^+=0}$$







$U^{[+]}$ Future pointing (SIDIS)

$U^{[-]}$ Past pointing (Drell-Yan)

Implication of time reversal symmetry (again)

$$\Phi^{[\pm]*}(k;P,S) = i\gamma^1\gamma^3\Phi^{[\mp]}(\tilde{k};\tilde{P},\tilde{S})i\gamma^1\gamma^3$$

$$f_1^{[+]}(x,k_T^2) = f_1^{[-]}(x,k_T^2)$$

Time-reversal even

$$\boxed{f_{1T}^{\perp[+]}(x,k_T^2) = -f_{1T}^{\perp[-]}(x,k_T^2)}$$

Time-reversal odd

Sign-change relation for the Sivers function:
a striking consequence of **the symmetries** of QCD

Do we really need QCD in the proof ?

$$U(0,\xi) = \mathcal{P}\, e^{-ig\int_0^{\xi} ds^{\mu} A^a_{\mu}(s) T^a}$$

In this proof, **nothing would change** if we considered **U(1) symmetry** instead of SU(3) !

We'd consider QED bound states and Wilson lines: a **QED-induced Sivers effect**

!

Why considering a QED-induced Sivers effect ?

PROS:

- QED is Abelian and perturbative at low energies: the PDFs for QED bound states are **calculable**
- QED measurements are **extremely precise**: up to 11 significant figures for alpha
- The sign change relation, a feature of the geometry of gauge theories, could be tested **with low energy experiments**

CONS:

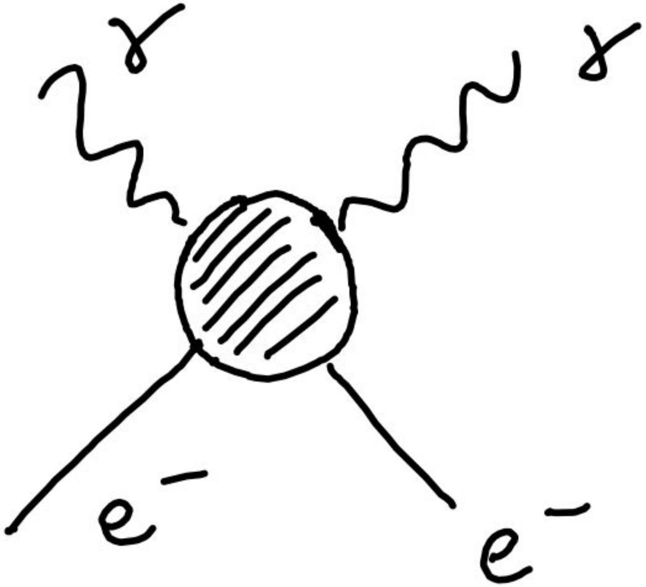
not a strong interaction … a QED structure effect might be **extremely weak** and difficult to measure

…

2. Atomic Compton scattering

Compton scattering

$\gamma e^- \to \gamma e^-$

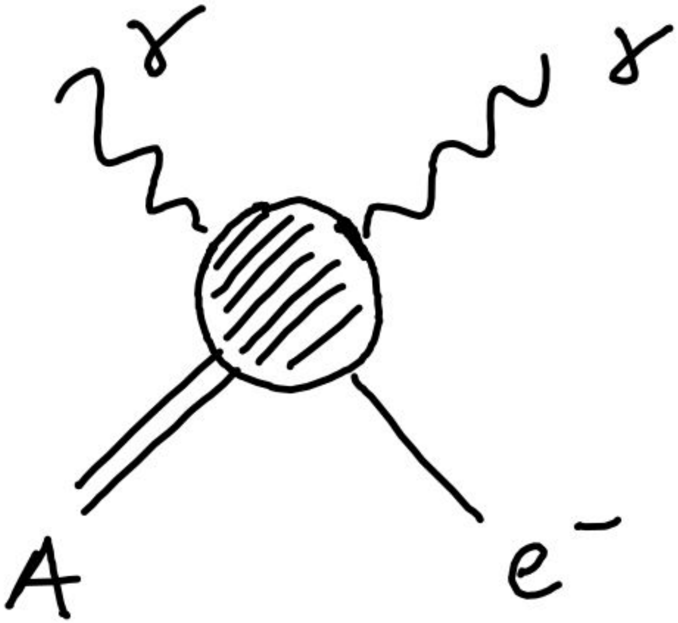


Atomic Compton scattering

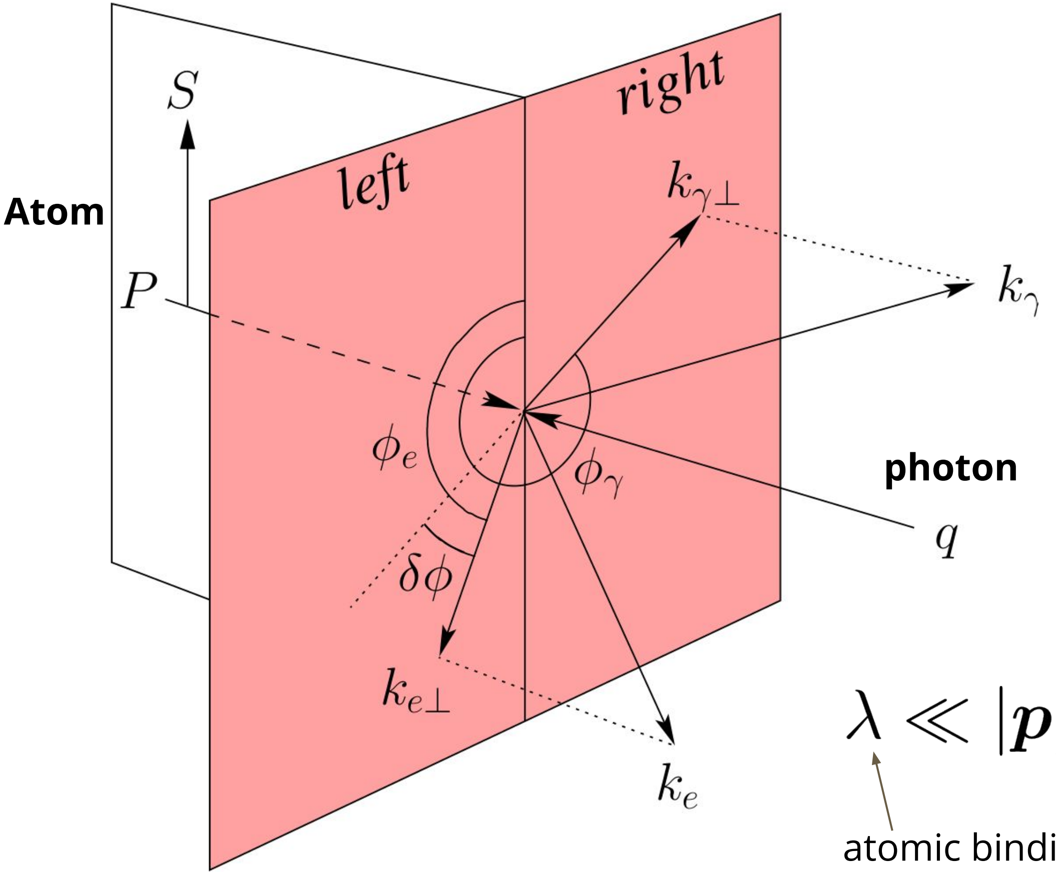
$$\gamma A \to \gamma e^-$$

Kinematics?

Cross section?



Atomic Compton scattering: kinematics



$$\boldsymbol{p}_\perp = \boldsymbol{k}_{\gamma\perp} + \boldsymbol{k}_{e\perp}$$

Transverse momentum imbalance

keV electron momentum

$$\lambda \ll |\boldsymbol{p}_\perp| \ll |\boldsymbol{k}_{\gamma\perp} - \boldsymbol{k}_{e\perp}| \approx \sqrt{s}$$

atomic binding (**eV**)

MeV photon beam

Atomic Compton scattering: cross section

Unpolarized TMD e/A distribution

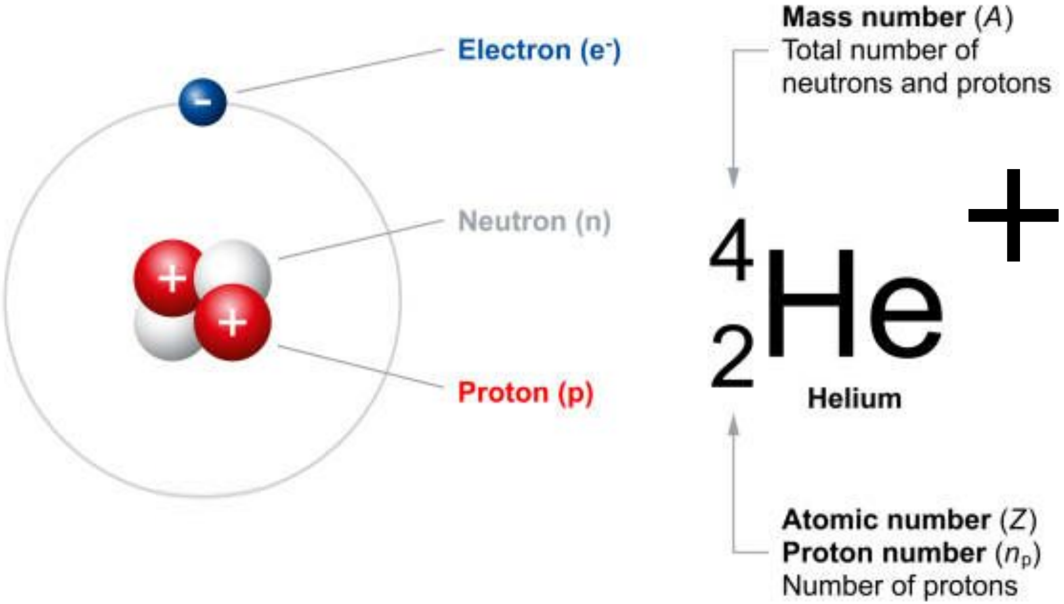
$$\frac{d\overline{\sigma}_{\mathrm{ACS}}}{dx_\perp\, d\eta_\gamma\, d\eta_e\, d\phi_\gamma\, d\phi_e} = \frac{\alpha_e^2}{64}\,\frac{x_\perp^2}{4x_e}\,\overline{f}(m_e,\hat{s})\left\{\boxed{f_1^{e/A}(x_e,p_\perp^2)} + \frac{|\boldsymbol{k}_{\gamma\perp}|\,|\boldsymbol{S}_\perp|}{M}\,\sin(\delta\phi)\,\cos(\phi_\gamma)\,\boxed{f_{1T}^{\perp\,e/A}(x_e,p_\perp^2)}\right\}$$

“free” Compton
scattering

Sivers TMD e/A distribution

3. Atomic bound states

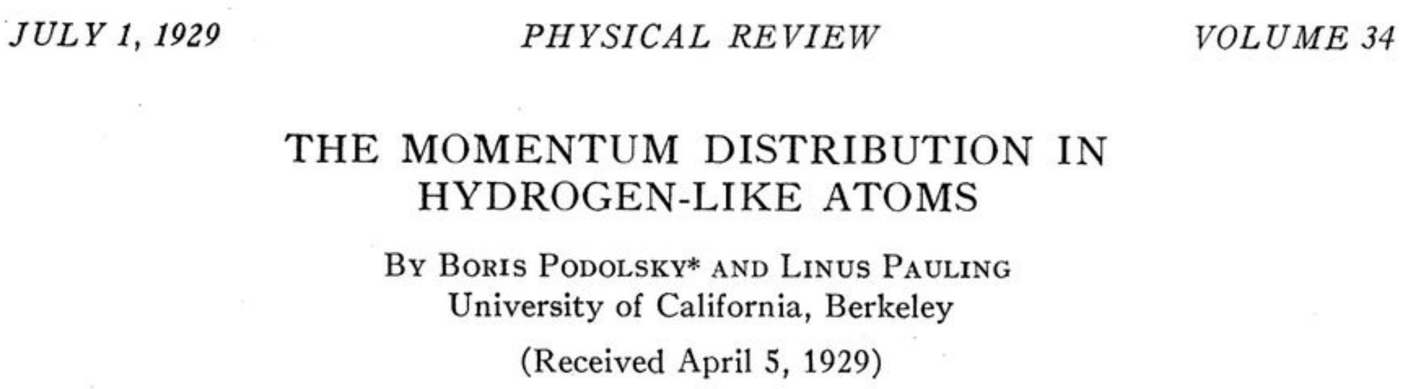
Ionized Helium-4



Hydrogen-like system:
Scalar nucleus,
only one “parton” (the electron)

Tool:
Quantum mechanics
(Schrodinger equation)

Electron wavefunctions



JULY 1, 1929 PHYSICAL REVIEW VOLUME 34

THE MOMENTUM DISTRIBUTION IN
HYDROGEN-LIKE ATOMS

By Boris Podolsky* and Linus Pauling
University of California, Berkeley

(Received April 5, 1929)

$$\psi_{nlm}(|\boldsymbol{p}|,\theta,\phi) = \left\{\frac{1}{(2\pi)^{\frac{1}{2}}} e^{+im\phi}\right\}\left\{\left(\frac{(2l+1)(l-m)!}{2(l+m)!}\right)^{\frac{1}{2}} P_l^{\,m}(\cos\theta)\right\}$$

$$\times\left\{-\frac{(-i)^l\,\pi 2^{2l+4} l!}{(\gamma h)^{\frac{3}{2}}}\left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}}\frac{\zeta^l}{(\zeta^2+1)^{l+2}} C^{l+1}_{n-l-1}\left(\frac{\zeta^2-1}{\zeta^2+1}\right)\right\}$$

Eigenfunctions of L^2, Lz, S^2, Sz

$$\Psi_{n\ell m_\ell m_s}(|\boldsymbol{p}|,\theta,\phi) = \psi_{n\ell m_\ell}(|\boldsymbol{p}|,\theta,\phi)\,\chi_{m_s}$$

Separable solutions

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

Total spin of the atom

Electron OAM

Electron spin

We want to mimic the case of the proton: $j = 1/2 \quad , \quad m_j = \pm 1/2$

Eigenfunctions of J², Jz

$$m_j = m_\ell + m_s$$

$$\Psi^{z\uparrow} = -\frac{1}{\sqrt{3}}\,\psi_{210}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\uparrow} + \sqrt{\frac{2}{3}}\,\psi_{211}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\downarrow}\,,$$

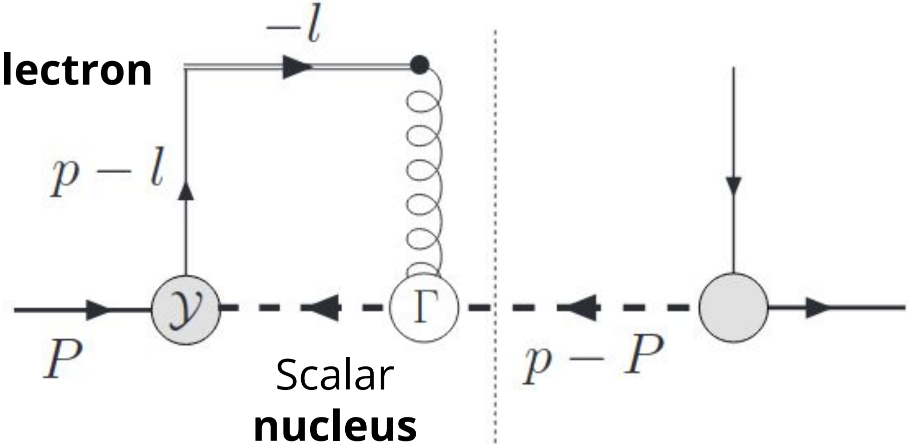
Eigenfunctions of J^2, Jz

$$m_j = m_\ell + m_s$$

$$\begin{aligned}
\Psi^{z\uparrow} &= -\frac{1}{\sqrt{3}}\,\psi_{210}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\uparrow} + \sqrt{\frac{2}{3}}\,\psi_{211}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\downarrow}\,, \\
\Psi^{z\downarrow} &= -\sqrt{\frac{2}{3}}\,\psi_{21-1}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\uparrow} + \frac{1}{\sqrt{3}}\,\psi_{210}(|\boldsymbol{p}|,\theta,\phi)\,\chi_z^{\downarrow}\,.
\end{aligned}$$

Then one needs to **rotate the polarization** from the z to the y axis

A spectator model-like approach



Wilson line: **Coulomb** interaction

electron

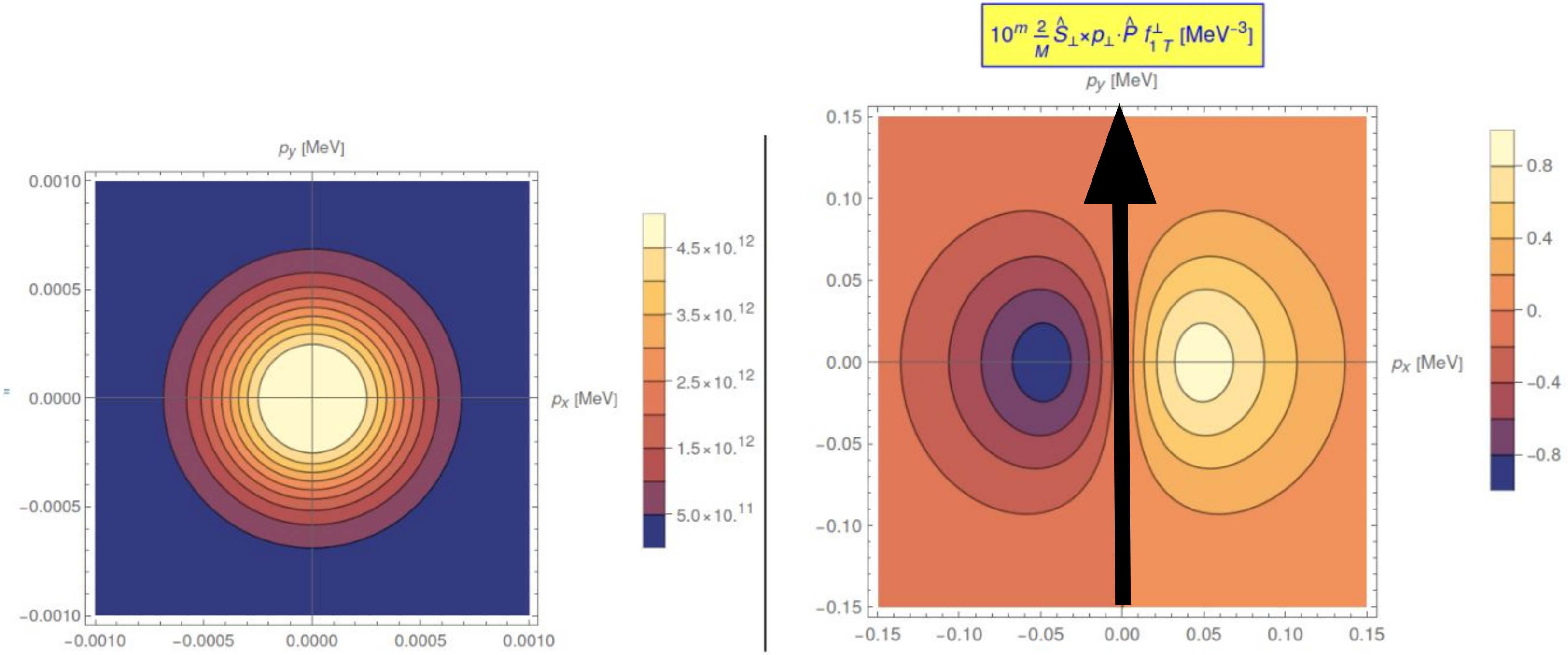
atom

Scalar **nucleus** (spectator)

https://inspirehep.net/literature/789754

4. Results

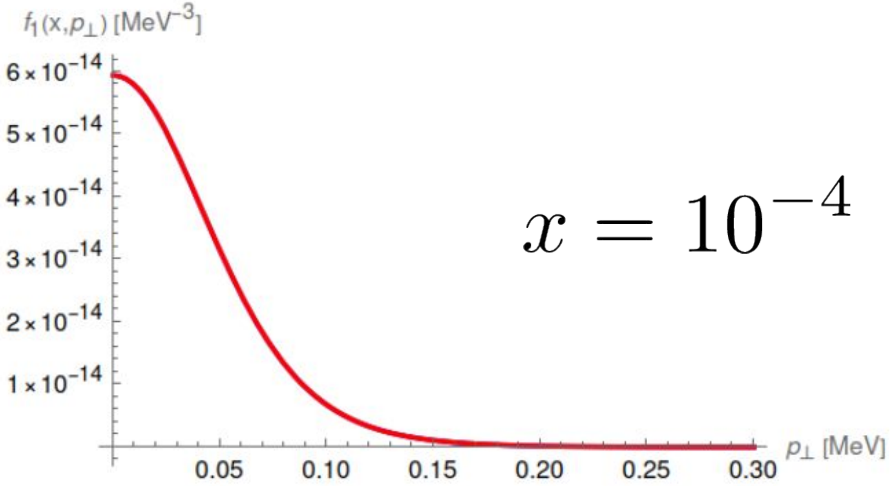
Unpolarized vs Sivers TMDs



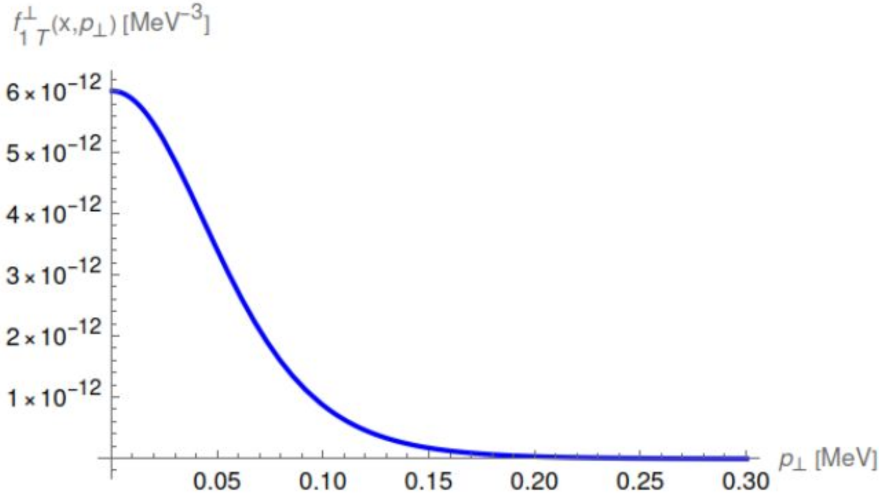
(a) (b)

FIG. 5: Contour 2D plots for the electron distributions in the transverse plane. (a) Contour plot for the unpolarized distribution, $10^m f_1^e(x, \boldsymbol{p}_\perp^2)$. (b) Contour plot for the polarized term in the correlation function, $\frac{2}{M}(\hat{\boldsymbol{S}}_\perp \times \boldsymbol{p}_\perp \cdot \hat{P}) f_{1T}^{\perp e}(x, \boldsymbol{p}_\perp^2)$. We choose the value $x = 10^{-4}$ in both plots. For both distributions, the 2D plot contains a magnification factor 10^m, with $m = 14$ in (a) and $m = 16$ in (b).

Unpolarized vs Sivers TMDs



$$x = 10^{-4}$$



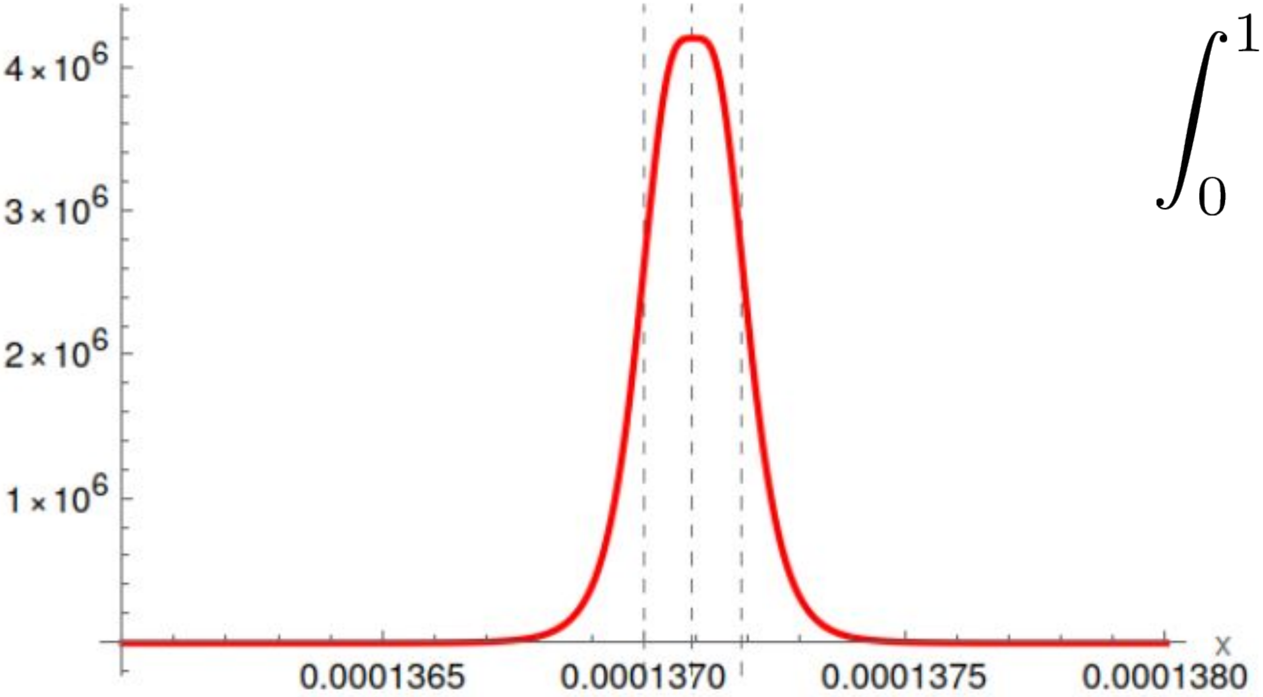
Ratio pol./unpo.: 2 to 4 orders of magnitude depending on the x value

Electron collinear distribution

$f_1^e(x)$ $x_0 \approx m_e/M_{He}$

Number sum rule:
only one parton

$$\int_0^1 dx\, f_1^e(x) = 1$$

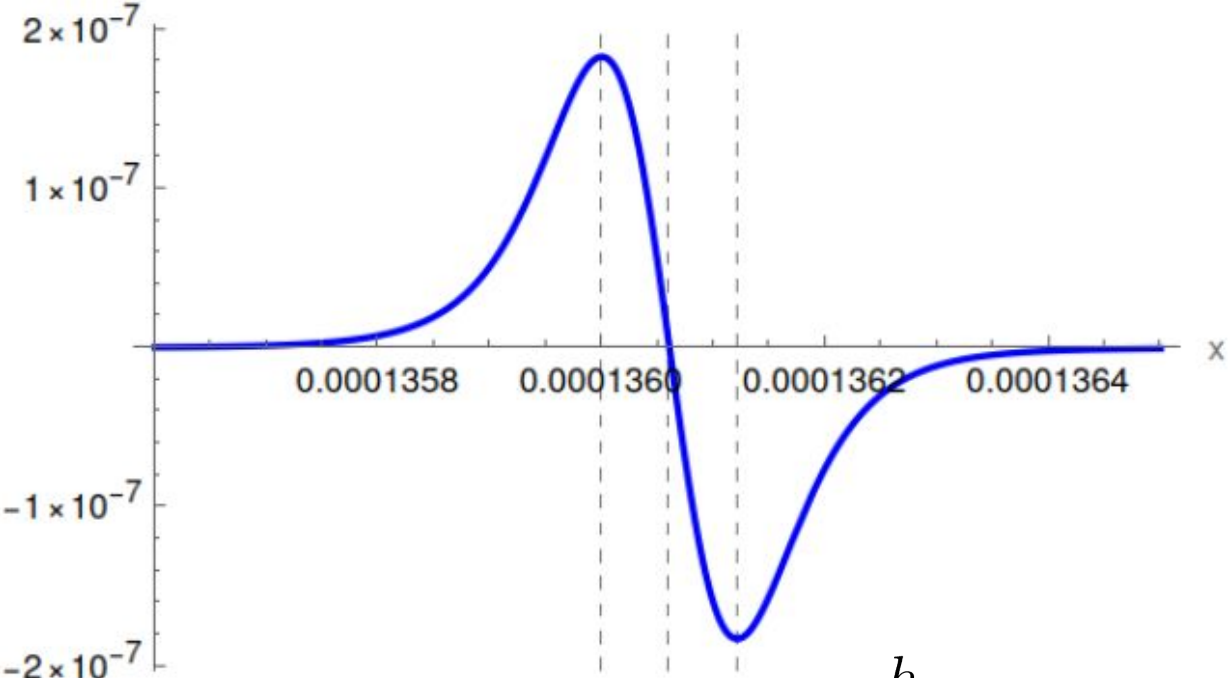


(a)

First moment of the Sivers function

$$f_{1T}^{\perp(1)e}(x) \qquad \bar{x} = m_e/M_{He}$$

Position of the node

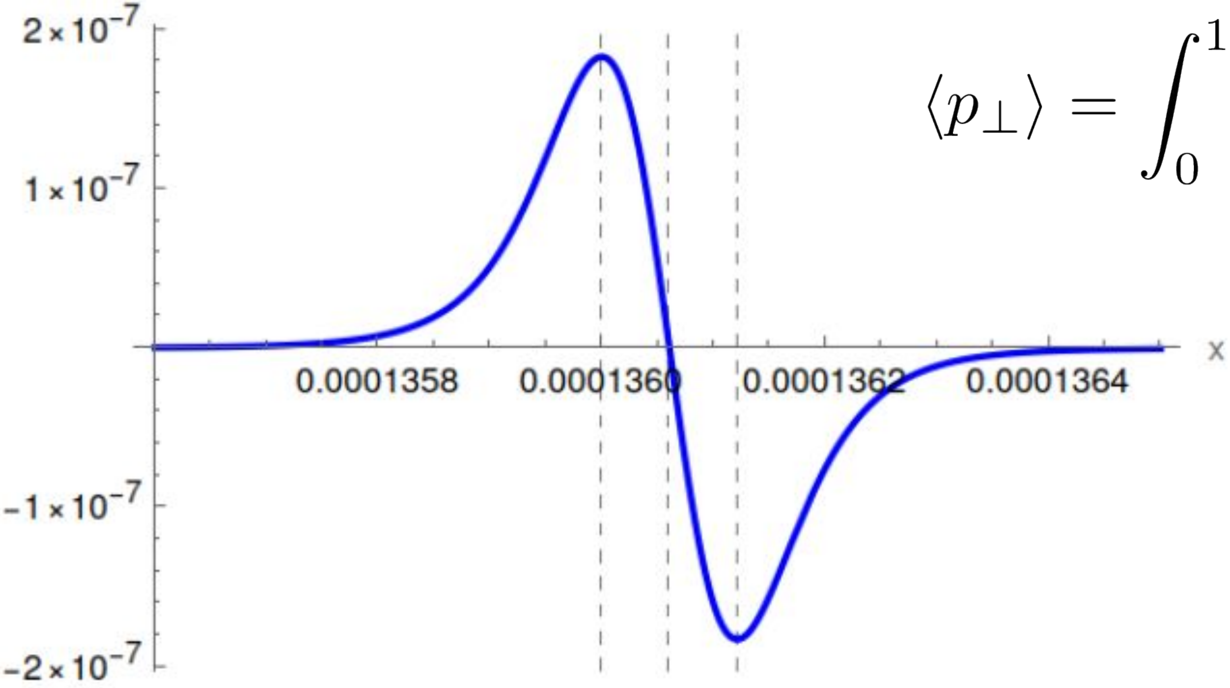


$$x_{1,2} = \bar{x} \pm \frac{h}{2\sqrt{7}a_0\pi M}$$

Position of the peaks

First moment of the Sivers function

$$f_{1T}^{\perp(1)e}(x)$$



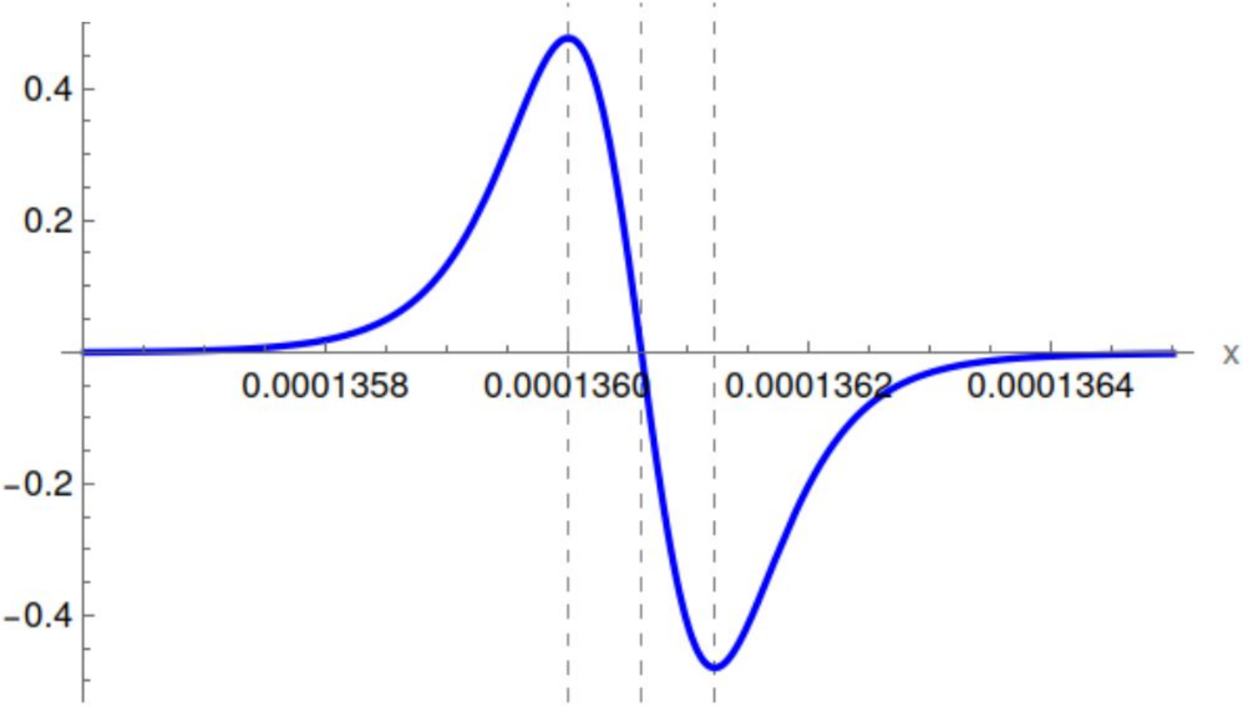
Burkardt sum rule

$$\langle p_\perp \rangle = \int_0^1 f_{1T}^{\perp(1)e}(x) = 0$$

The observable

$M^{\perp(1)}(x) \quad [\text{MeV}]$

$\bar{x} = m_e/M_{He}$ Position of the node



$$x_{1,2} = \bar{x} \pm \frac{h}{2\sqrt{7}a_0 \pi M}$$

Position of the peaks

Conclusions and outlook

We have devised a **QED analogue of the Sivers effect in QCD**

There are certainly ways to improve the treatment from the theoretical point of view.

As the QCD one, it probes the symmetries of the theory: the same conceptual relevance

It might be possible **to test the sign change relation with much simpler experimental devices**

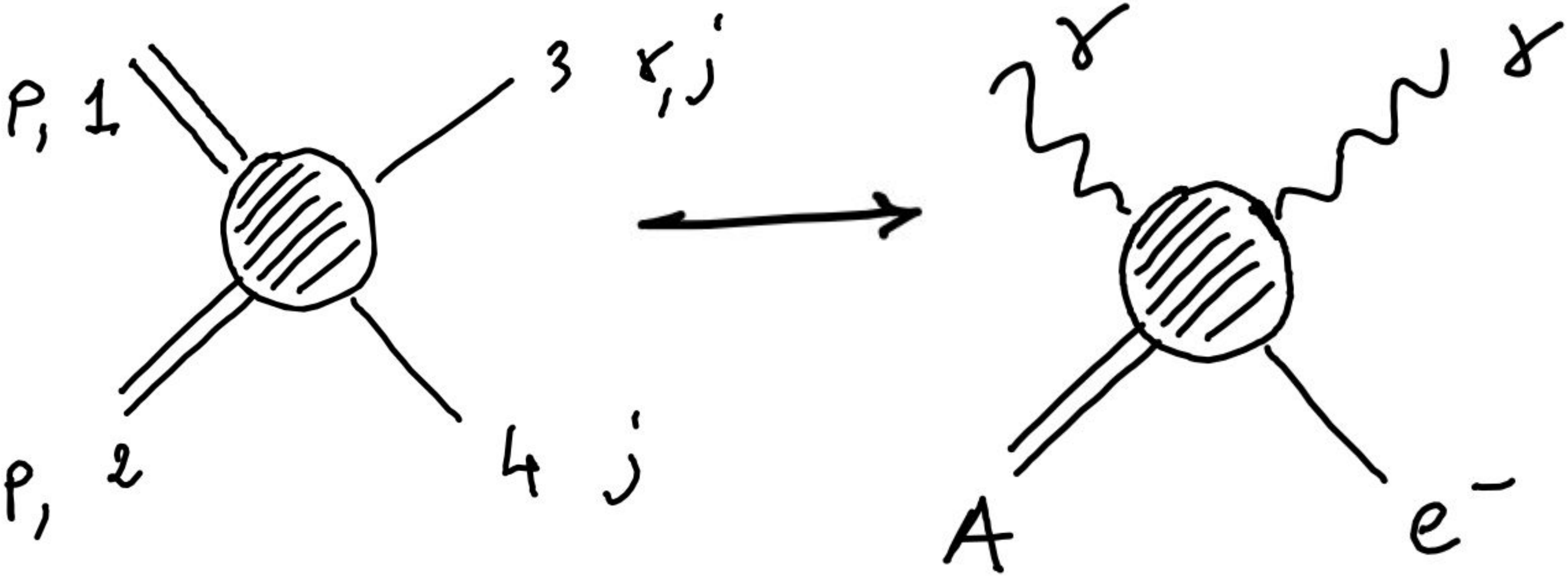
We need atomic physicists to **measure** this

Backup

Similarity with the photon-jet case

$pp \to \gamma j X, jjX$

$\gamma A \to \gamma e^-$



Time reversal symmetry

$a^\mu = \left(a^0,\, \vec{a}\right)\,, \qquad \tilde{a}^\mu = \left(a^0,\, -\vec{a}\right)$ ← let's consider this definition

$$z^\mu \longrightarrow -\tilde{z}^\mu$$

$$P^\mu \longrightarrow \tilde{P}^\mu$$

$$S^\mu \longrightarrow \tilde{S}^\mu$$

$$n_\pm \longrightarrow -n_\mp$$

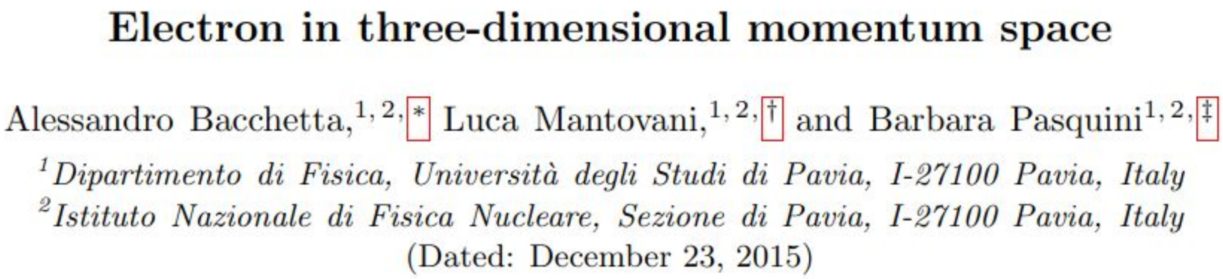
$$\psi(\xi) \longrightarrow \mathscr{T}\,\psi(\xi)\mathscr{T}^\dagger = \Lambda_{\mathscr{T}}\,\psi(-\tilde{\xi})\,, \quad \Lambda_{\mathscr{T}} = -i\gamma_5 C = i\gamma^1\gamma^3$$

$$\gamma^\mu \longrightarrow \mathscr{T}\,\gamma^\mu\,\mathscr{T}^\dagger = \Lambda_{\mathscr{T}}\,\gamma^\mu\,\Lambda^\dagger_{\mathscr{T}} = \gamma^*_\mu$$

The action on the quark field is the one that leaves the QCD lagrangian invariant under time reversal transformation (symmetry)

Existing work in this direction

https://inspirehep.net/literature/1390132



Electron in three-dimensional momentum space

Alessandro Bacchetta,[1, 2, *] Luca Mantovani,[1, 2, †] and Barbara Pasquini[1, 2, ‡]

[1] *Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy*
[2] *Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy*
(Dated: December 23, 2015)

Electron and photon in a dressed electron: Sivers function is zero

Target: a dressed electron, **not a bound state**

From Schrodinger to LF wavefunctions

$$x = \frac{p^+}{P^+} \overset{TRF}{=} \frac{E_e + p_z}{M} =$$

$$= \frac{\sqrt{m_e^2 + |\boldsymbol{p}_e|^2} + p_z}{M} \approx \frac{m_e\sqrt{1+\alpha^2} + p_z}{M} \approx \frac{m_e + p_z}{M}$$

Bohr
momentum:
$p \approx m_e\,\alpha$

https://inspirehep.net/literature/1841546 (Paul Hoyer)

A spectator model-like approach

$$f_1^e(x, \boldsymbol{p}_\perp^2) = \frac{M}{4} \sum_{\lambda=\pm} \left(|\Psi_\lambda^{y\uparrow}(x, \boldsymbol{p}_\perp)|^2 + |\Psi_\lambda^{y\downarrow}(x, \boldsymbol{p}_\perp)|^2 \right)$$

Sum over electron chirality states

Change to light-cone coordinates for a non-relativistic system

$$\frac{2\,(\hat{\boldsymbol{S}}_\perp \times \boldsymbol{p}_\perp) \cdot \hat{\boldsymbol{P}}}{M}\, f_{1T}^\perp(x, \boldsymbol{p}_\perp^2) =$$

$$= \frac{M}{4} \int d\boldsymbol{p}'_\perp\, G(x, \boldsymbol{p}_\perp, \boldsymbol{p}'_\perp) \sum_{\lambda=\pm} \left[\Psi_\lambda^{y\uparrow\,\dagger}(x, \boldsymbol{p}_\perp)\, \Psi_\lambda^{y\uparrow}(x, \boldsymbol{p}'_\perp) - \Psi_\lambda^{y\downarrow\,\dagger}(x, \boldsymbol{p}_\perp)\, \Psi_\lambda^{y\downarrow}(x, \boldsymbol{p}'_\perp) \right] + \mathrm{hc}$$

QED gauge-link contribution

$$\operatorname{Im} G(x, \boldsymbol{p}_\perp, \boldsymbol{p}'_\perp) = -\frac{e^2}{2(2\pi)^2} \frac{1}{(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp)^2}$$

Analytic results: unpolarized

$$f_1^e(x,\boldsymbol{p}_\perp^2) = \frac{M}{3\pi^2}\left(\frac{2\,\hbar}{a_0}\right)^7 \frac{(xM-m_e)^2+\boldsymbol{p}_\perp^2}{\left((xM-m_e)^2+\boldsymbol{p}_\perp^2+\left(\frac{\hbar}{a_0}\right)^2\right)^6}$$

$$f_1^e(x) = \int d^2\boldsymbol{p}_\perp\, f_1^e(x,\boldsymbol{p}_\perp^2) = \frac{64\,M\,a_0\,h^7\,(h^2+20\,a_0\,\pi^2\,(m_e-Mx)^2)}{15\,(h^2+4\,a_0^2\,\pi^2\,(m_e-Mx)^2)^5}$$

Analytic results: Sivers

$$f_{1T}^{\perp e}(x, \boldsymbol{p}_\perp^2) = \frac{\alpha}{12\pi^2}\left(\frac{2\hbar}{a_0}\right)^7 \frac{M(m_e - xM)\left(\boldsymbol{p}_\perp^2 + 2(xM - m_e)^2 + 2\left(\frac{\hbar}{a_0}\right)^2\right)}{\left(\boldsymbol{p}_\perp^2 + (xM - m_e)^2 + \left(\frac{\hbar}{a_0}\right)^2\right)^5 \left((xM - m_e)^2 + \left(\frac{\hbar}{a_0}\right)^2\right)^2}$$

$$f_{1T}^{\perp (1)\, e}(x) = \frac{8 a_0 \alpha h^7 (m_e - xM)}{3M\left(h^2 + 4a_0^2\pi^2(m_e - xM)^2\right)^4}$$

Unpolarized vs Sivers TMDs

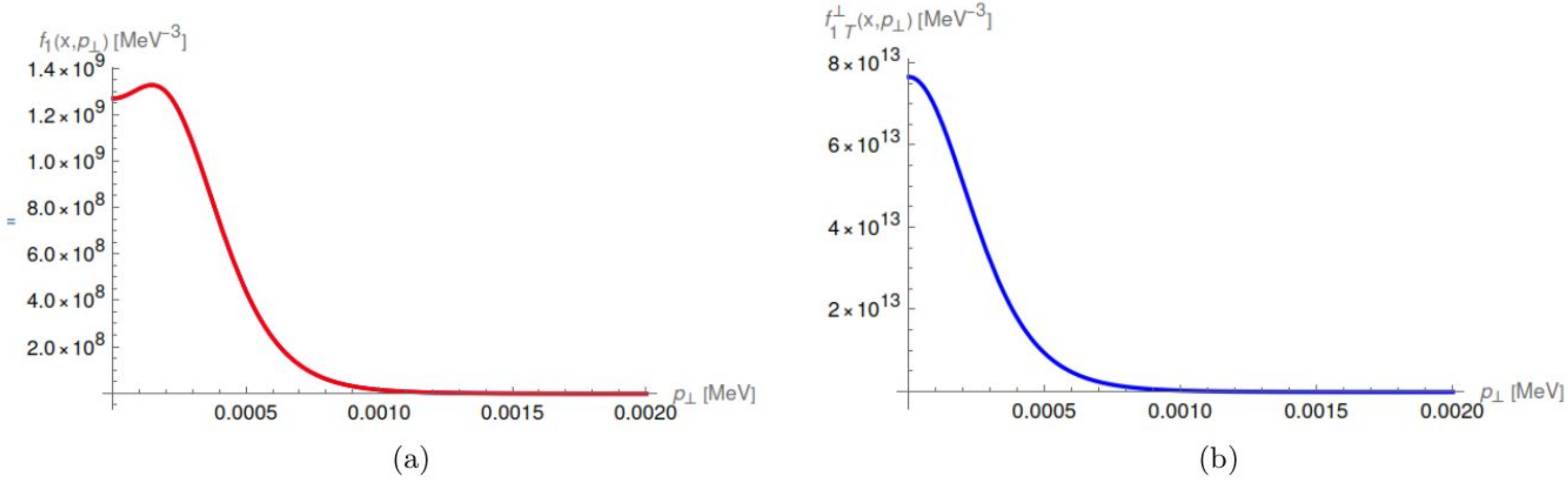
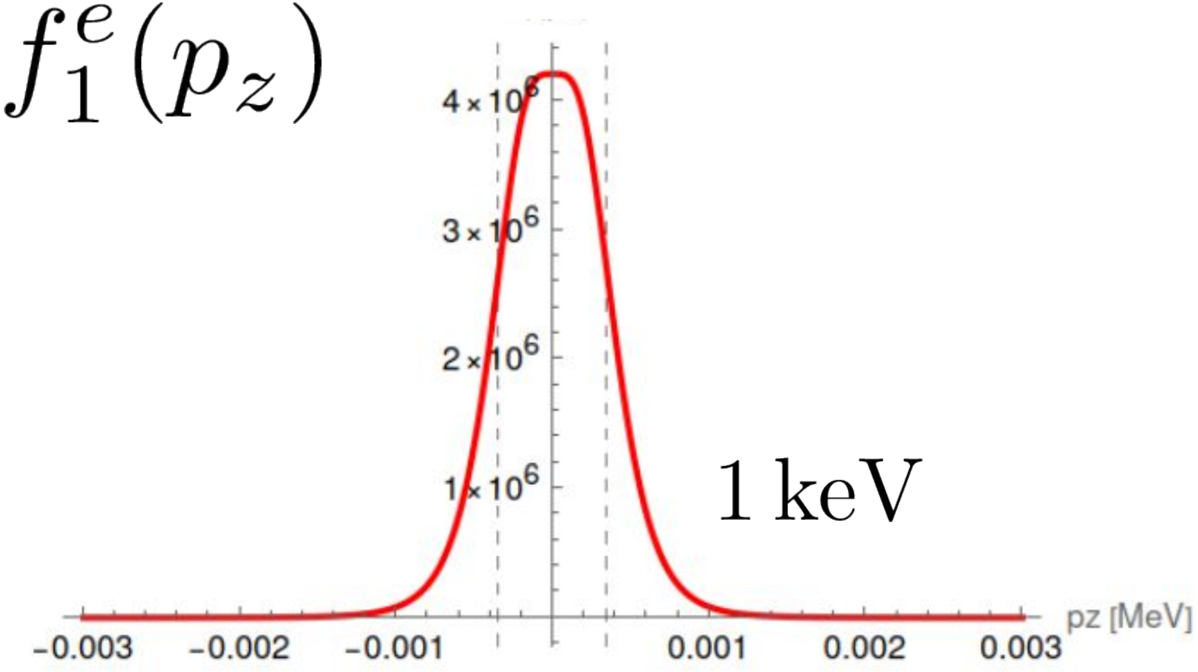


FIG. 7: (a) The transverse momentum dependence of the unpolarized distribution $f_1^e(x, \boldsymbol{p}_\perp^2)$ at $x = x_2$ and (b) the Sivers TMD PDF $f_{1T}^{\perp e}(x, \boldsymbol{p}_\perp^2)$ at $x = x_2$. To compare the two distributions we choose a value of x that maximizes/minimizes the first transverse moment of the Sivers TMD PDF.

Electron collinear distribution

$$f_1^e(p_z)$$



$1\,\mathrm{keV}$

(a)

Isolating the Sivers function

$$M^{\perp}(x_e, \boldsymbol{p}_{\perp}^2) = \int_0^{2\pi} \underline{d\phi_\gamma \int_0^{2\pi} d\phi_e \, \sin(\delta\phi) \, \cos(\phi_\gamma)} \, \frac{d\overline{\sigma}_{\mathrm{ACS}}}{dx_{\perp} \, d\eta_\gamma \, d\eta_e \, d\phi_\gamma \, d\phi_e}$$

$$= \pi^2 \, \frac{\alpha_e^2}{64} \, \frac{x_{\perp}^2}{4x_e} \, \overline{f}(m_e, \hat{s}) \, \frac{|\boldsymbol{k}_{\gamma\perp}| \, |\boldsymbol{S}_{\perp}|}{M} \, \boxed{f_{1T}^{\perp \, e/A}(x_e, \boldsymbol{p}_{\perp}^2)} \, .$$

$$M^{\perp \, (1)}(x_e) = \int d^2\boldsymbol{p}_{\perp} \, \boldsymbol{p}_{\perp}^2 \, M^{\perp}(x_e, \boldsymbol{p}_{\perp}^2) =$$

$$= \frac{M \pi^2 \, \alpha_e^2}{32 \, x_e \, s} \, \overline{f}(x_e, \hat{s}) \, |\boldsymbol{k}_{\gamma\perp}|^3 \, |\boldsymbol{S}_{\perp}| \, \boxed{f_{1T}^{\perp \, (1) \, e/A}(x_e)}$$