

# Recent developments in the light strange sector

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**Jefferson Lab**  
*Thomas Jefferson National Accelerator Facility*



  
**OLD DOMINION**  
UNIVERSITY

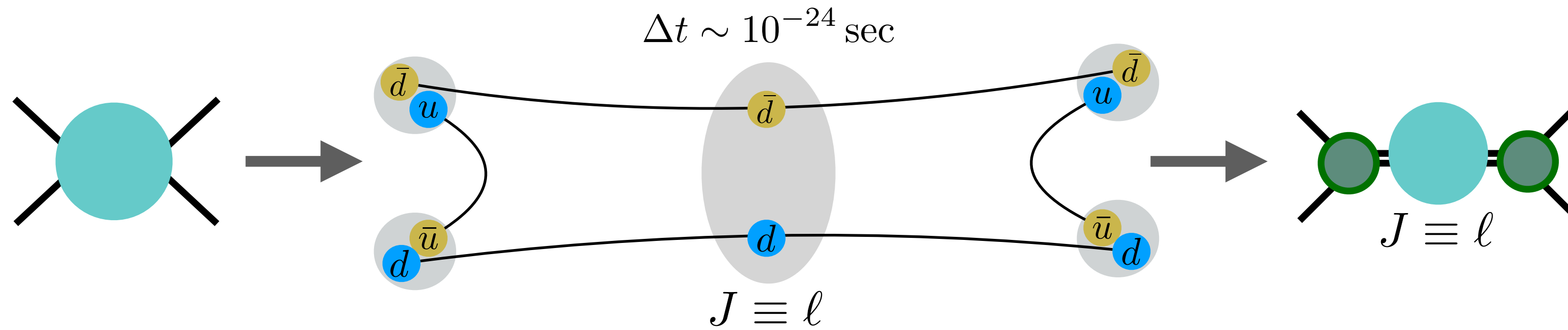
Arkaitz Rodas

8th KLF Collaboration Meeting  
6-May-2026

# Spectroscopy in QCD

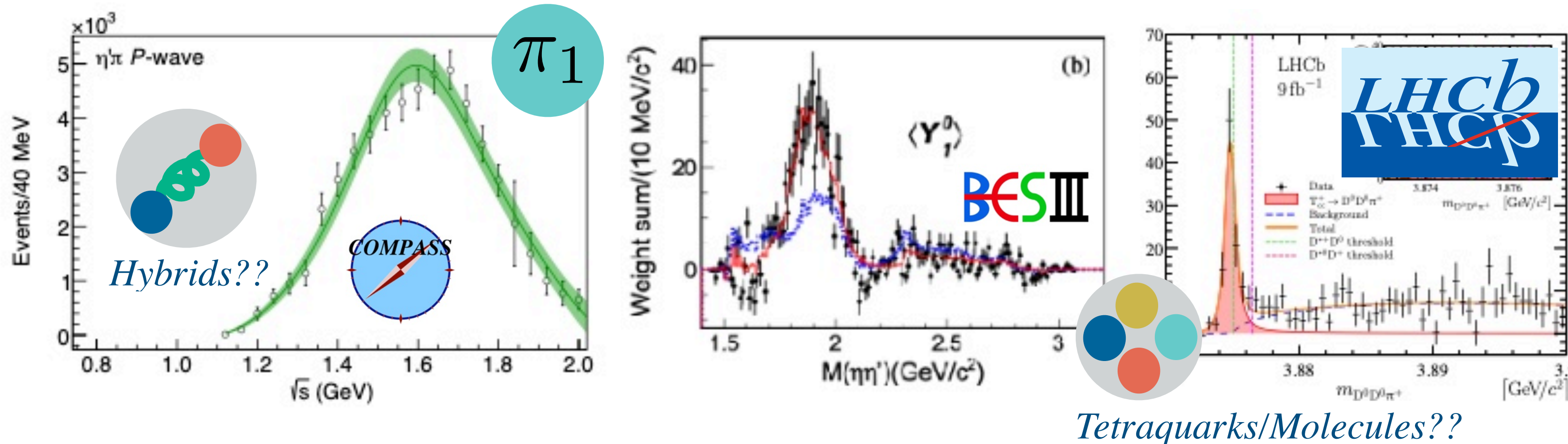
## How do quark and gluons combine inside unstable hadrons?

We need a combination of lattice QCD, theory/phenomenology, and experiment to answer this question



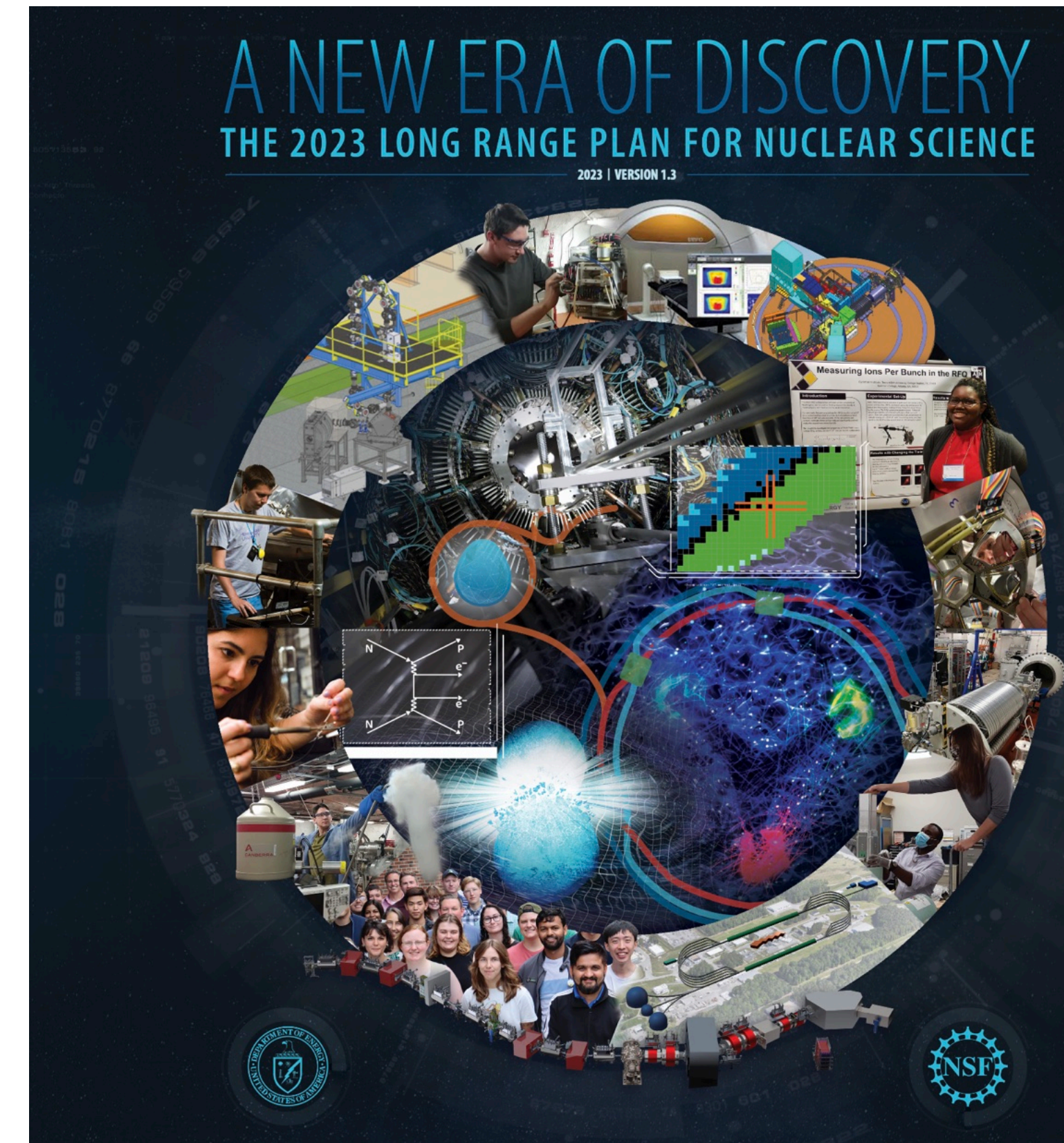
*“hadron spectroscopy explores the possible bound combinations of quarks and gluons allowed by the interactions of QCD”*

## Confirm existence of elusive hadrons



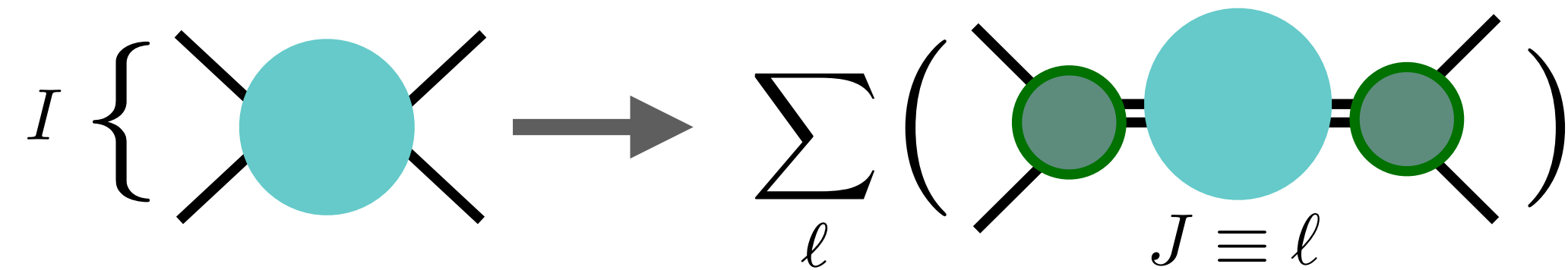
Guide searches

Go beyond standard analyses to understand their nature (Lattice QCD)



# Scattering at hadronic energies: Resonances

## □ Determine the spectrum



$$T^I(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \underline{t_{\ell}^I(s)} P_{\ell}(\cos \theta_s)$$

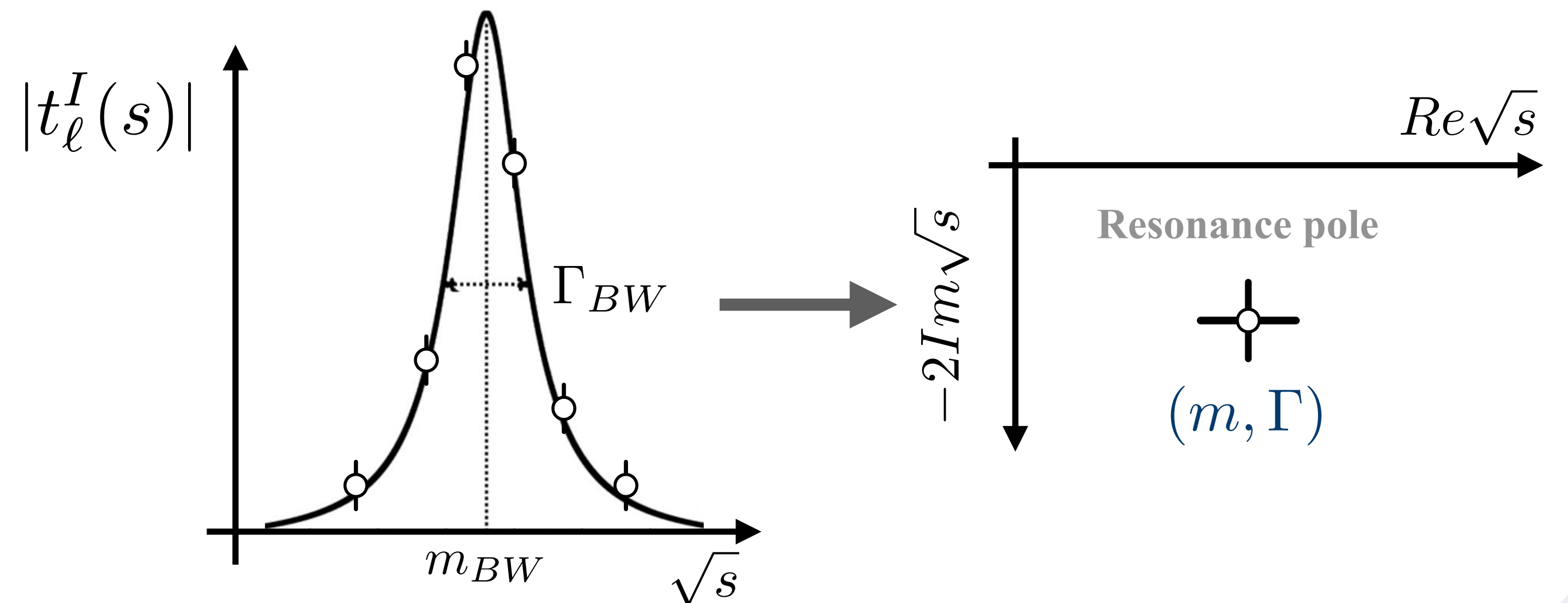
*Partial wave, unstable hadrons live here*

Assume we have scattering data for well-defined angular momentum

Assume the resonance is narrow and isolated

$$t_{\ell}^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s} \Gamma_{BW}}{m_{BW}^2 - s - i\sqrt{s} \Gamma_{BW}}$$

*Pole at  $\sqrt{s_p} \sim (m_{BW} - i\Gamma_{BW}/2)$*



# Scattering at hadronic energies: Resonances

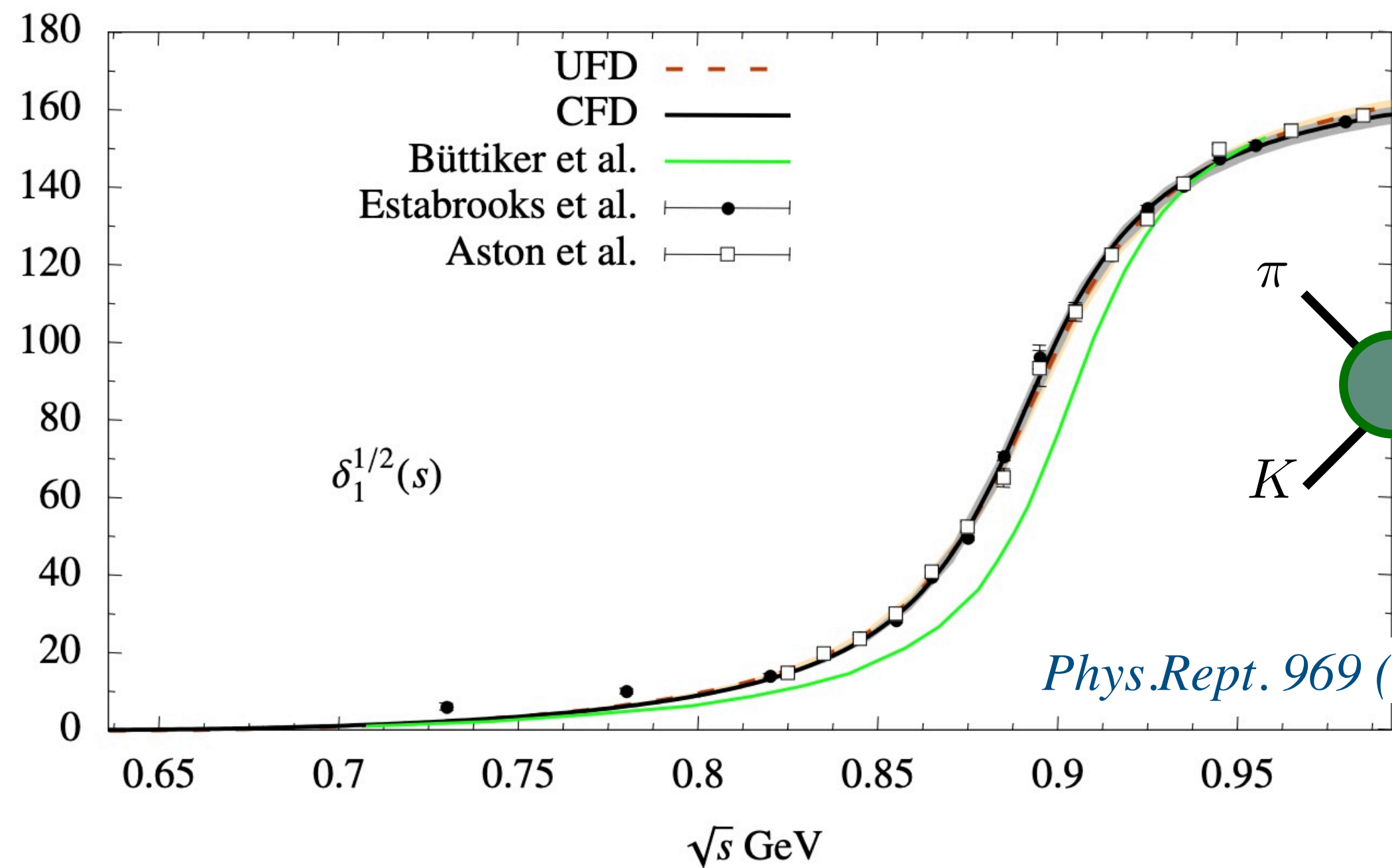
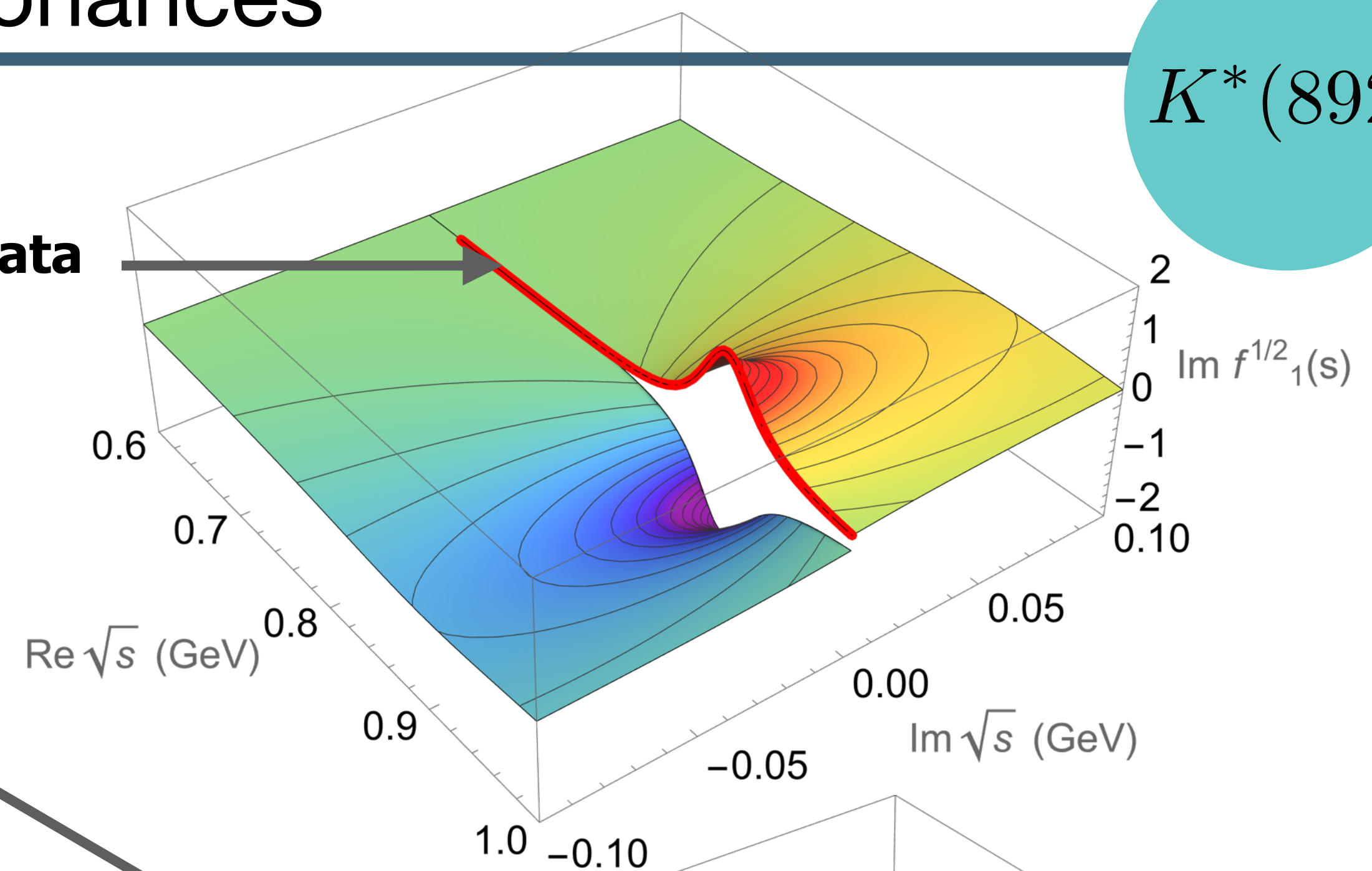
$K^*(892)$

The BW has worked wonders in many cases

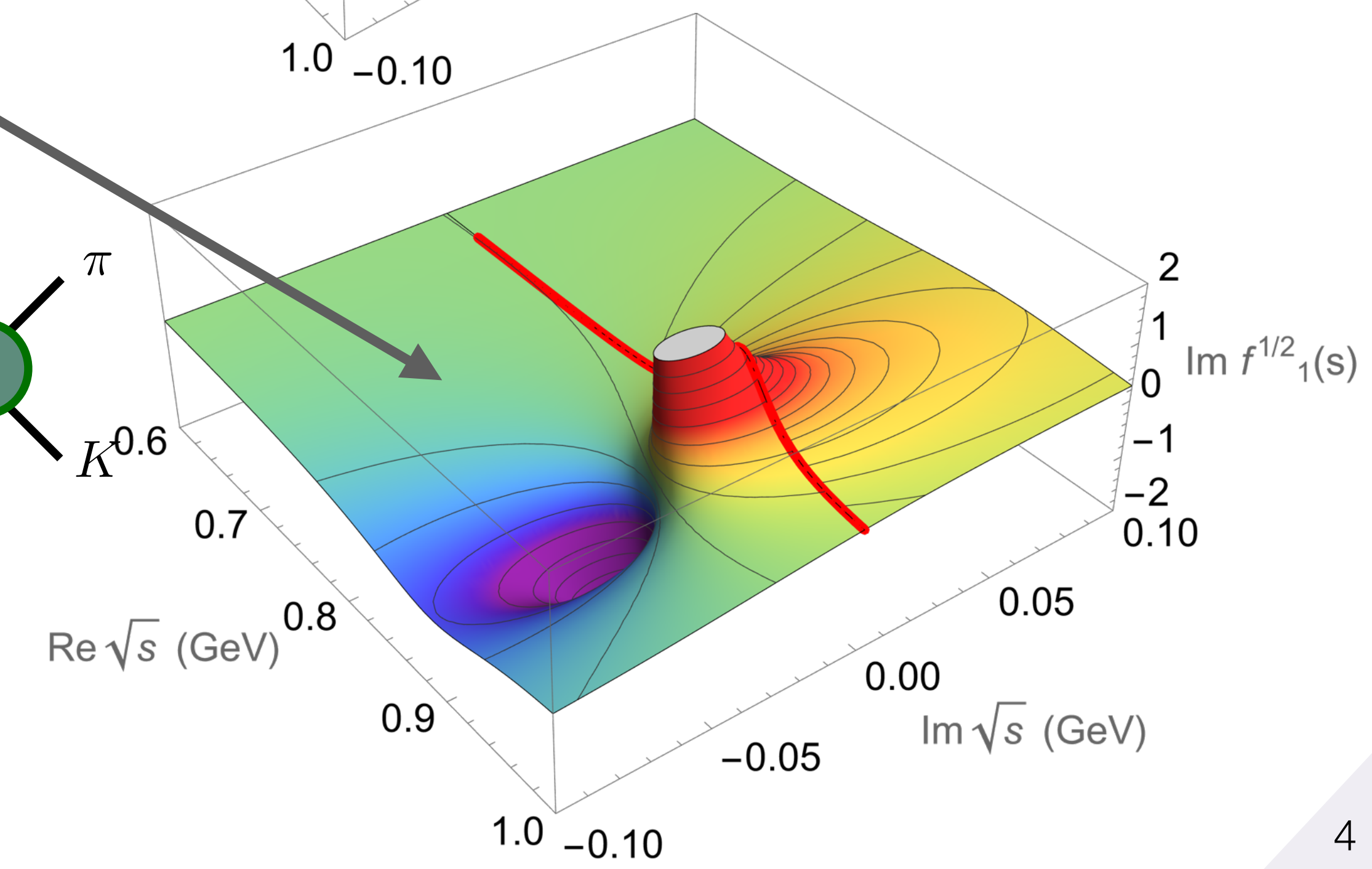
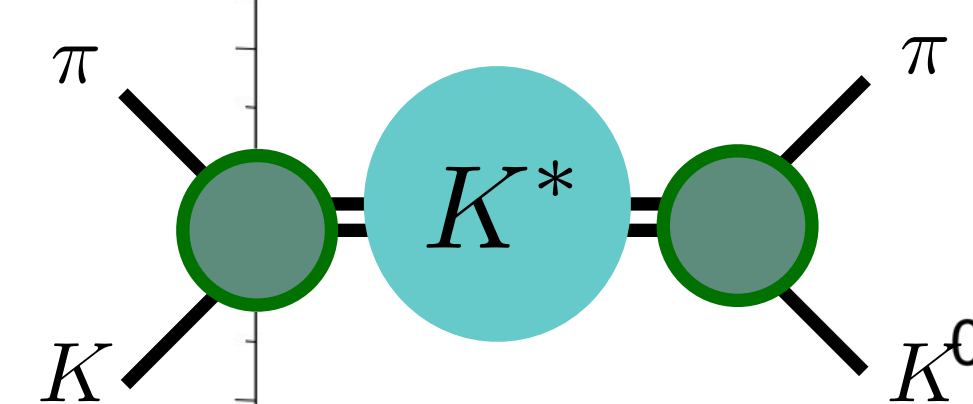
$$t_\ell(s) \sim \frac{\sqrt{s}\Gamma(s)}{m^2 - s - i\sqrt{s}\Gamma(s)}$$

$$\sqrt{s_p} = m + i\Gamma/2$$

Data



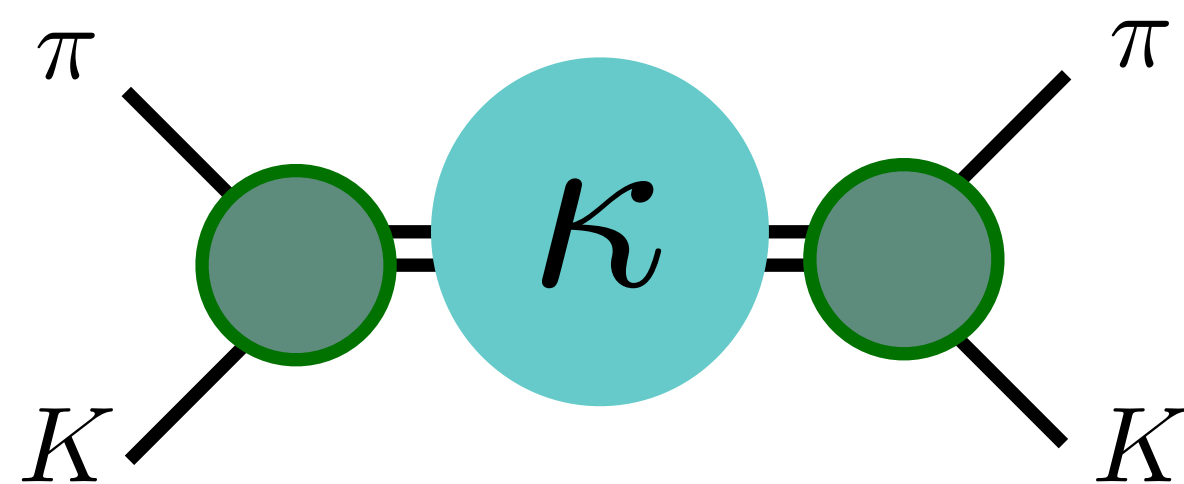
*Phys.Rept. 969 (2022) 1-126*



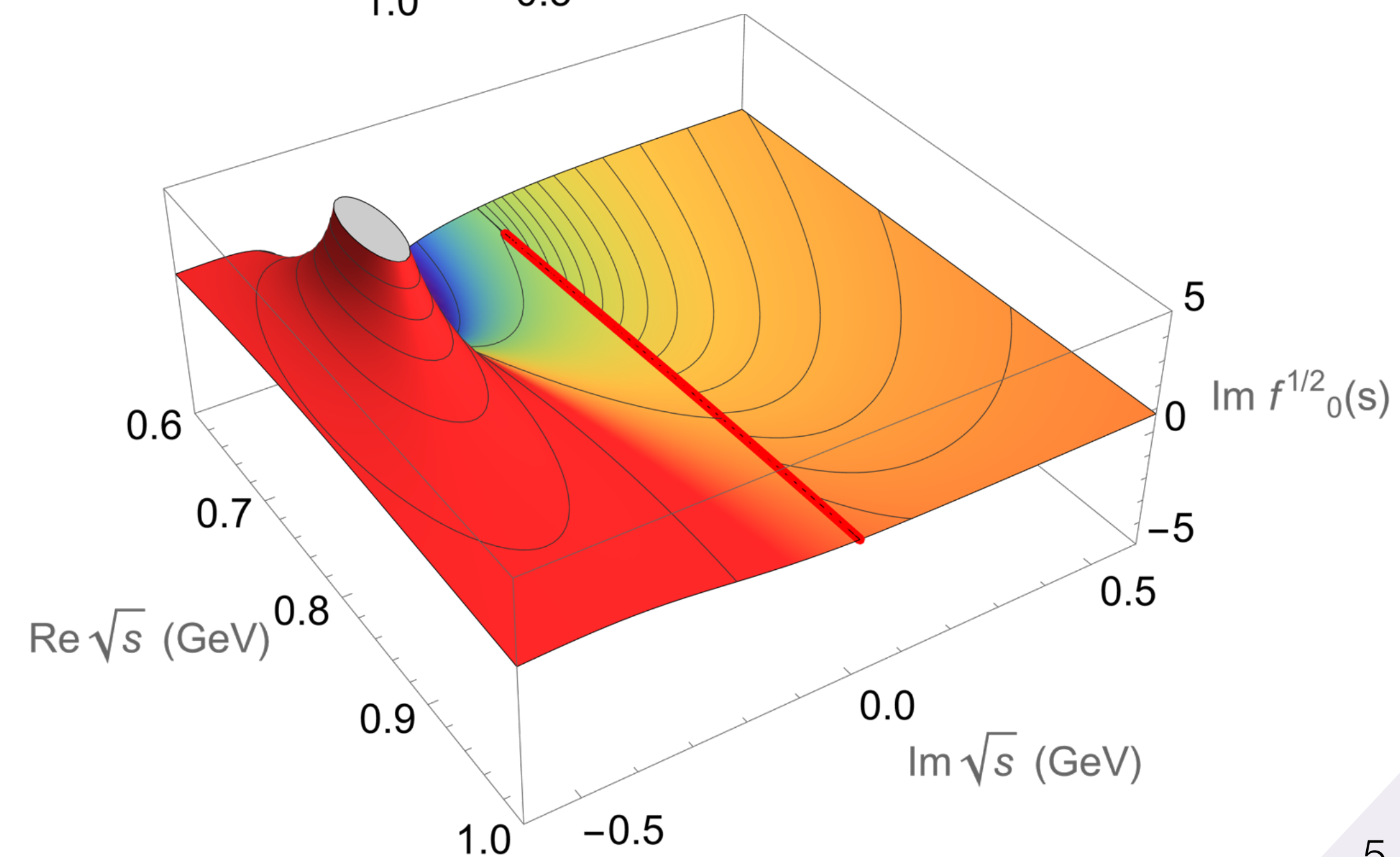
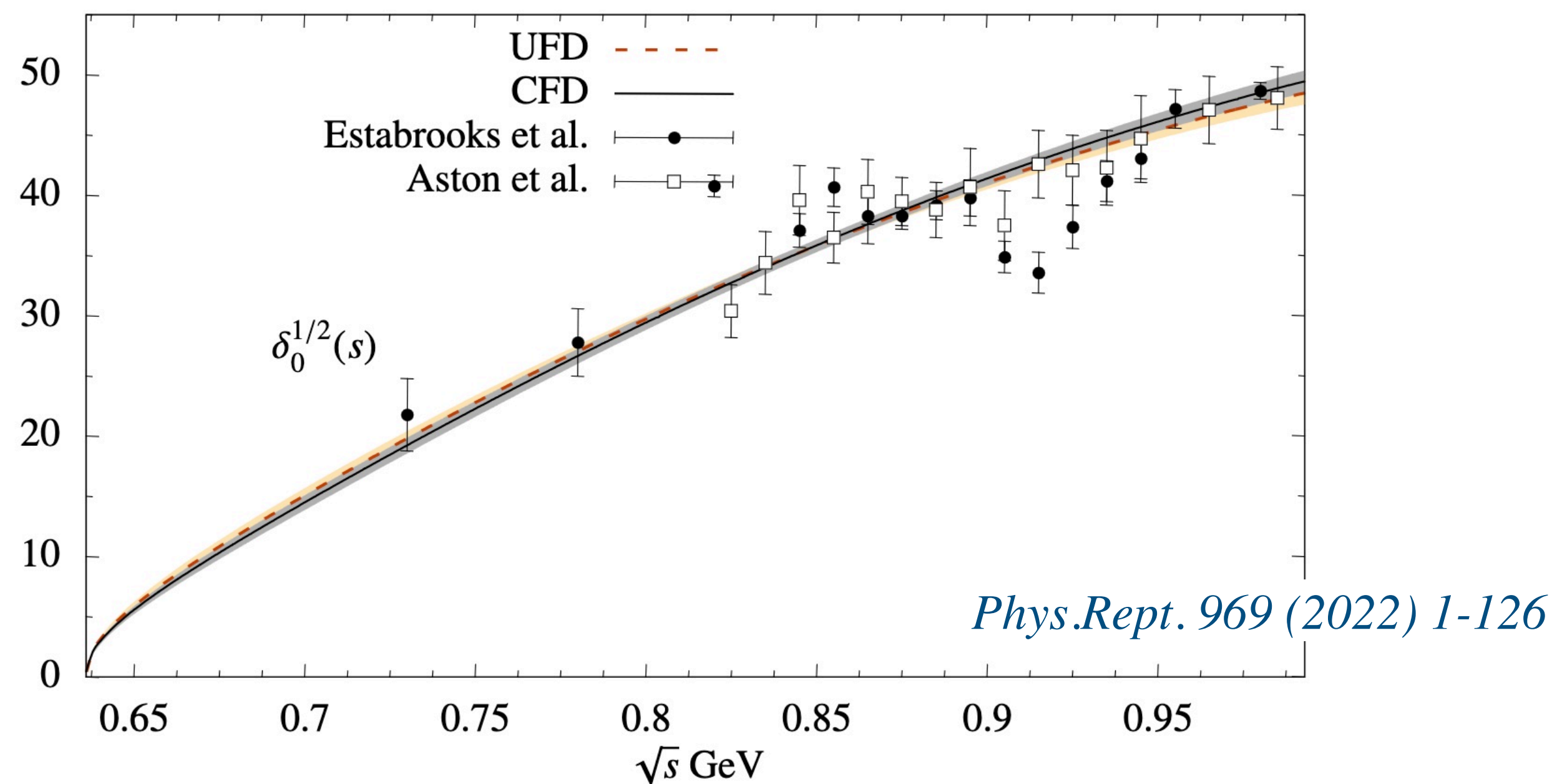
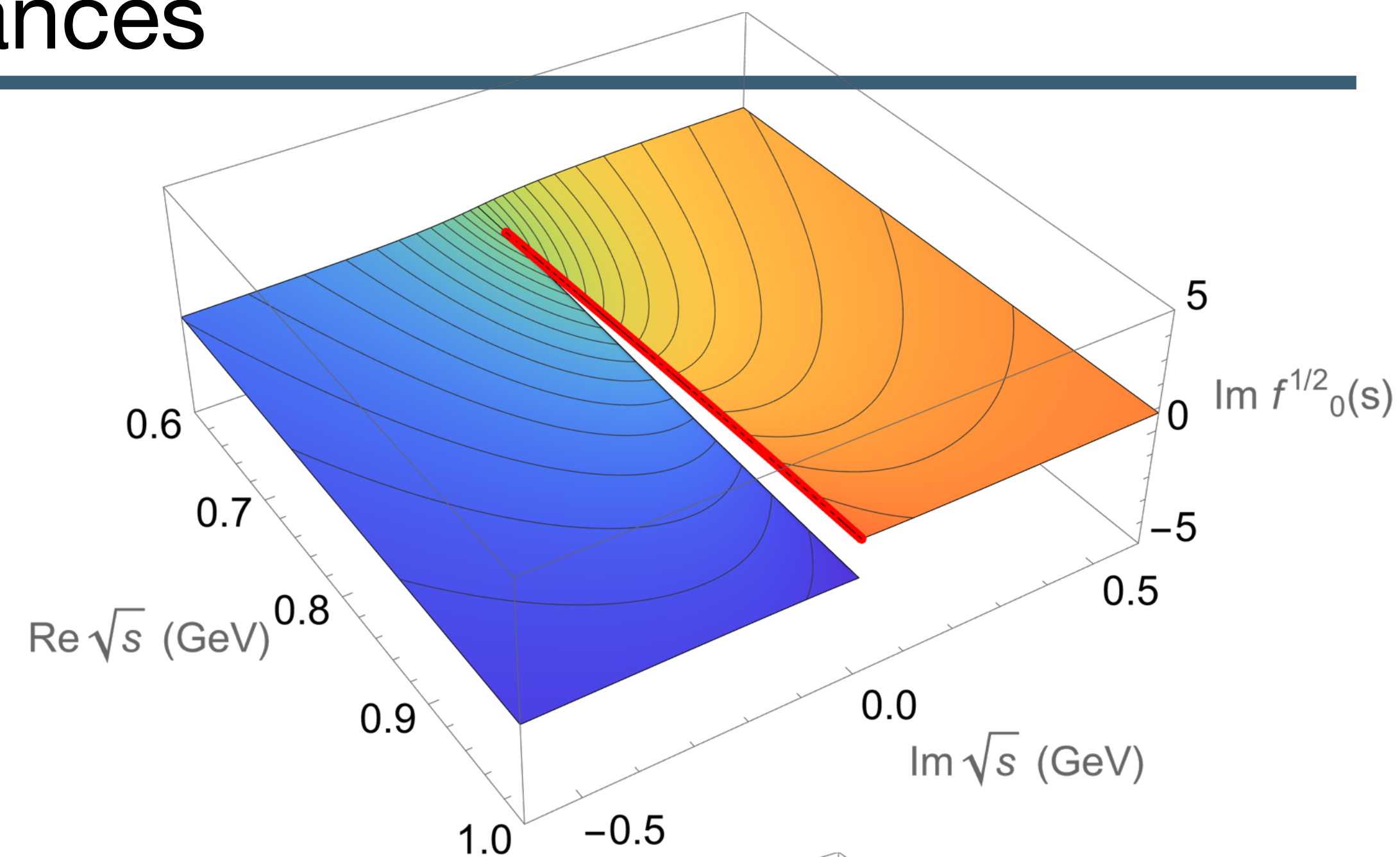
# Scattering at hadronic energies: Resonances

**Definitely not a BW**

$$t_\ell(s) \sim ??$$



**What's this??**



# The “data”

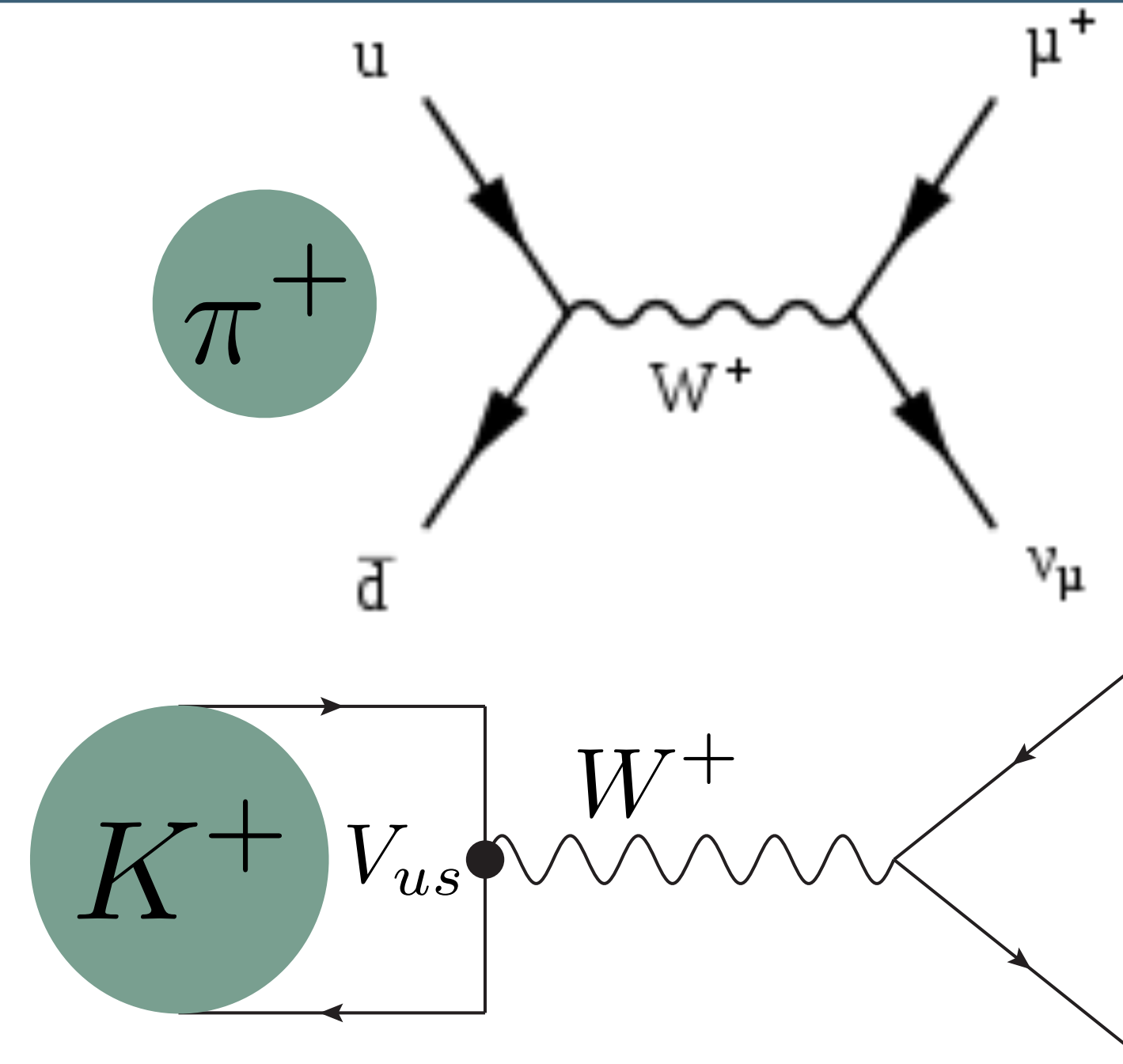
## Mesons decay

One beam?? (maybe)

Not two beams

$$c\tau_{\pi^+} = 7.8 \text{ m}$$

$$c\tau_K = 3.7 \text{ m}$$

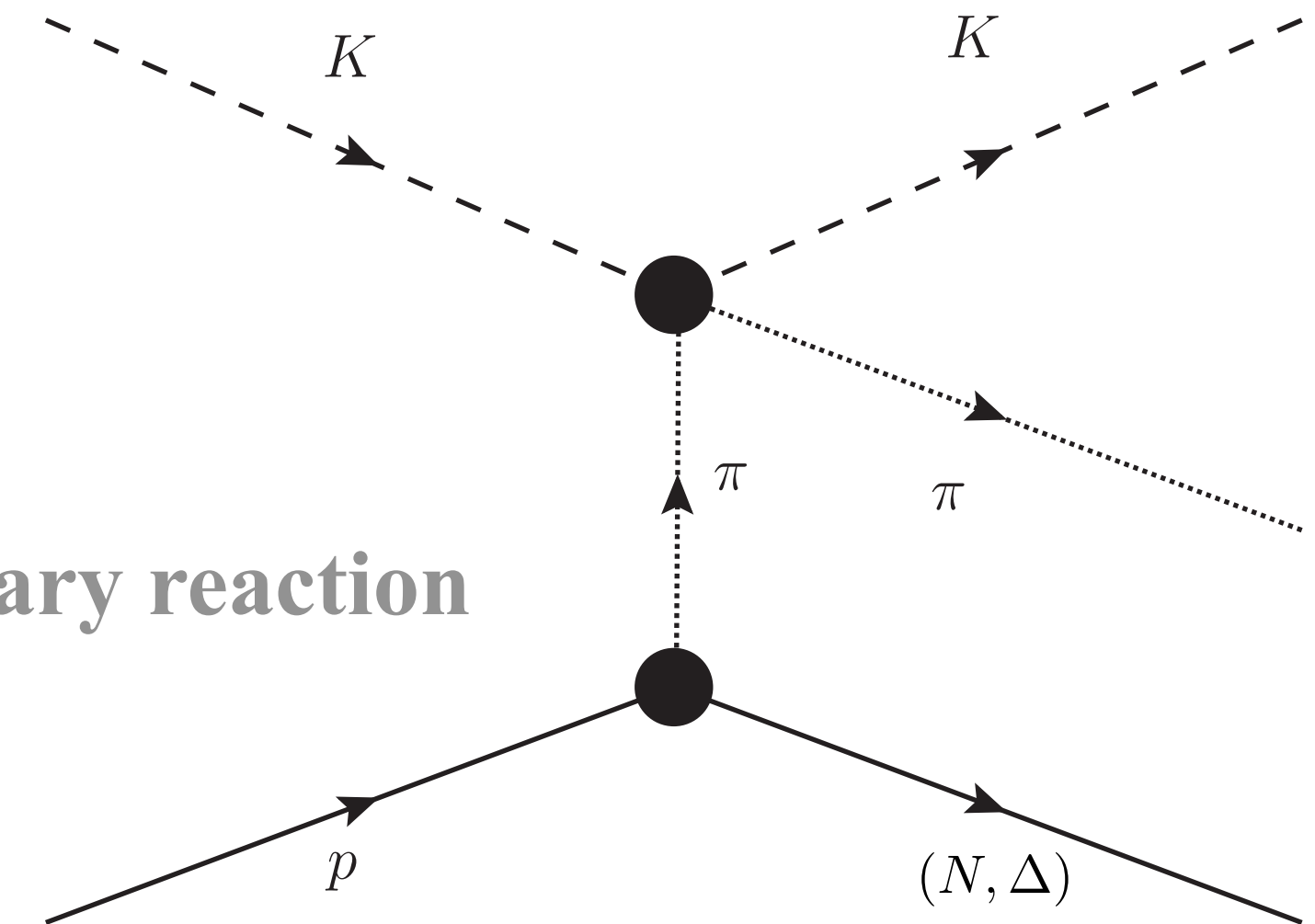


## “Data” is not data

For meson-meson data must be always modeled

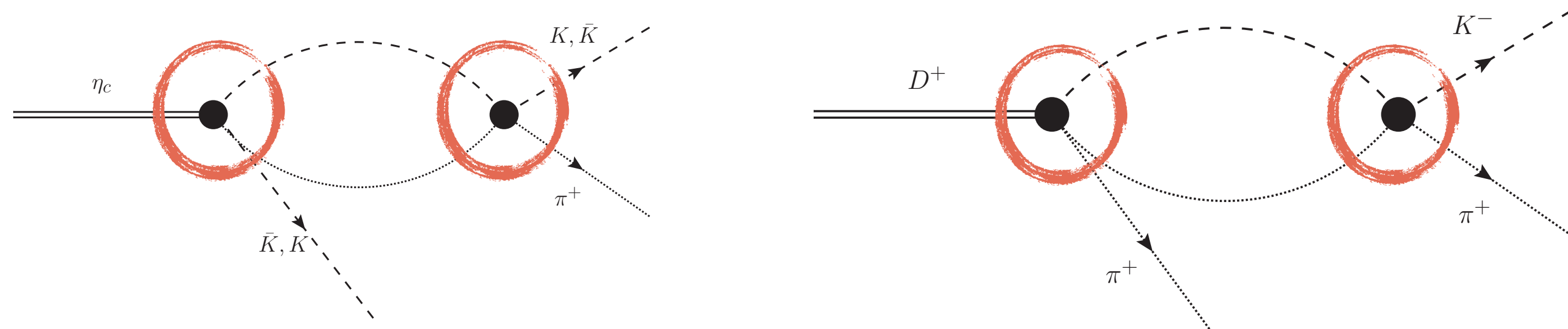
Some systematic uncertainties unknown

Customary reaction

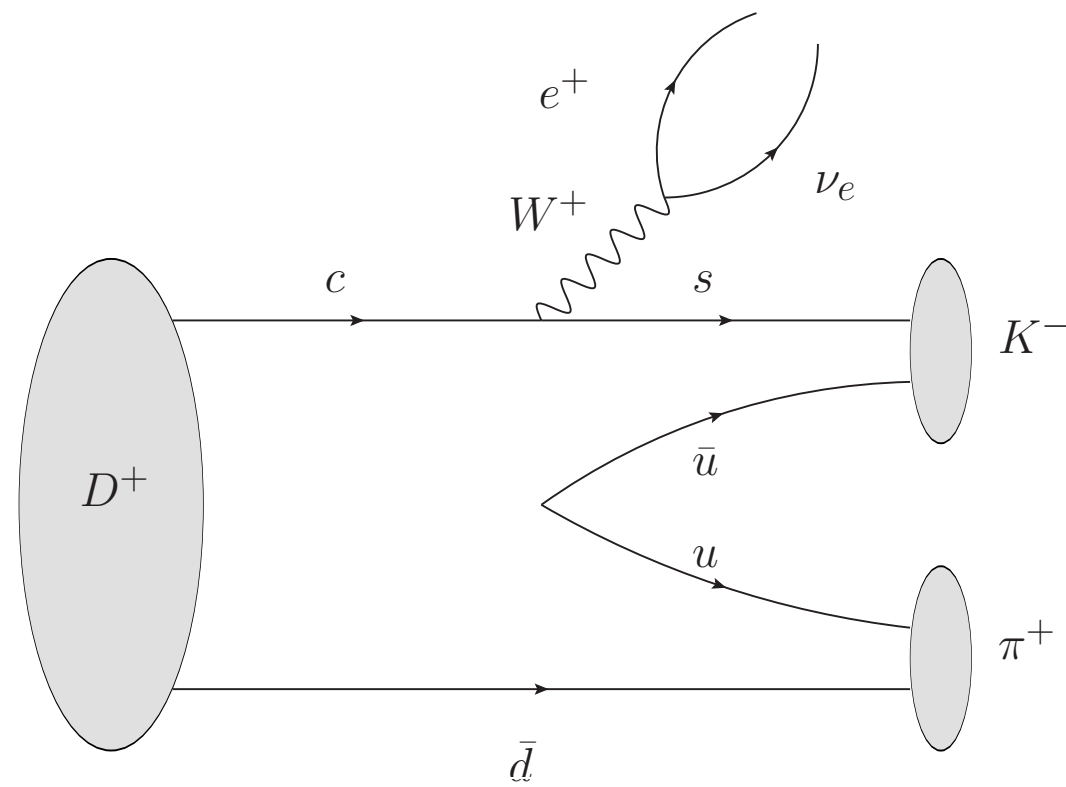


# The "data"

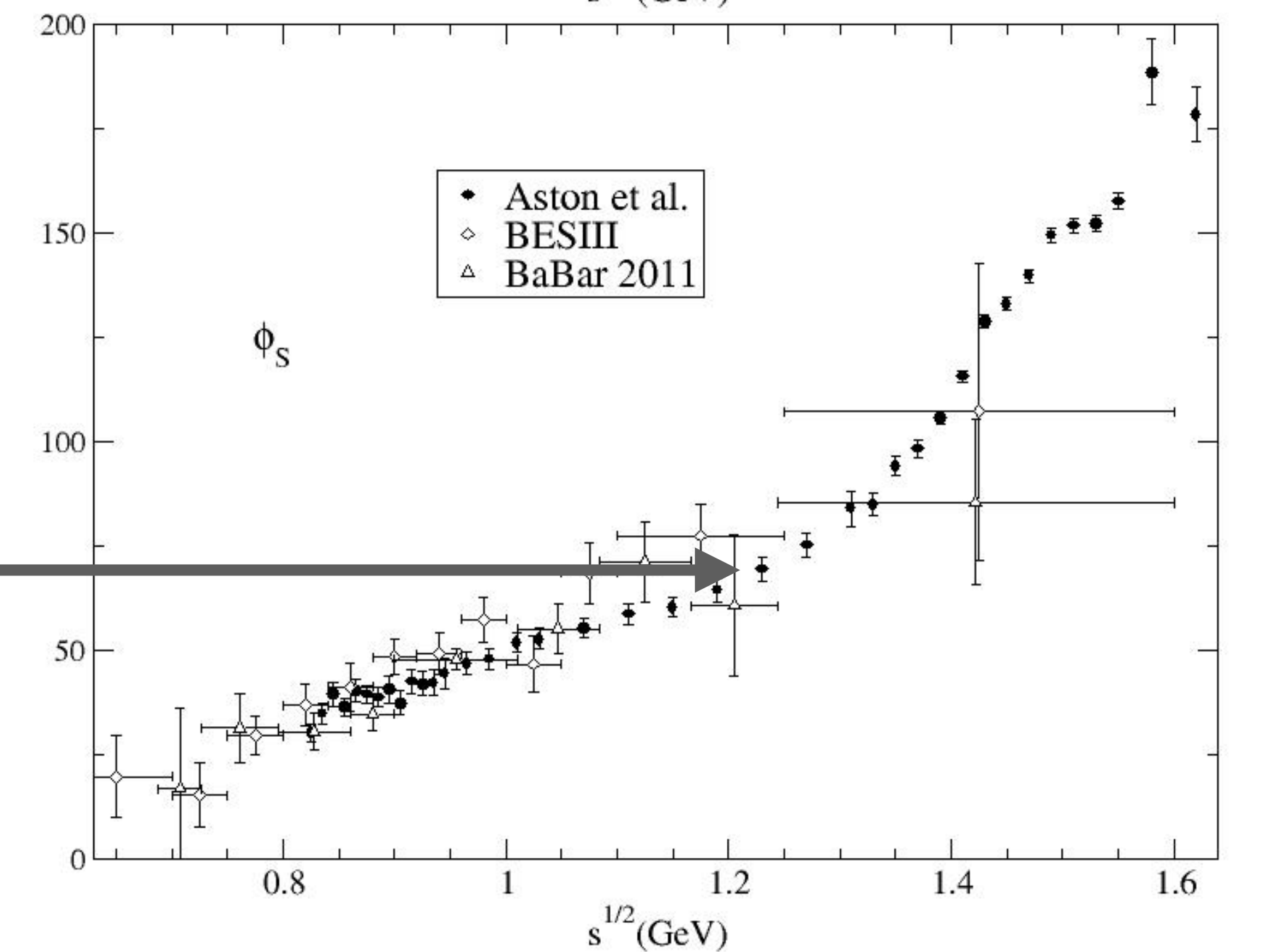
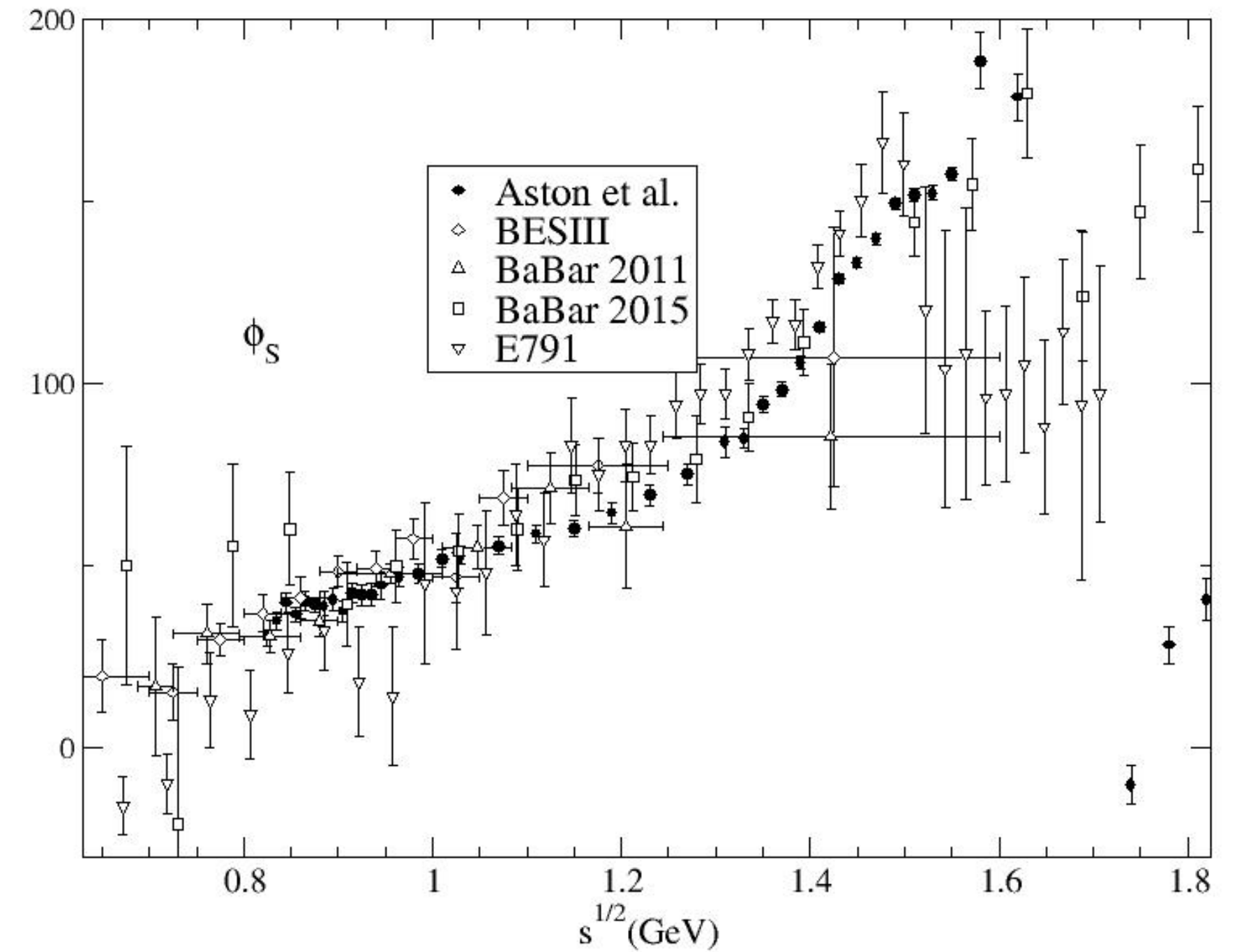
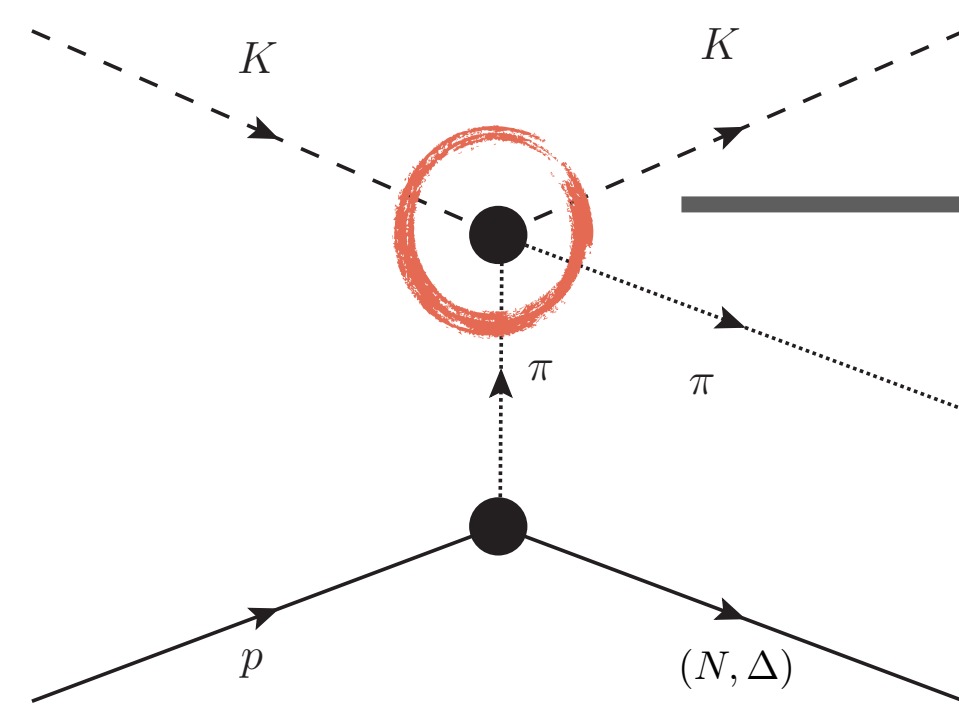
**Hadronic decays require heavier modeling**



**Semileptonic decays do not have great statistics**



**Production offers most precise results, by far**



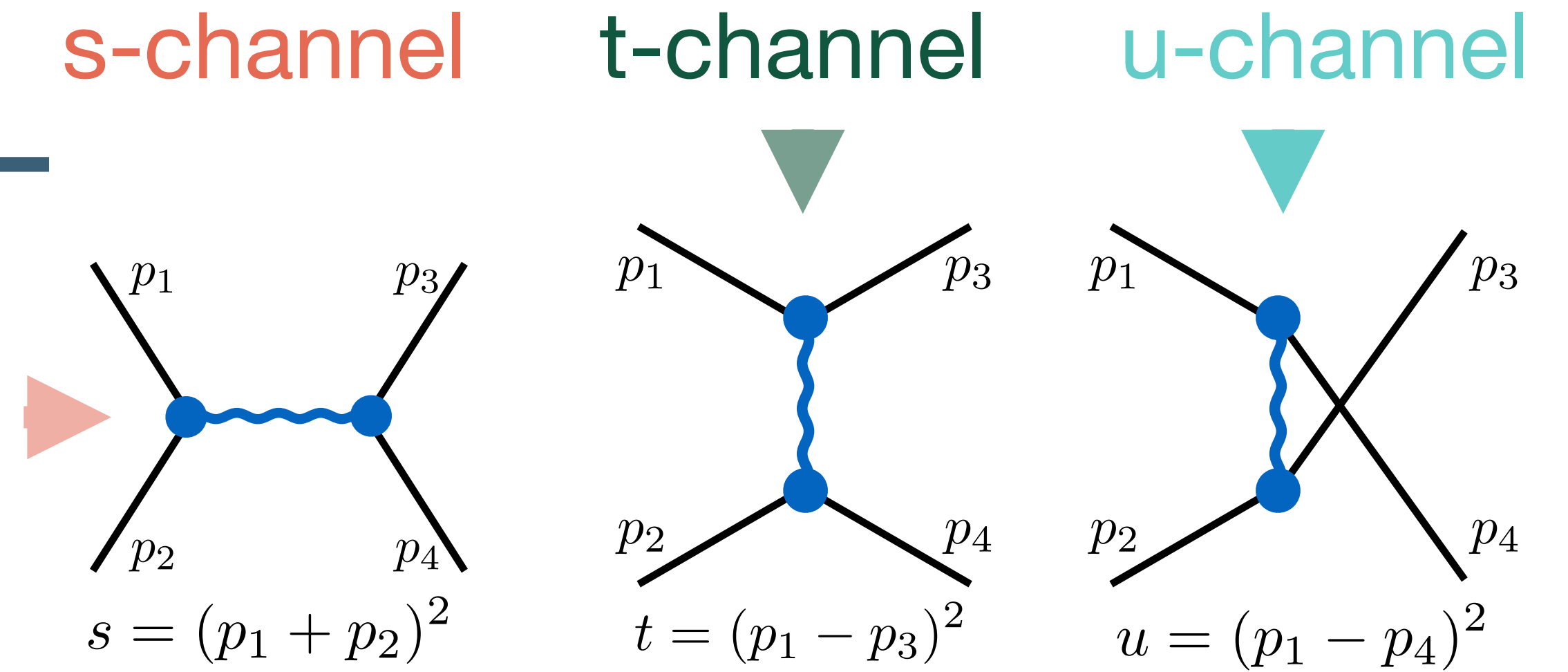
# Best theoretical technique: Dispersion relations

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# Dispersion relations: $\pi\pi$ example

Particles and anti-particles are related

Crossing symmetry relates amplitudes in different channels and isospins



$$F^0(s, t) = \frac{1}{3}F^{(0)}(s, t) + F^{(1)}(s, t) + \frac{5}{3}F^{(2)}(s, t),$$

$$F^1(s, t) = \frac{1}{3}F^{(0)}(s, t) + \frac{1}{2}F^{(1)}(s, t) - \frac{5}{6}F^{(2)}(s, t),$$

$$F^2(s, t) = \frac{1}{3}F^{(0)}(s, t) - \frac{1}{2}F^{(1)}(s, t) + \frac{1}{6}F^{(2)}(s, t)$$

Mandelstam hypothesis (all related to a singular analytic amplitude)

$$F^{I=0}(s, t, u) = 3F(s, t, u) + F(t, s, u) + F(u, t, s),$$

$$F^{I=1}(s, t, u) = F(t, s, u) - F(u, t, s),$$

$$F^{I=2}(s, t, u) = F(t, s, u) + F(u, t, s)$$

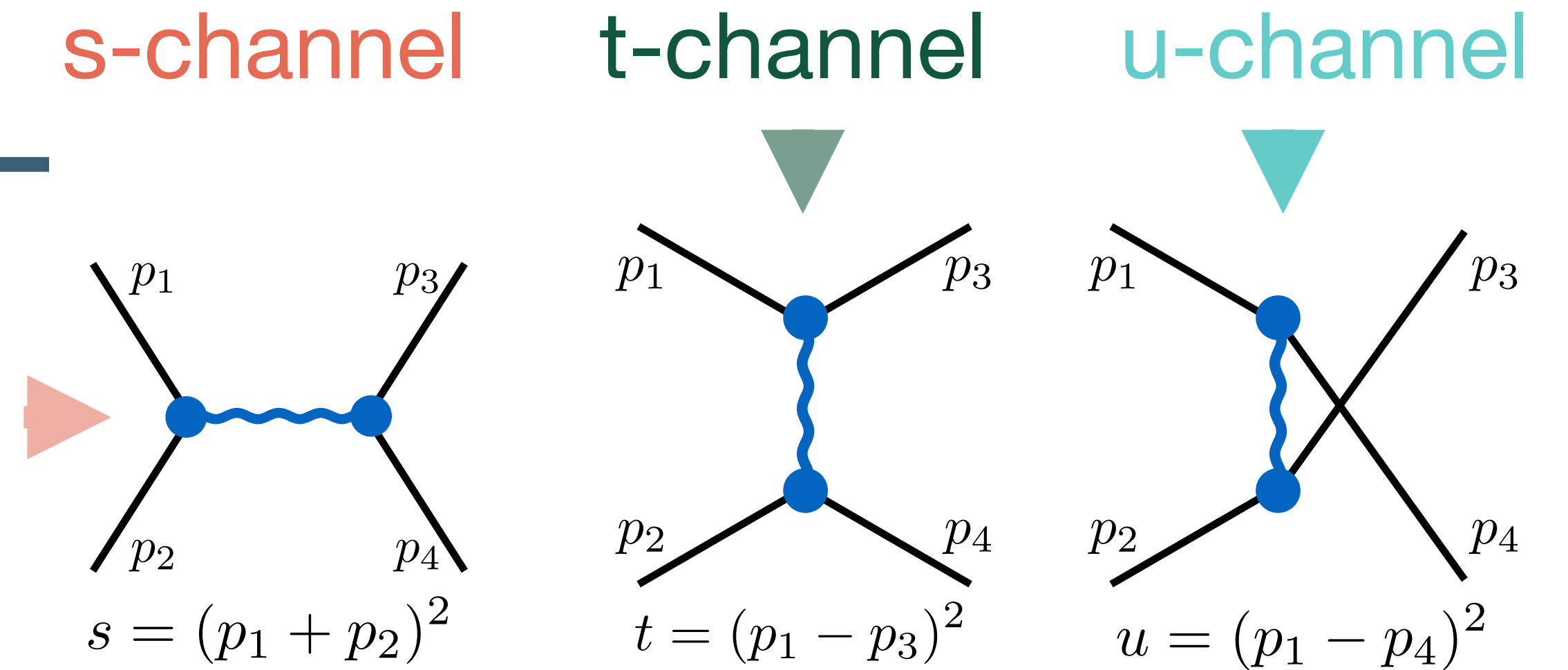
Dispersion relations offer a framework, by construction, to impose crossing symmetry

$$\text{Re } \tilde{f}_\ell^I(s) = \text{ST}_\ell^I(s) + \text{DT}_\ell^I(s) + \sum_{\ell' I'} \text{P.V.} \int_{4m_\pi^2}^{\infty} ds' K_{\ell, \ell'}^{I, I'}(s, s') \text{Im } f_{\ell'}^{I'}(s')$$

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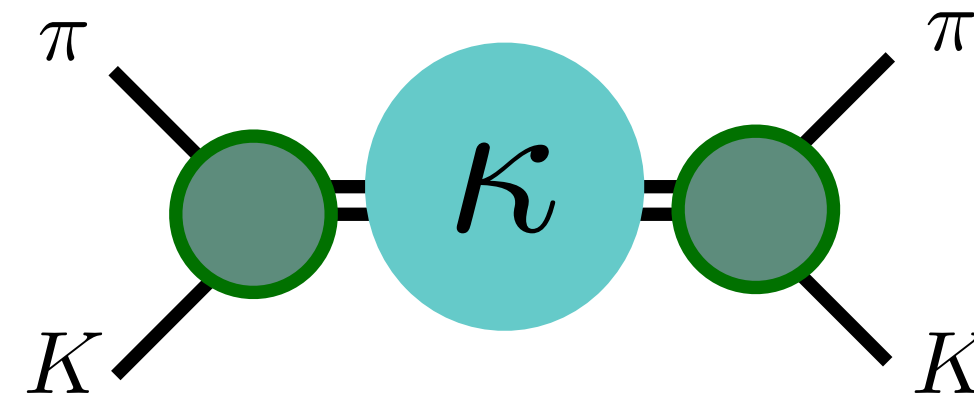
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*The PW we want*

*All the relevant PWs contribute*

# Main problem: The $\kappa$

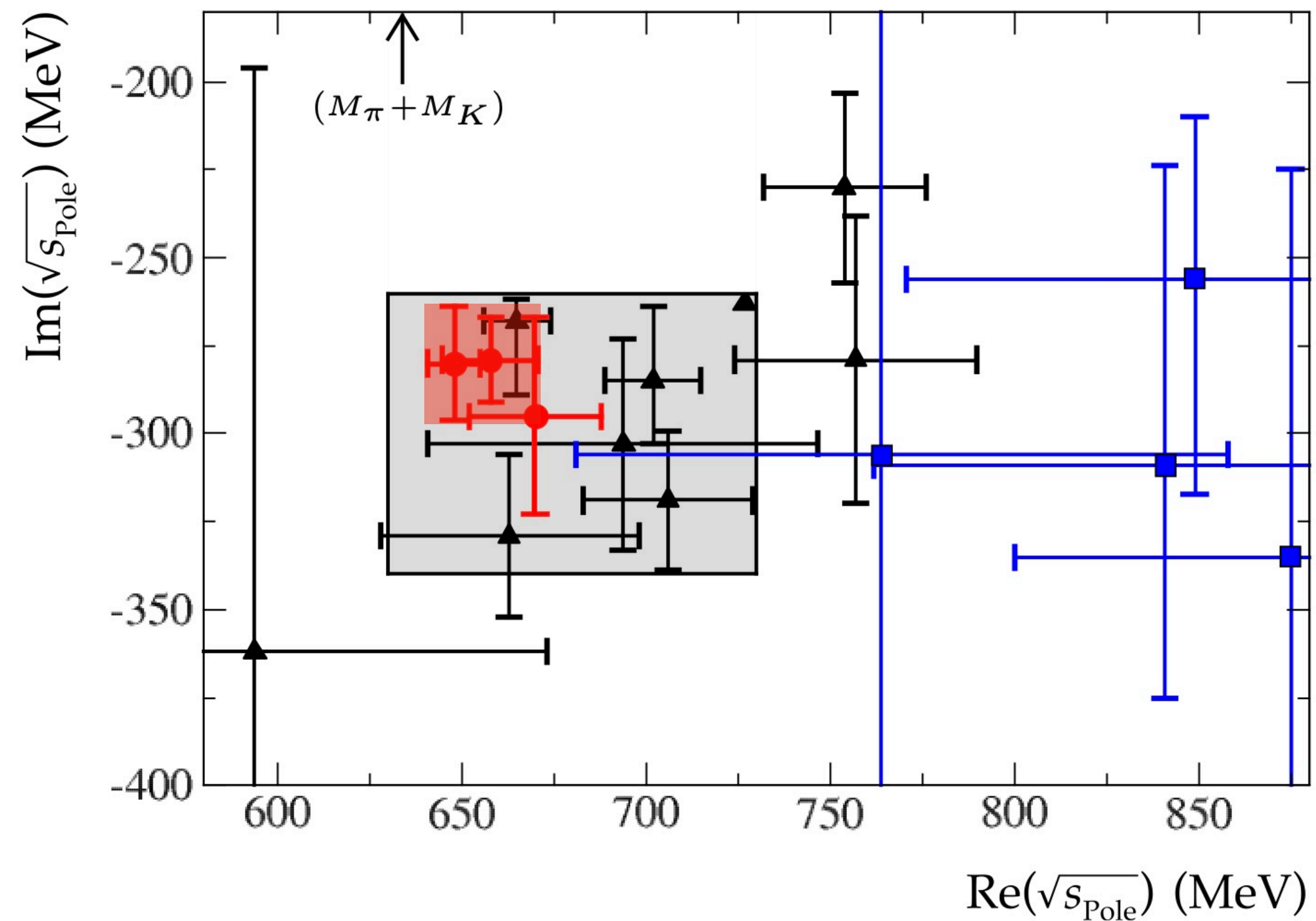
Over 50 years of debate!!



PDG then

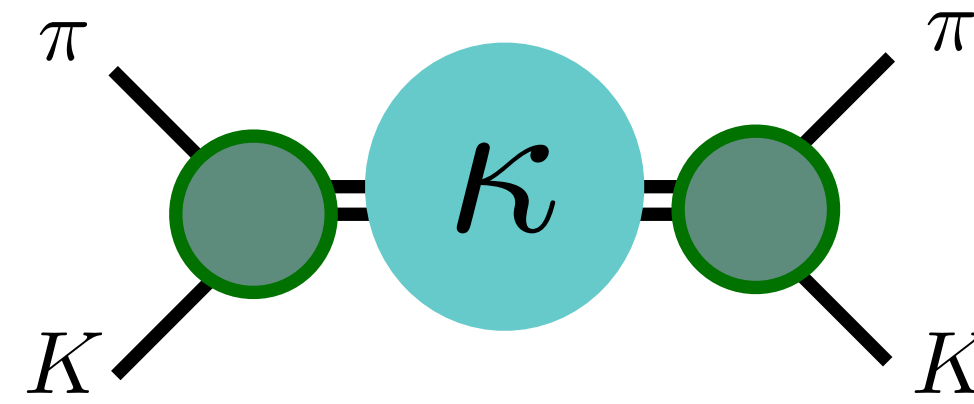
“We are beginning to think that  $\kappa$  should be classified along with flying saucers, the Loch Ness Monster, and the Abominable Snowman.”

PDG now



# Main problem: The $\kappa$

Over 50 years of debate!!



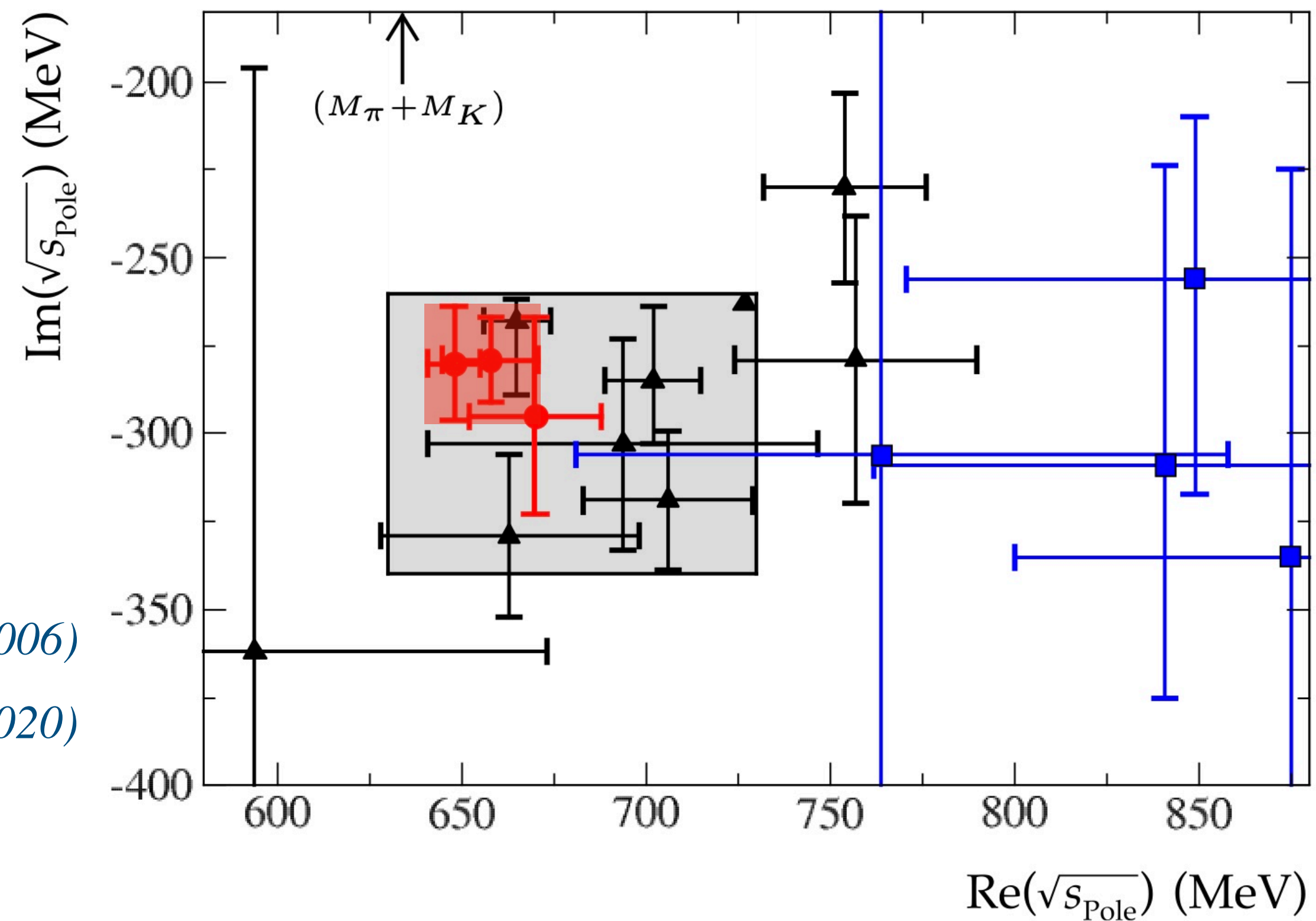
## PDG then

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## PDG now

*Eur.Phys.J.C* 48 (2006)

*Phys.Rev.Lett.* 124 (2020)

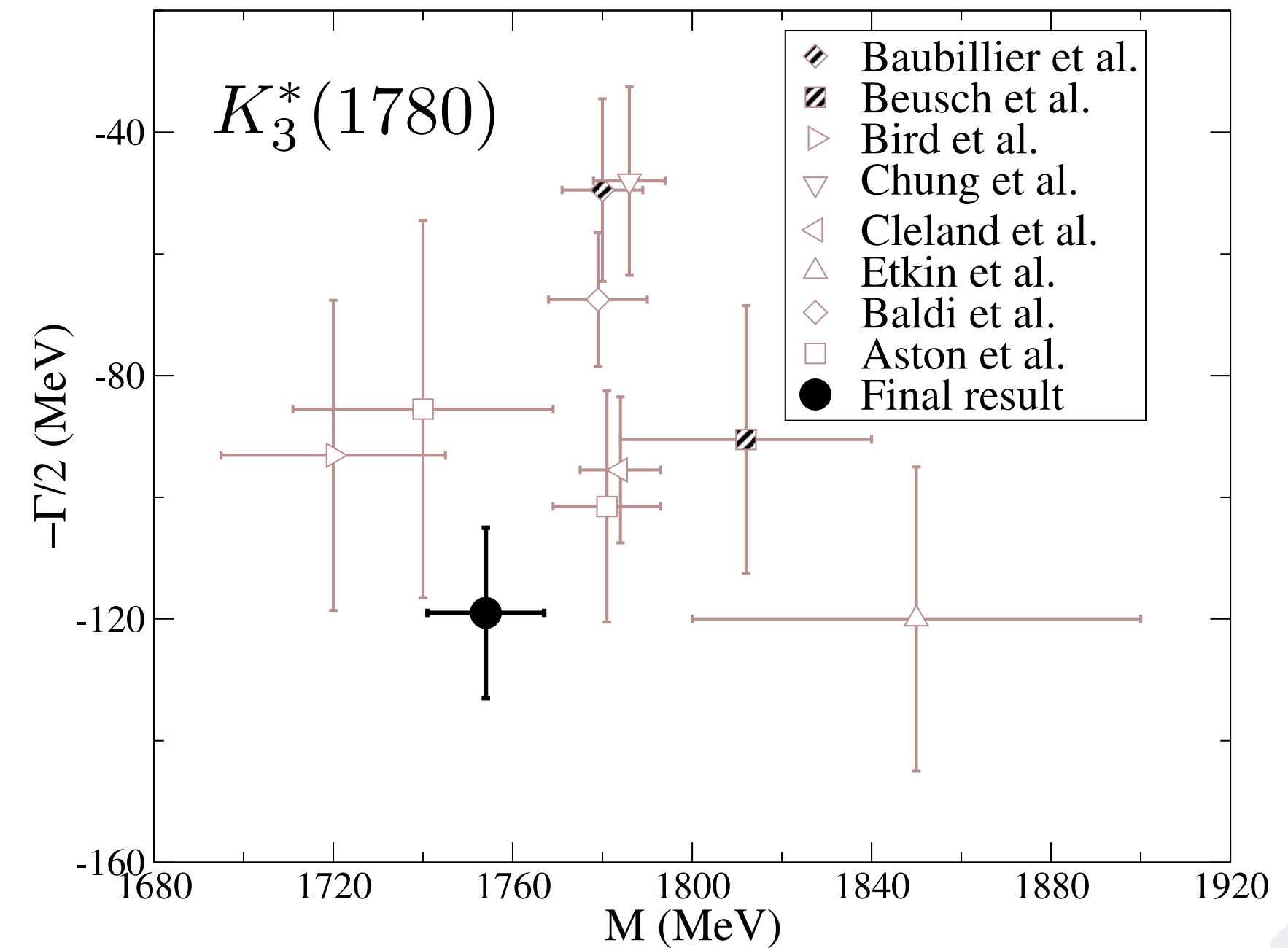
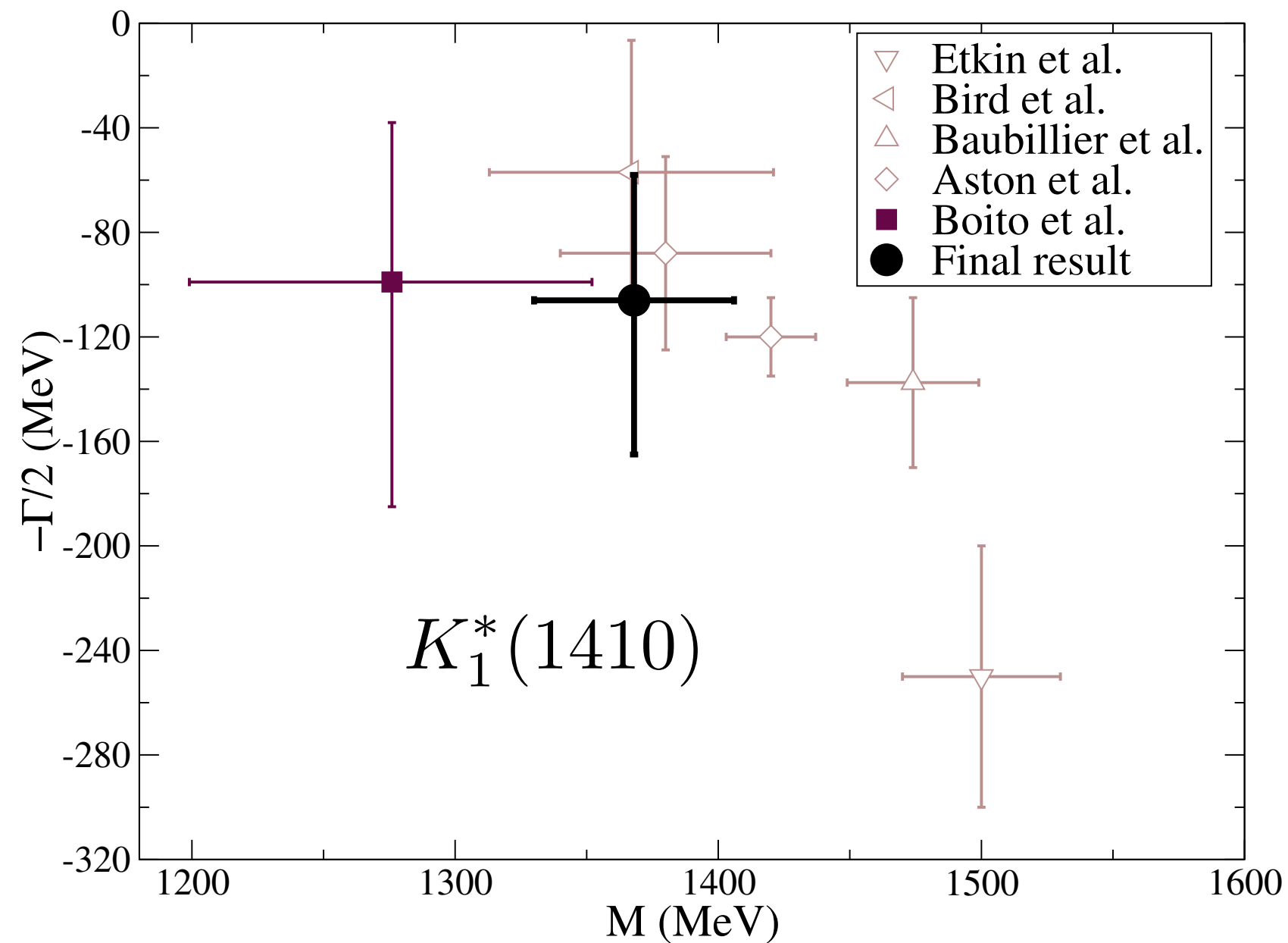
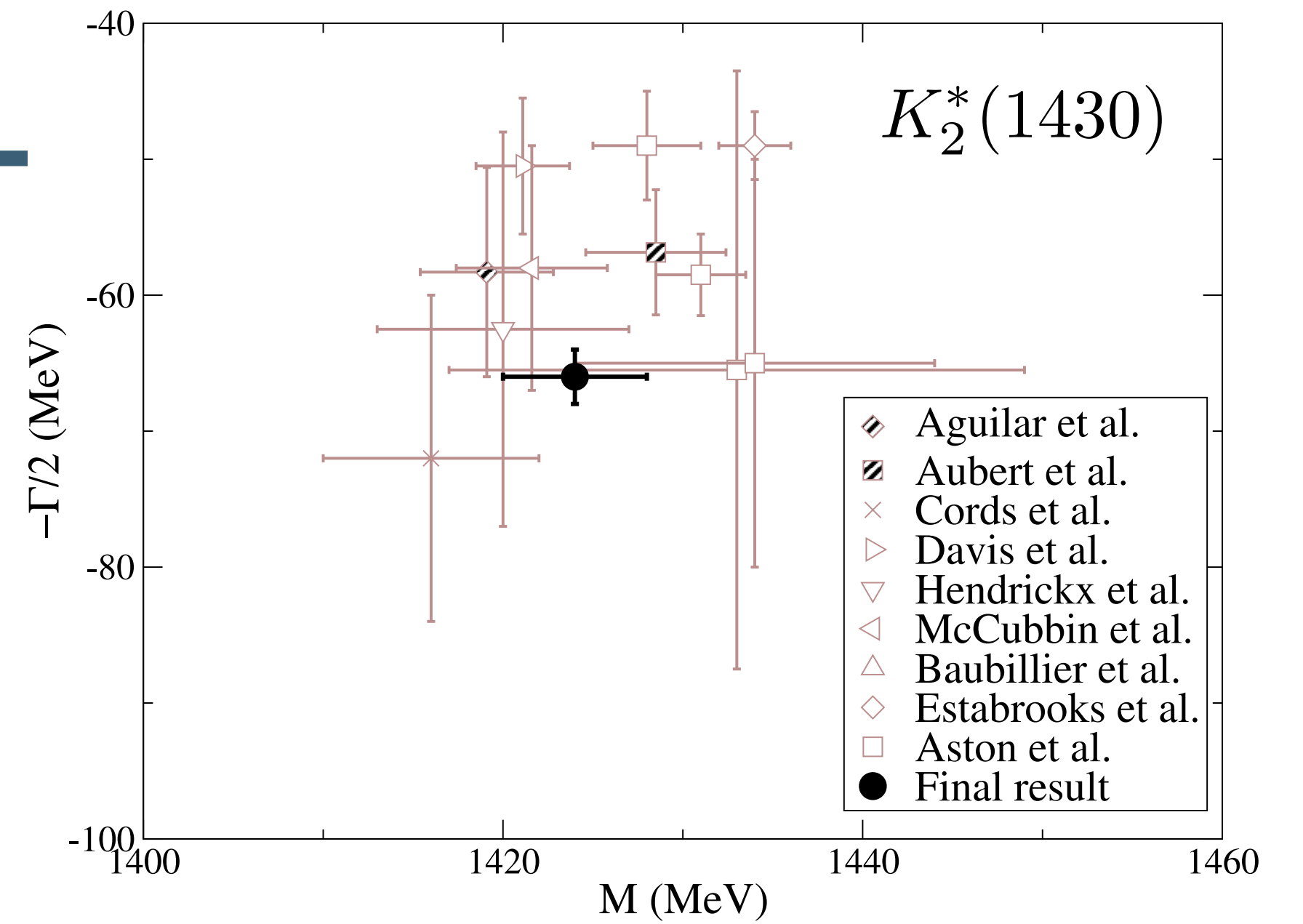
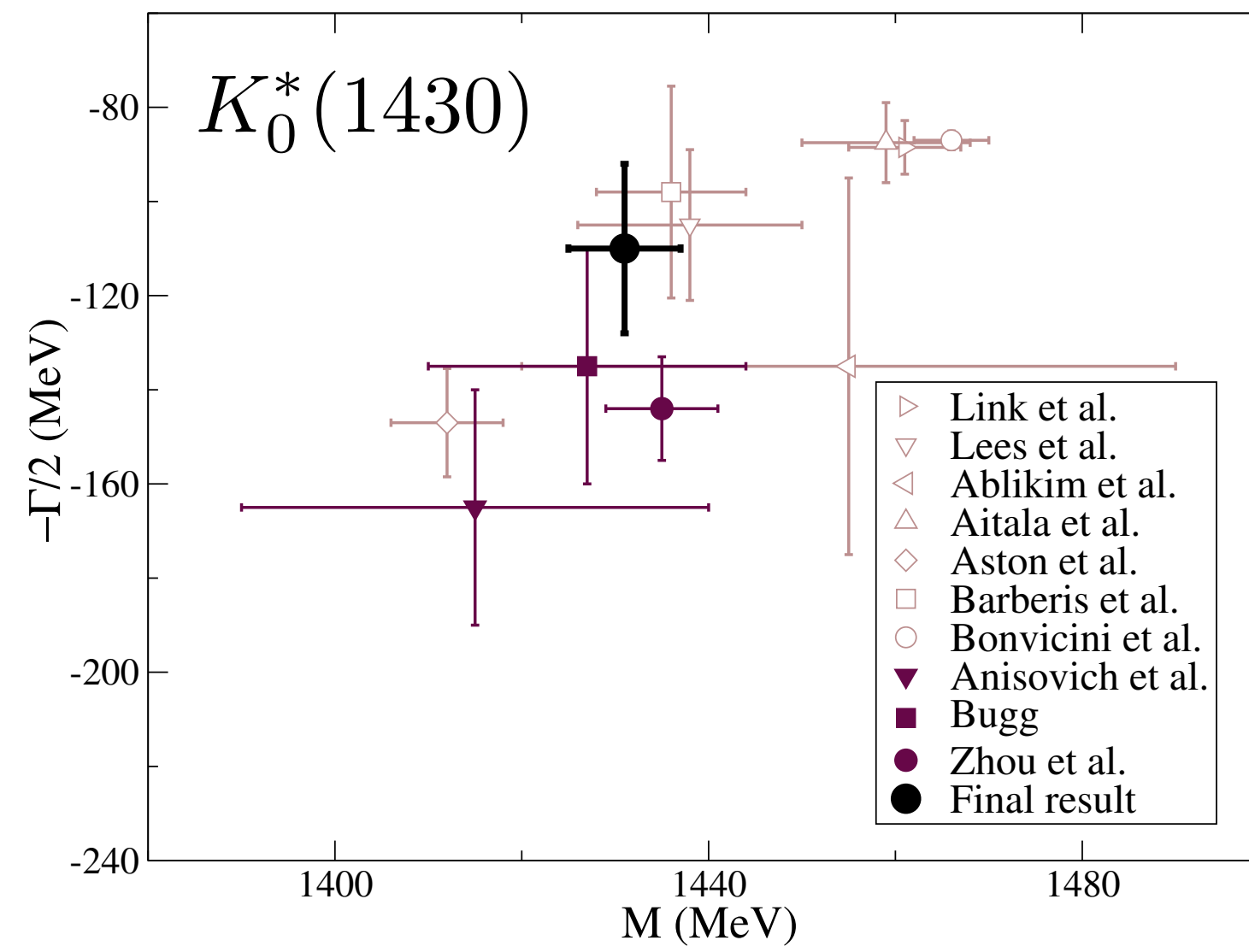


# Main problem: Other resonances

Heavier  $K^*$  are poorly known

Different production mechanisms lead to not very consistent results

Scattering should provide a clean extraction of these states



# Recent applications: Heavier particles

$K_0^*$

Second lightest resonance

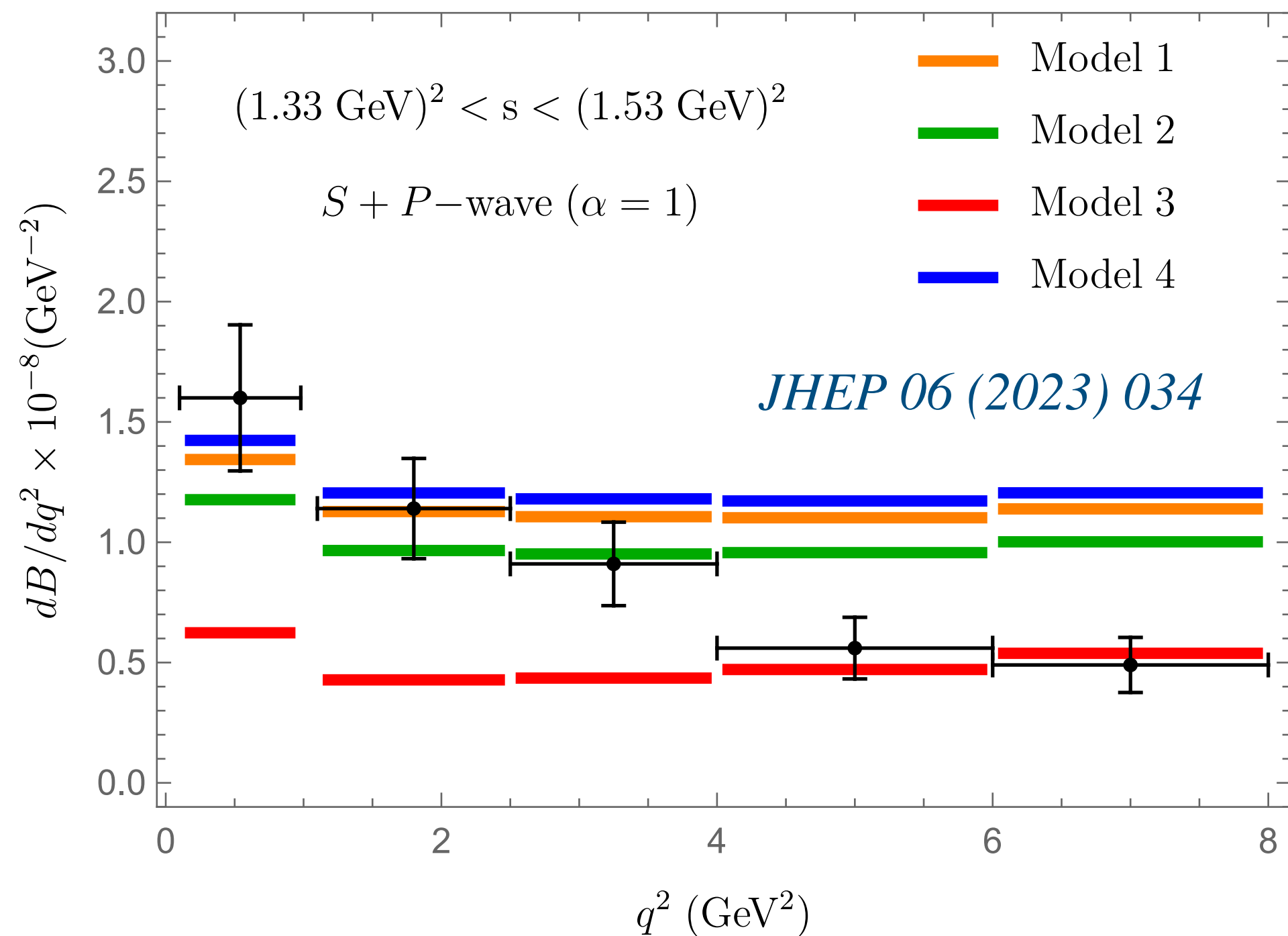
Extremely broad  $\rightarrow$  extremely short-lived

Correlated with chiral symmetry-breaking phenomena

Not well-understood  $\rightarrow$  new observables ??

## Input to hadron physics observables

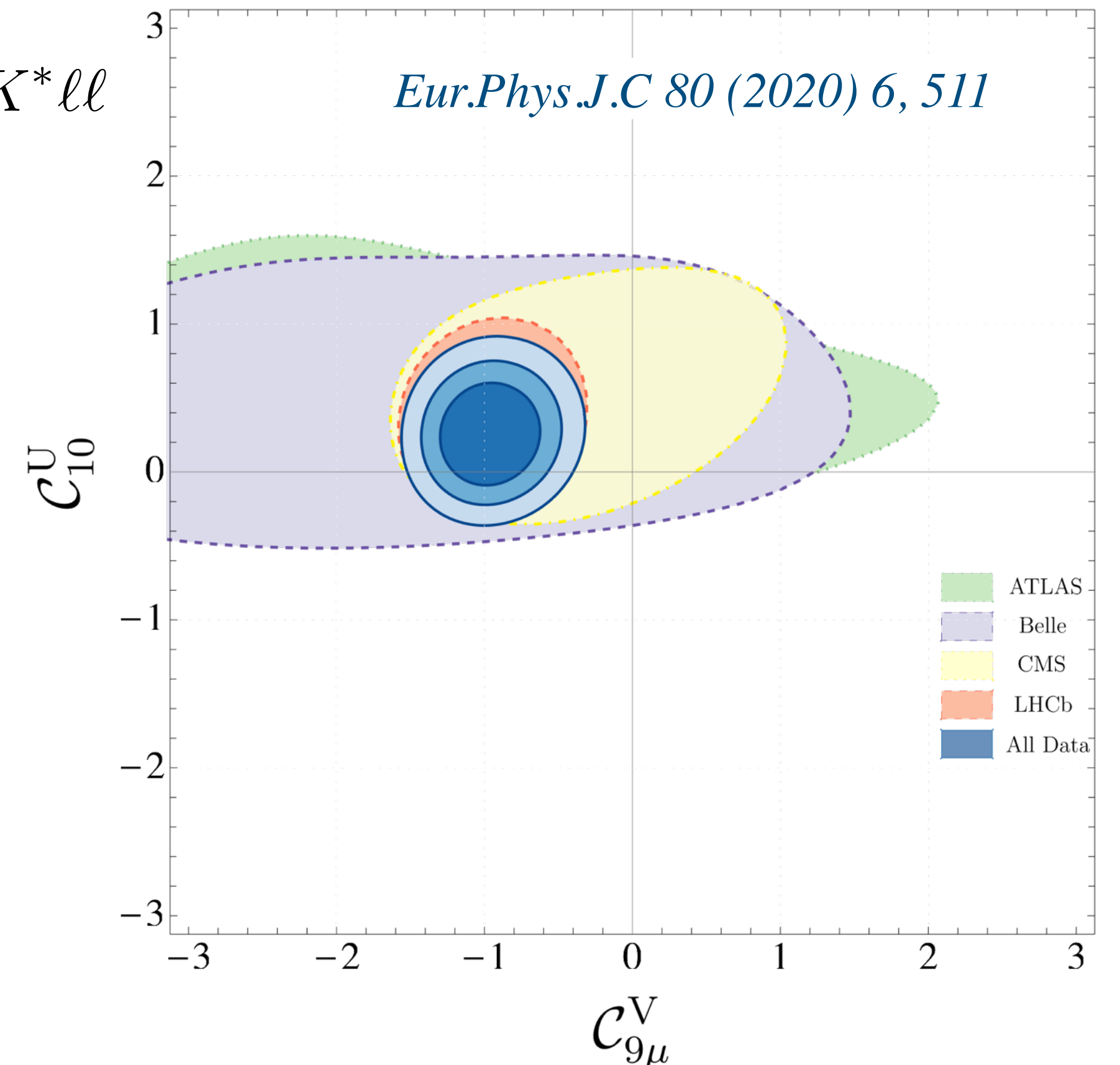
$B \rightarrow K\pi$



$B \rightarrow K^* \ell \ell$

$K_0^*$

$K^*$



# Recent applications: Heavier particles

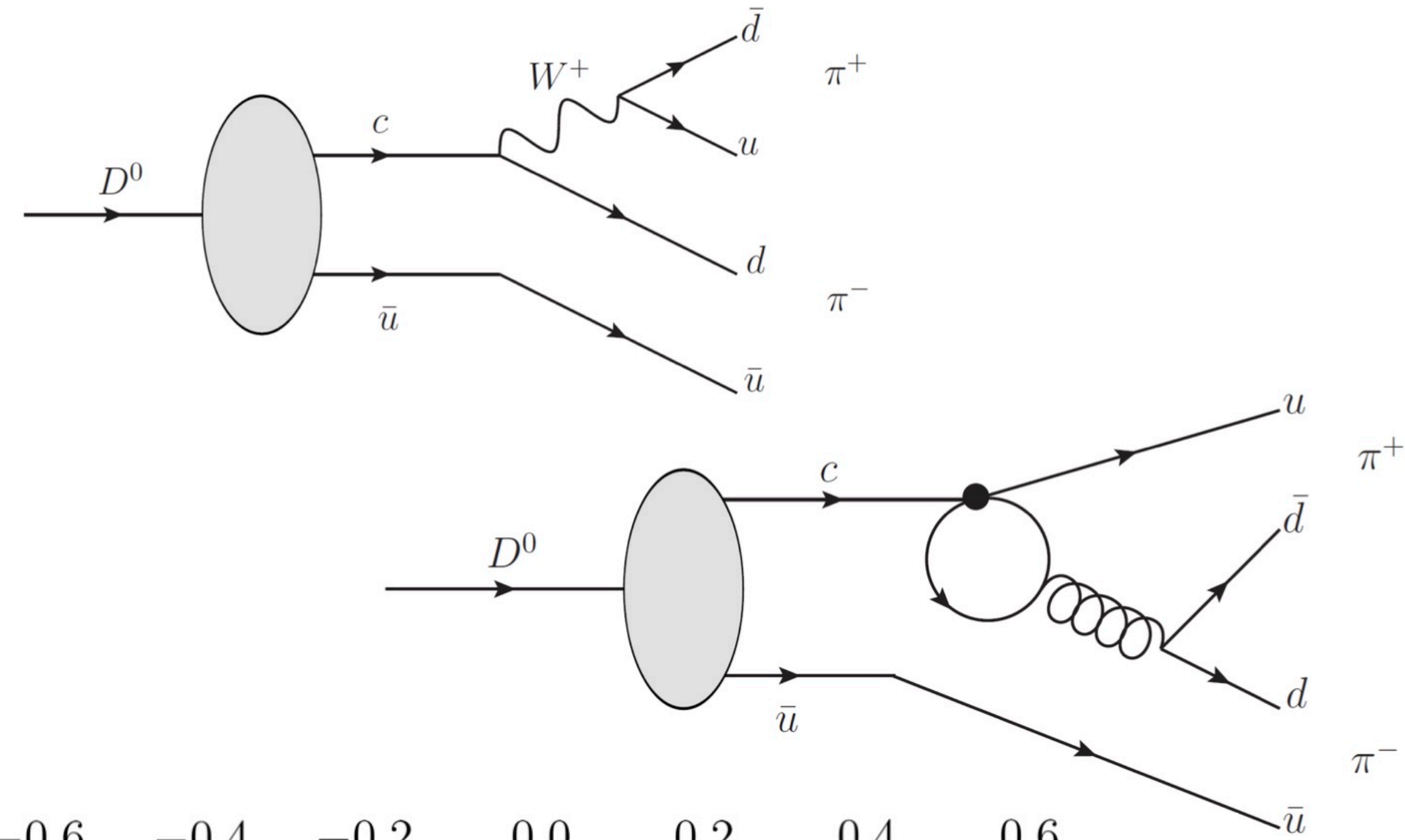
## Understanding decays of charmonia and bottomonia

Lineshapes dominated by final state interactions (FSI)

$$D^0 \rightarrow \pi^+ \pi^- \quad D^0 \rightarrow K^+ K^-$$

$$B^\pm \rightarrow K^\pm \pi^+ \pi^- \quad B^\pm \rightarrow \pi^\pm K^+ K^-$$

$$B^+ \rightarrow \pi^+ \pi^+ \pi^-$$



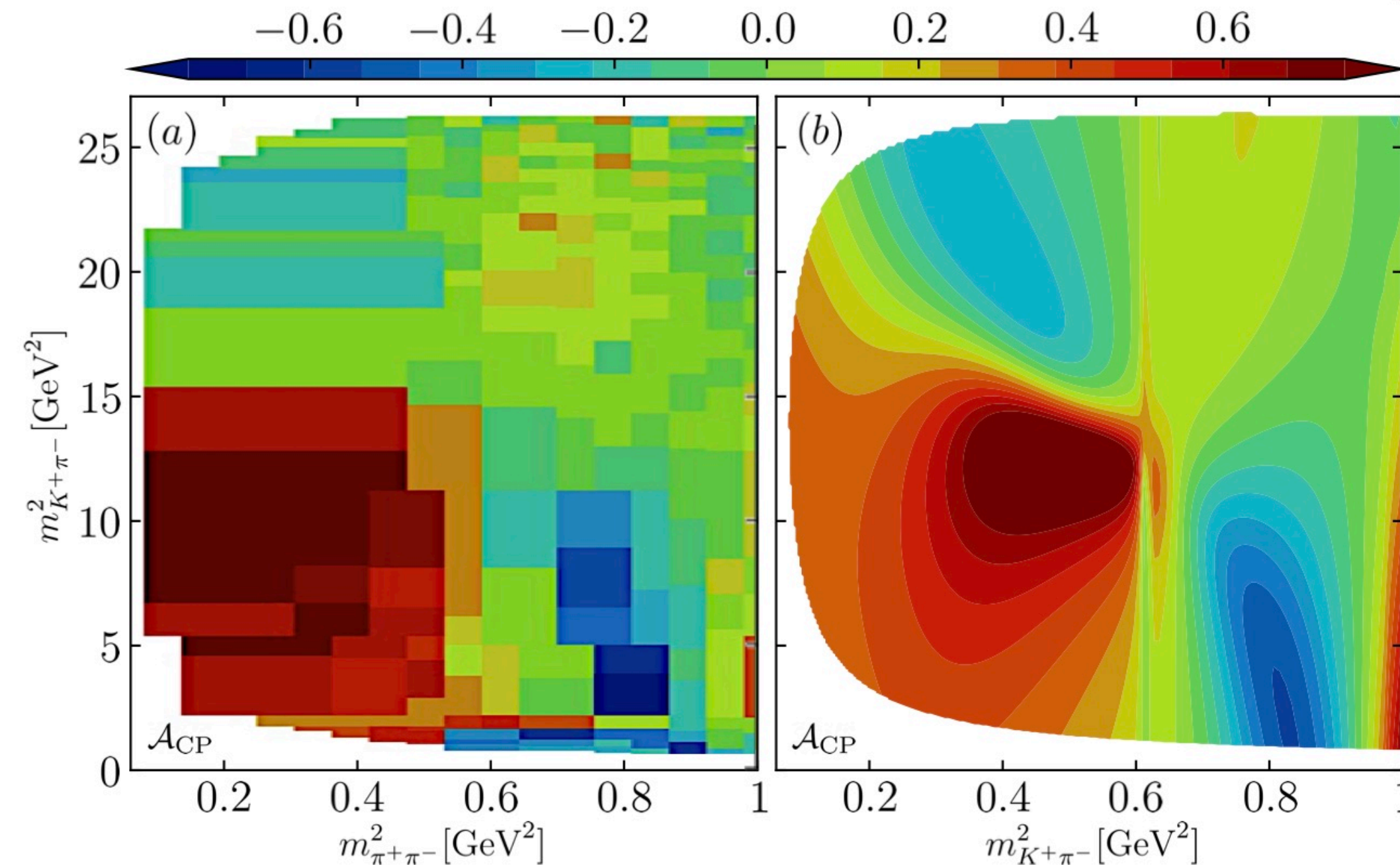
## Understanding CP violation in this sector

Largest violating signal in these decays

*Phys.Rev.Lett.* 130 (2023) 20, 201901

*Phys.Rev.Lett.* 136 (2026) 11, 111901

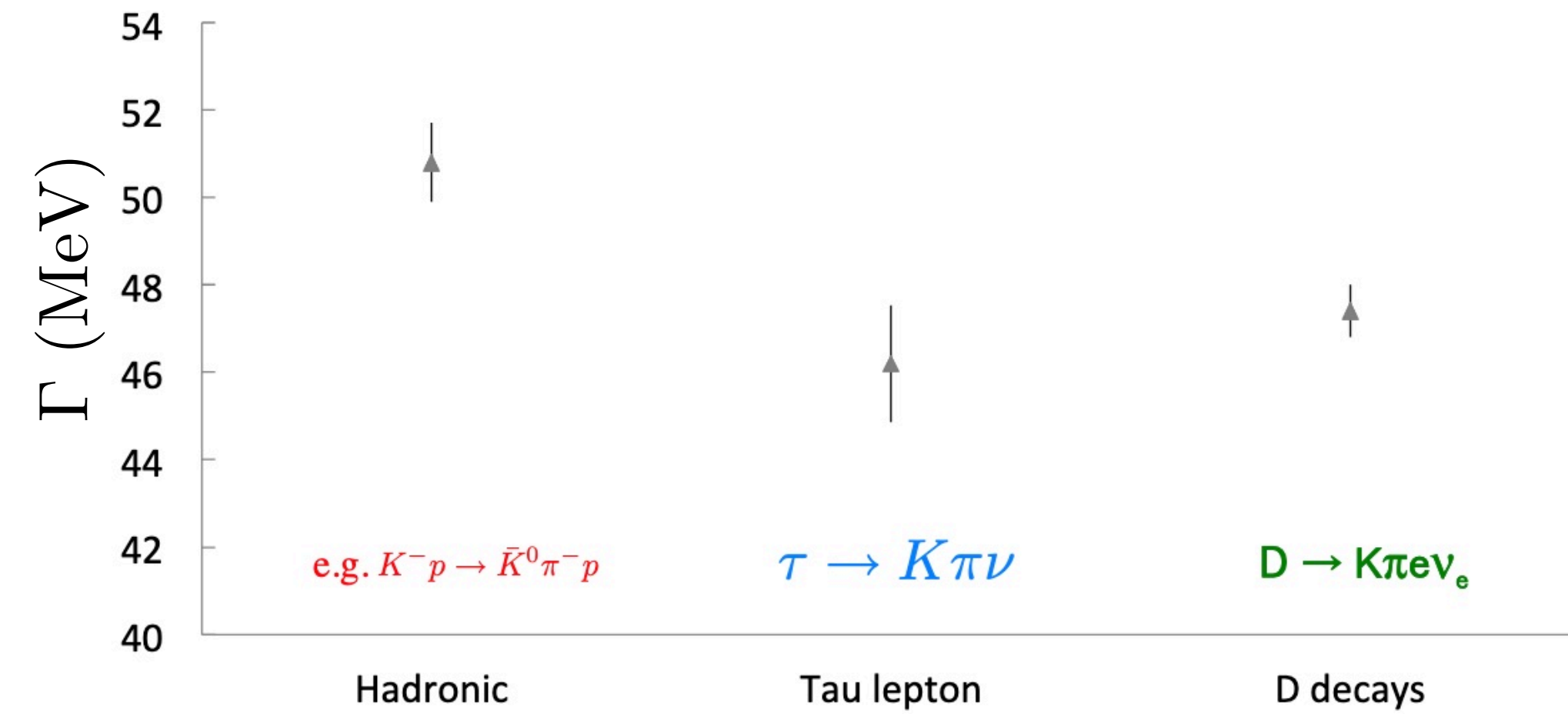
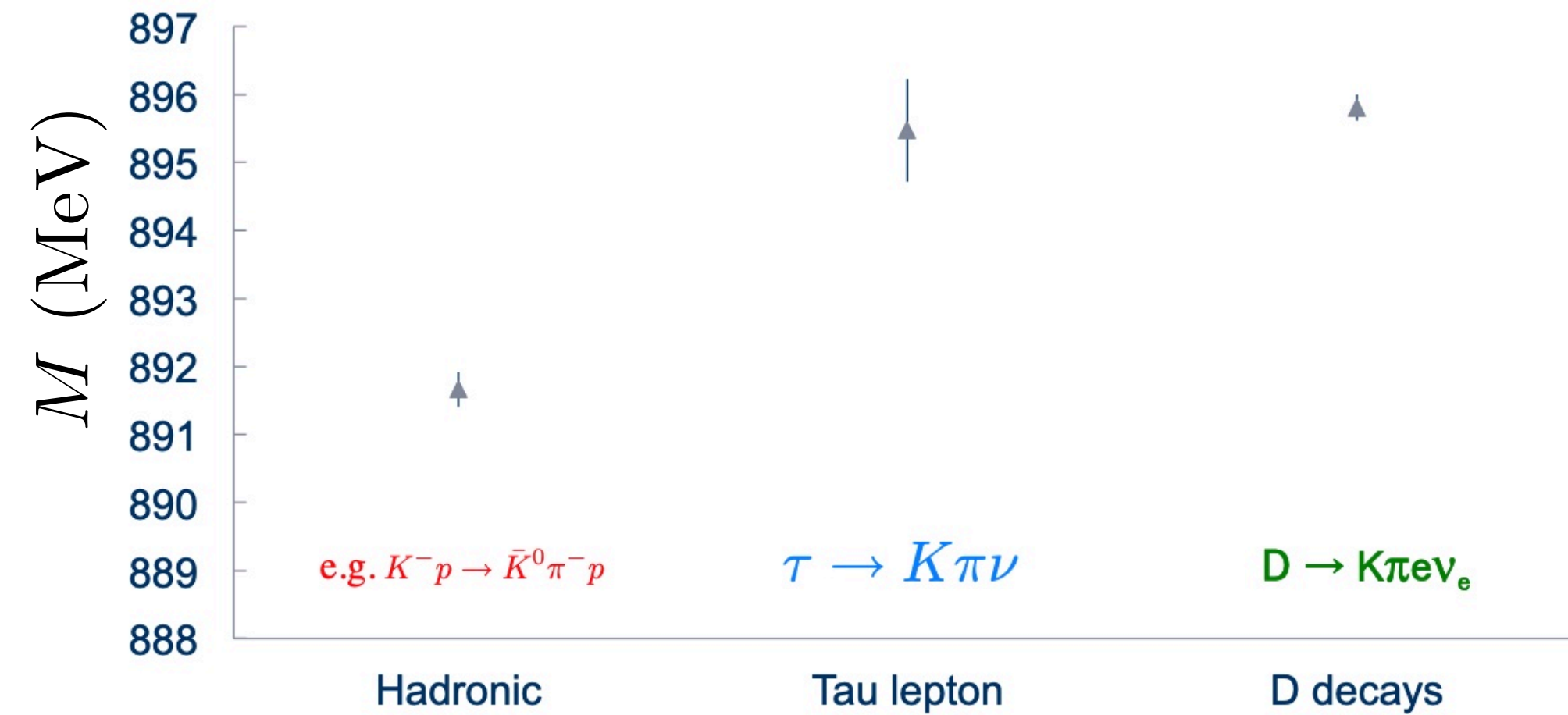
## Other decays related to precision physics



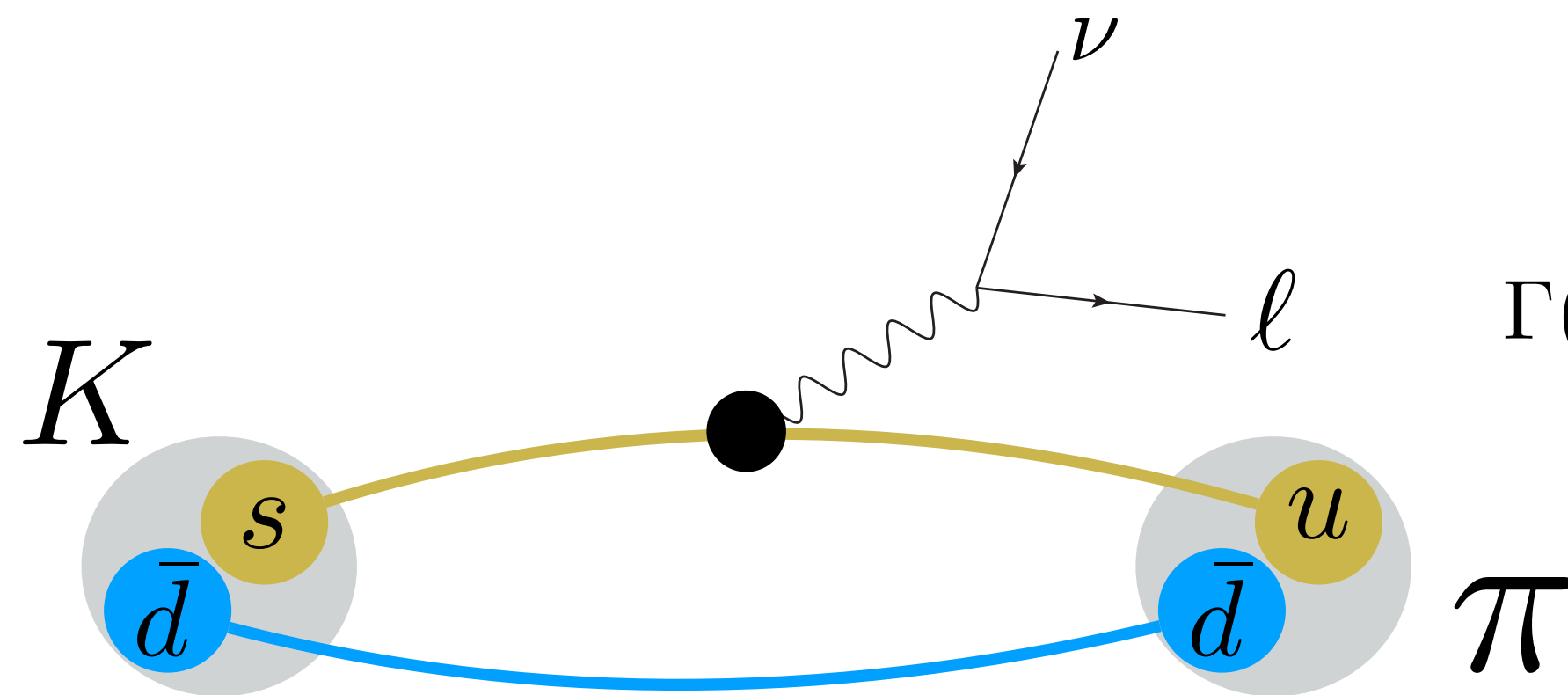
$$B^\pm \rightarrow K^\pm \pi^+ \pi^-$$

# Other applications: EW physics

Not even the narrow  $K^*(892)$  is so well known



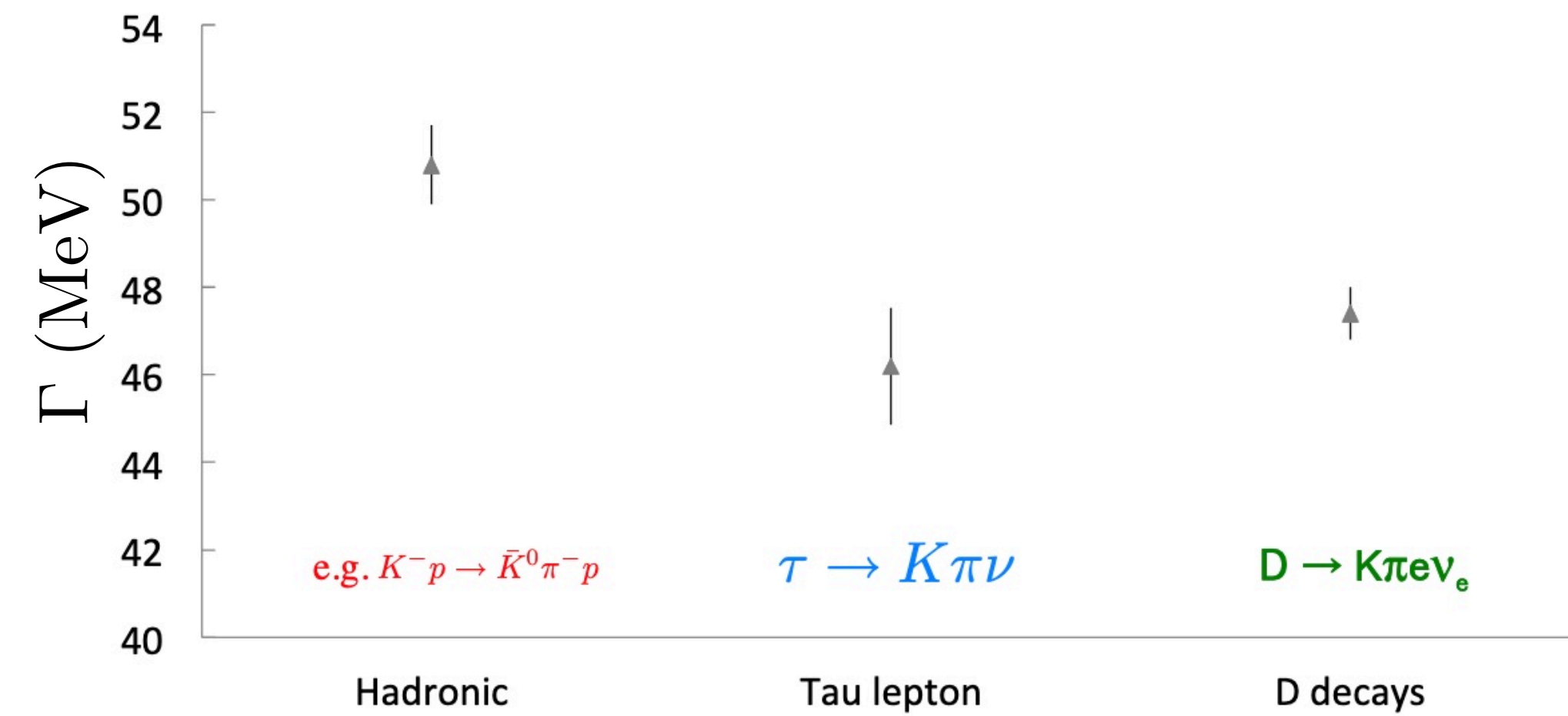
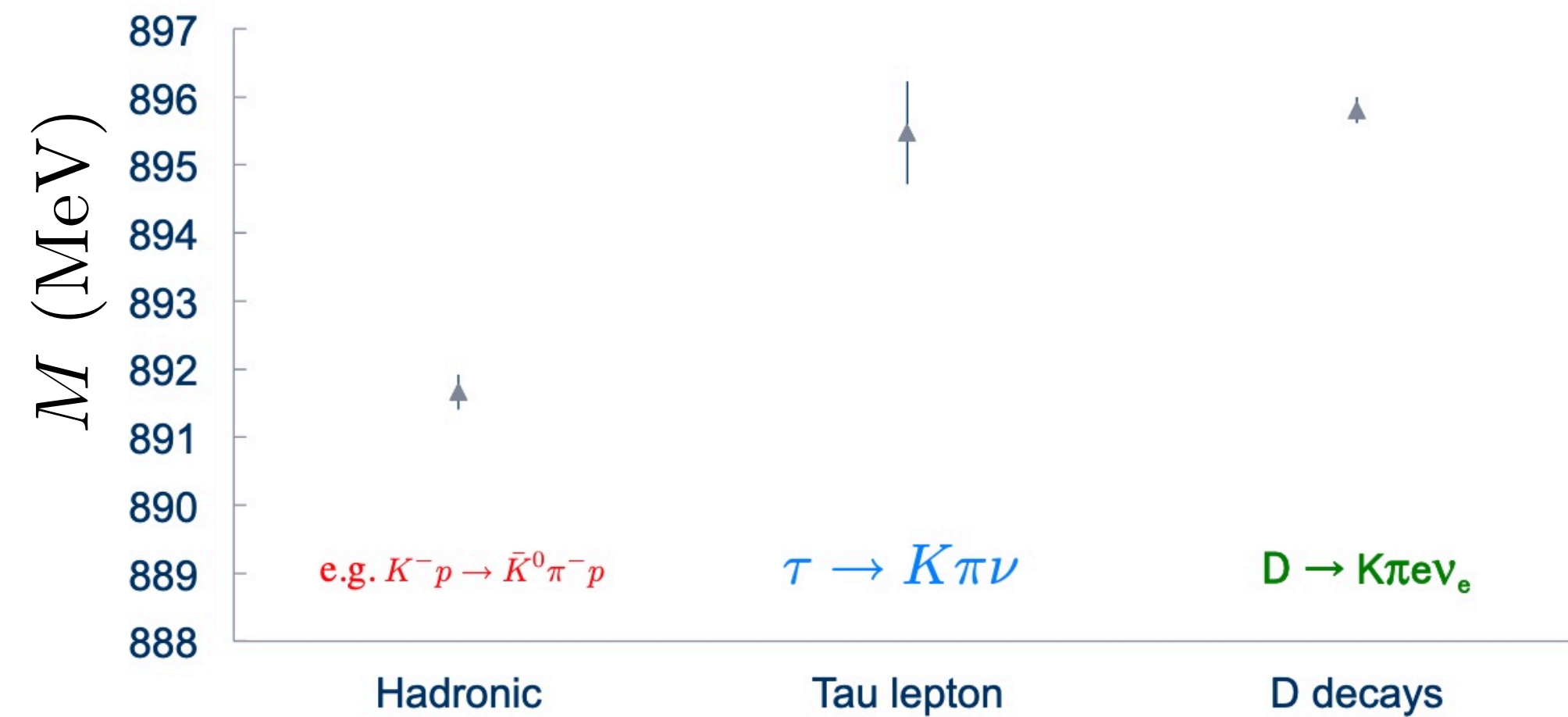
Flavor determinations and BSM searches often rely on hadronic determinations as input



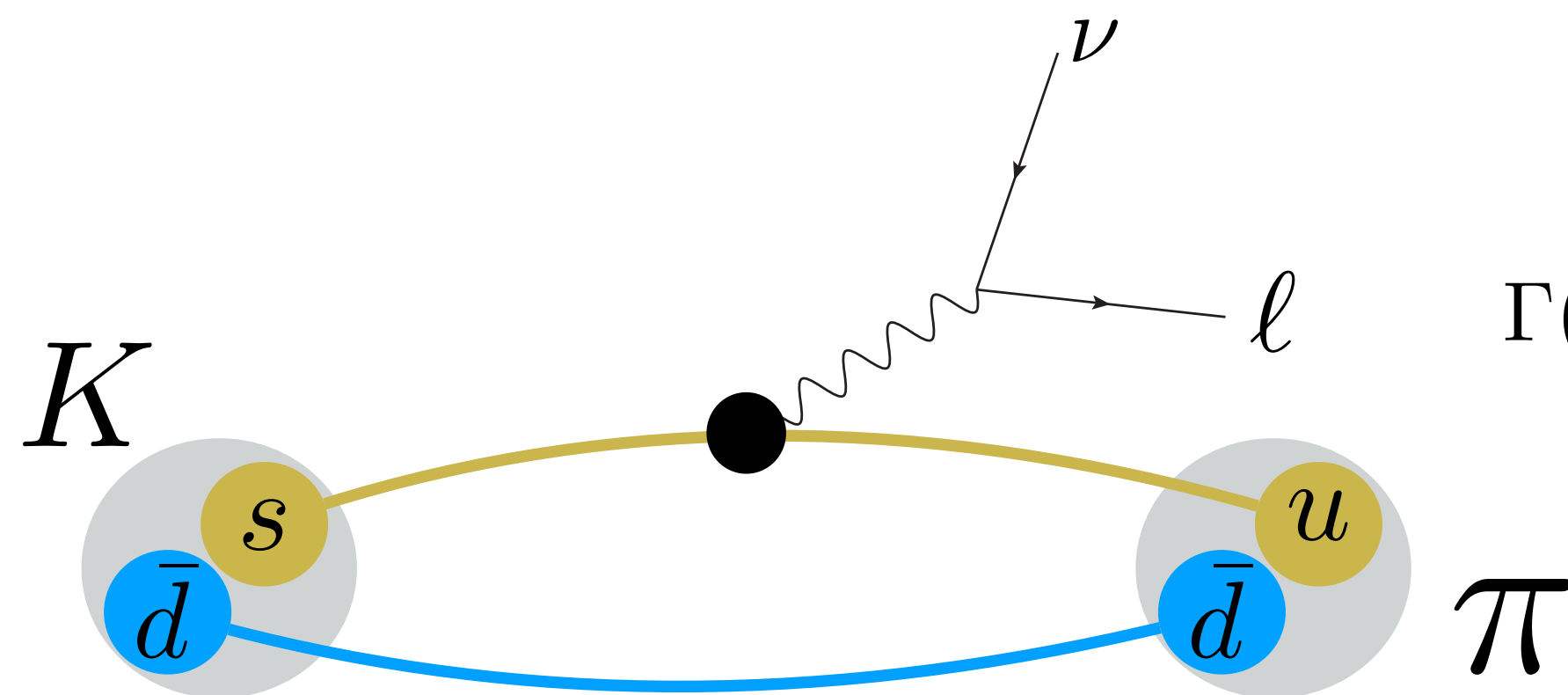
$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192 \pi^3} C_K^2 S_E^K |V_{us}|^2 \left| f_+^{K^0 \pi^-}(0) \right|^2 I_K^l \left( 1 + \delta_{\text{EM}}^{Kl} + \delta_{\text{SU}(2)}^{K\pi} \right)^2$$

# Other applications: EW physics

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Flavor determinations and BSM searches often rely on hadronic determinations as input



$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192 \pi^3} C_K^2 S_E^K \overset{\text{CKM}}{\left( |V_{us}|^2 \right)} \left| f_+^{K^0 \pi^-}(0) \right|^2 \overset{\text{Vector+Scalar}}{I_K^l} \left( 1 + \delta_{\text{EM}}^{Kl} + \delta_{\text{SU}(2)}^{K\pi} \right)^2$$

**Vector+Scalar  $\pi K$  form factors**

# Recent applications: Femtoscopy

**ALICE femtoscopy program is devoted to resonance structure studies**

*Phys.Lett.B 856 (2024) 138915*

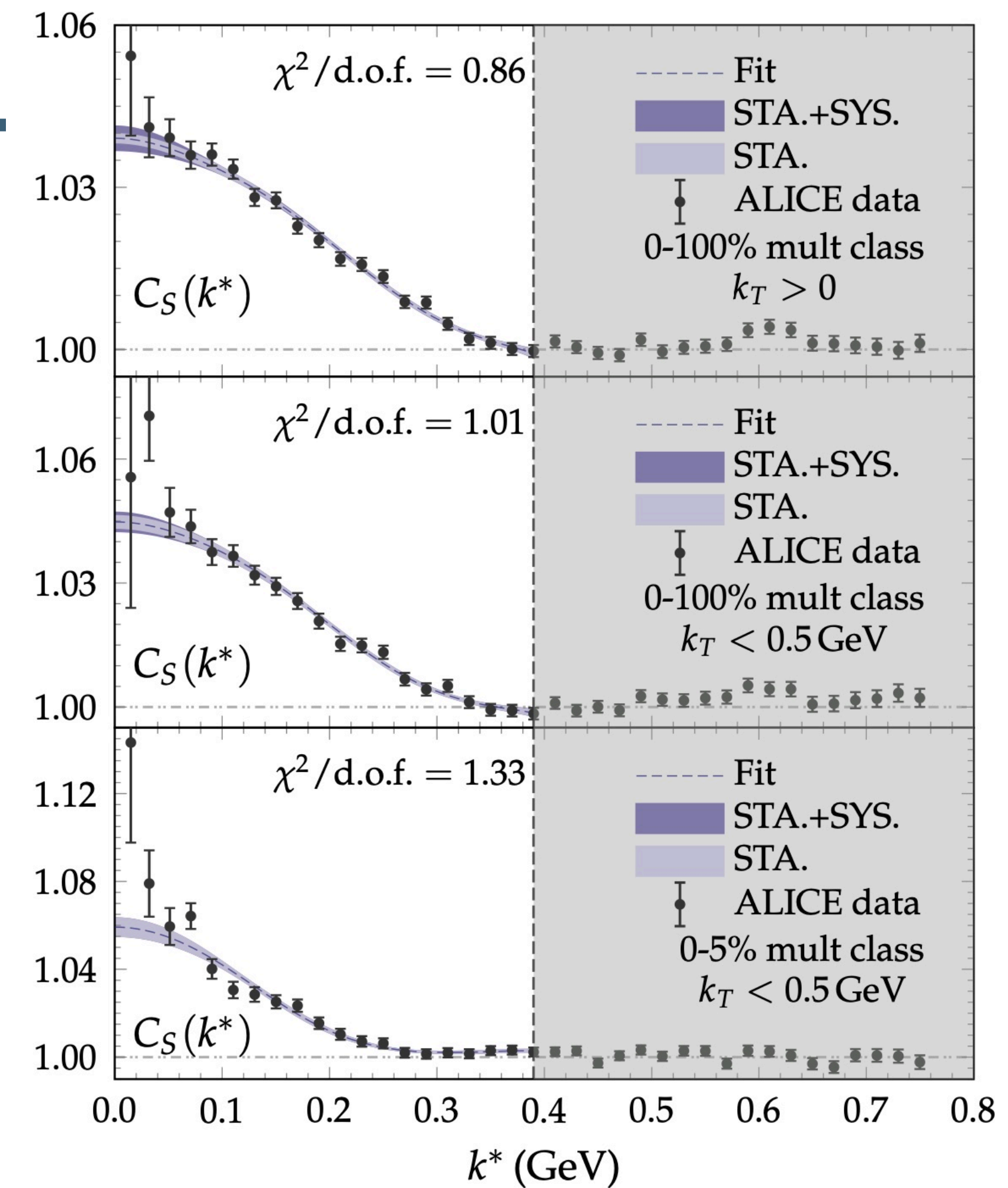
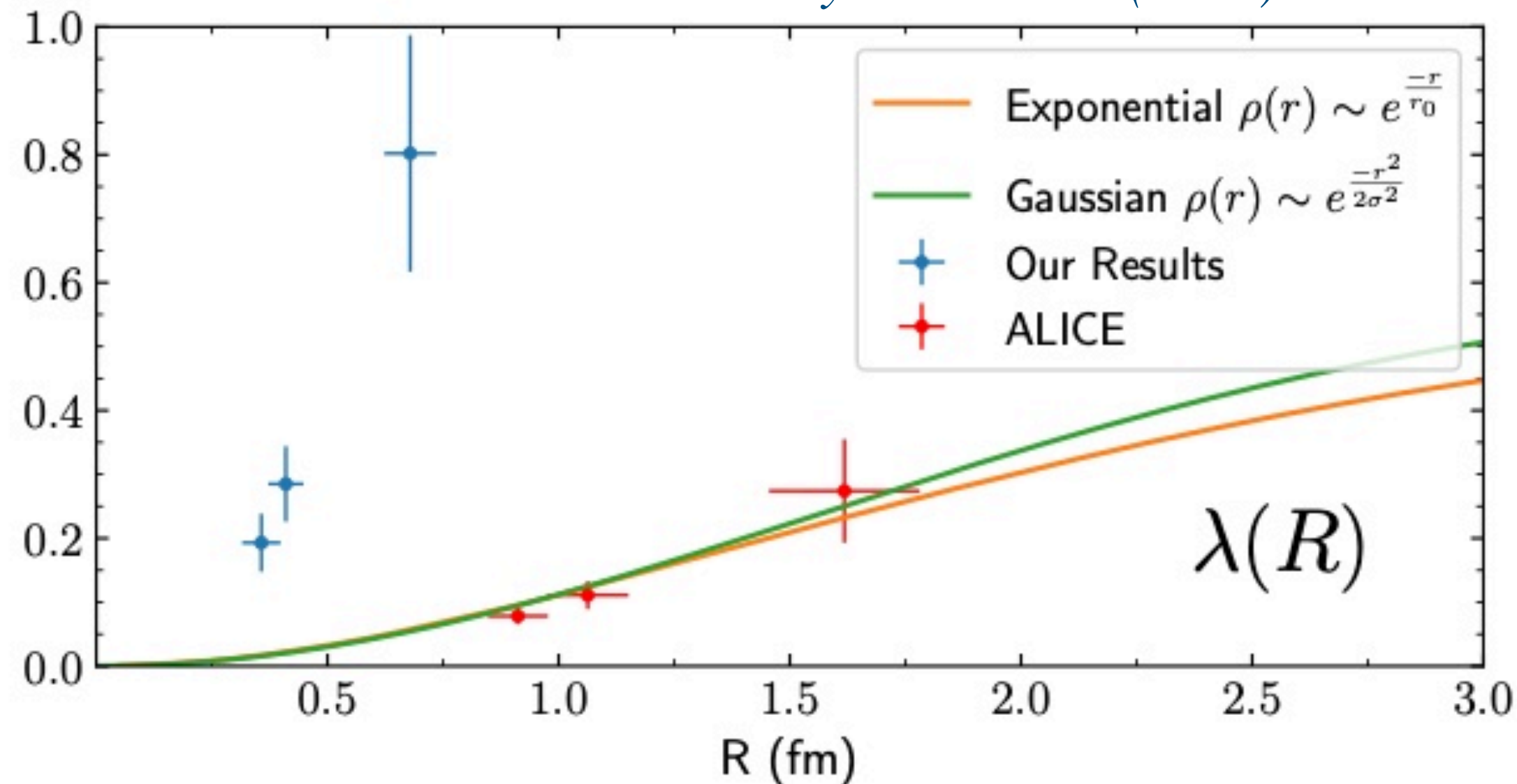
Understanding these correlations “gives\*” an idea of the size of the source producing them

$$C(k^*) = \int d^3r S(r) |\psi^*(k^*, r)|^2$$

$$S(r) \sum_i \left| \delta_{if} \psi^*(k^*, r) + \int dp \psi^*(p, k^*) f^{if}(p, k^*) \right|^2$$

*$\pi K$  scattering amplitude*

*Phys.Lett.B 866 (2025) 139552*



ALICE claimed evidence for a  $\kappa$  tetraquark, using naive amplitude analysis

The correct theory shows no ordinary  $q\bar{q}$  or tetraquark

## **Light meson spectroscopy is crucial to hadron physics**

“Exotic” matter identification and understanding

Heavier hadron decays at higher energy colliders

CP-violating processes, diffraction, Primakoff, etc.

Precision and BSM physics studies

## **We do what we can, but data is not great**

Unresolved tensions

Unkown systematics

Uncovered energy regions

# Analyticity: Dispersion relations

**Our amplitude has two branch-cuts**

**Use Cauchy's theorem over contour C, assume that circular parts go to zero**

$$T(s, t, u) = \frac{1}{2\pi i} \int_C \frac{T(s', t, u'(s', t))}{s' - s} ds' + \text{maybe subtractions}$$

**How is this useful?? → “hooks” are given by (Schwarz reflection principle)**

$$T(s + i\epsilon, t, u - i\epsilon) - T(s - i\epsilon, t, u + i\epsilon) = 2i \operatorname{Im} T(s, t, u) \longrightarrow \text{data}$$

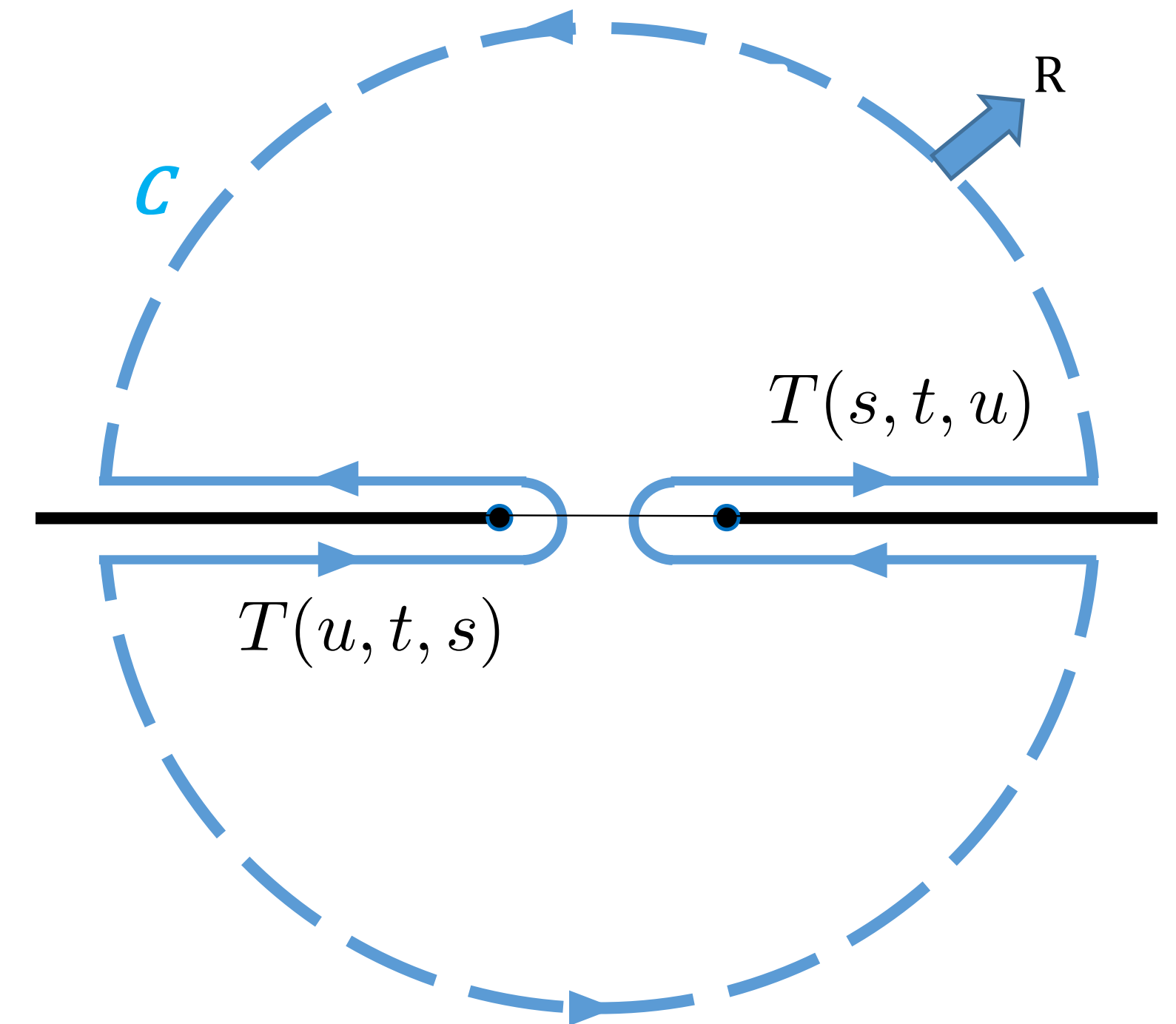
**Project the integral into partial waves to get your dispersion relations (ex. Roy eqs.):**

$$\frac{t_\ell^I(s)}{\quad} \rightarrow \frac{\tilde{t}_\ell^I(s)}{\quad} = \tau_\ell^I(s) + \sum_{I', \ell'} \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s', s) \operatorname{Im} t_{\ell'}^{I'}(s')$$

Initial fit to data

Final dispersive output

$s$  – plane (fixed  $t$ )



# Roy vs Steiner equations

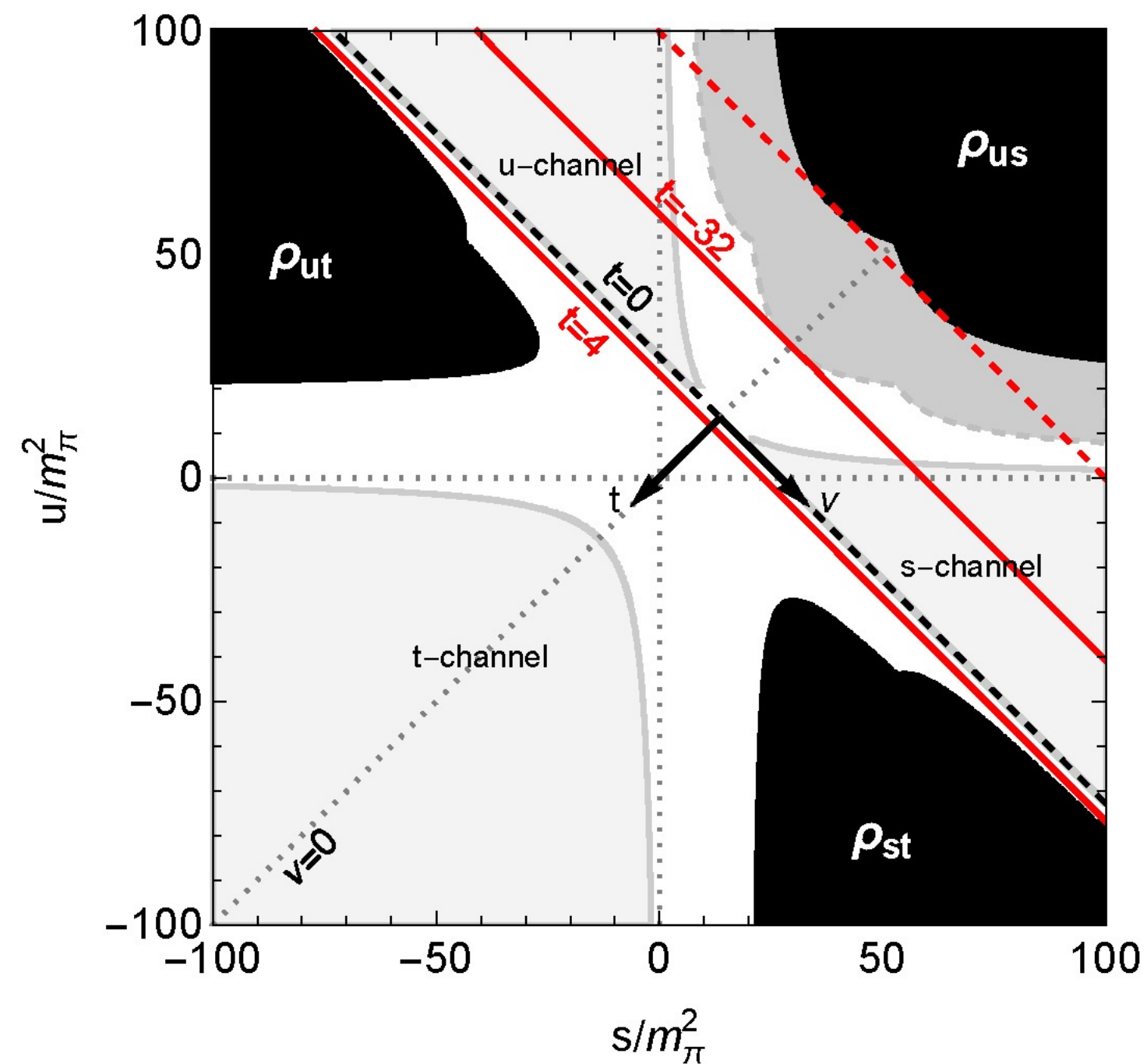
Both impose exact crossing in s-u, not everywhere

**Roy eqs:** *Roy Phys.Lett. B 36 (1971)*

Fixed t

Limited validity region when masses are unequal

Used for  $\pi\pi$



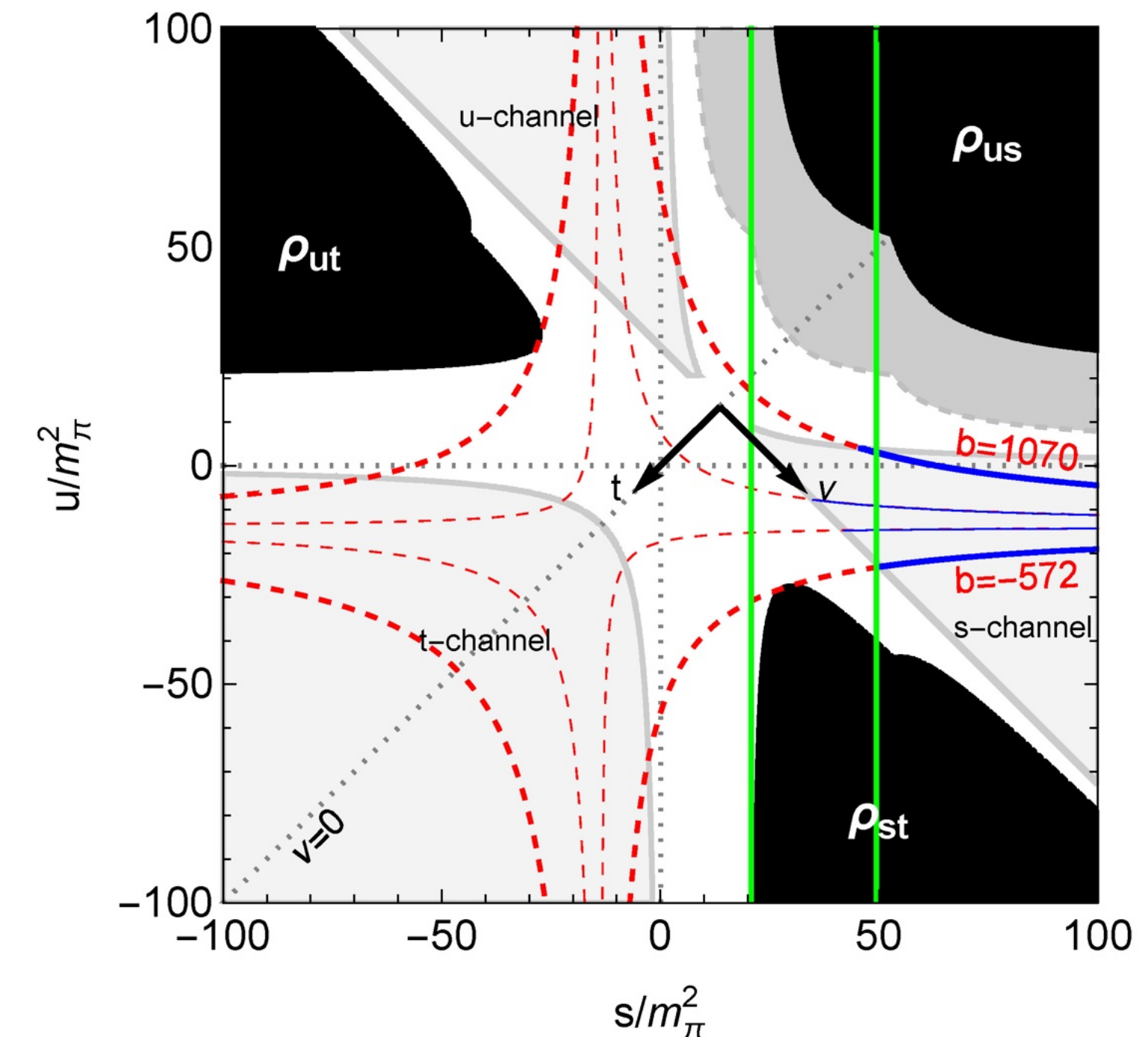
**Hite/Steiner eqs:** *Nuovo Cim.A 18 (1973)*

$$(s - a)(u - a) = b$$

More cumbersome than fixed-t

Regge contributions tend to be more suppressed

Used for  $\pi K$  and  $\pi N$



# DRs: How to ?

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## 1- Select a relevant basis of partial waves (at relatively low energy)

$$T^I(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) t_{\ell}^I(s) P_{\ell}(\cos \theta_s) \quad t_{\ell}^I(s) \sim q^{\ell}(s)$$

## 2- Fit them

## 3- Select/build a Regge description of high-energy region

$$\int_{4m_{\pi}^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s', s) \text{Im} t_{\ell'}^{I'}(s') = \int_{4m_{\pi}^2}^{\Lambda} ds' \dots + \underbrace{\int_{\Lambda}^{\infty} ds' \dots}_{\text{Regge}}$$

## 4- Define a sensible approach to solve/constrain these DRs

Should we solve them?

Should we use them as constrain?

Should we use them to select?

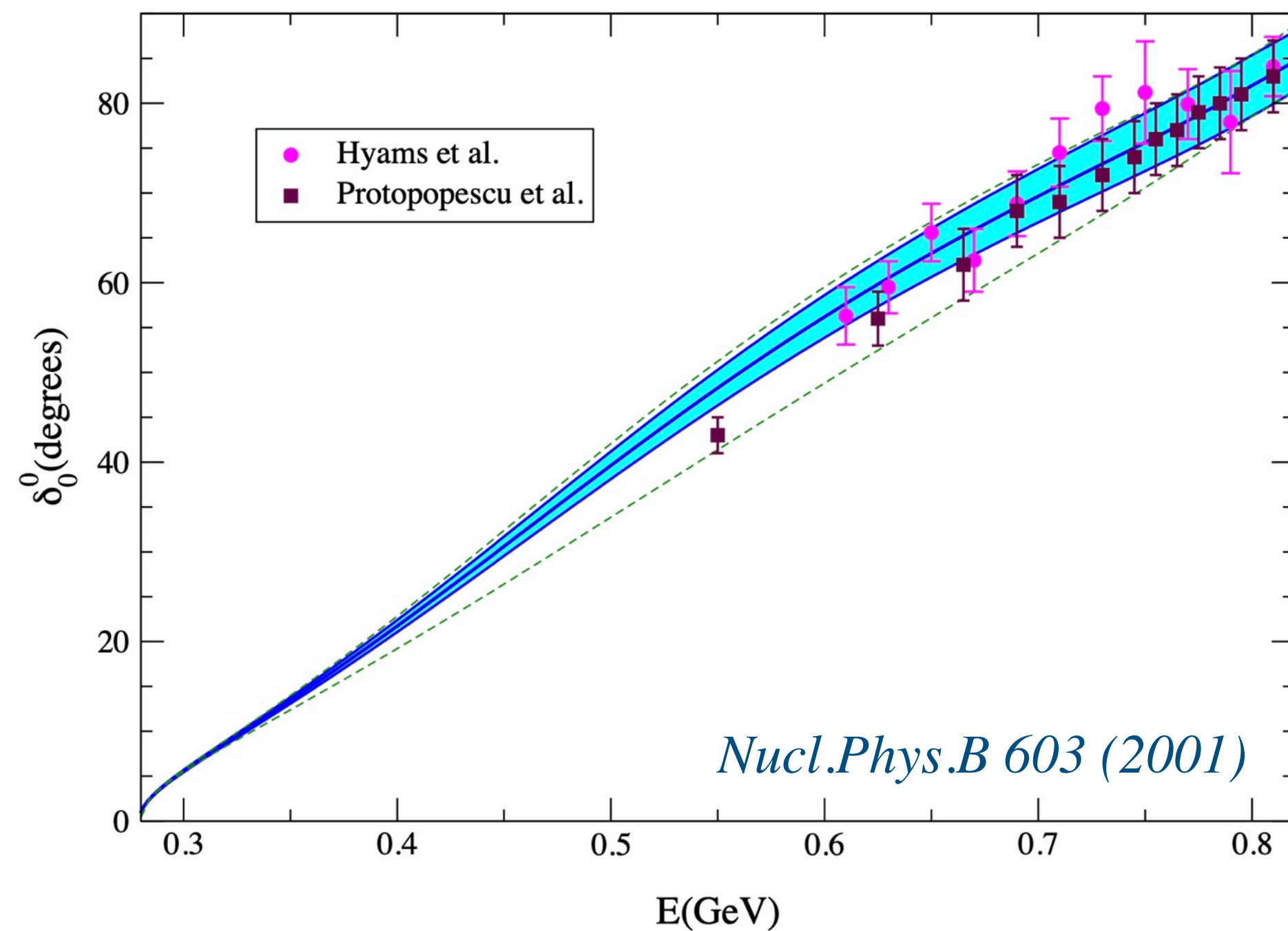
## 5- Use your DR to obtain observables !!

$$t_{\ell}^I(s) \rightarrow \tilde{t}_{\ell}^I(s)$$

# DRs: Solving vs constraining

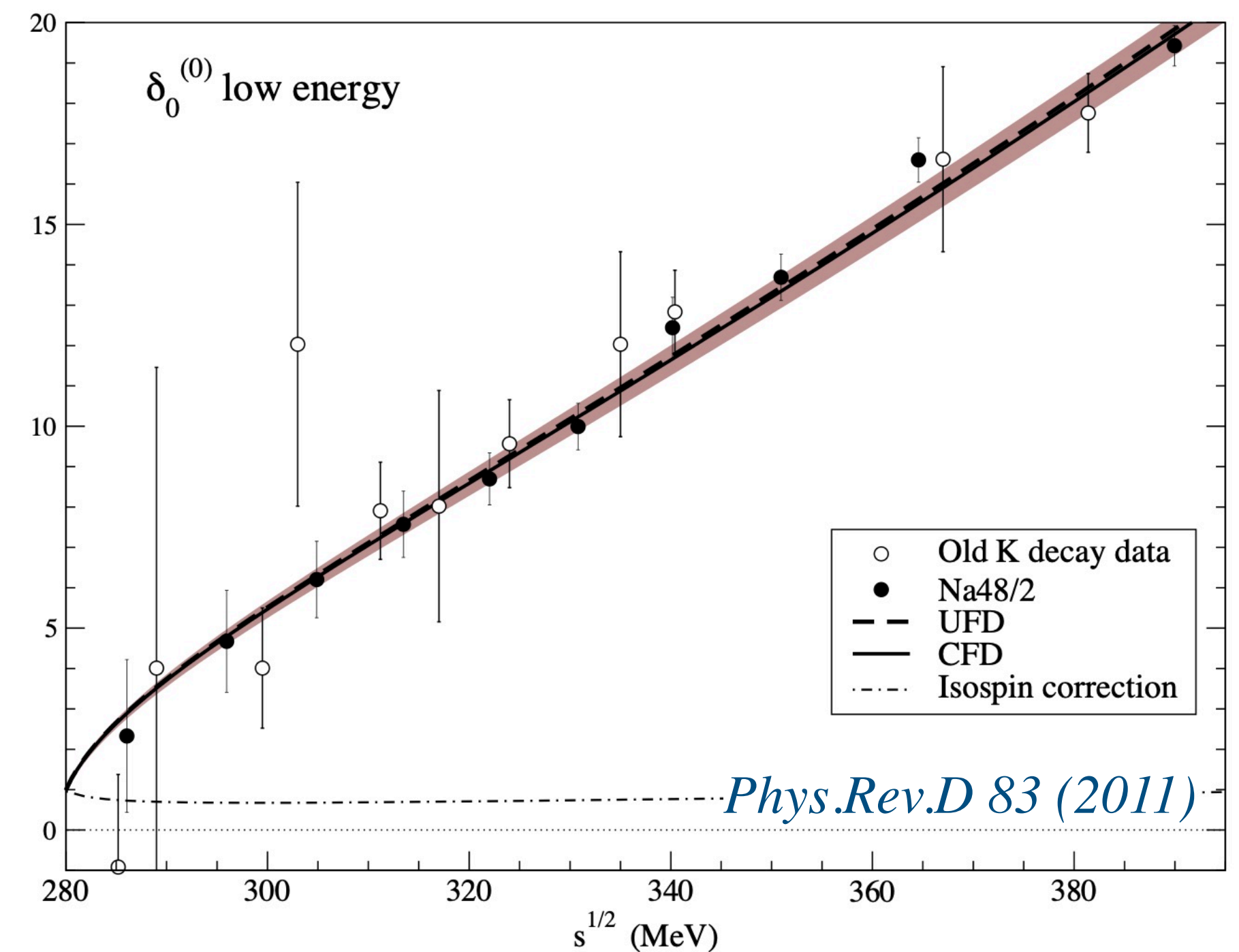
## Solving:

- ✓ Unique solution
- ✓ Very accurate unitary fulfilment
- ✓ Phase shifts are a “prediction”
- ✗ Boundary conditions have to be set



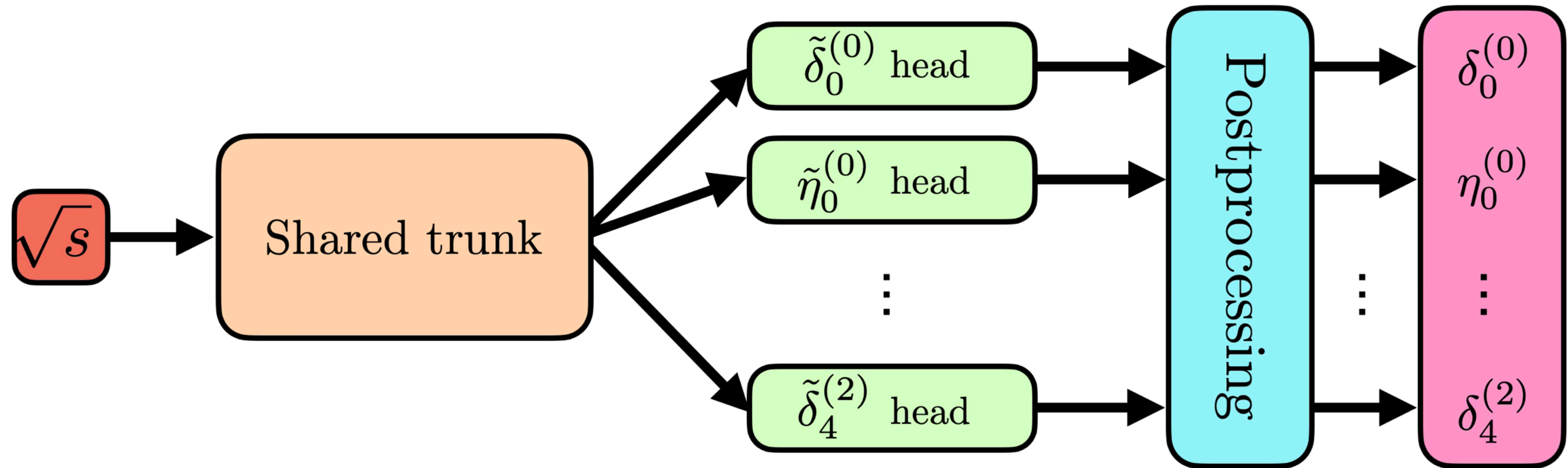
## Constraining:

- ✓ Data plays a central role
- ✓ Can select between incompatible experimental results
- ✗ Unitarity is slightly violated



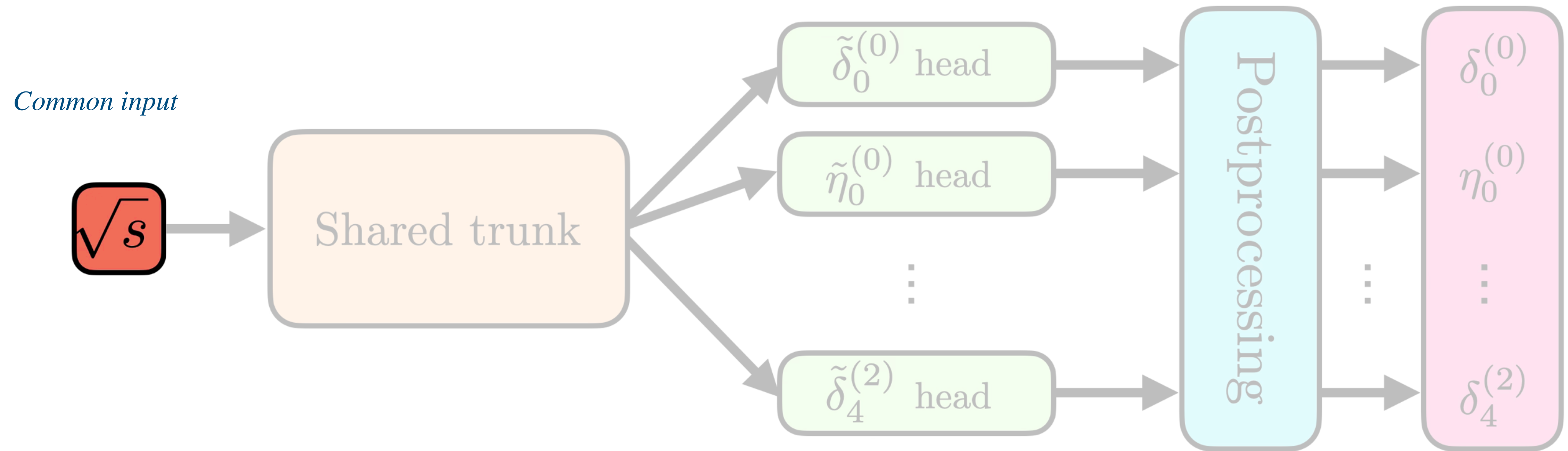
# $\pi\pi$ scattering: NN

## Hybrid multi-tasking architecture



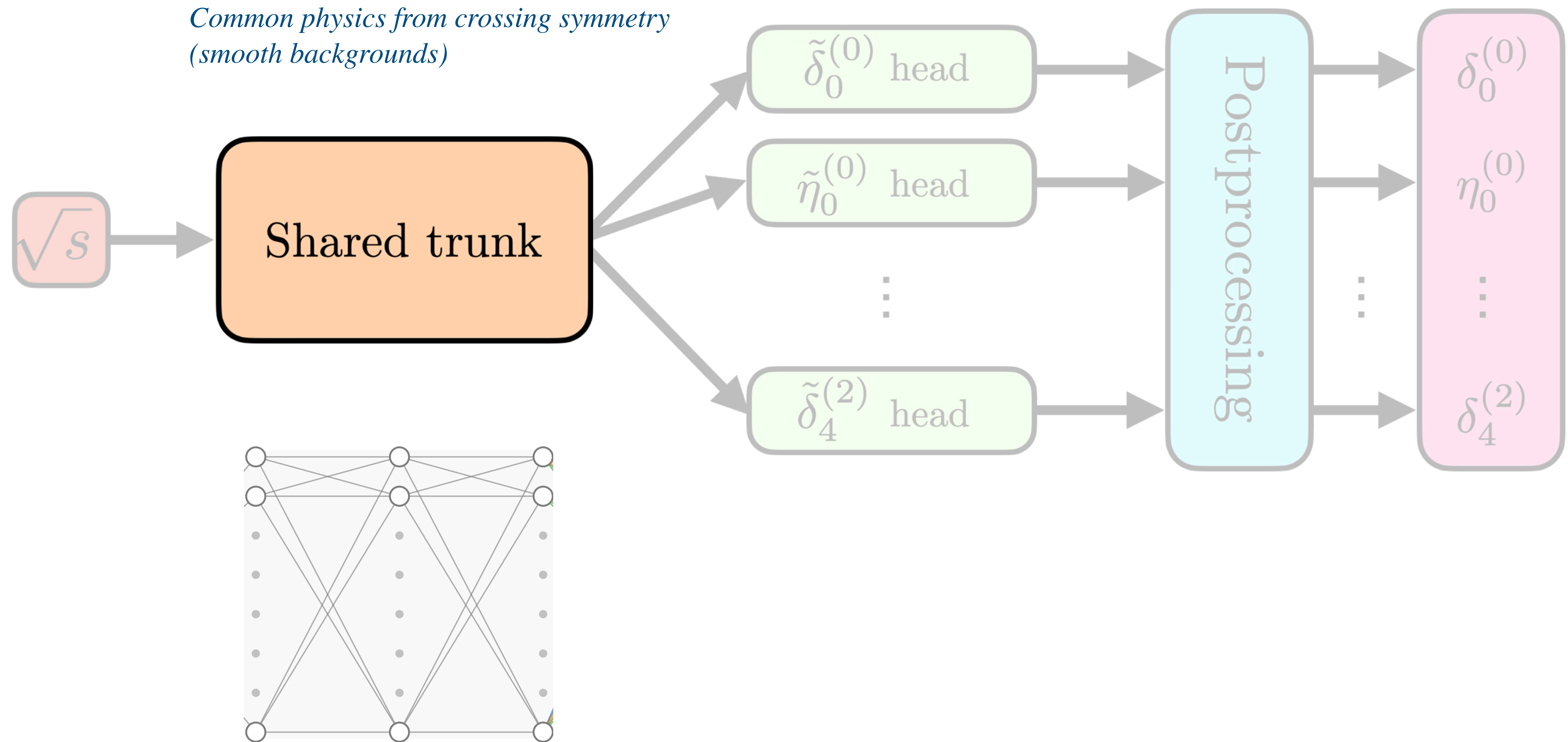
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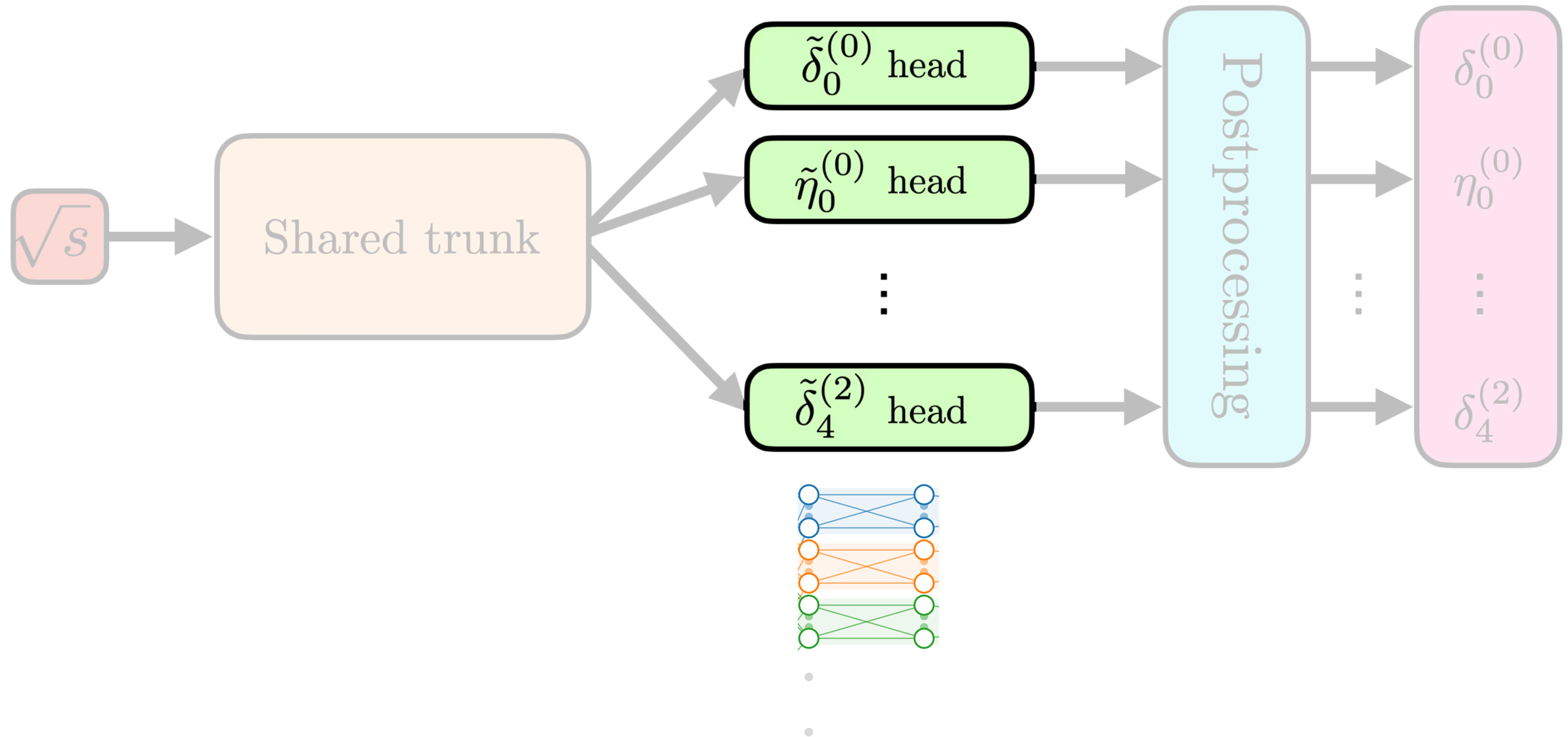
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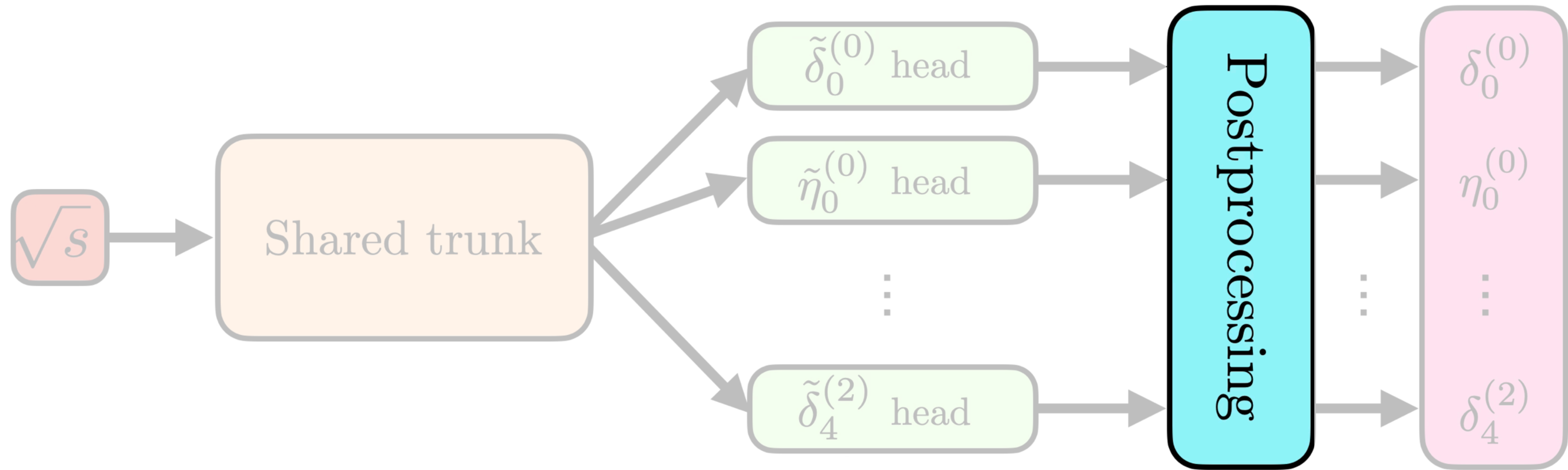
*Local dynamic effects (low correlation)*



# $\pi\pi$ scattering: NN

## Hybrid multi-tasking architecture

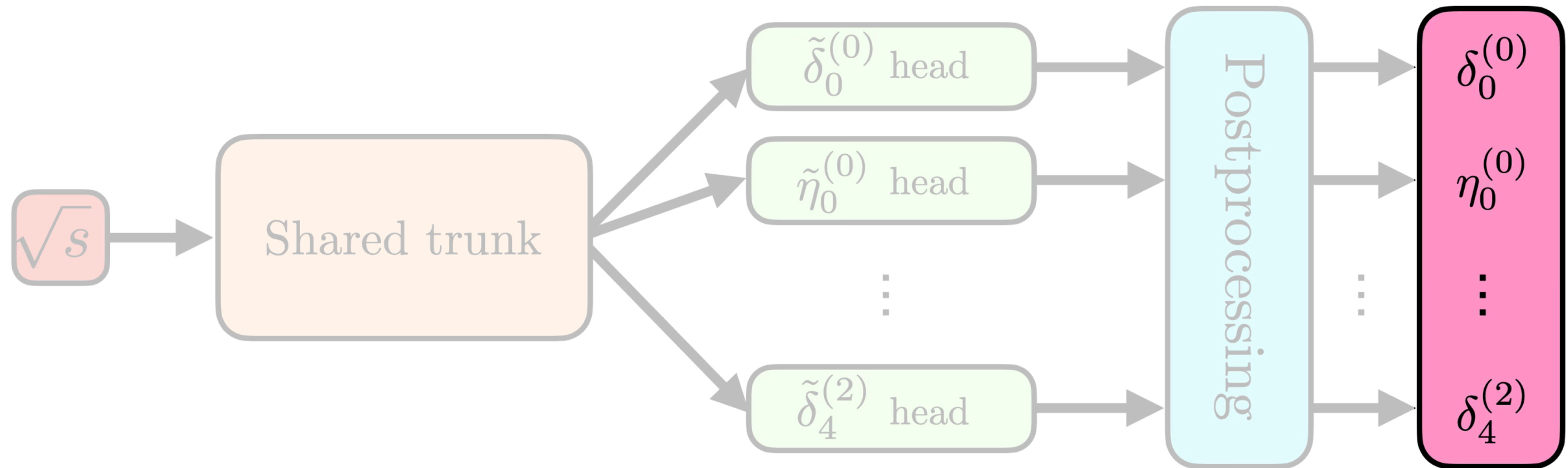
*Enforcing kinematic behaviors and unitarity*



# $\pi\pi$ scattering: NN

## Hybrid multi-tasking architecture

*NN lineshapes*



# $\pi\pi$ scattering: data grouping

We have: flexible NN that fits data, while respecting S-matrix principles, for “all” waves, at once

Perfect “model” to asses if data is compatible or incompatible

*Not possible with customary PWA models:*

- *PWs are disconnected, but data lives in all at once*
- *Not flexible enough, could be data problems, could be model problems*
- *Model testing does not satisfy S-matrix for each test*

## 1- Data reweighting and resampling

*We now use this framework to group data by “overall distance”*

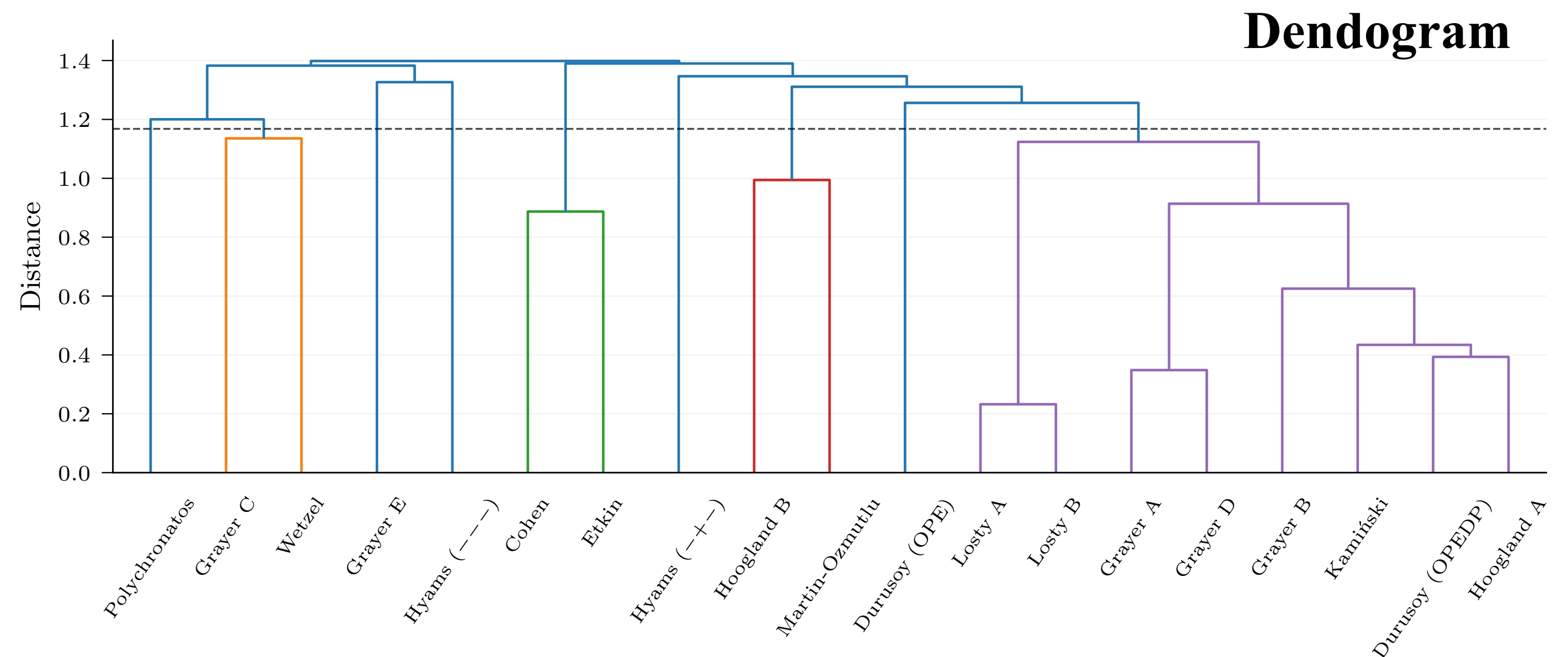
*Start by reweighting experiments and refitting*

*Determine how experiment  $I$  affects experiment  $j$*

$$z_{ij} = \frac{\langle \chi_{\text{disc},i}^2 \rangle_j - \langle \chi_{\text{base}}^2 \rangle_j}{\sqrt{s_{\text{base},j}^2/n_{\text{base}} + s_{\text{disc},j}^2/n_{\text{disc}}}}$$

*Symmetrize (made up) to create response matrix*

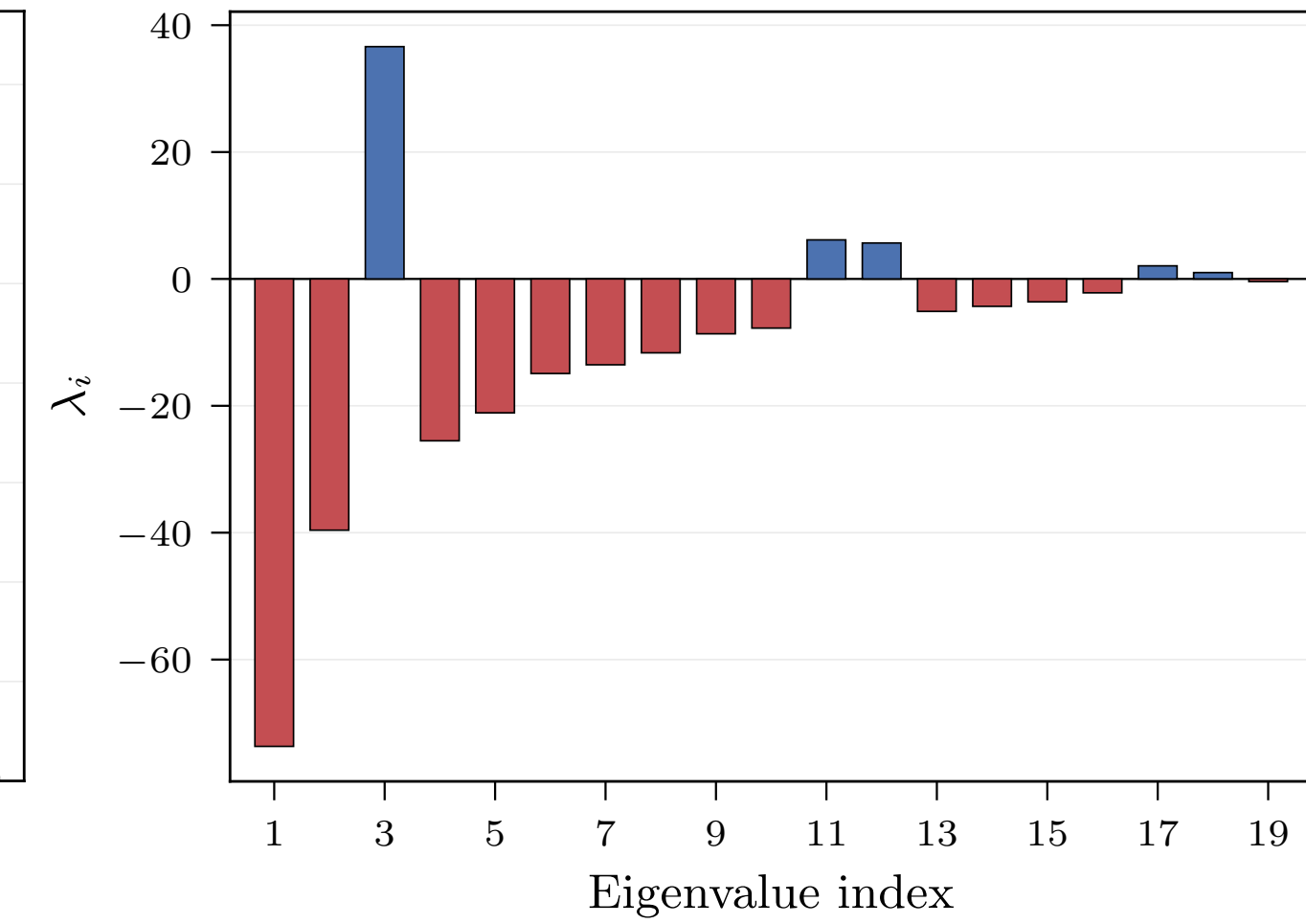
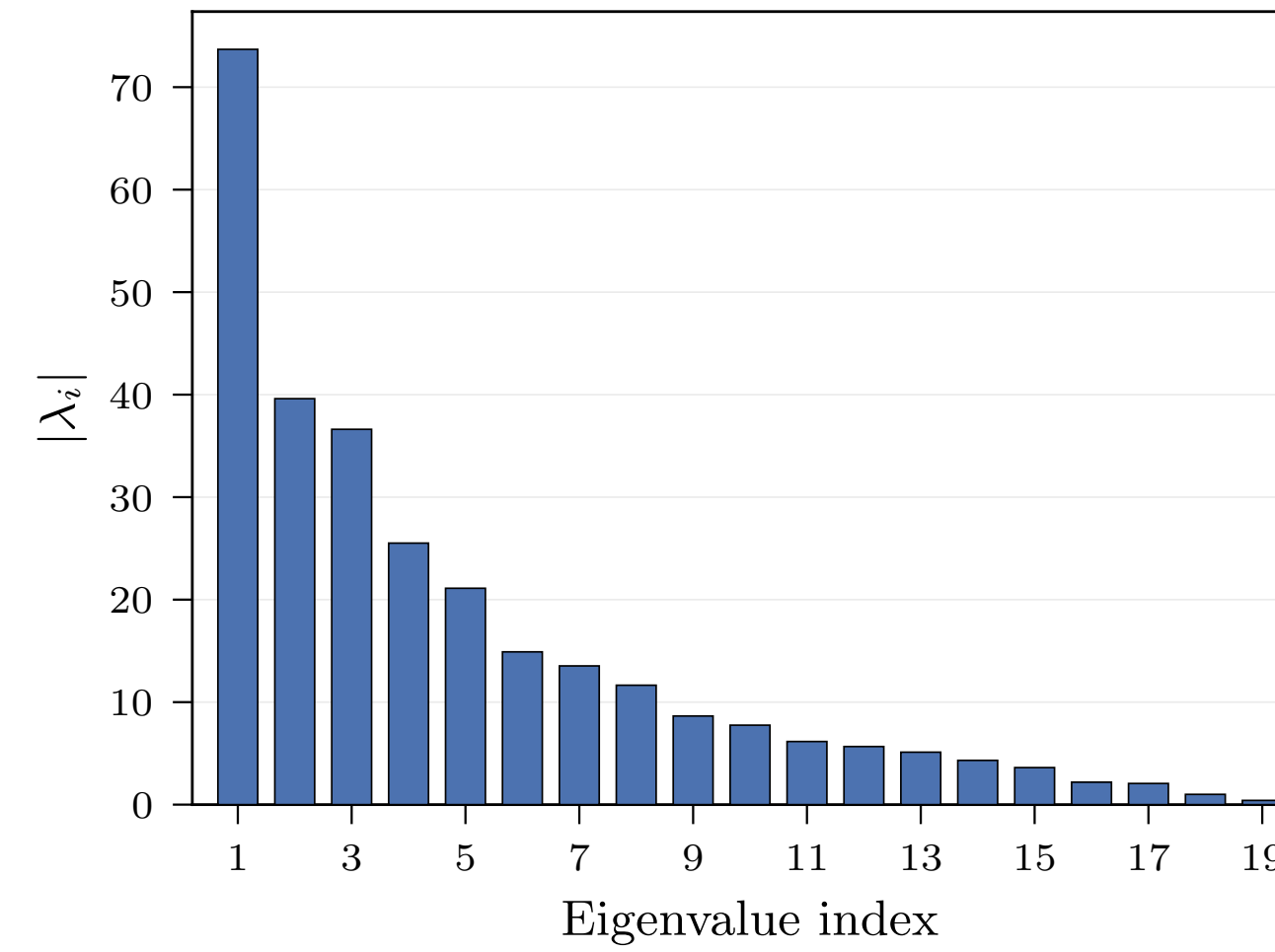
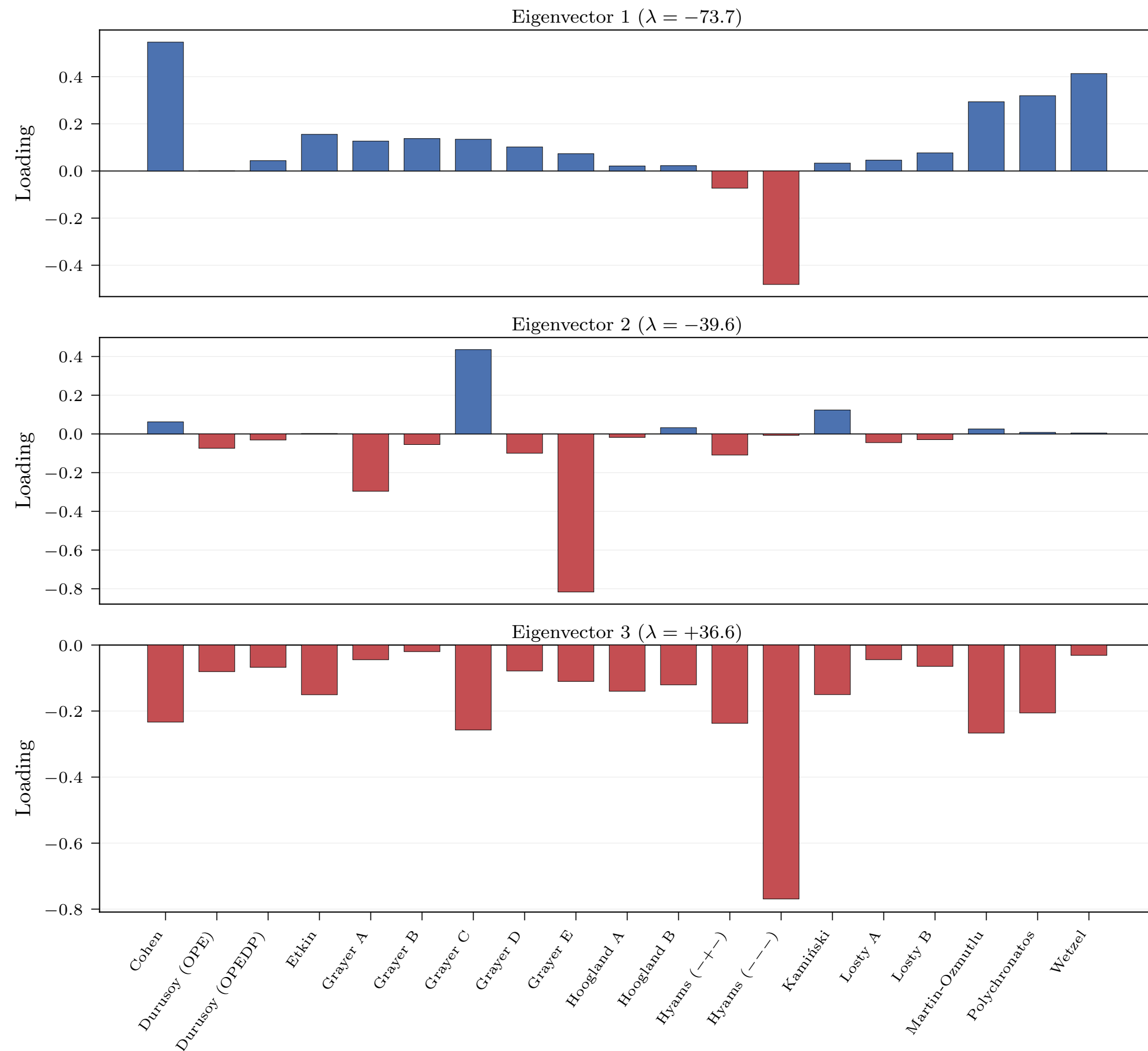
$$C_{ij} = \frac{z_{ij} + z_{ji}}{2}$$



# $\pi\pi$ scattering: data pruning

## 2- Principal component analysis (distillation)

*Reduce  $C_{ij}$  dimensions by picking most relevant eigenvectors*



*Exp. contribution to main eigenvectors*

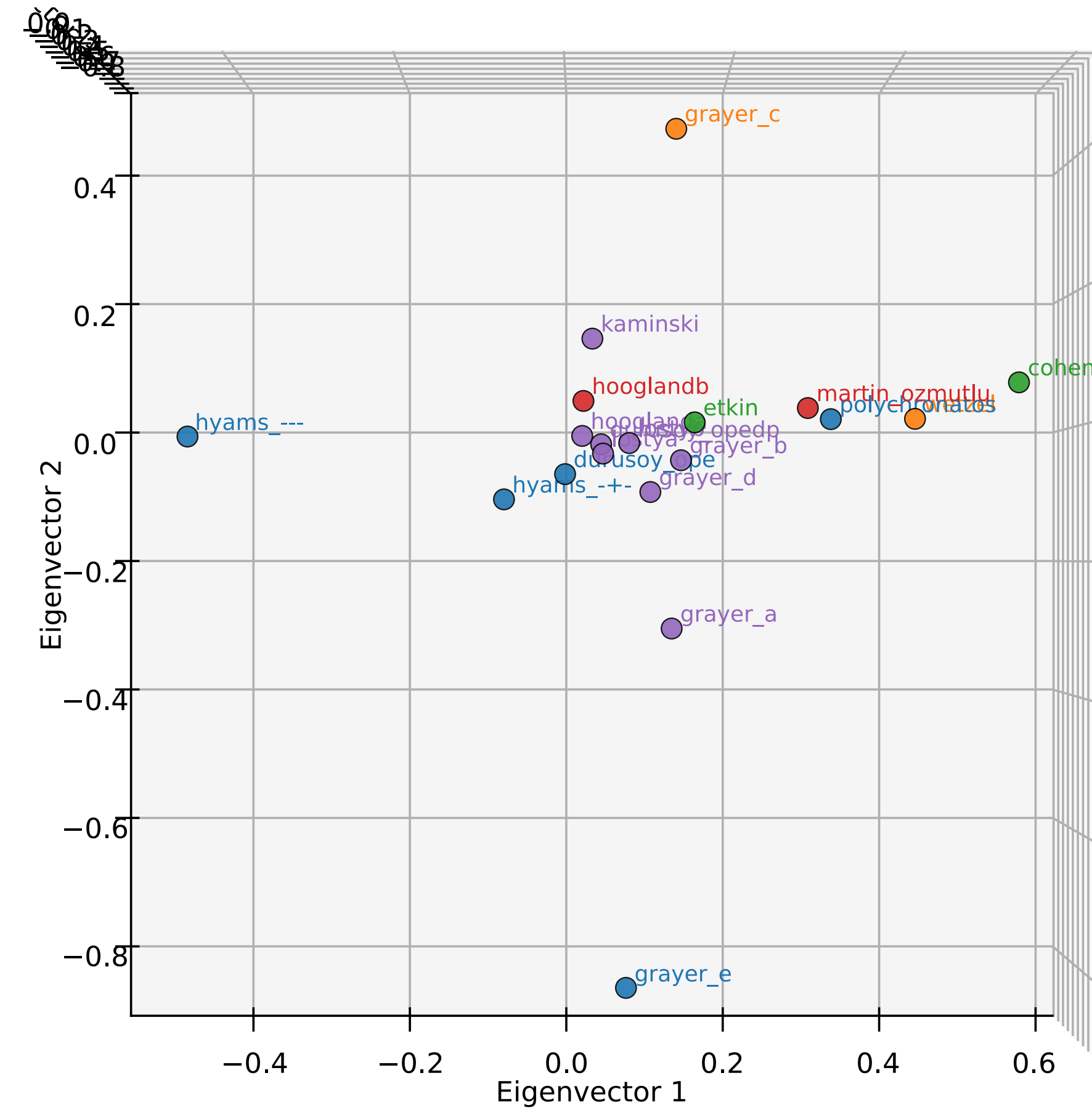
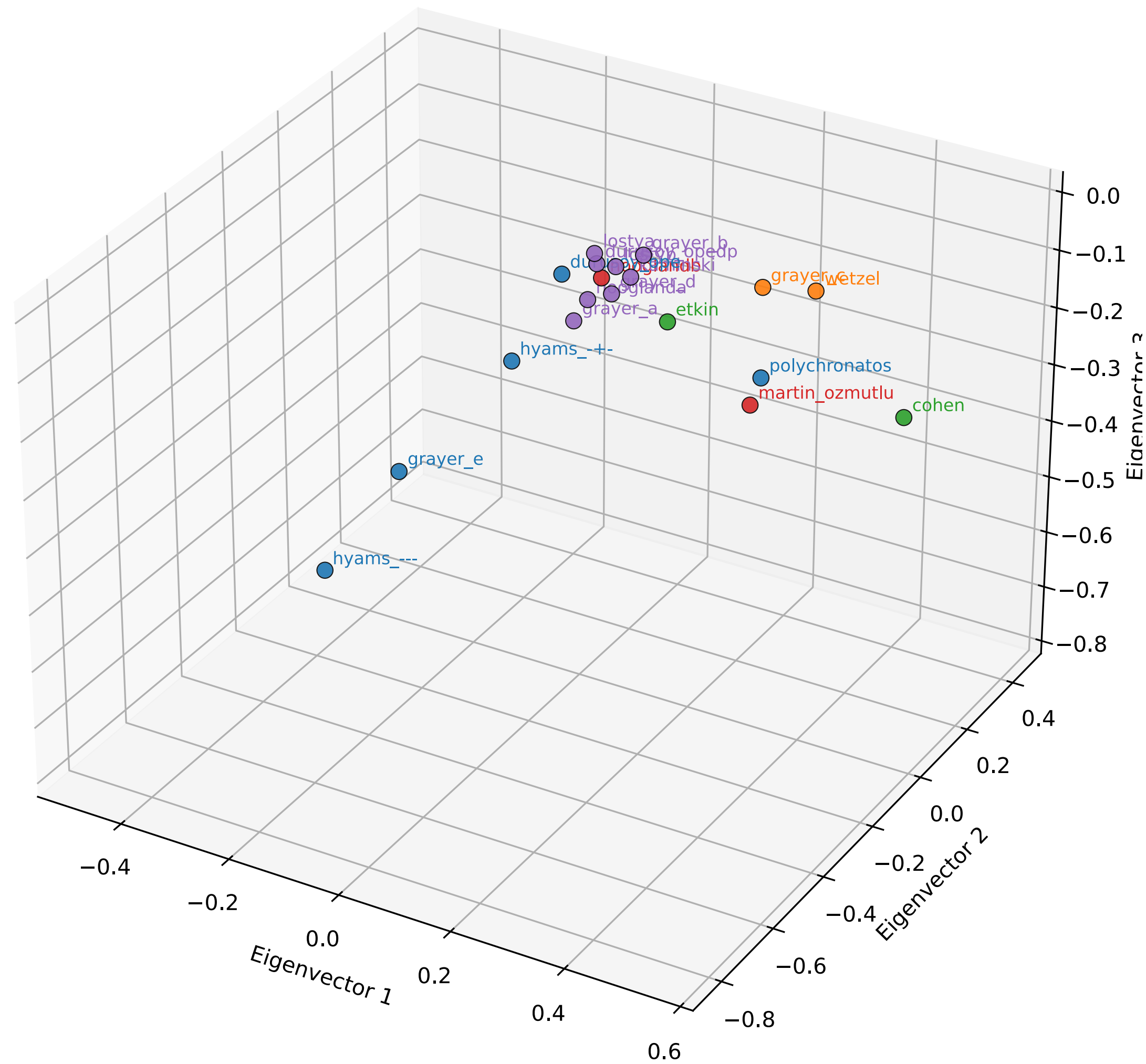
*We create a smaller dimensional space in which we place the experiments given by these coefficients, and measure their euclidean distances*

*We grouped them in the dendrogram by distance*

# $\pi\pi$ scattering: data pruning

## 2- Principal component analysis (distillation)

*In 3D, we can view the grouping process*



# $\pi\pi$ scattering: data pruning

## 3- Now, we study the compatibility between groups

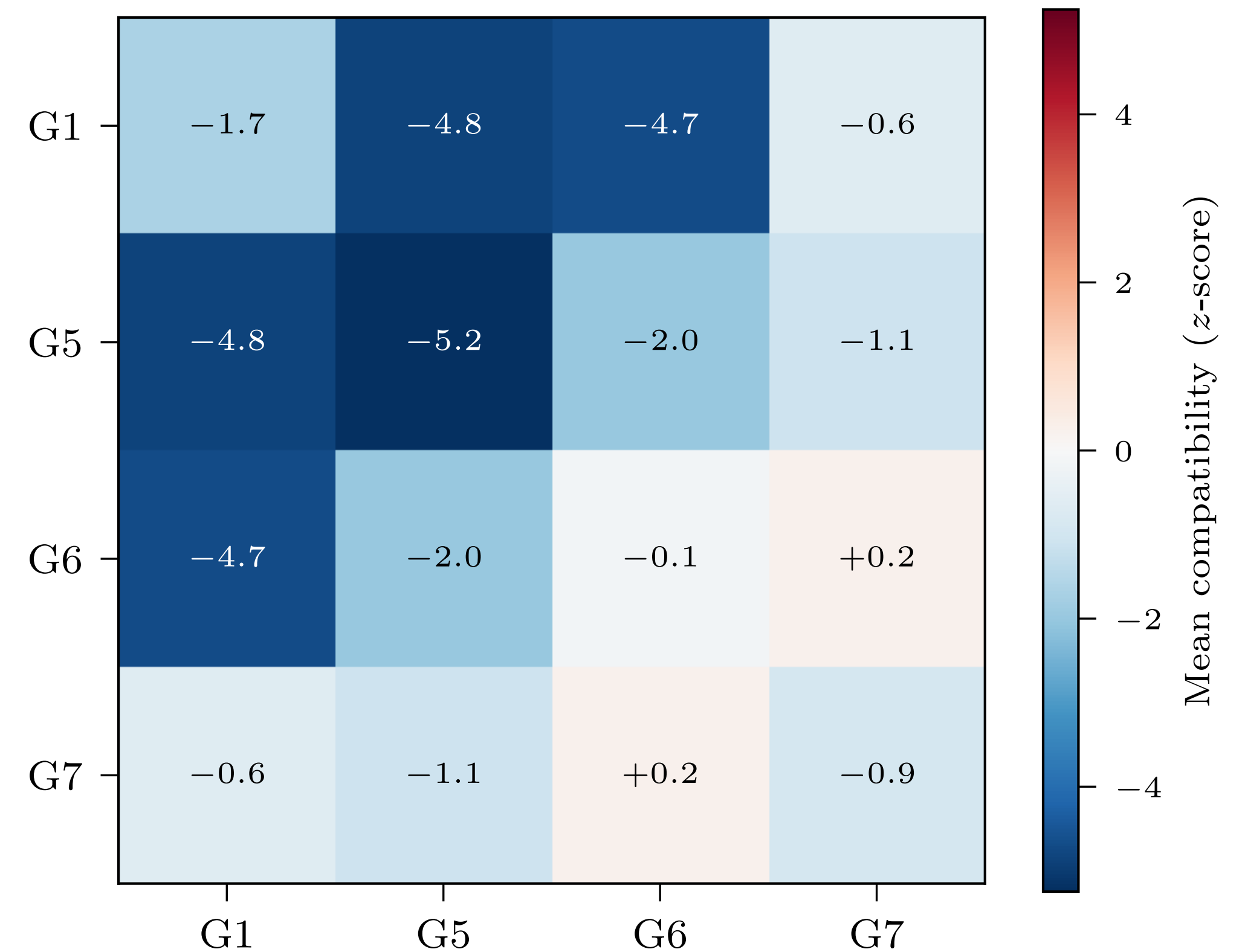
*Define new response in reduced dimensionality*

$$\bar{C}_{l,l} = \frac{2}{|G_l|(|G_l| - 1)} \sum_{\substack{i,j \in G_l \\ i < j}} C_{ij}$$

$$\bar{C}_{l,m} = \frac{1}{|G_l||G_m|} \sum_{i \in G_l} \sum_{j \in G_m} C_{ij}$$

## 4- Pruning/selection

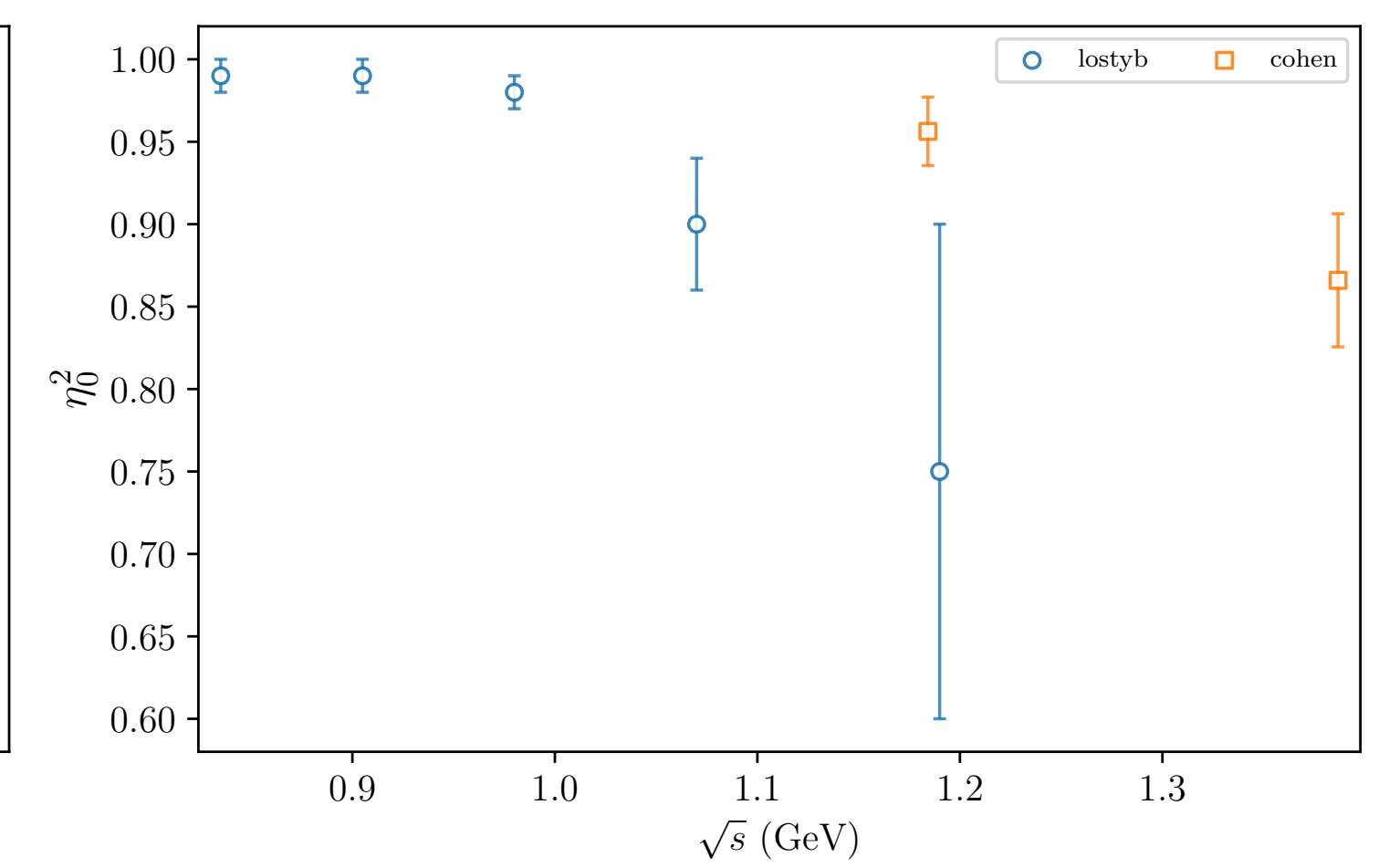
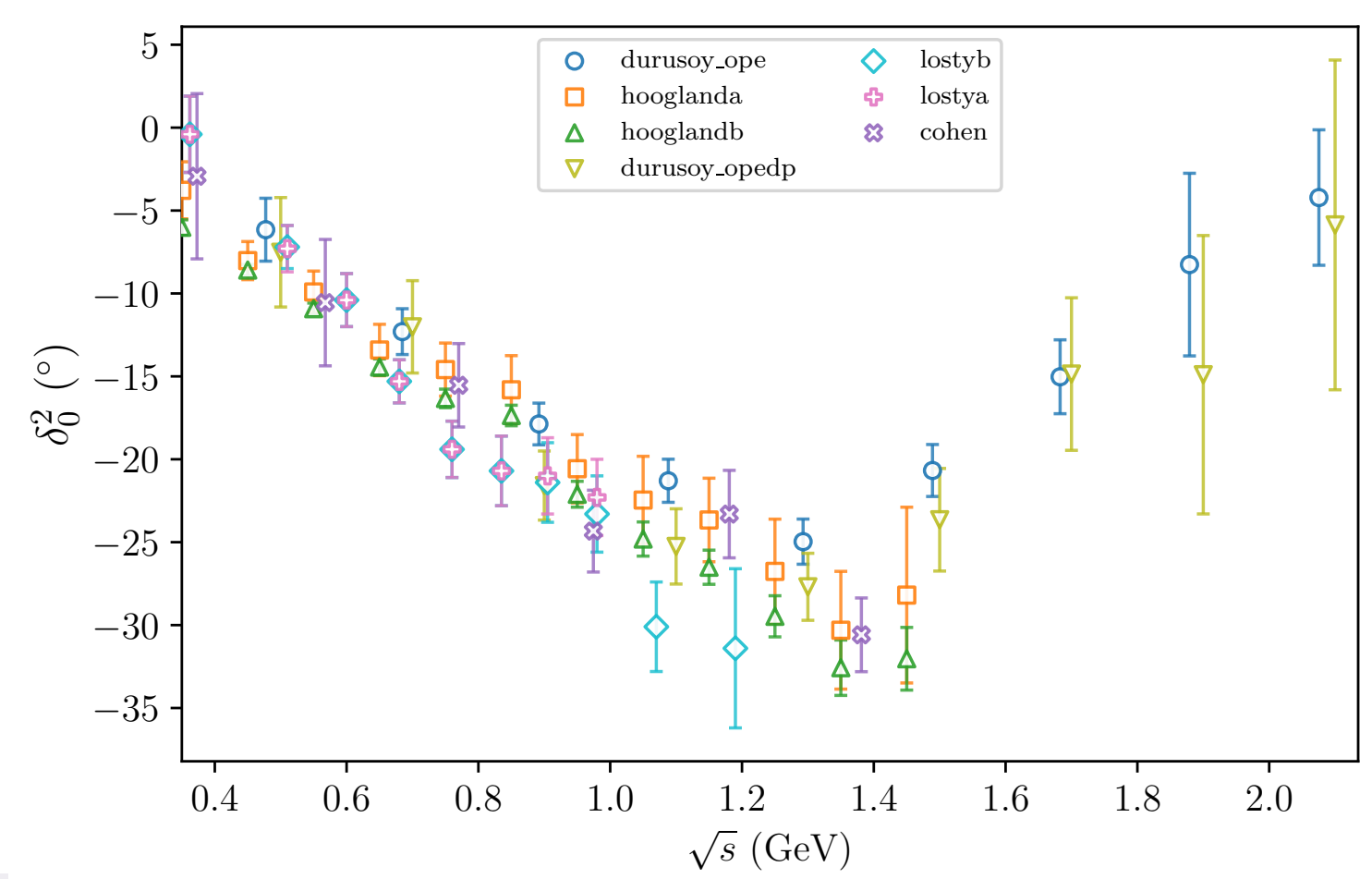
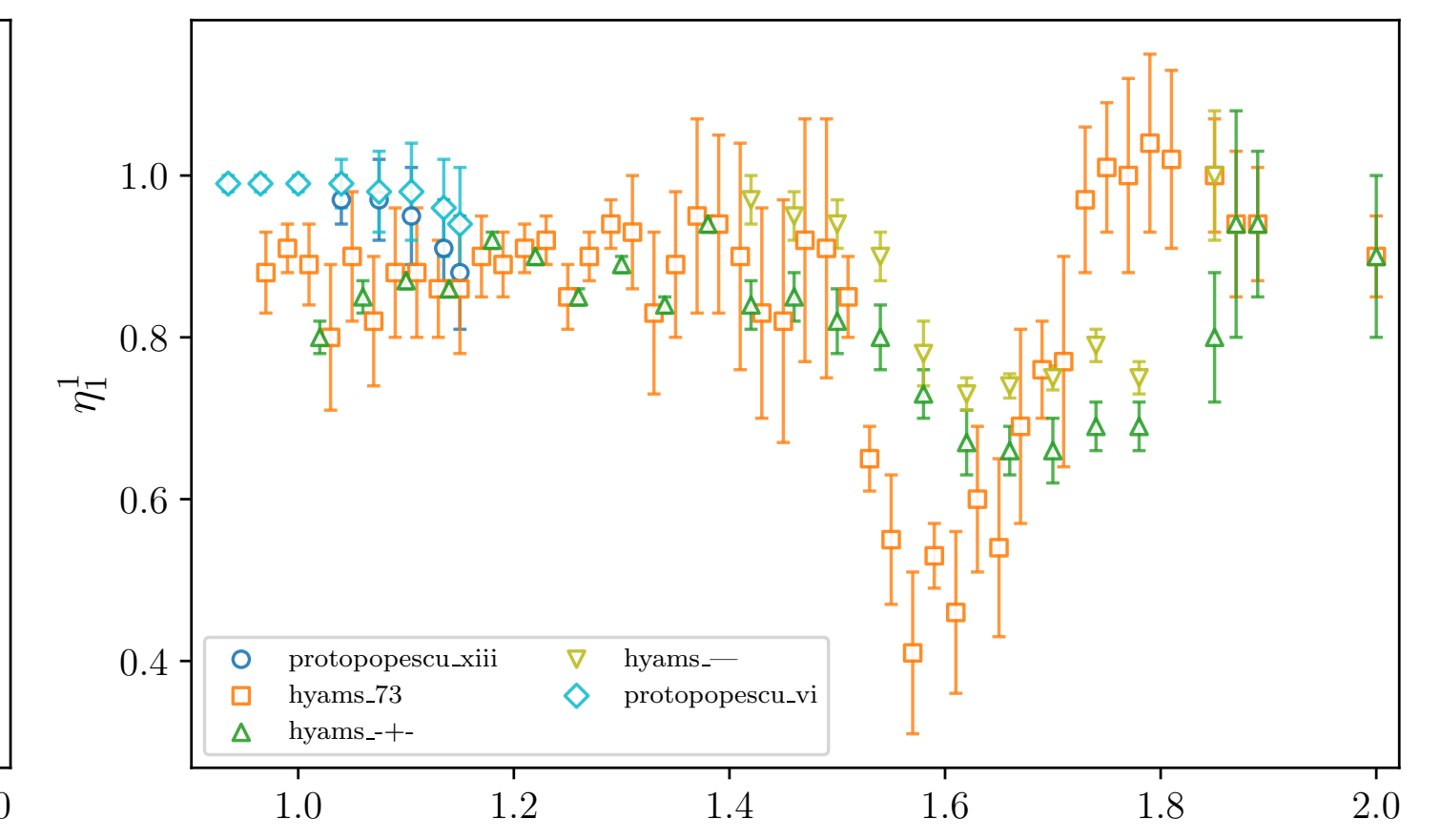
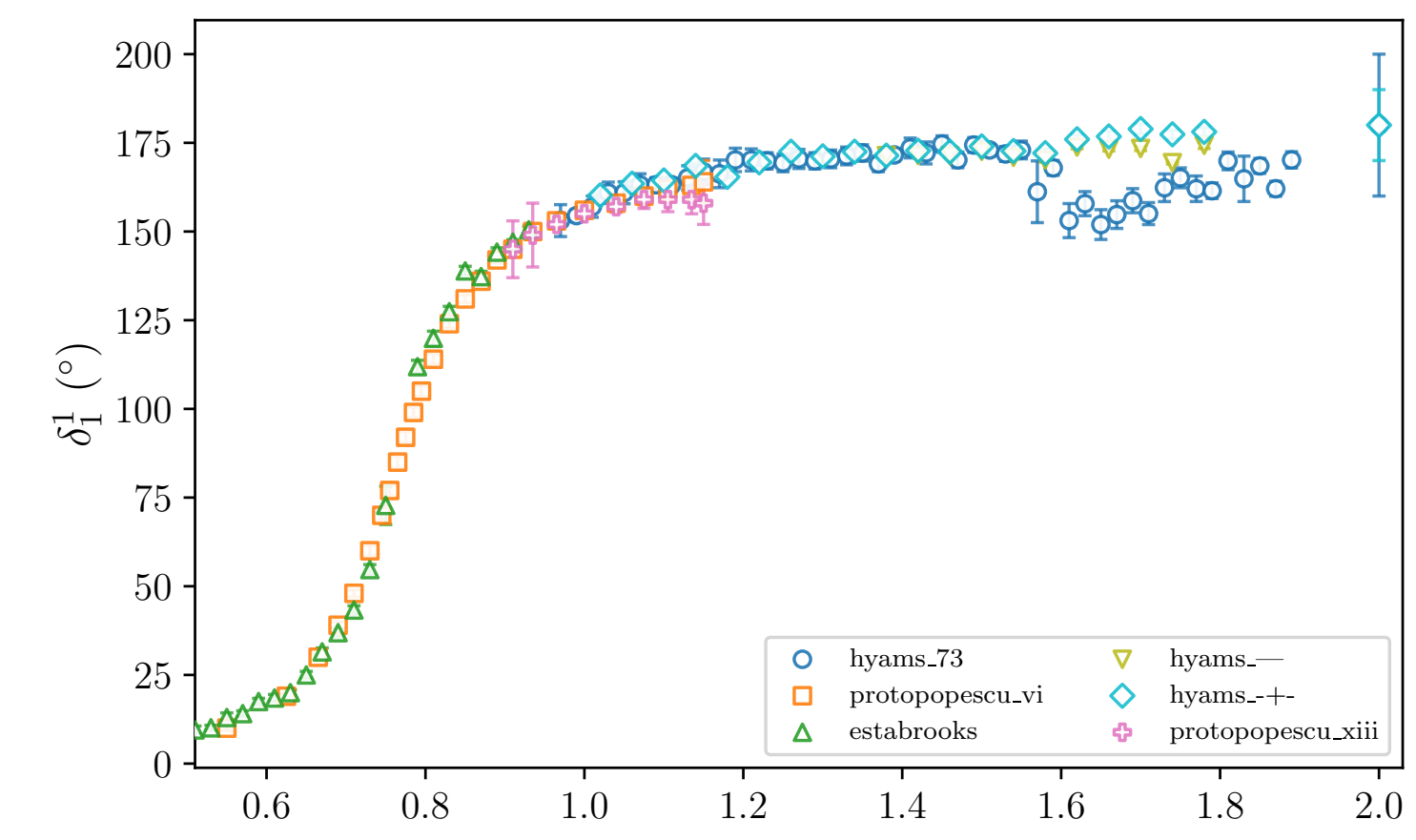
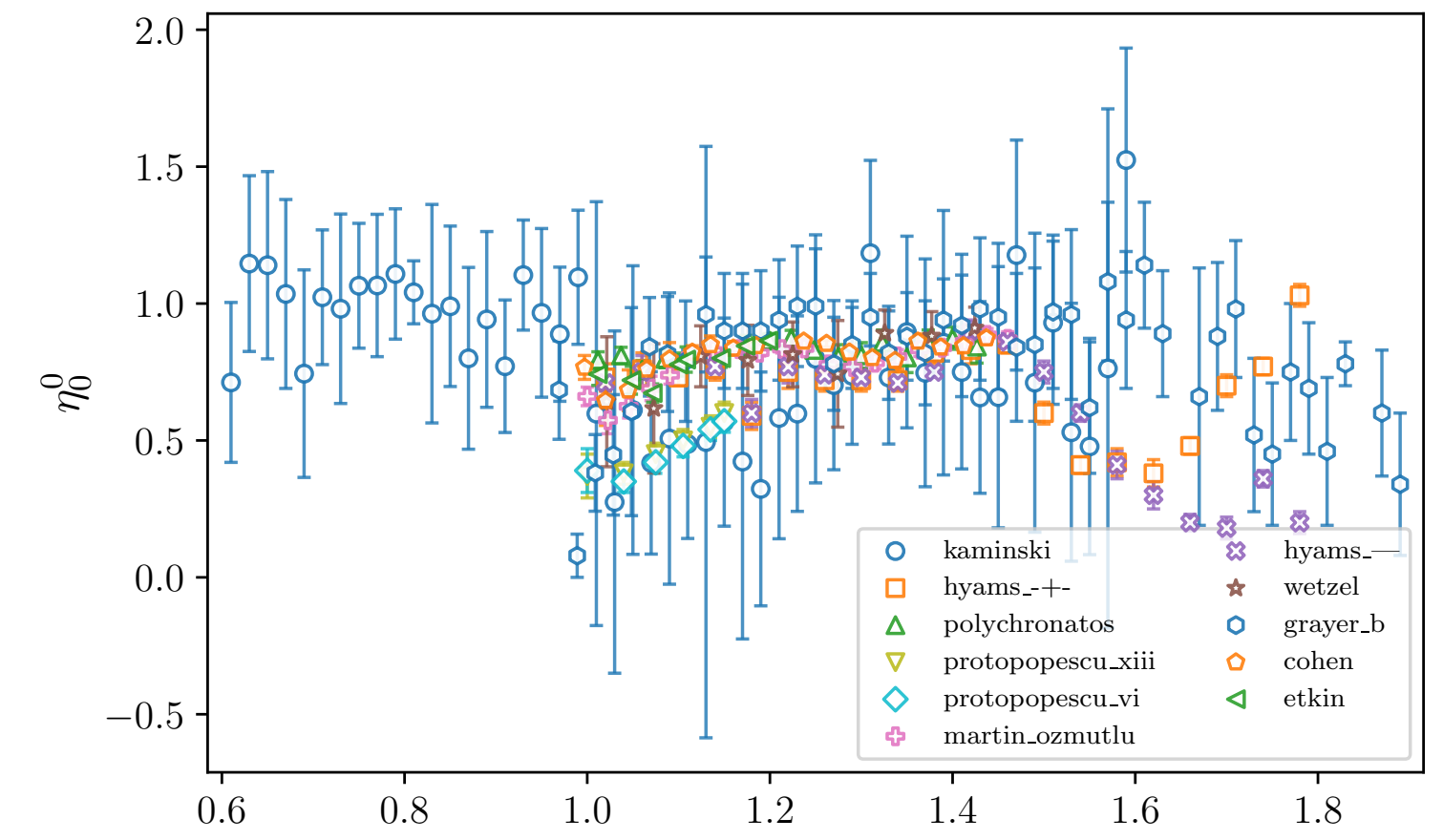
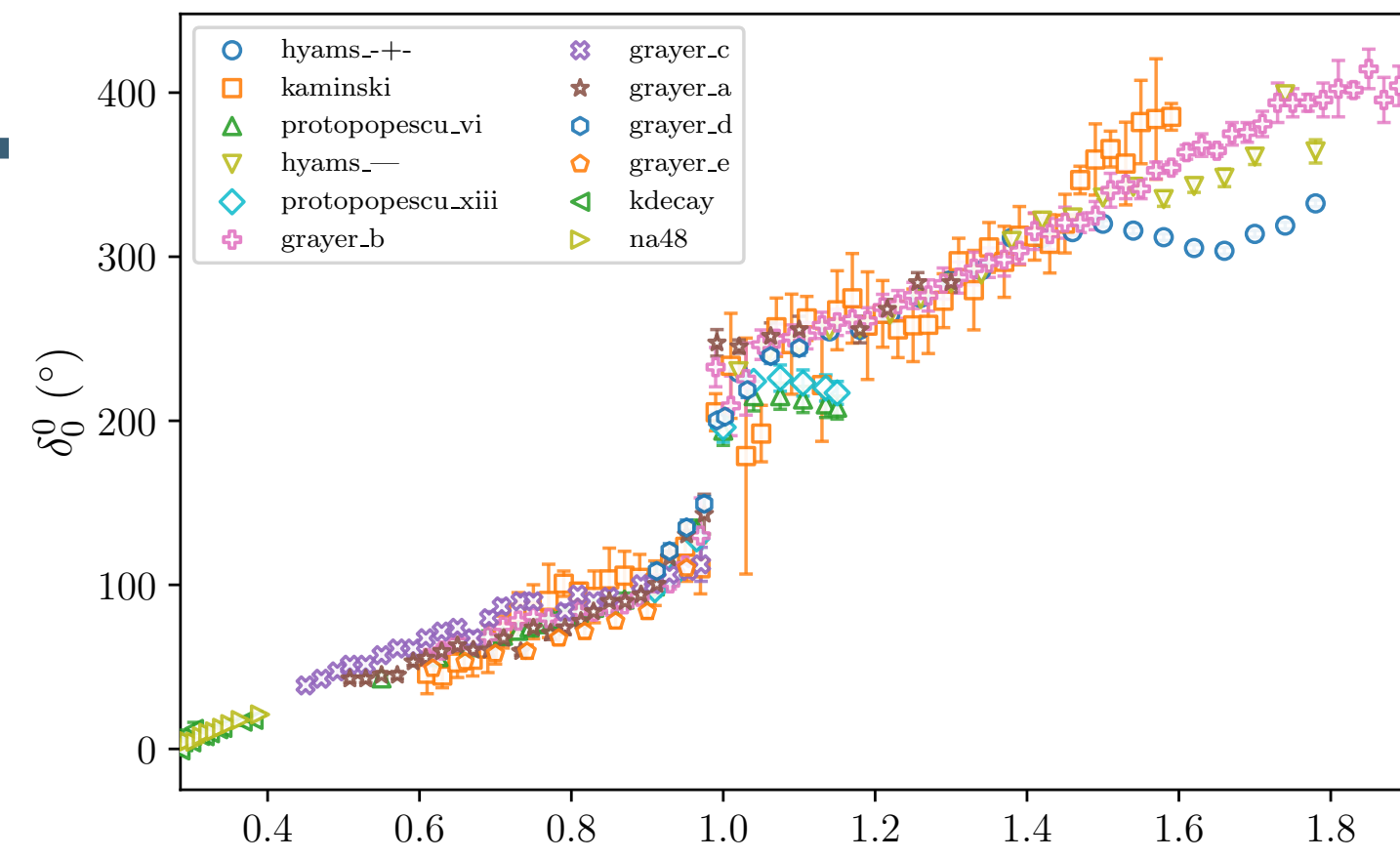
*Remove groups iteratively until responses are “within one sigma”*



# $\pi\pi$ scattering: data

Data collected for 7 partial waves

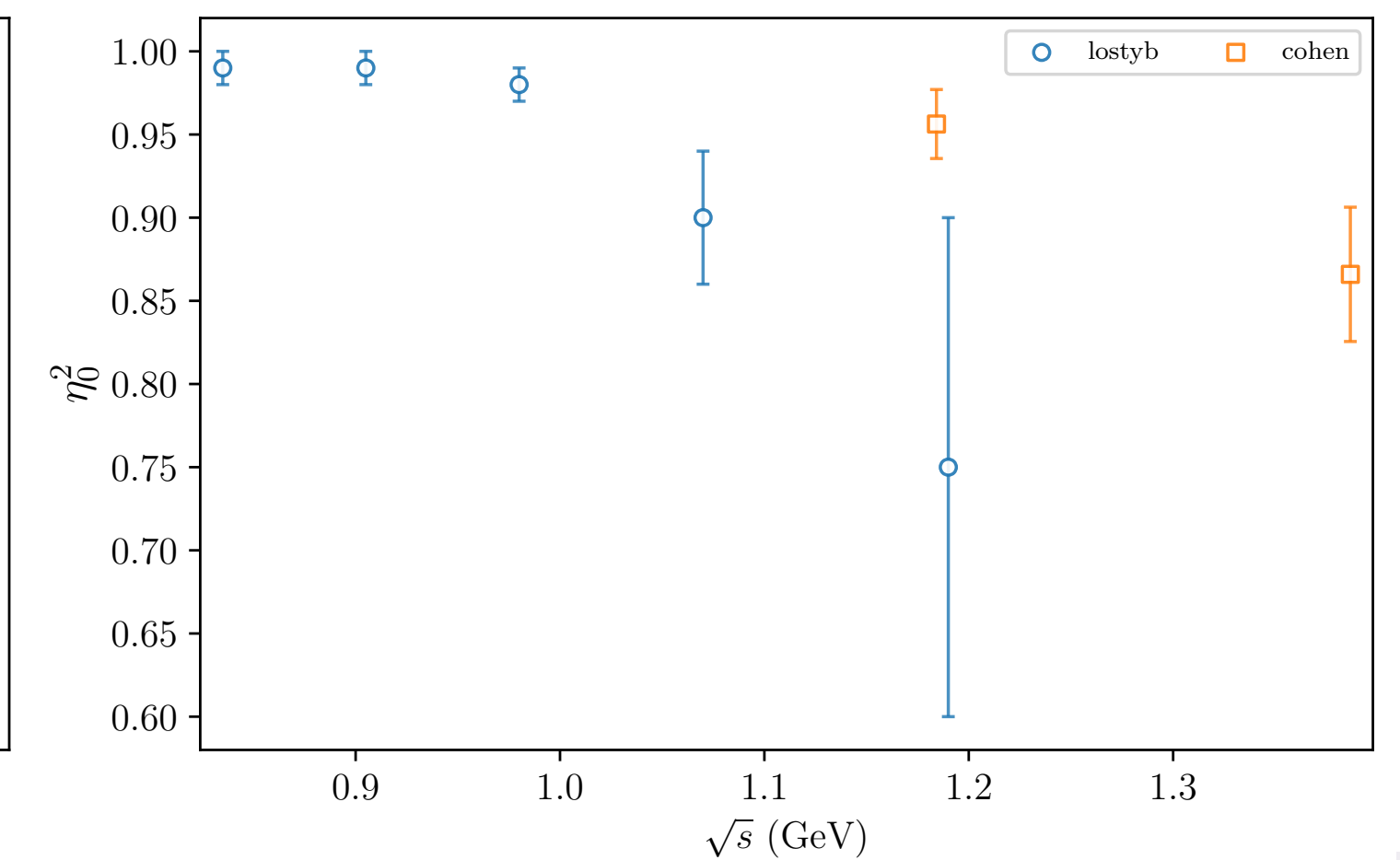
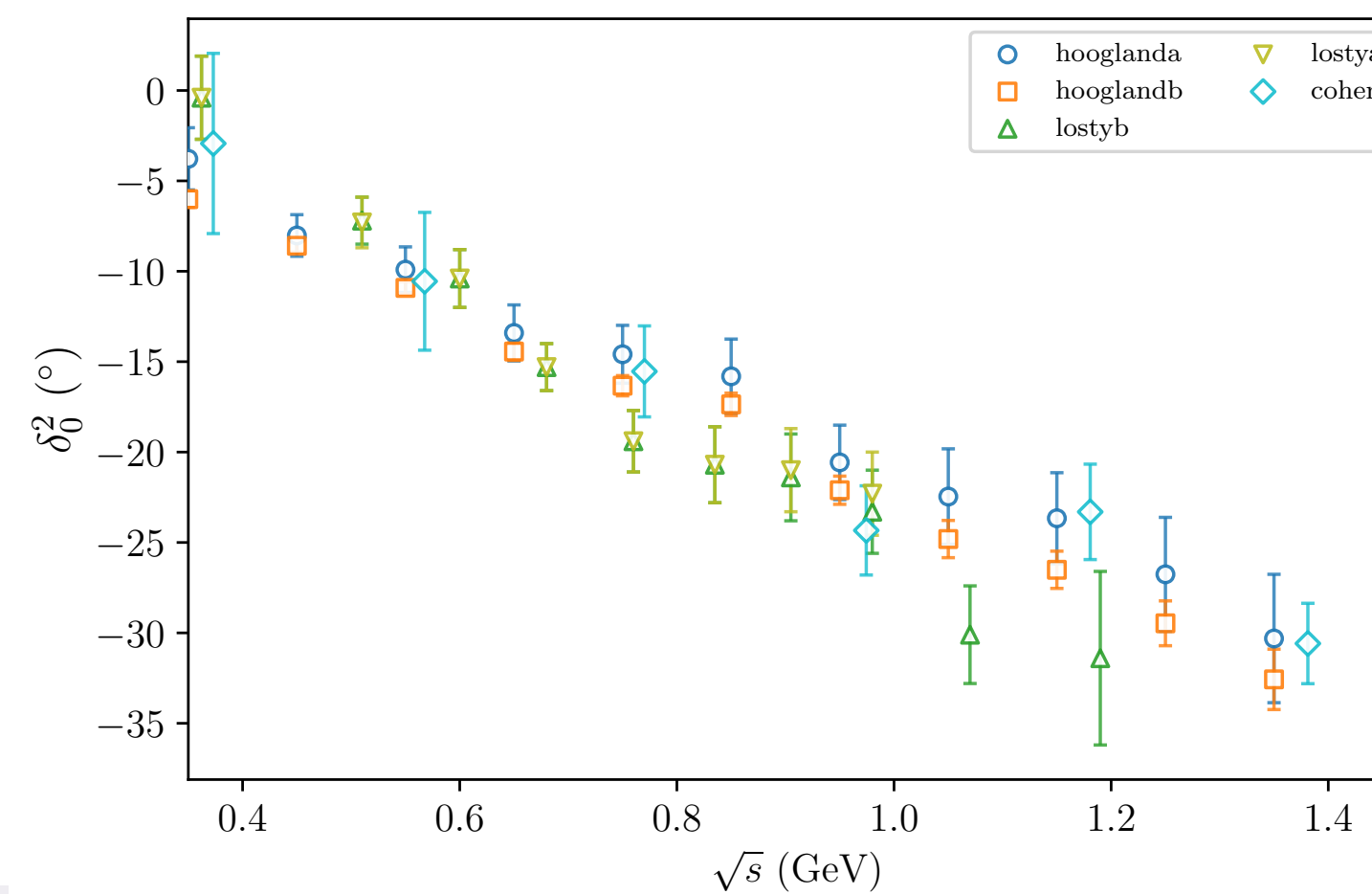
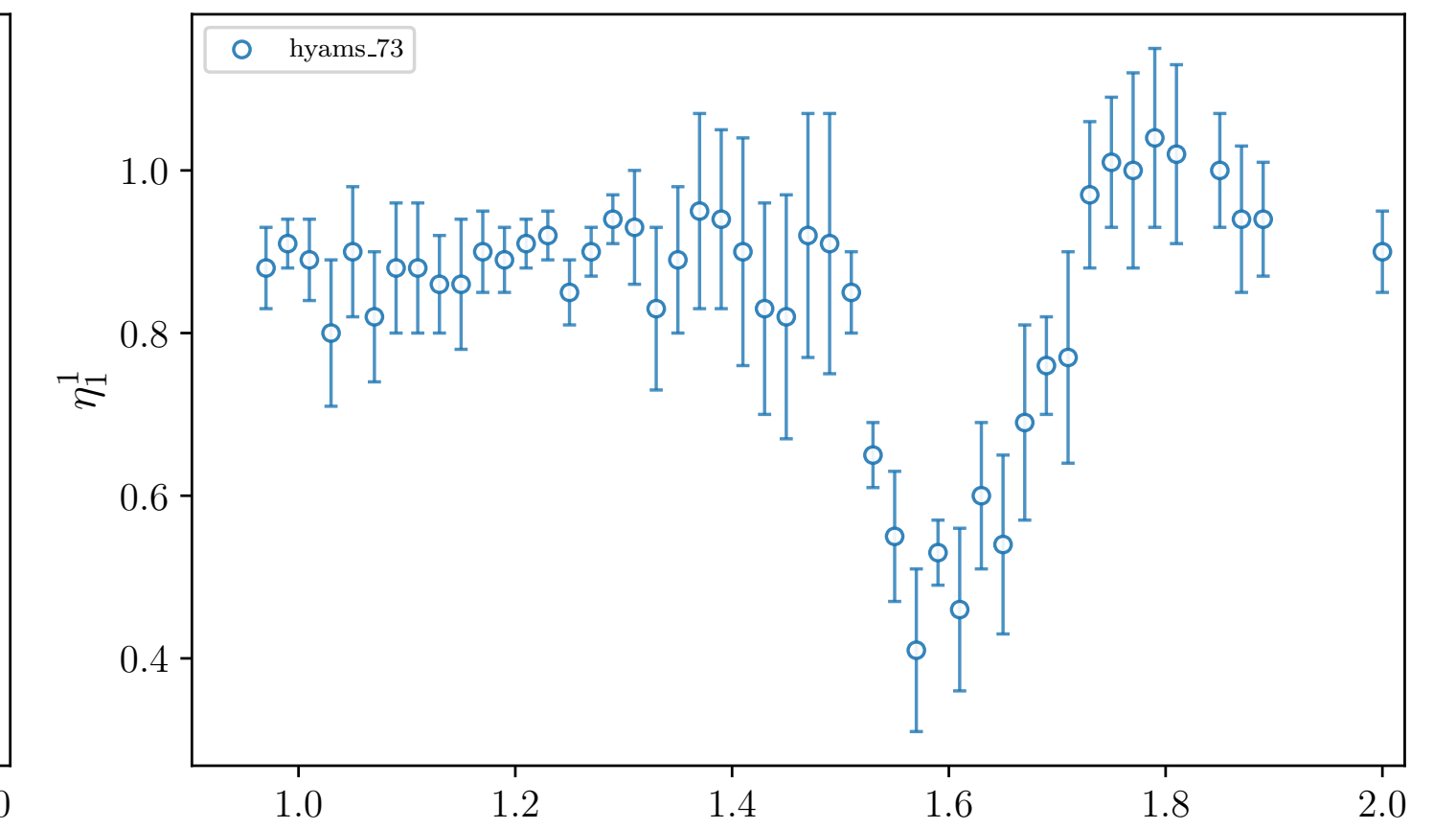
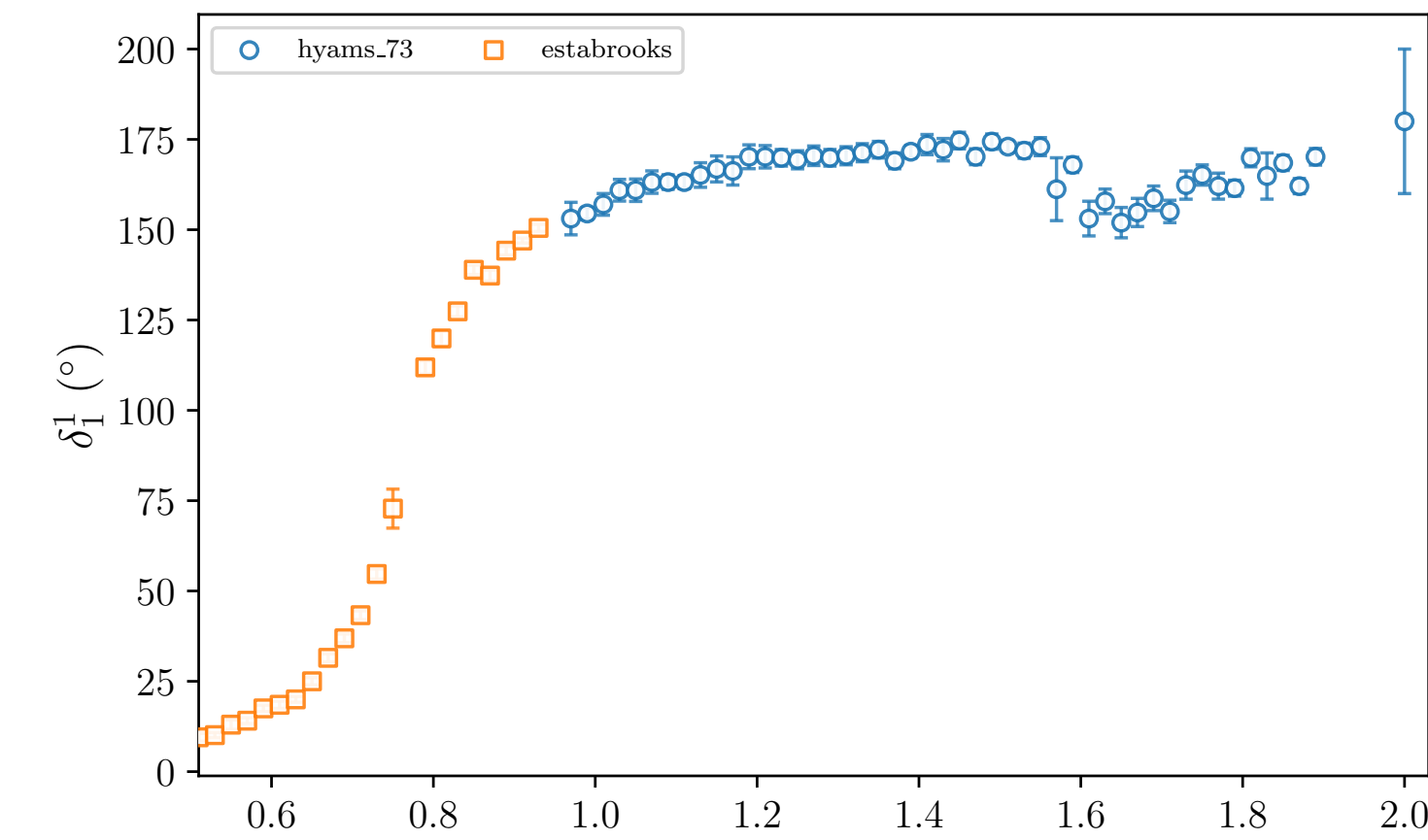
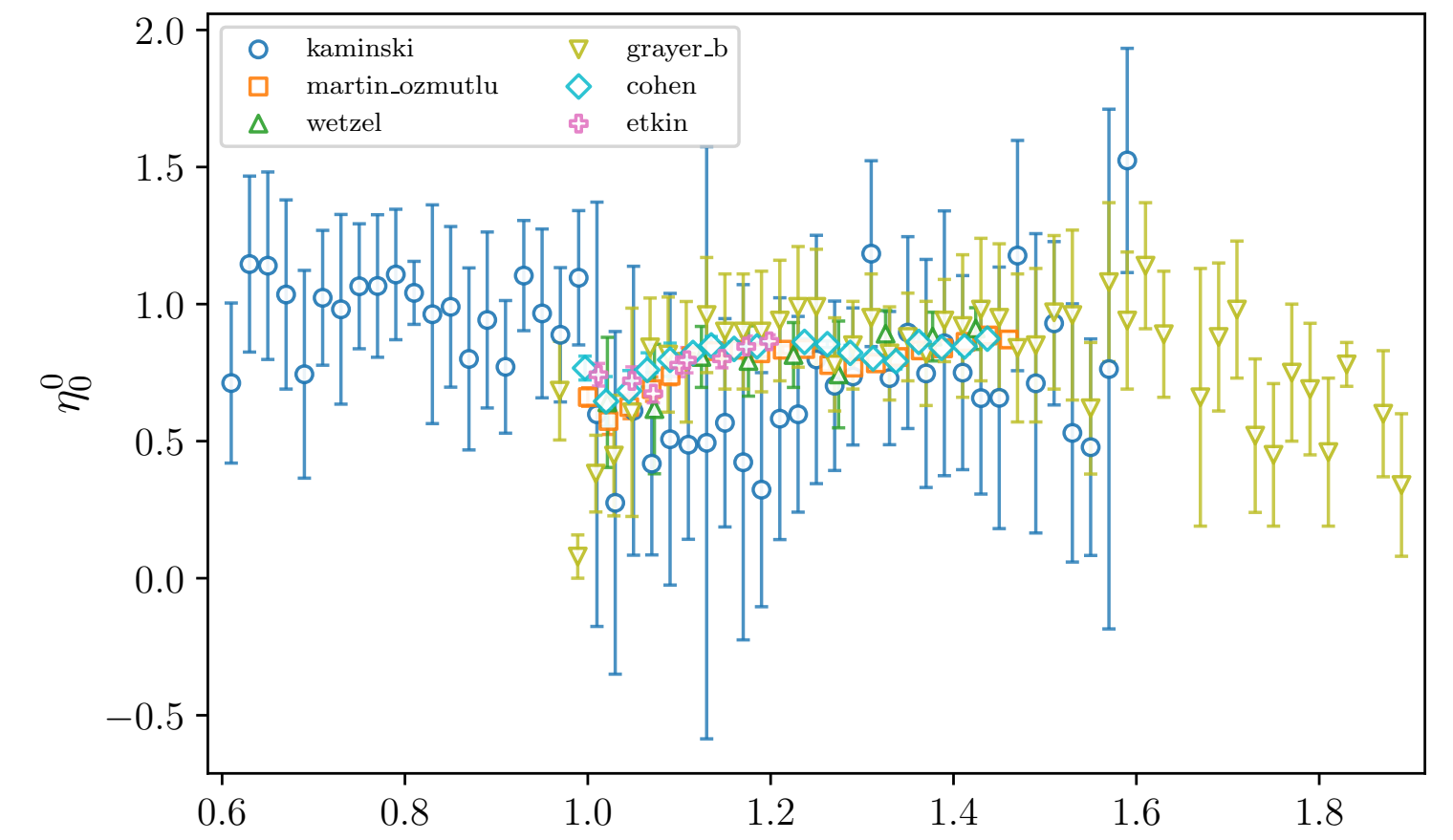
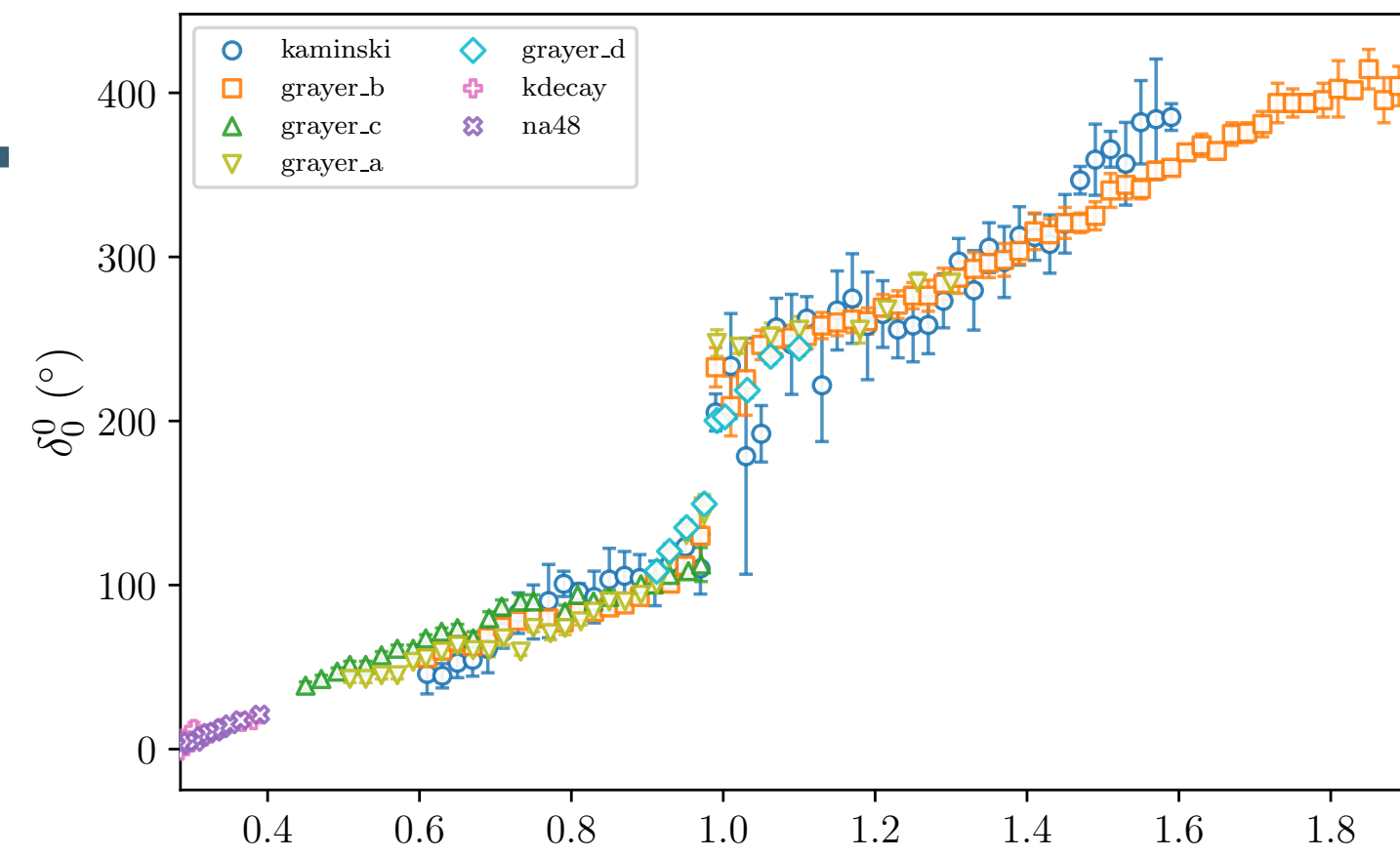
Over 1000 initial data points



# $\pi\pi$ scattering: data

## Data collected for 7 partial waves

Over 600 final data points in the fitting region



# $\pi\pi$ scattering: results

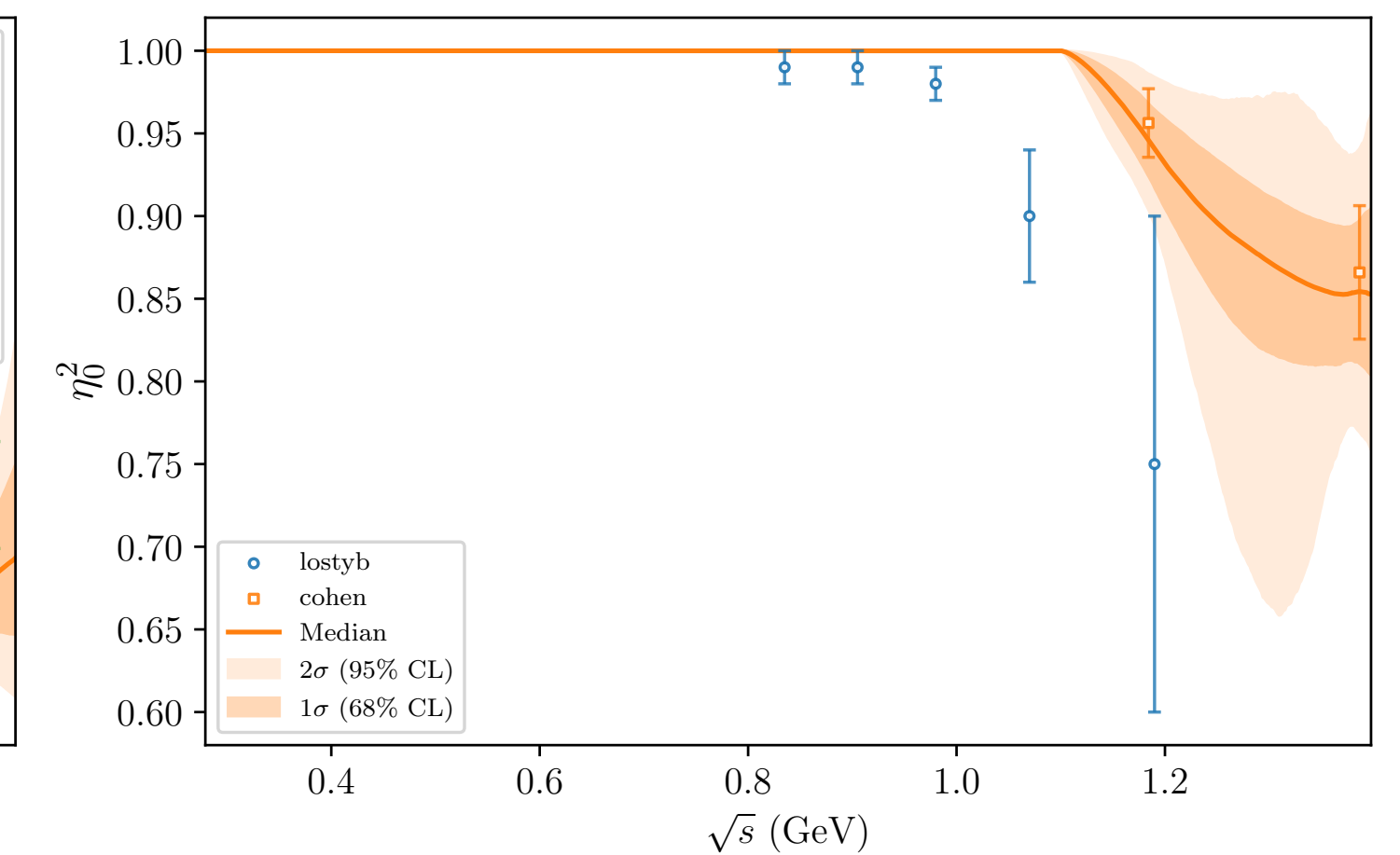
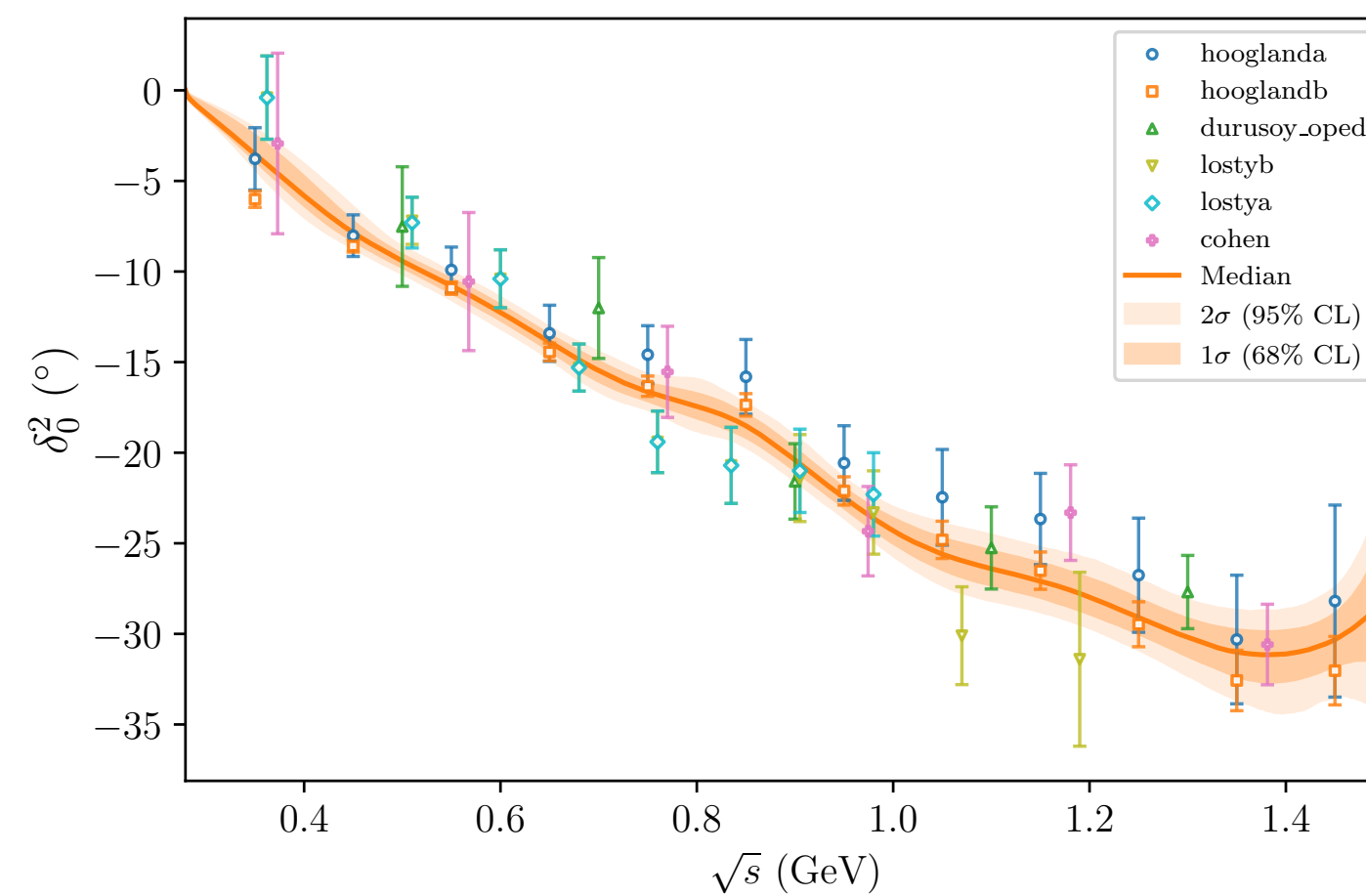
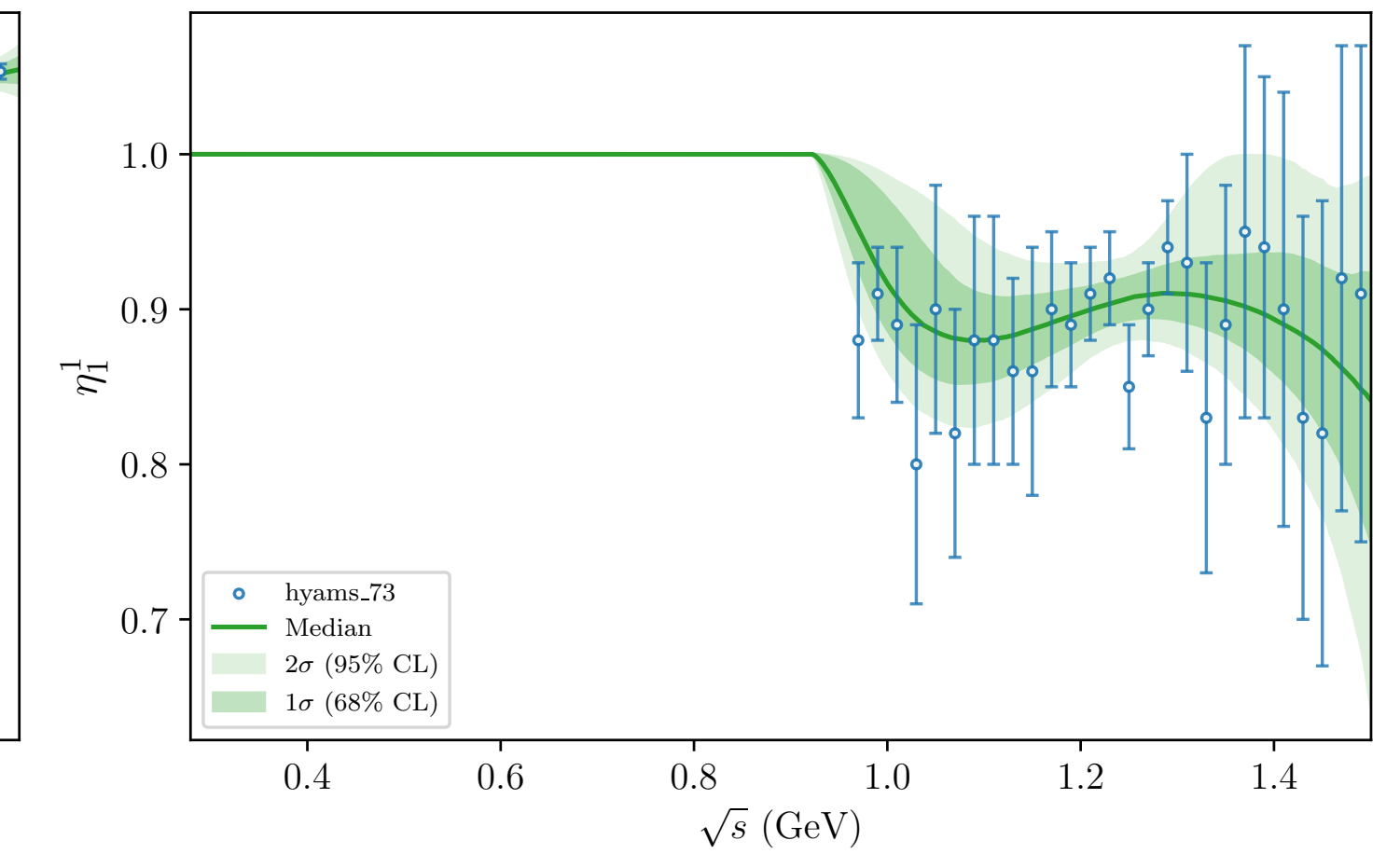
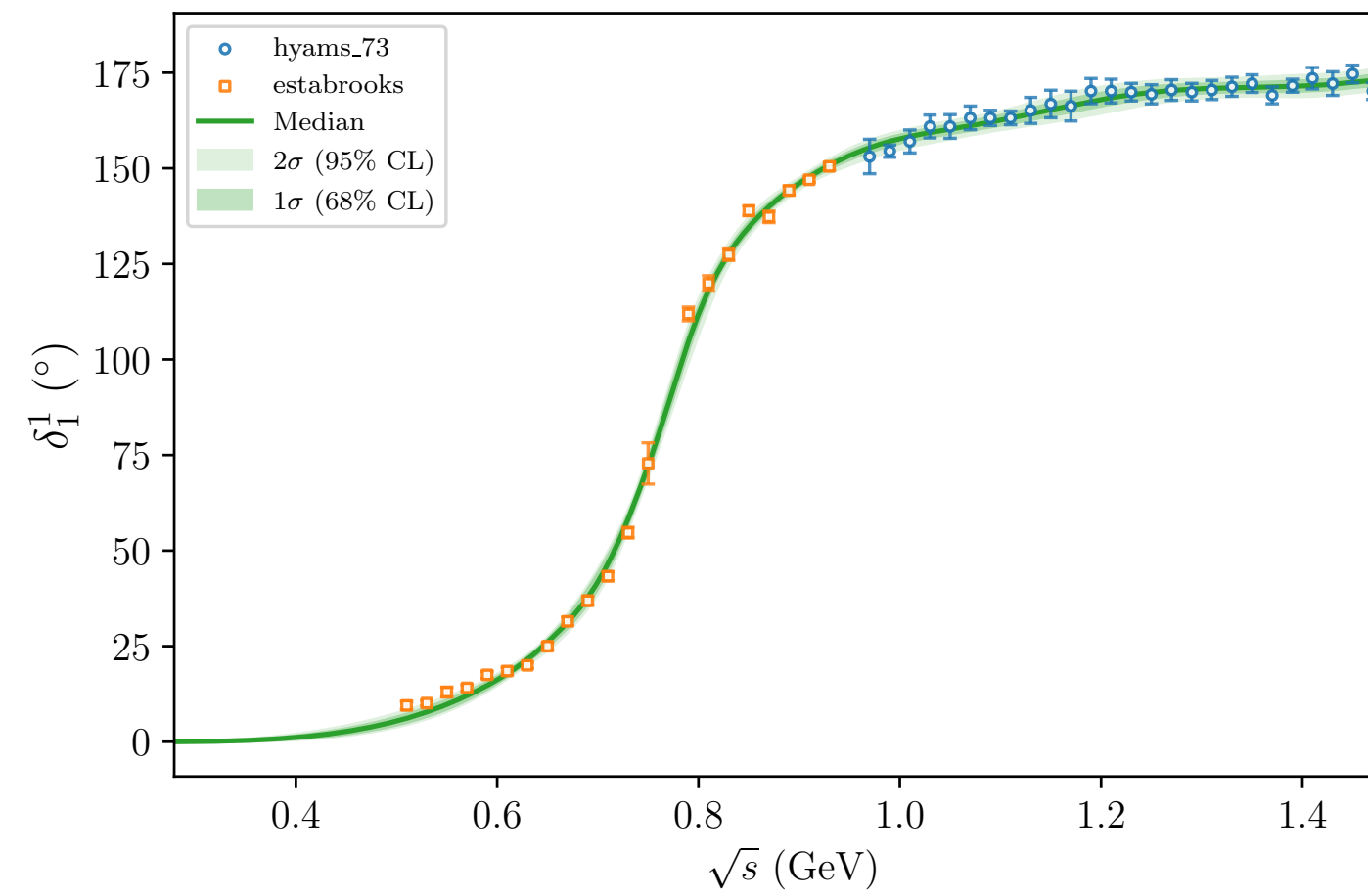
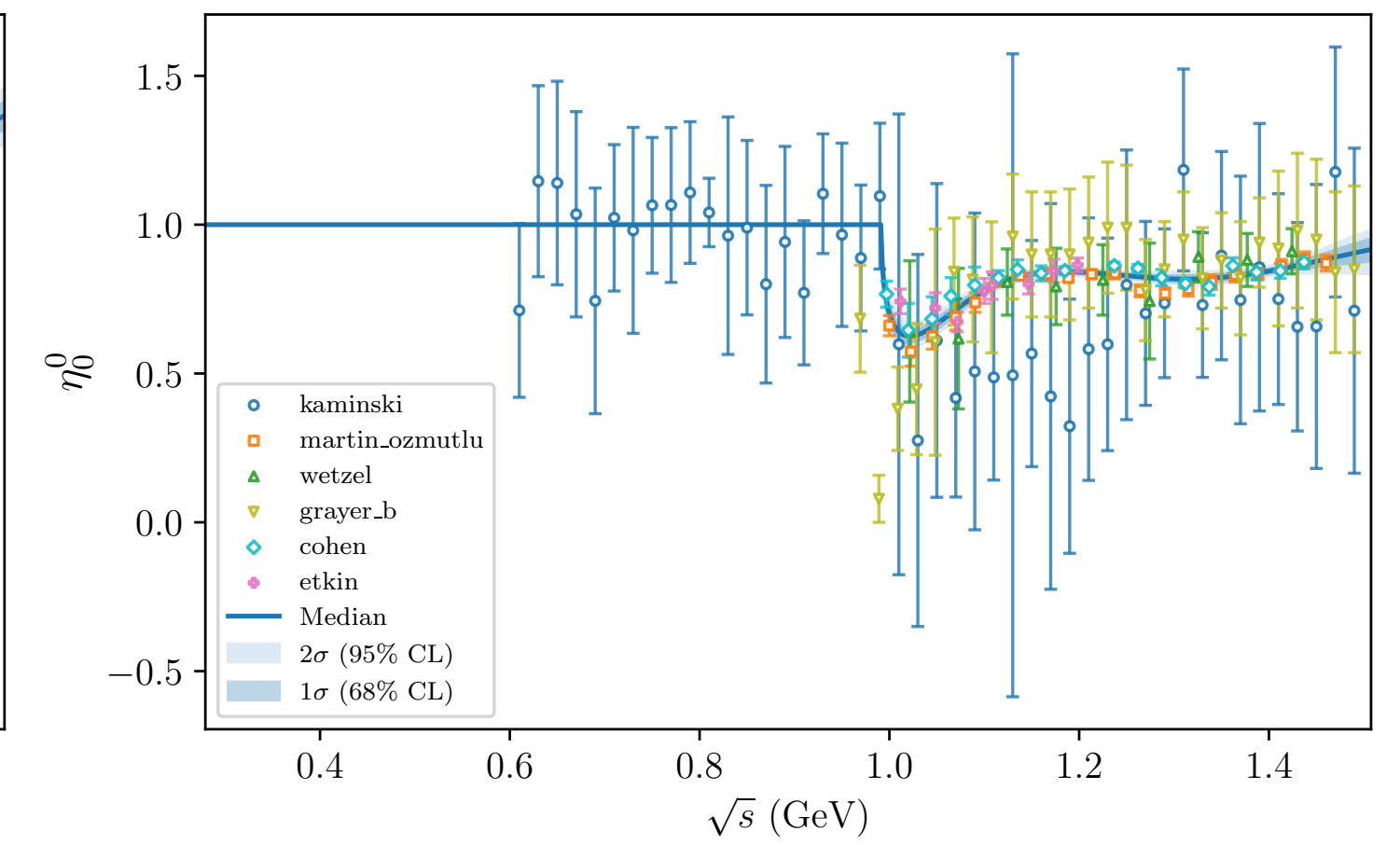
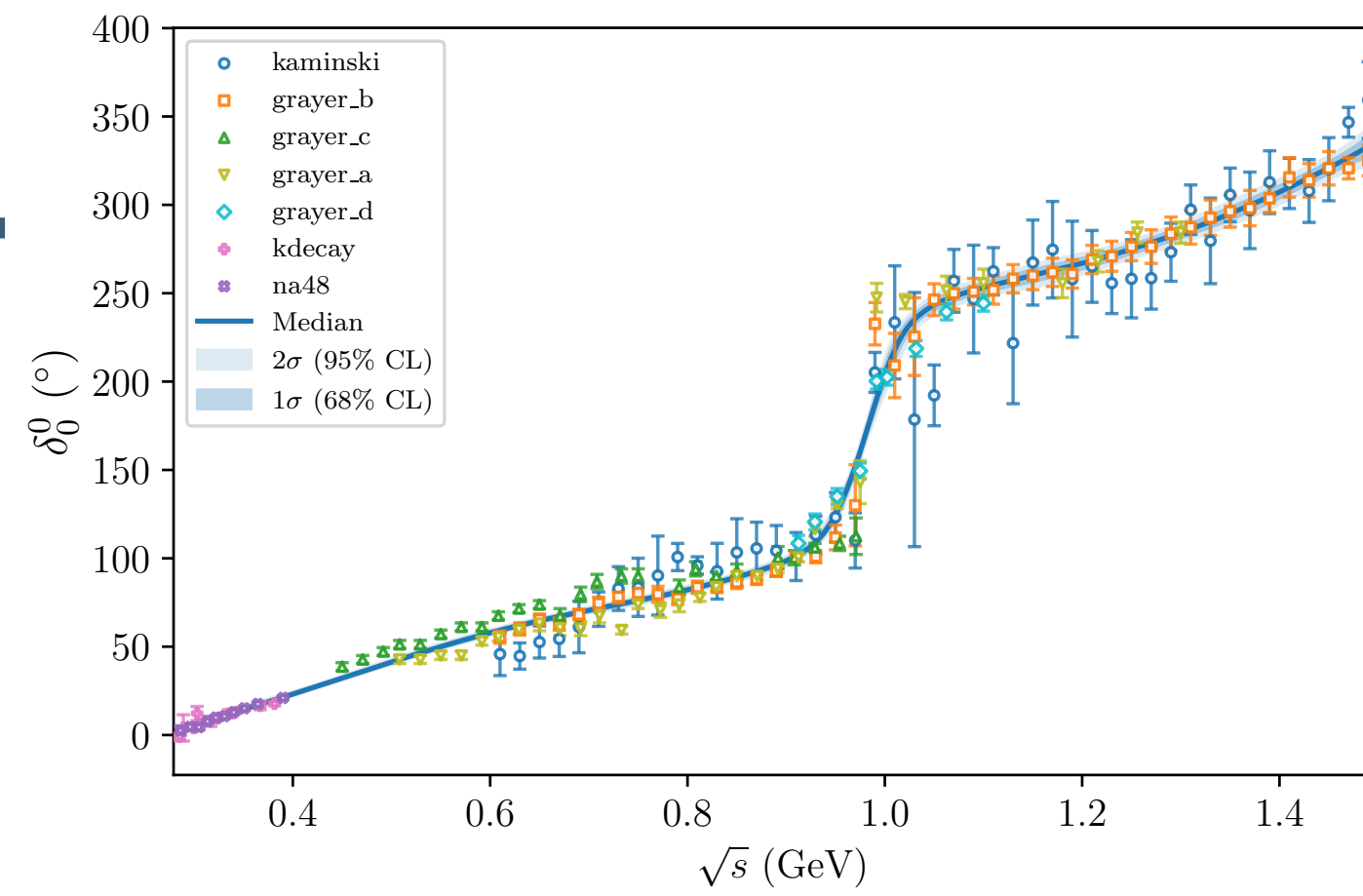
## Data collected for 7 partial waves

Over 600 final data points in the fitting region

## Very good data description

Fits produce  $\chi^2/\text{ndof} \sim 3$

$2\sigma$  band calculated from 5k resamples



# $\pi\pi$ scattering: results

Data collected for 7 partial waves

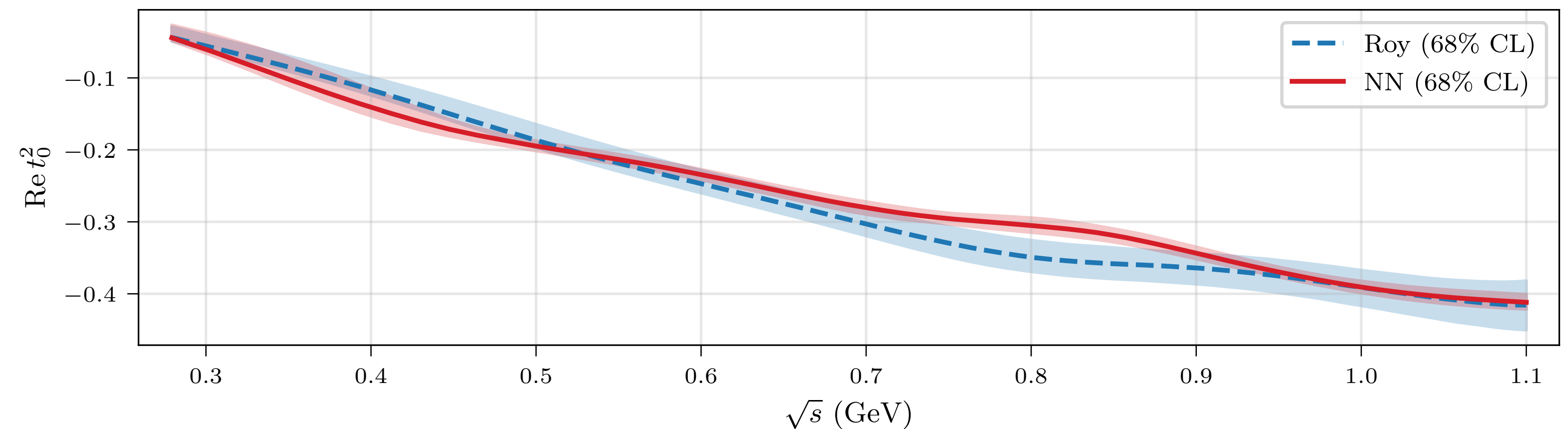
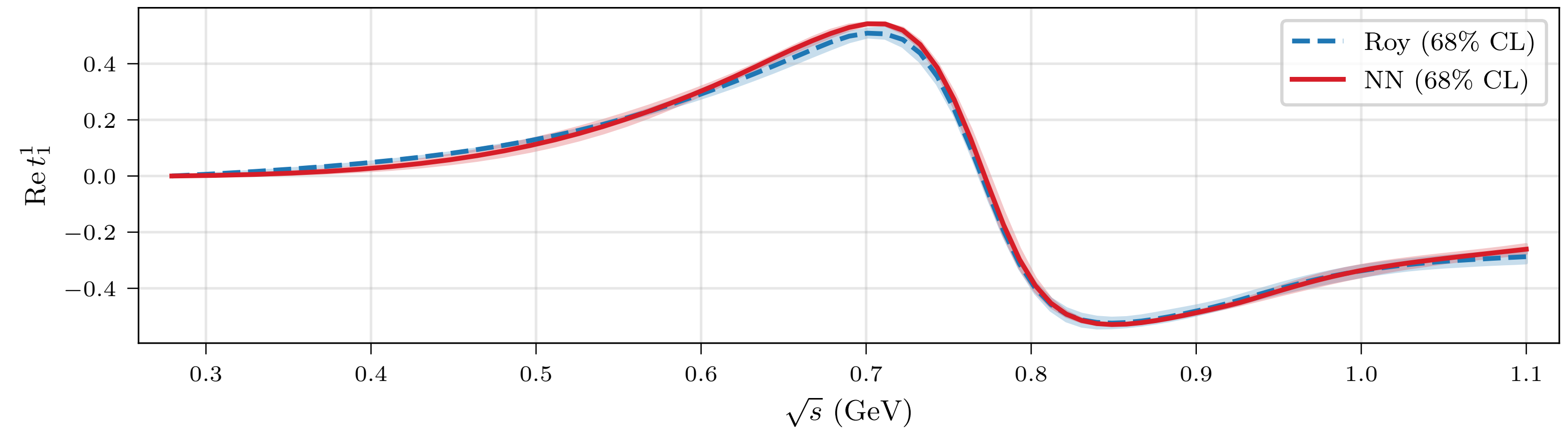
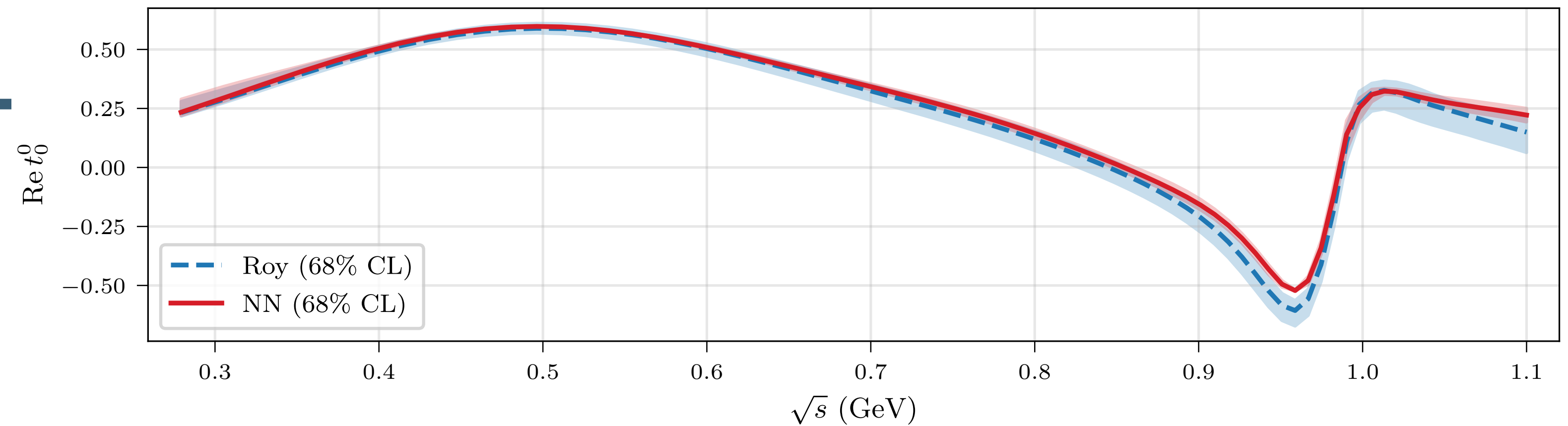
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Dispersion relations are satisfied numerically



# $\pi\pi$ scattering: results

## Data collected for 7 partial waves

Over 600 final data points in the fitting region

## Very good data description

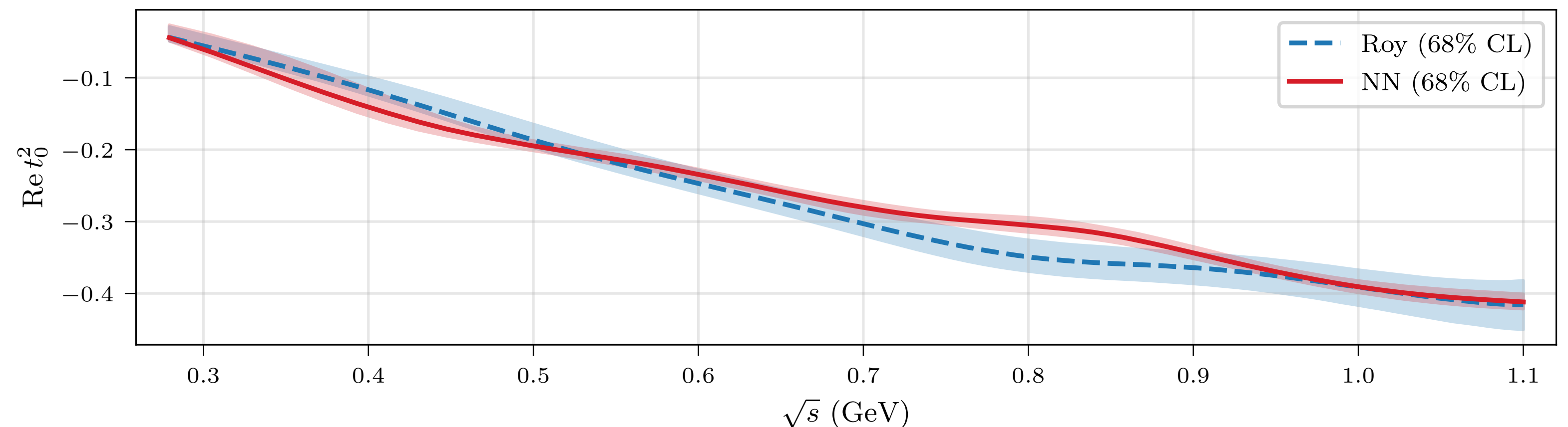
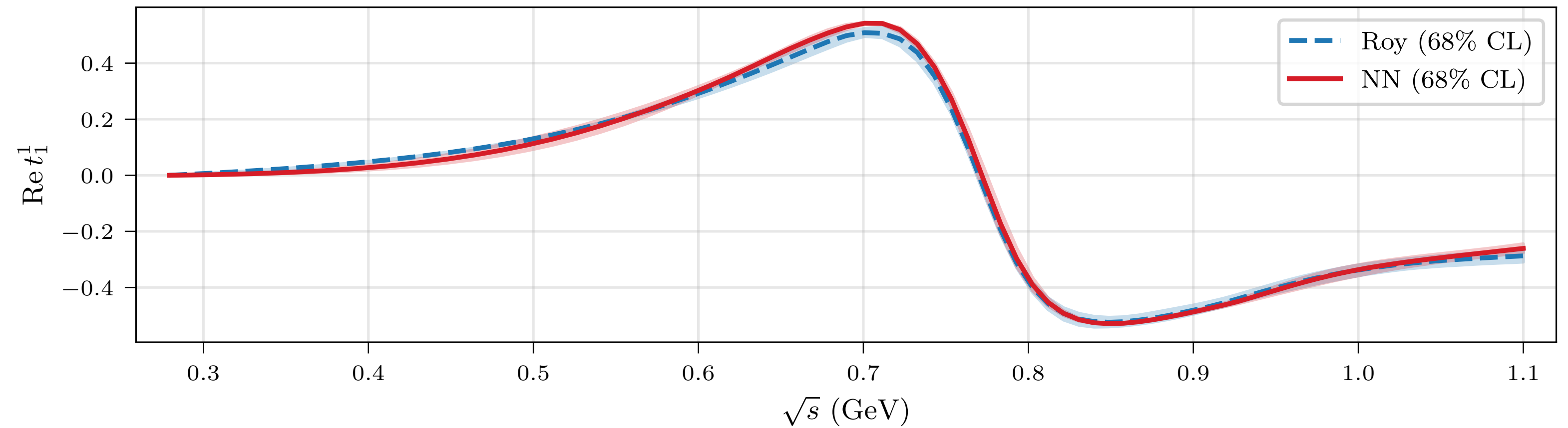
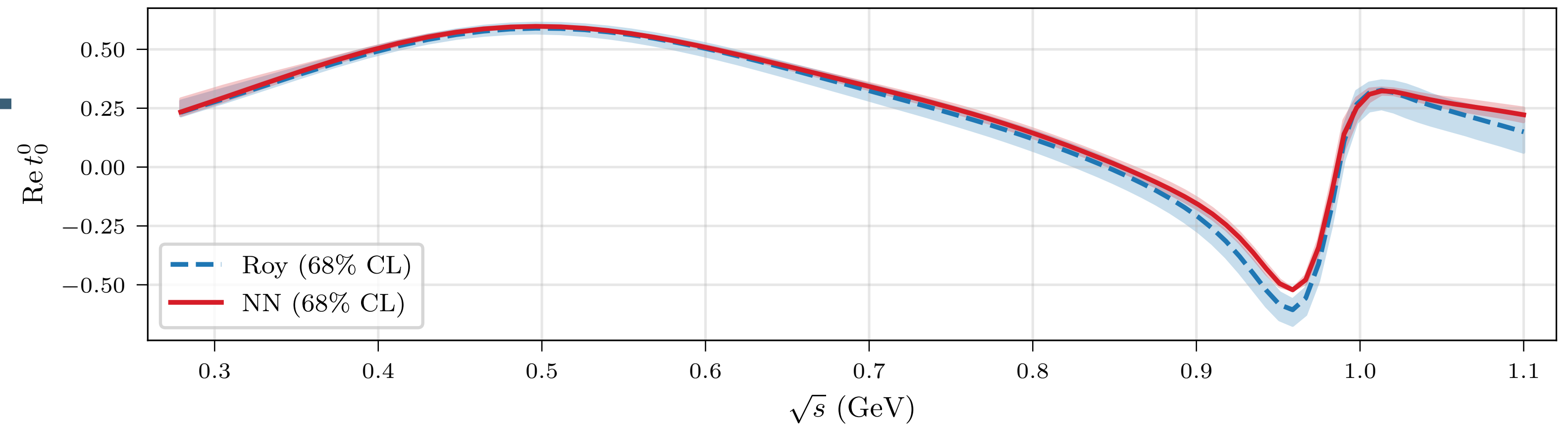
Fits produce  $\chi^2/\text{ndof} \sim 3$

$2\sigma$  band calculated from 5k resamples

## Dispersion relations are satisfied numerically

## Stable under architecture perturbations

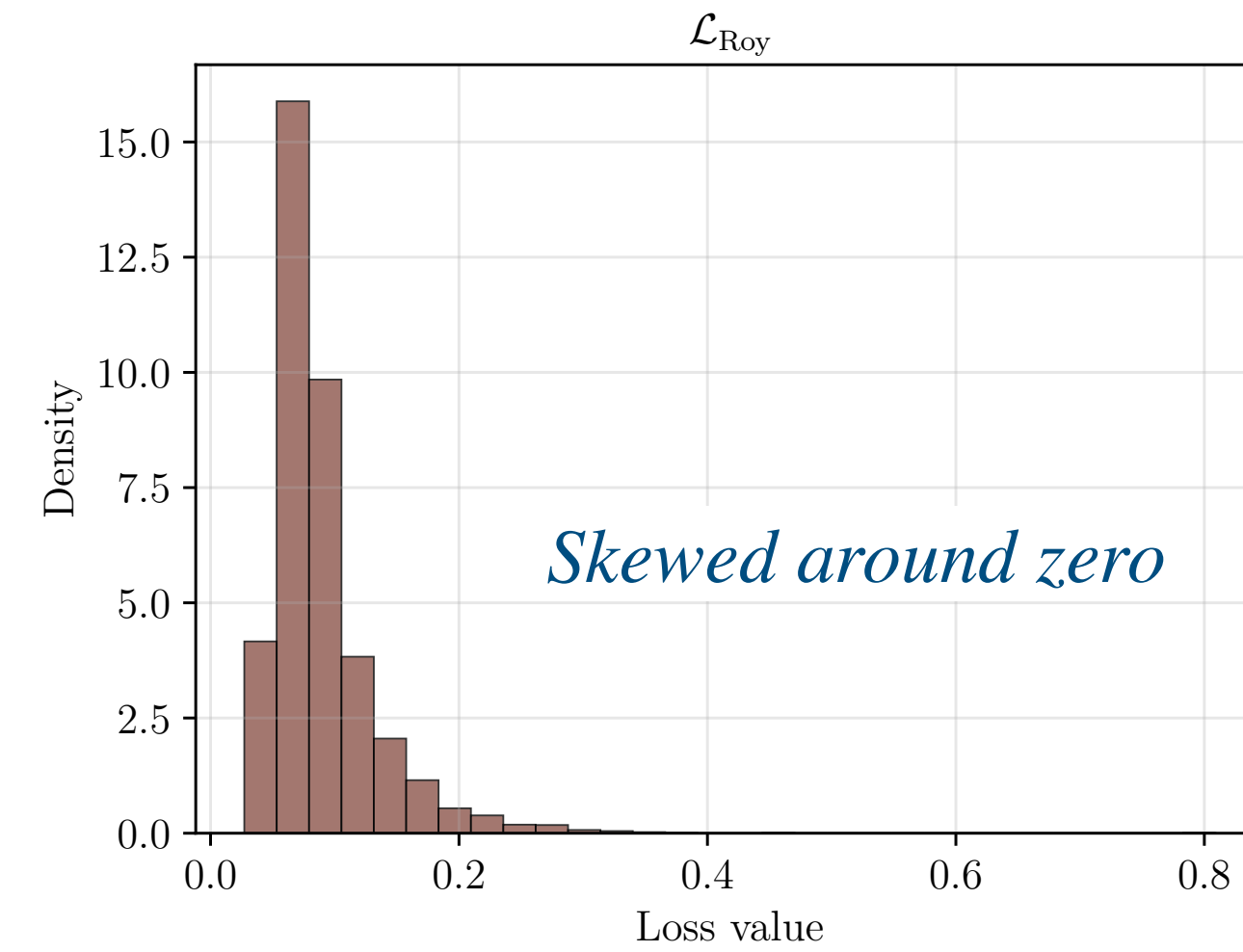
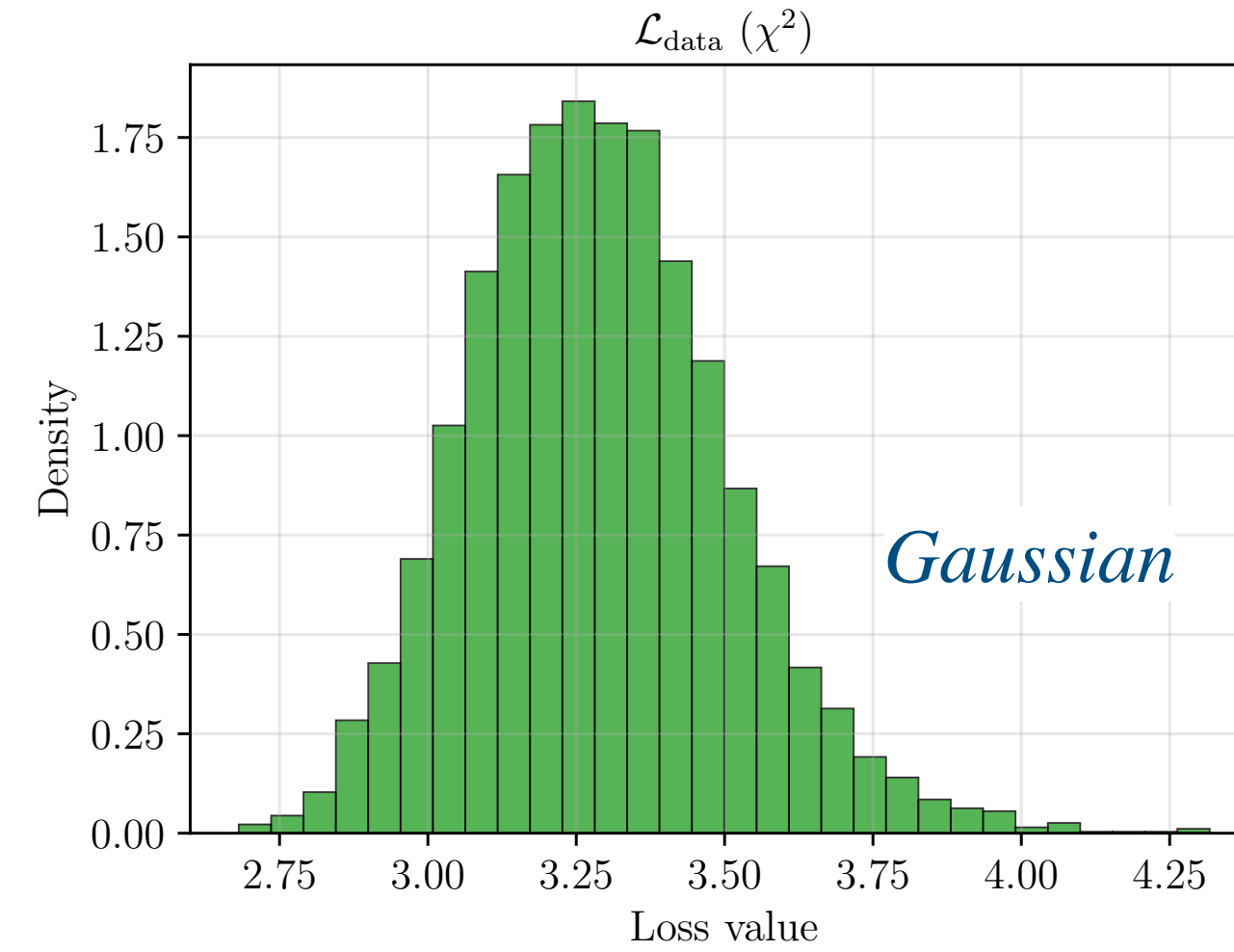
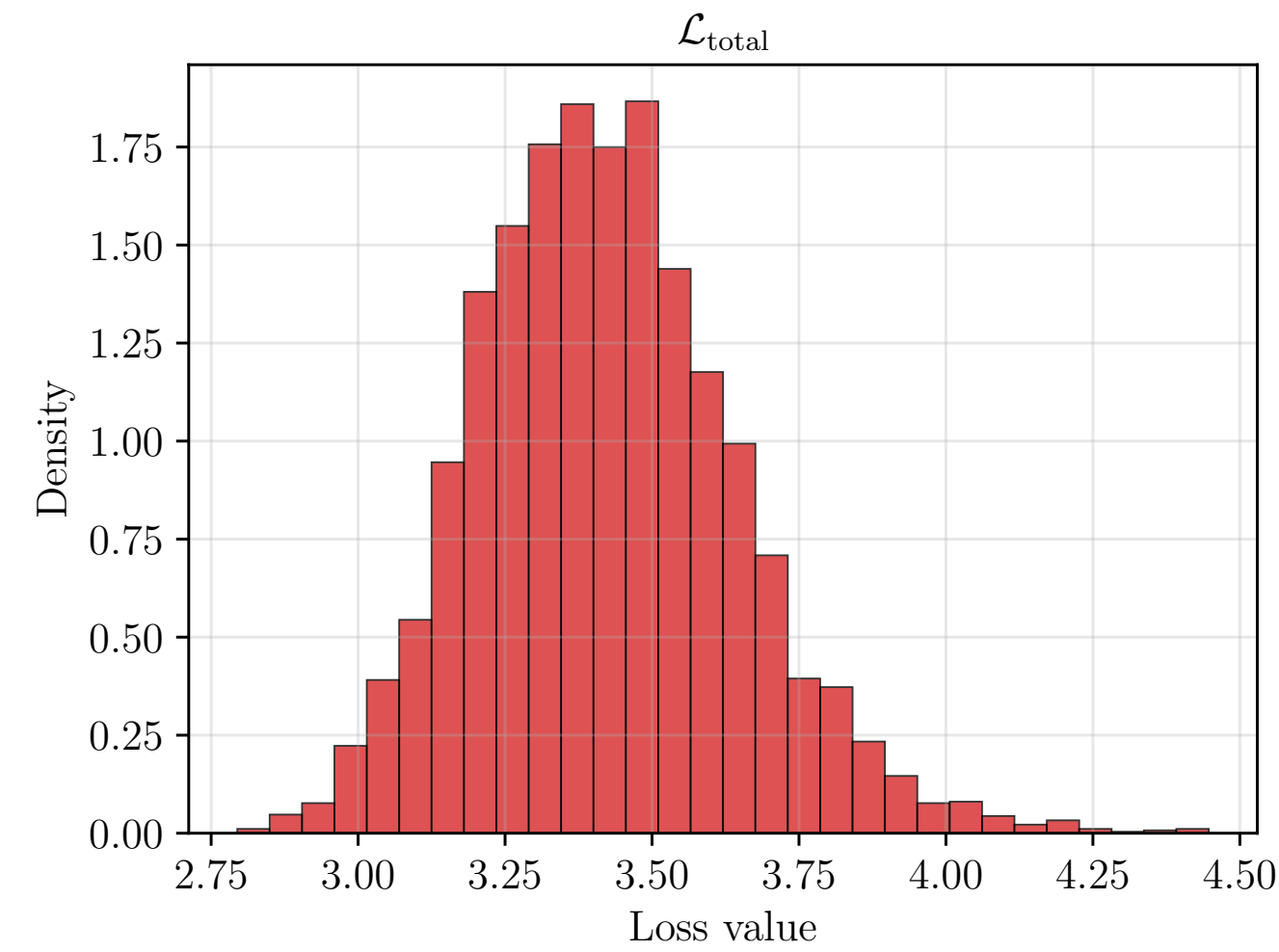
Rank	Source	Trunk	$N_{\text{par}}$	$\mathcal{L}_{\text{total}}$
1	Sweep	[8, 12], GELU	2332	2.29
2	Sweep	[16, 8], SiLU	2142	2.32
3	Sweep	[12], GELU	1742	2.33
4	Sweep	[16], SiLU	980	2.36
5	Sweep	[16, 16], SELU	2714	2.39
6	Sweep	[12, 8, 4], SELU	1376	2.40
7	Sweep	[8, 12], GELU	1658	2.46
8	Prod.	[8, 12], tanh/SiLU	1156	2.46
9	Sweep	[12, 12], GELU	1204	2.47
10	Sweep	[16, 8], GELU	1162	2.49



# $\pi\pi$ scattering: results

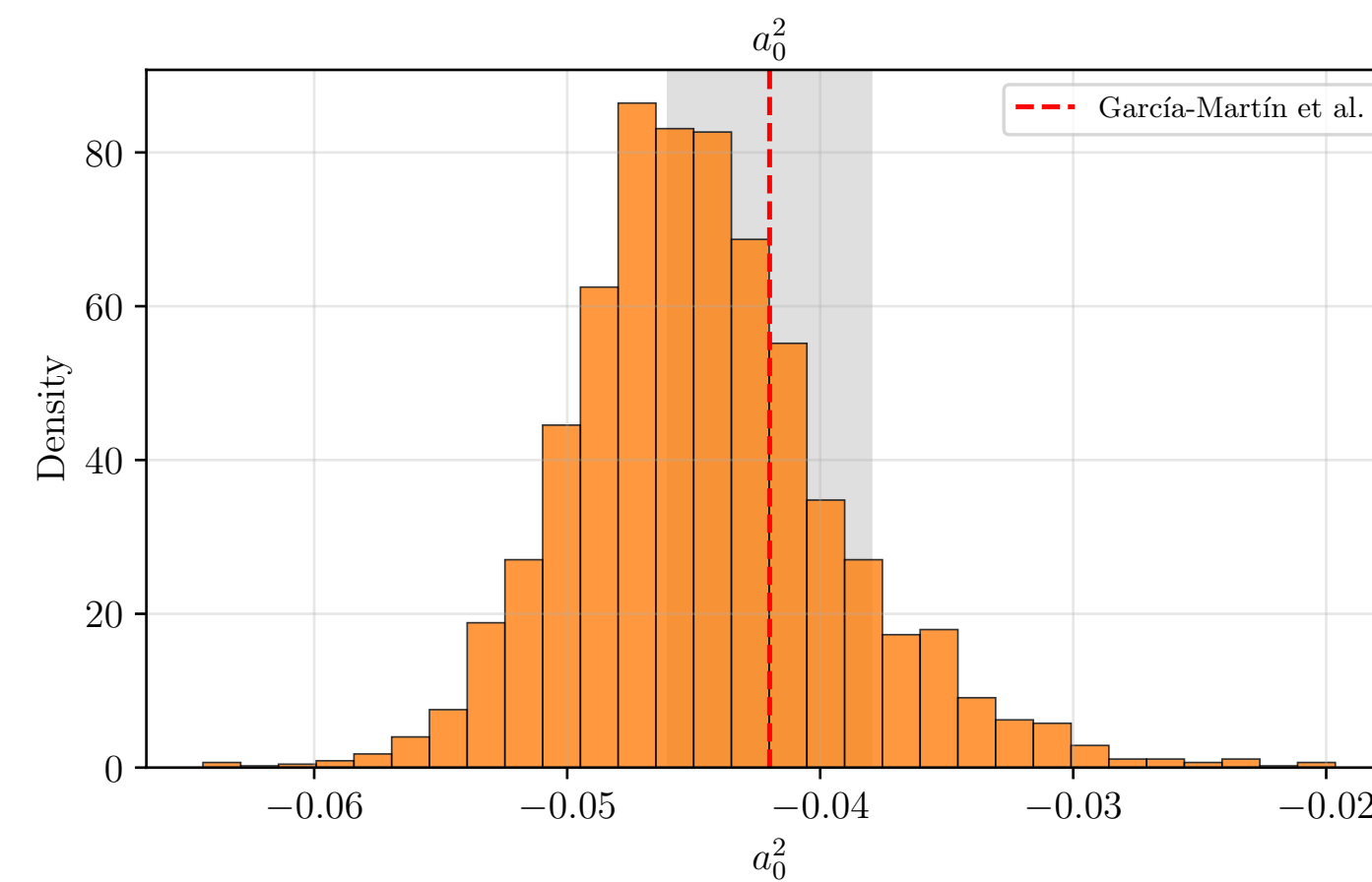
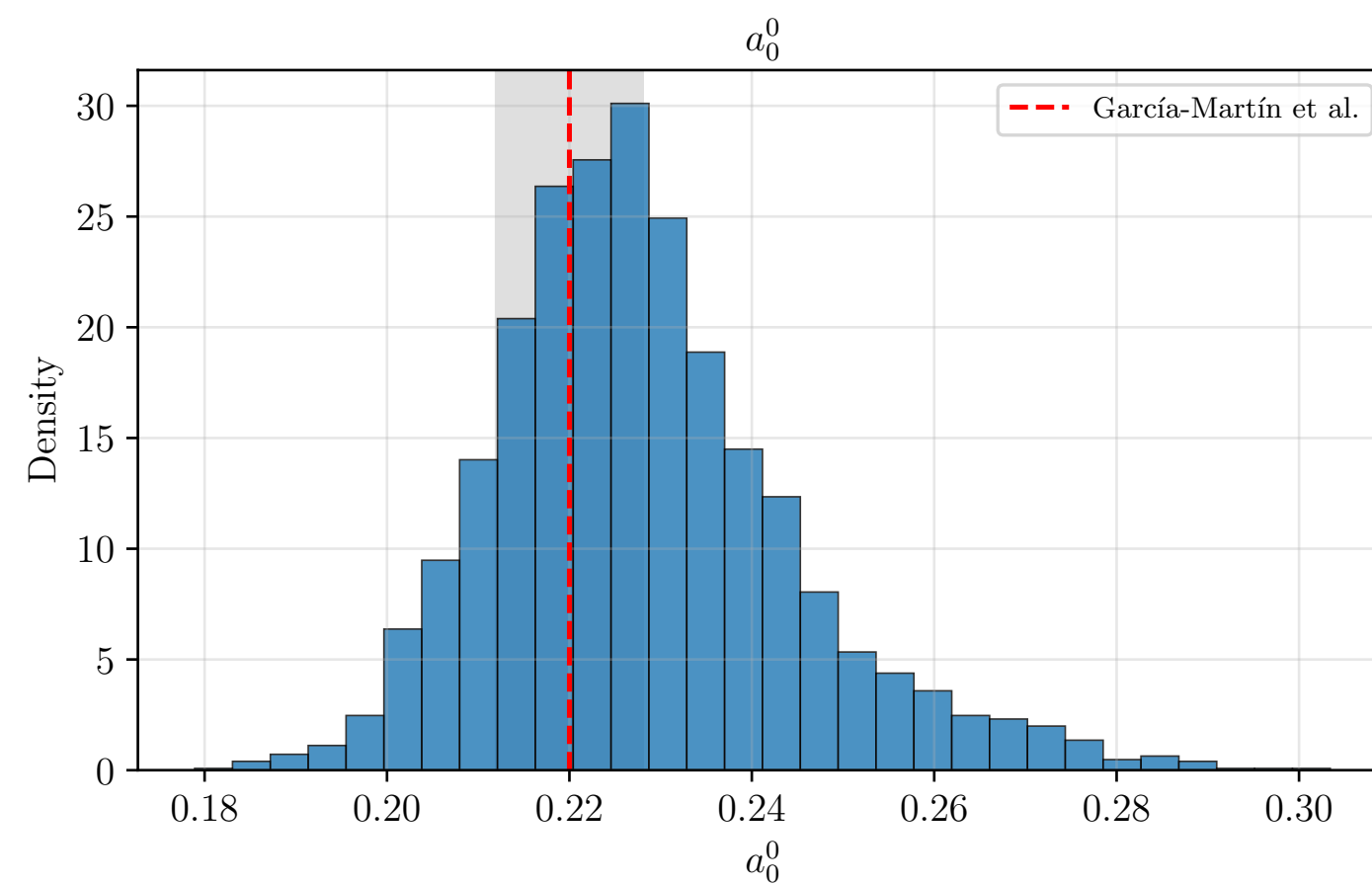
## Final residues are reasonable statistically

Loss dominated by experimental data

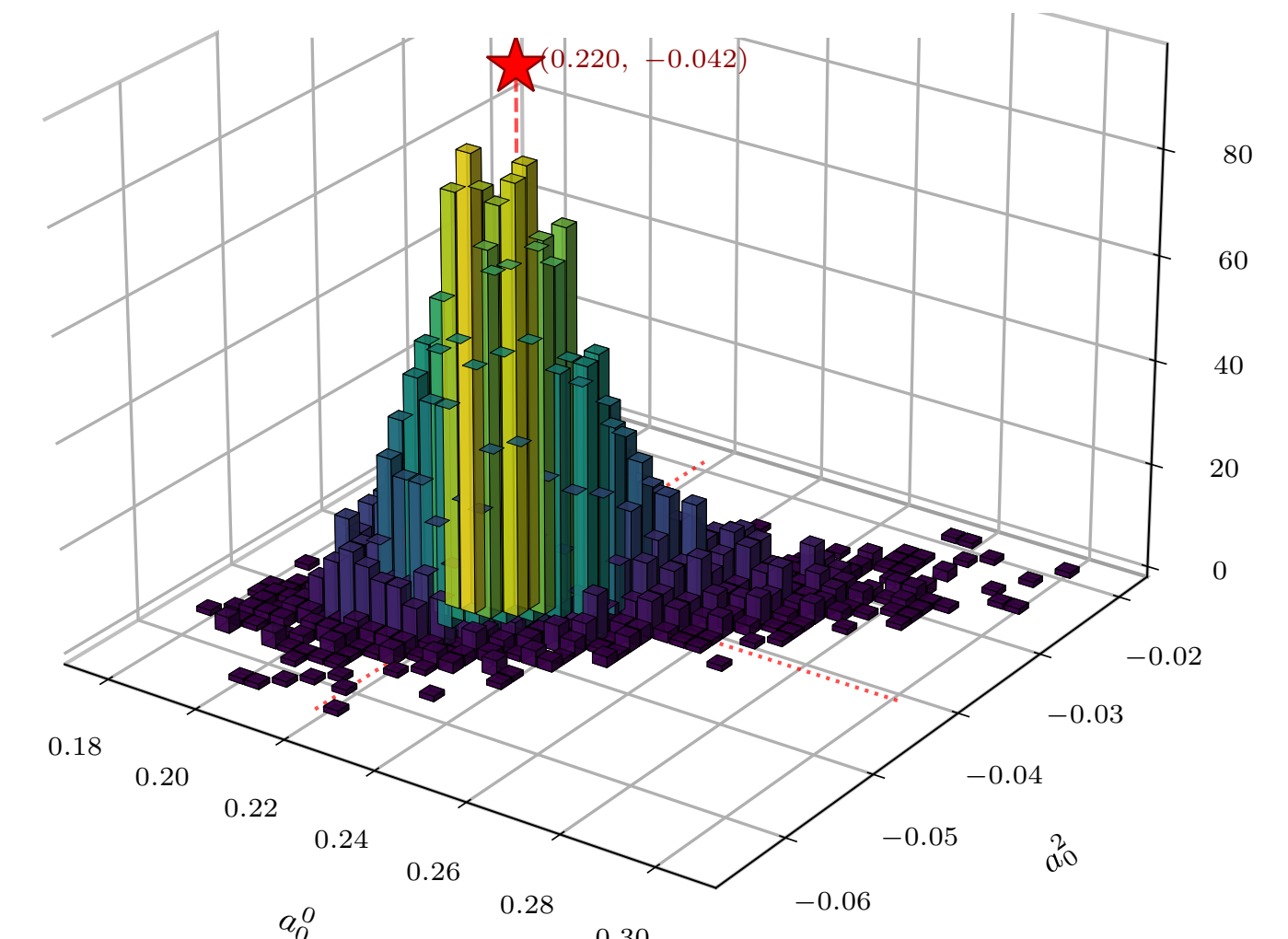


## Scattering lengths compatible with most competitive results

Our errors are substantially larger



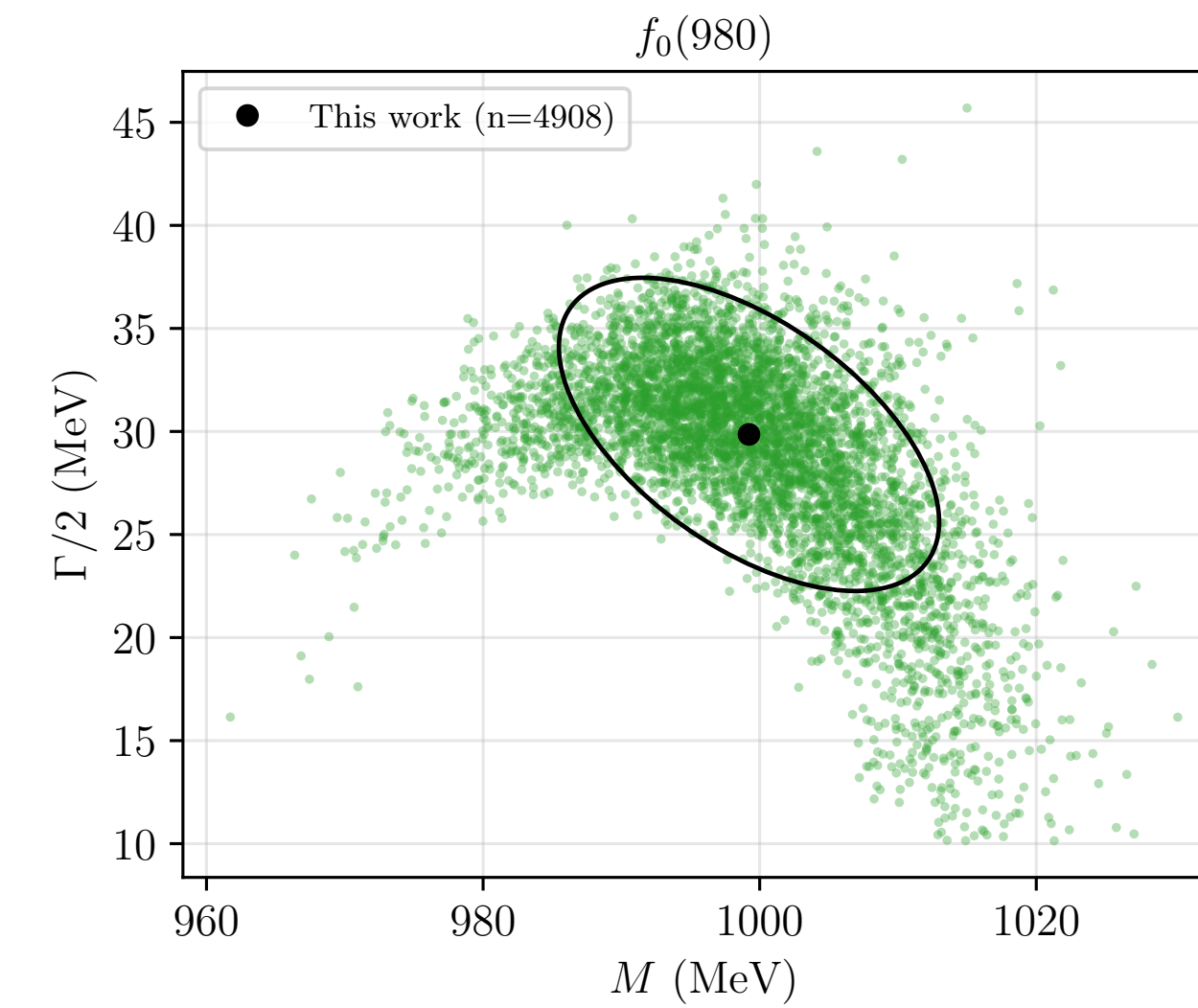
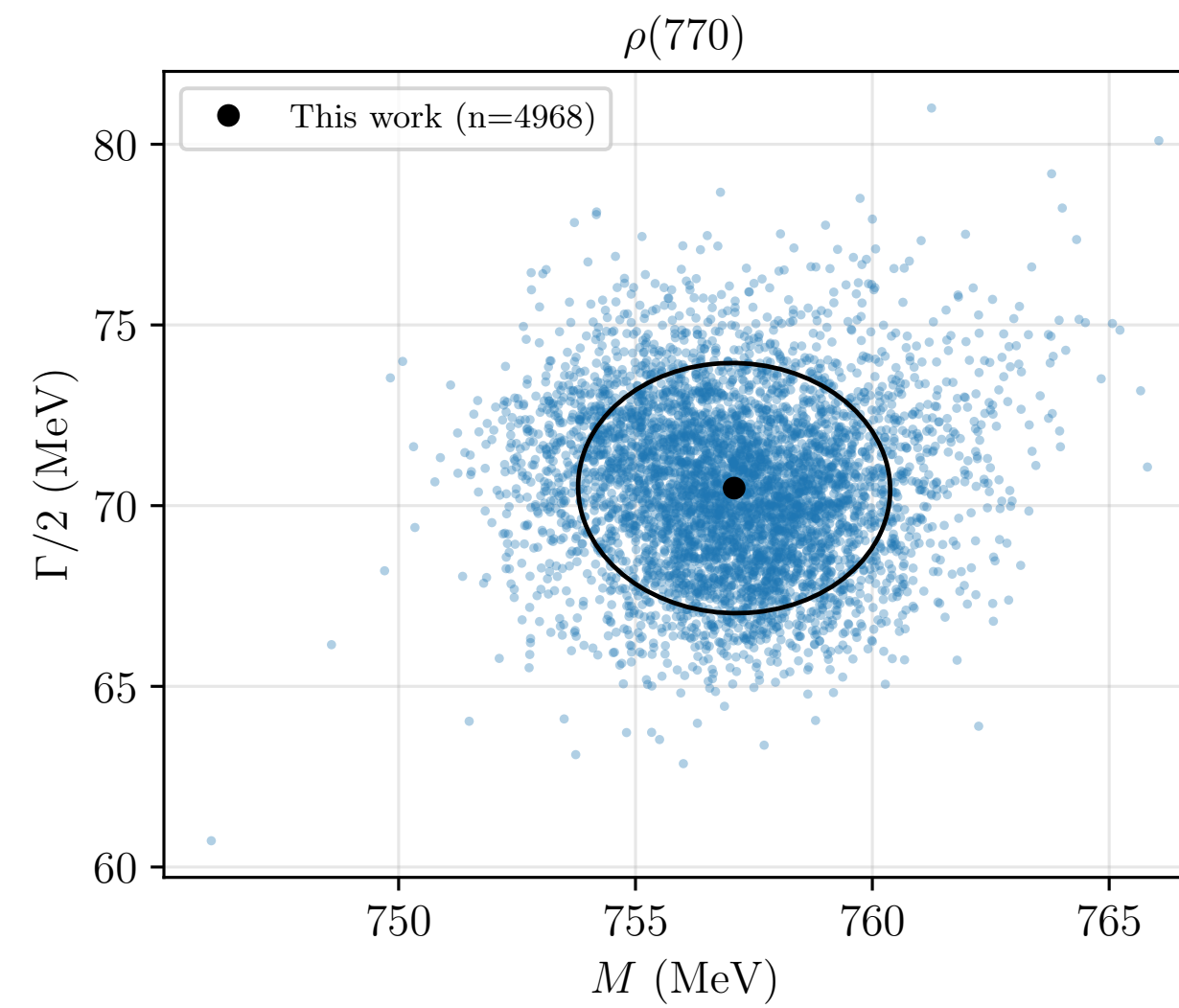
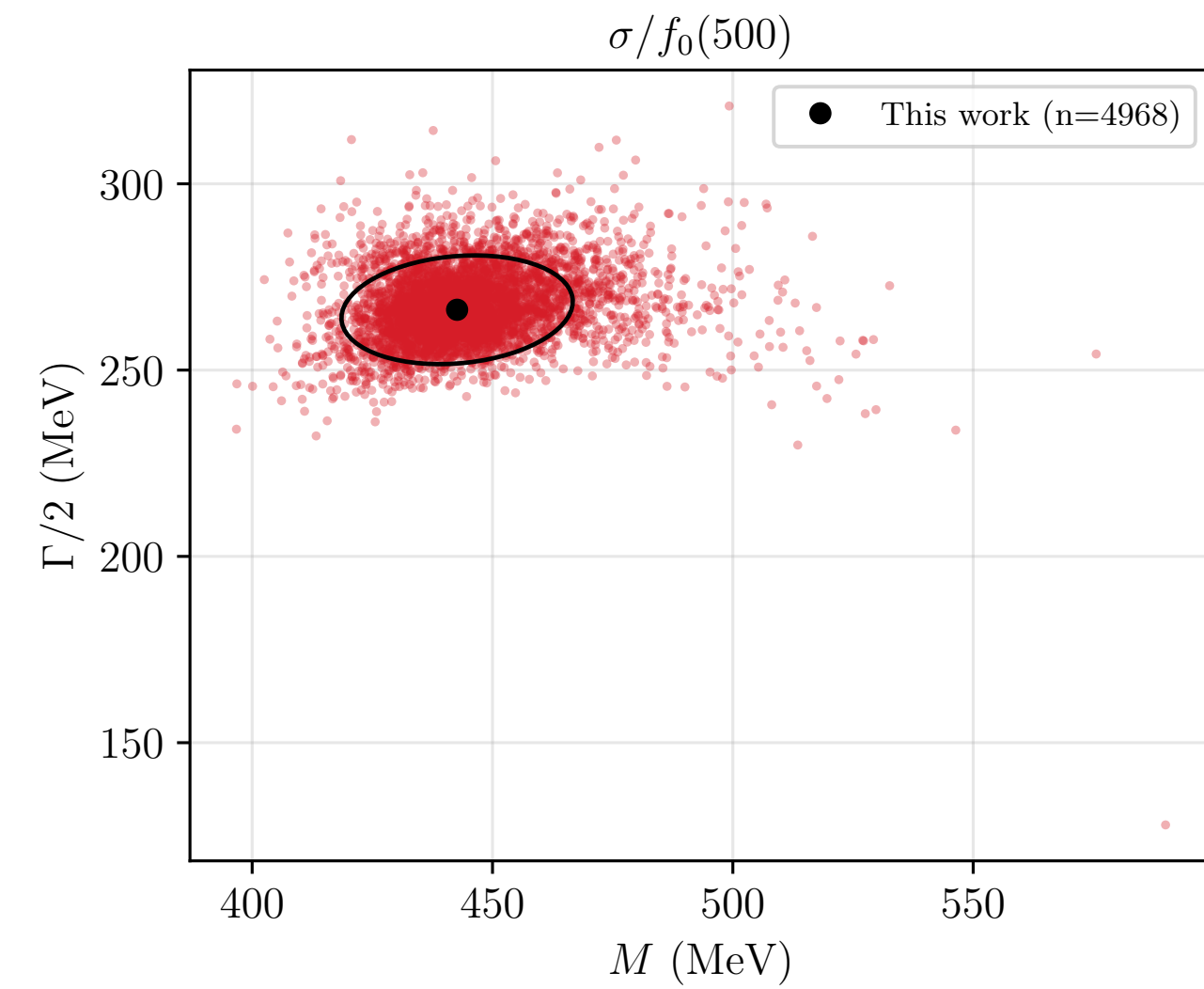
★ García-Martín et al.



# $\pi\pi$ scattering: results

## Pole positions are stable

Accurate and competitive with most well-known dispersive analyses



## We also extracted couplings

