Mass/gravitational FFs from threshold J/ψ production

... and (how) are they related to the E.M. FFs?

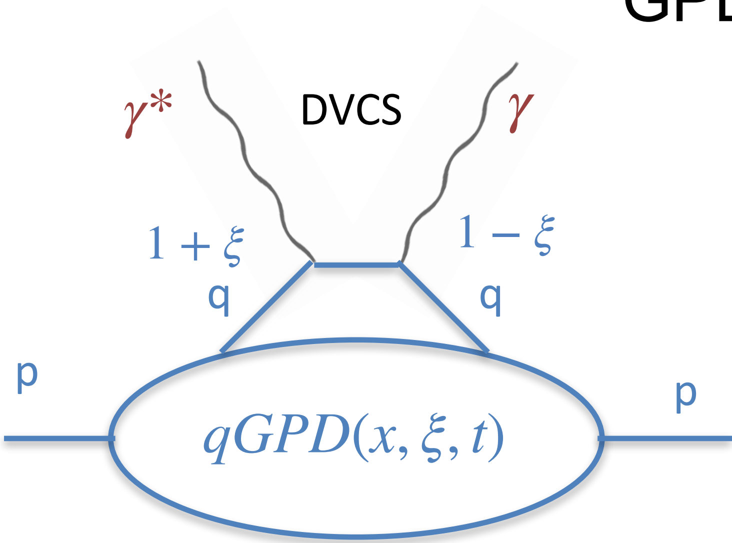
Lubomir Pentchev

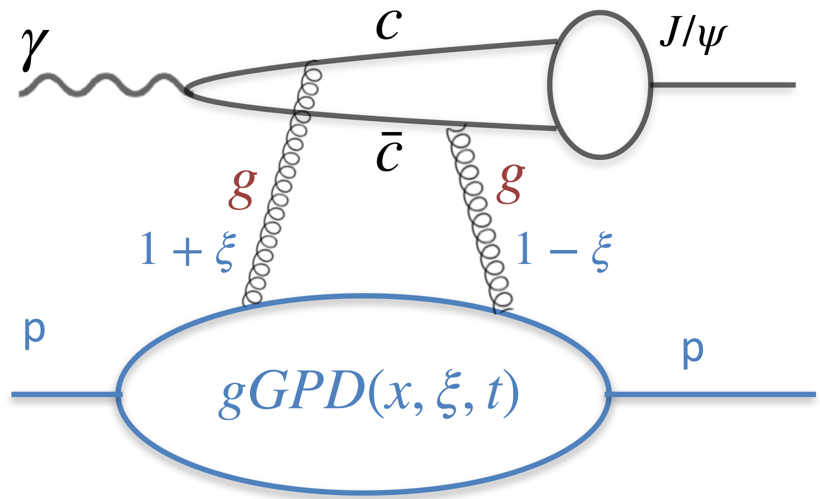
- Charmonium threshold production and Mass/mechanical/gravitational FFs (GFFs) of the proton
- "Rosenbluth" (kinematic) separation of the J/ψ near-threshold photo-production - experimental test of energy independence of corresponding FF functions
- Extracting gluon GFFs from the data using two theoretical approaches
- How the extracted gluon GFFs compare to lattice results

- GFFs and E.M. FFs as moments of GPD - some relations b/n GFFs and E.M.FFs

- What can be done with SBS

Uniqueness of threshold charmonium photoproduction - GPD approach





- Compton-like amplitudes, form-factors $\mathscr{A}_g(t), \mathscr{B}_g(t), \mathscr{C}_g(t)$, as in Deeply Virtual Compton scattering (DVCS)

However:

- gluon (not quark) exchange
- Threshold kinematics is very different: high momentum transfer t and skewness ξ
- In heavy-quark limit: $t \to \infty$ $\xi \to 1$

For $\xi \to 1$ expansion in x/ξ:

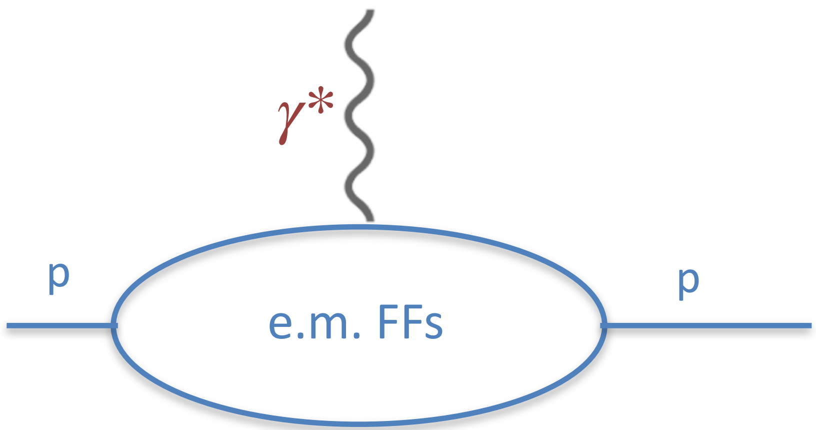
$d\sigma/dt = F(E_\gamma)\xi^{-4}[G_0(t) + \xi^2 G_2(t) + \xi^4 G_4(t)] + \ldots,$ $G_i(t)$ functions of $\mathscr{A}_g^{(2)}(t), \mathscr{B}_g^{(2)}(t), \mathscr{C}_g(t)$

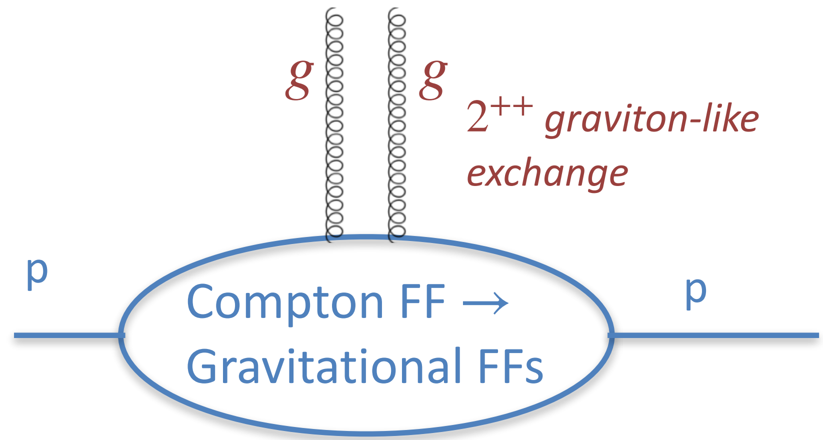
In leading-moment approximation:

gluon Gravitational Form Factors $A_g(t), B_g(t), C_g(t)$ are leading terms of $\mathscr{A}_g^{(2)}(t), \mathscr{B}_g^{(2)}(t), \mathscr{C}_g(t)$

GPD analysis by Guo, Ji, Liu, Yuan PRD 109 (2024)

Gluon Gravitational Form Factors





$$\left(\frac{d\sigma}{d\Omega}\right)_{ep\to ep} = \left(\frac{d\sigma}{d\Omega}\right)_M \frac{1}{(1+\tau)}\left[G_E^2(t) + \frac{\tau}{\epsilon}G_M^2(t)\right]$$

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p\to J/\psi p} = F(E_\gamma)\xi^{-4}[G_0(t) + \xi^2 G_2(t) + \xi^4 G_4(t)]$$

Model approach - fit dipole/tripole FFs (within some model) to data

$$G_E(t), G_M(t) \sim G_D(t) = \frac{1}{(1+t/0.71GeV^2)^2}$$

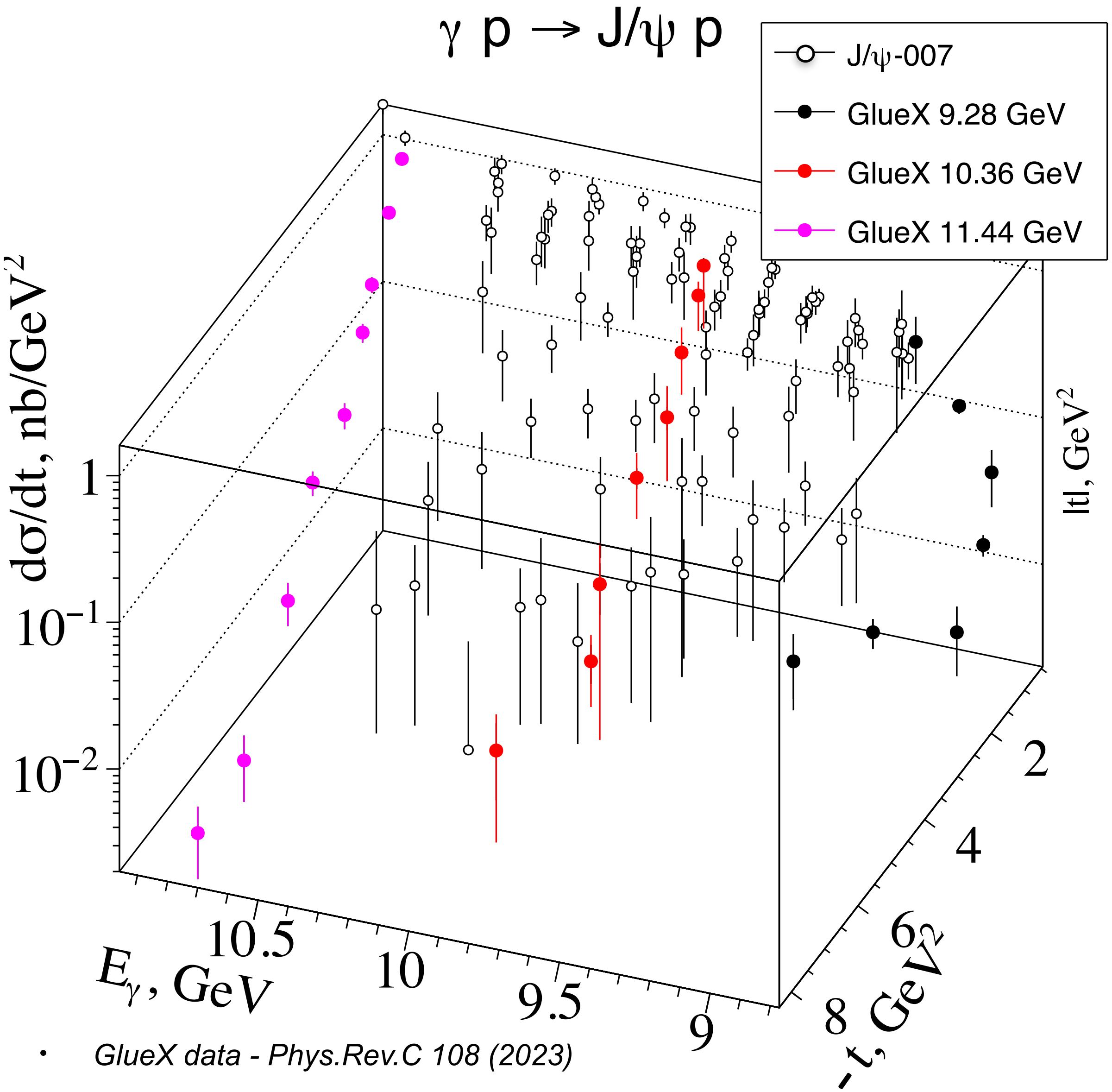
$$A_g(t), B_g(t), C_g(t) \sim \frac{1}{(1+t/m_i^2)^{2(3)}}$$

Rosenbluth separation - data-driven kinematic GFF extraction

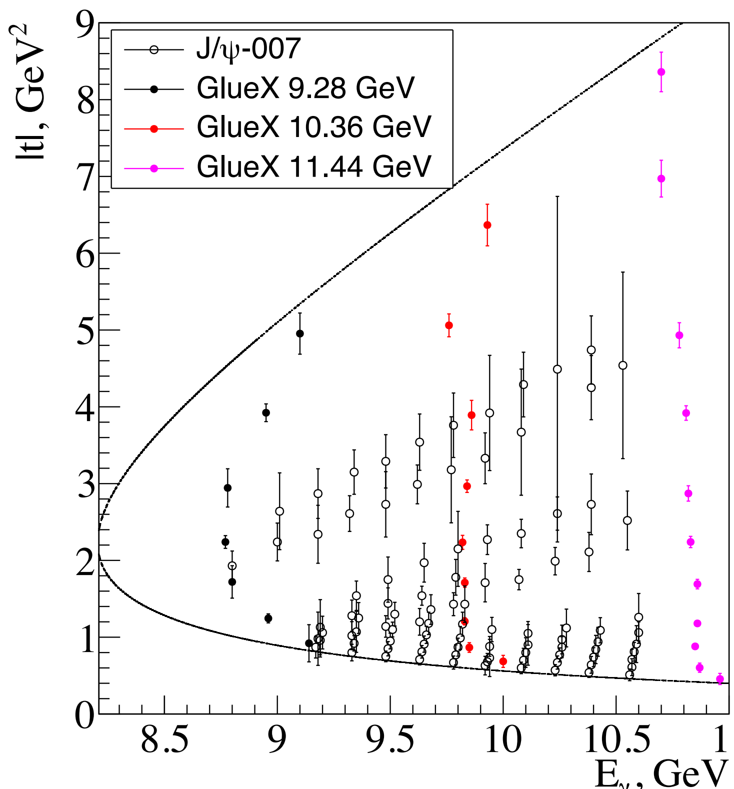
$$\sigma_R = \frac{d\sigma}{d\Omega}/\left(\frac{d\sigma}{d\Omega}\right)_M \frac{\epsilon(1+\tau)}{\tau} = \frac{\epsilon}{\tau}G_E^2(t) + G_M^2(t),$$

$$\sigma_{R0} = \frac{d\sigma}{dt}\frac{\xi^2}{F(E_\gamma)} = \xi^{-2}G_0(t) + G_2(t) + \xi^2 G_4(t)$$

Threshold J/ψ photoproduction - the data



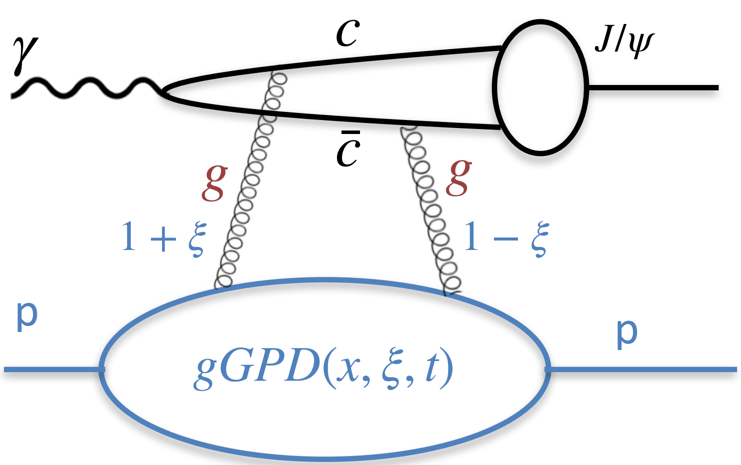
find features in the data consistent with some general model predictions

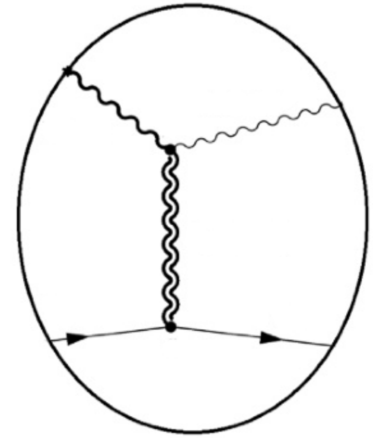


- *GlueX data - Phys.Rev.C 108 (2023)*

J/ψ-007 data - Nature 615 (2023)

Threshold charmonium photoproduction - GPD and holographic approaches





Witten diagram representation

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p\to J/\psi p} = F(E_\gamma)\xi^{-4}[G_0(t)+\xi^2 G_2(t)] + \ldots$$

$$G_0(t) = \left(\mathscr{A}_g^{(2)}(t)\right)^2 - \frac{t}{4m^2}\left(\mathscr{B}_g^{(2)}(t)\right)^2$$

$$G_2(t) = 2\mathscr{A}_g^{(2)}(t)\mathscr{C}_g(t) + 2\frac{t}{4m^2}\mathscr{B}_g^{(2)}(t)\mathscr{C}_g(t) - \left(\mathscr{A}_g^{(2)}(t) + \mathscr{B}_g^{(2)}(t)\right)^2$$

$$\mathscr{A}_g^{(2)}(t) = A_g(t)$$

$$\mathscr{B}_g^{(2)}(t) = B_g(t)$$

$$\mathscr{C}_g(t) = 4C_g(t)$$

$$\xi = \frac{M_{J/\psi}^2 - t}{2(s-m^2) - M_{J/\psi}^2 + t}$$

for high ξ values

leading moments

GPD analysis by Guo, Ji, Yuan PRD 109 (2024)

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p\to J/\psi p} = N(E_\gamma)[H_0(t)+\eta^2 H_2(t)] + \ldots$$

$$H_0(t) = A_g^2(t)$$

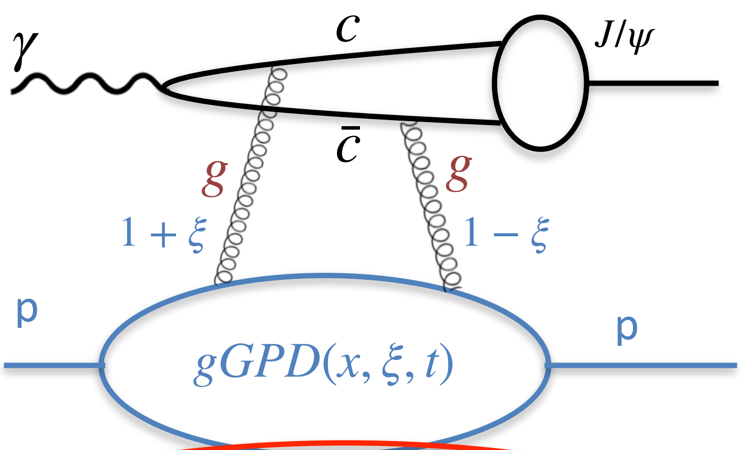
$$H_2(t) = 8A_g(t)C_g(t)$$

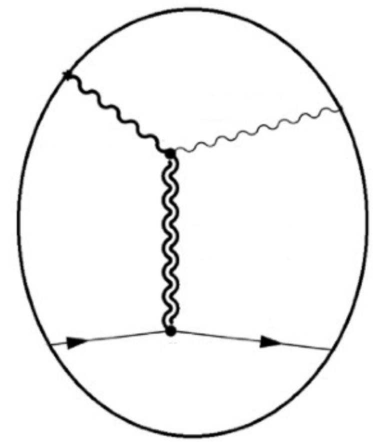
for large N_c and strong α_s

$$\eta = \frac{M_{J/\psi}^2}{2(s-m^2) - M_{J/\psi}^2 + t}$$

Holographic analysis by Mamo and Zahed PRD 106 (2022), PRD, PRD 101 (2020), Hatta and Yang PRD 98 (2018)

Threshold charmonium photoproduction - GPD and holographic approaches





Witten diagram representation

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p \to J/\psi p} = F(E_\gamma)\xi^{-4}[G_0(t)+\xi^2 G_2(t)] + \ldots$$

for high ξ values

$$G_0(t) = \left(\mathscr{A}_g^{(2)}(t)\right)^2 - \frac{t}{4m^2}\left(\mathscr{B}_g^{(2)}(t)\right)^2$$

$$G_2(t) = 2\mathscr{A}_g^{(2)}(t)\mathscr{C}_g(t) + 2\frac{t}{4m^2}\mathscr{B}_g^{(2)}(t)\mathscr{C}_g(t) - \left(\mathscr{A}_g^{(2)}(t) + \mathscr{B}_g^{(2)}(t)\right)^2$$

$$\mathscr{A}_g^{(2)}(t) = A_g(t)$$

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leading moments

$$\xi = \frac{M_{J/\psi}^2 - t}{2(s-m^2) - M_{J/\psi}^2 + t}$$

GPD analysis by Guo, Ji, Yuan PRD 109 (2024)

$$\left(\frac{d\sigma}{dt}\right)_{\gamma p \to J/\psi p} = N(E_\gamma)[H_0(t)+\eta^2 H_2(t)] + \ldots$$

for large N_c and strong α_s

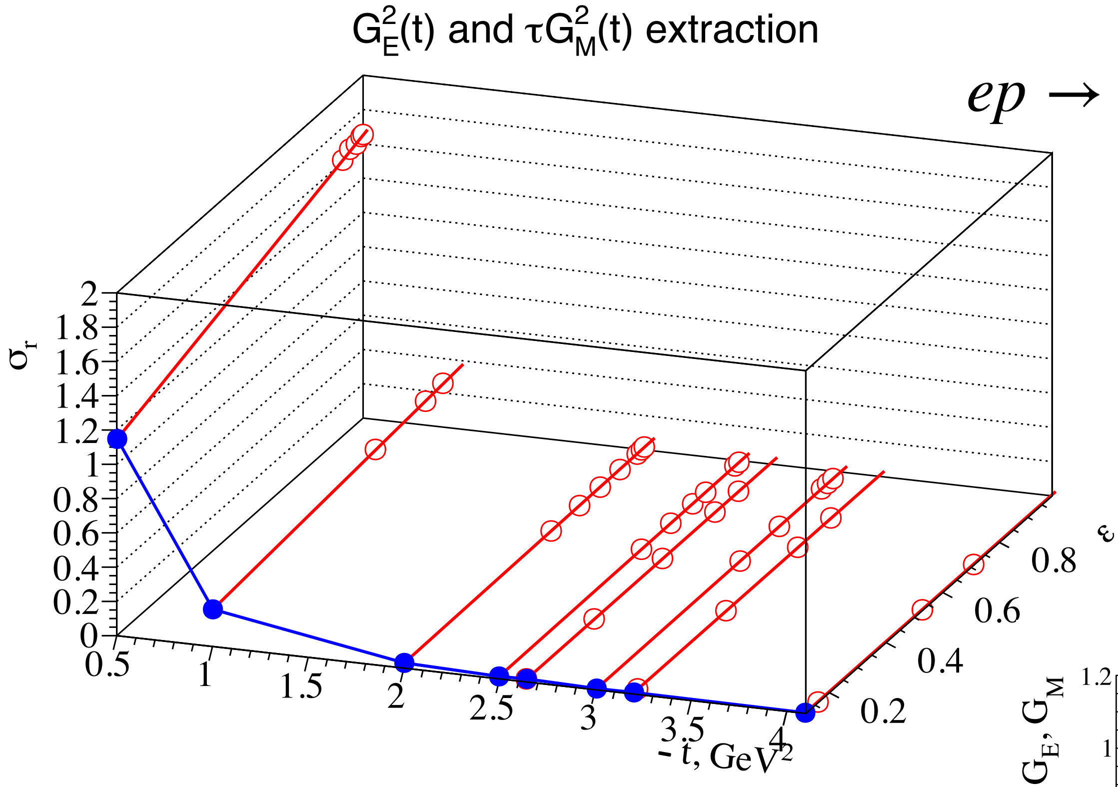
$$H_0(t) = A_g^2(t)$$

$$H_2(t) = 8A_g(t)C_g(t)$$

$$\eta = \frac{M_{J/\psi}^2}{2(s-m^2) - M_{J/\psi}^2 + t}$$

Holographic analysis by Mamo and Zahed PRD 106 (2022), PRD, PRD 101 (2020), Hatta and Yang PRD 98 (2018)

Rosenbluth separation

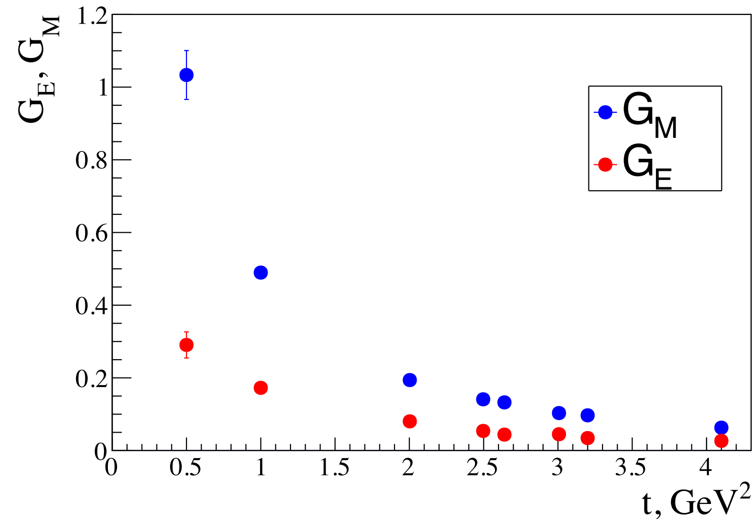


$ep \to ep$

Data from:

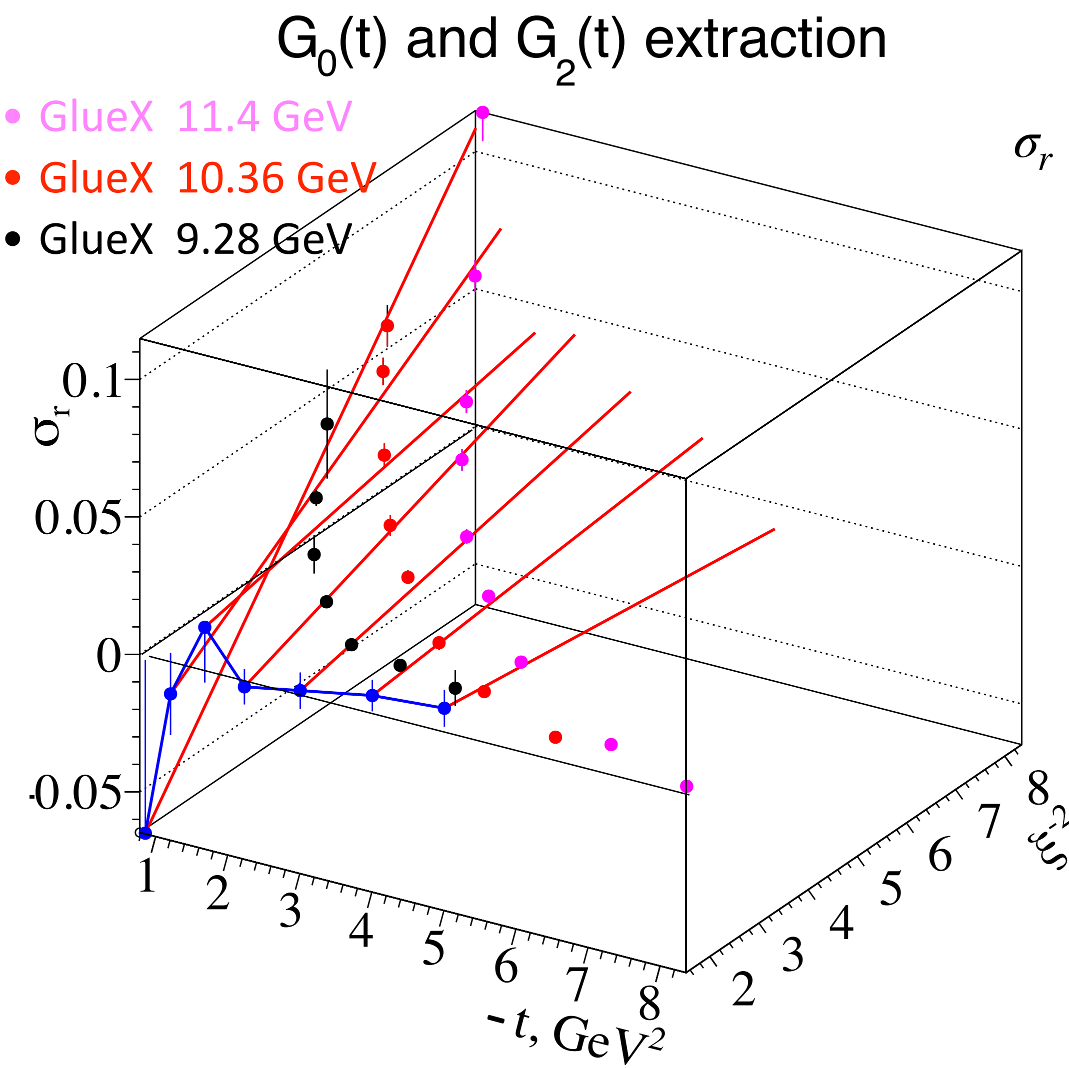
R.C.Walker et al. Phys.Rev.D 49, 11 (1994) - SLAC

I.A.Qattan et al. arXiv 2411.05201 (2024) - JLab



$$\sigma_R = \frac{d\sigma}{d\Omega} / \left(\frac{d\sigma}{d\Omega}\right)_M \varepsilon(1+\tau) = \varepsilon G_E^2(t) + \tau G_M^2(t)$$

Threshold J/ψ photoproduction - "Rosenbluth" separation



$$\sigma_r = \frac{d\sigma}{dt}\frac{\xi^2}{F(E_\gamma)} = \xi^{-2}G_0(t) + G_2(t)$$

$$G_0(t) = \left(\mathscr{A}_g^{(2)}(t)\right)^2 - \frac{t}{4m^2}\left(\mathscr{B}_g^{(2)}(t)\right)^2 > 0$$

Therefore $d\sigma/dt(E,t)$ at fixed t, must increase with energy, consistent with the data!

Testing Energy Independence of Form Factor Functions

$G_0(t)$ - GPD

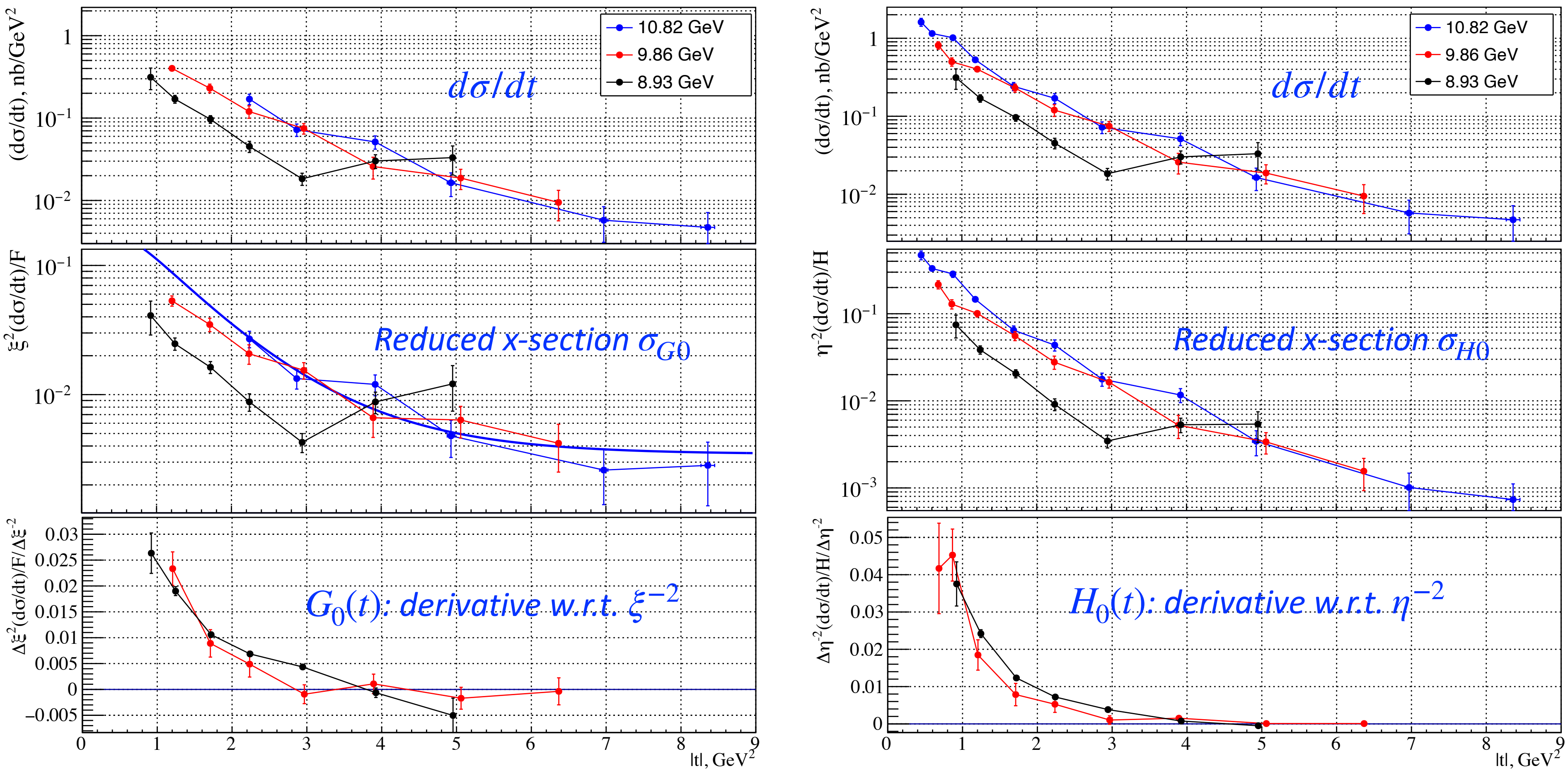
$$\sigma_{G0} = \frac{d\sigma}{dt}\frac{\xi^2}{F(E_\gamma)} = \xi^{-2}G_0(t) + G_2(t) \qquad \xi > 0.4$$

$$G_0(t) = \left[\sigma_{R0}(E_i,t) - \sigma_{R0}(E_j,t)\right] / \left[\xi^{-2}(E_i,t) - \xi^{-2}(E_j,t)\right]$$

$H_0(t)$ - Holography

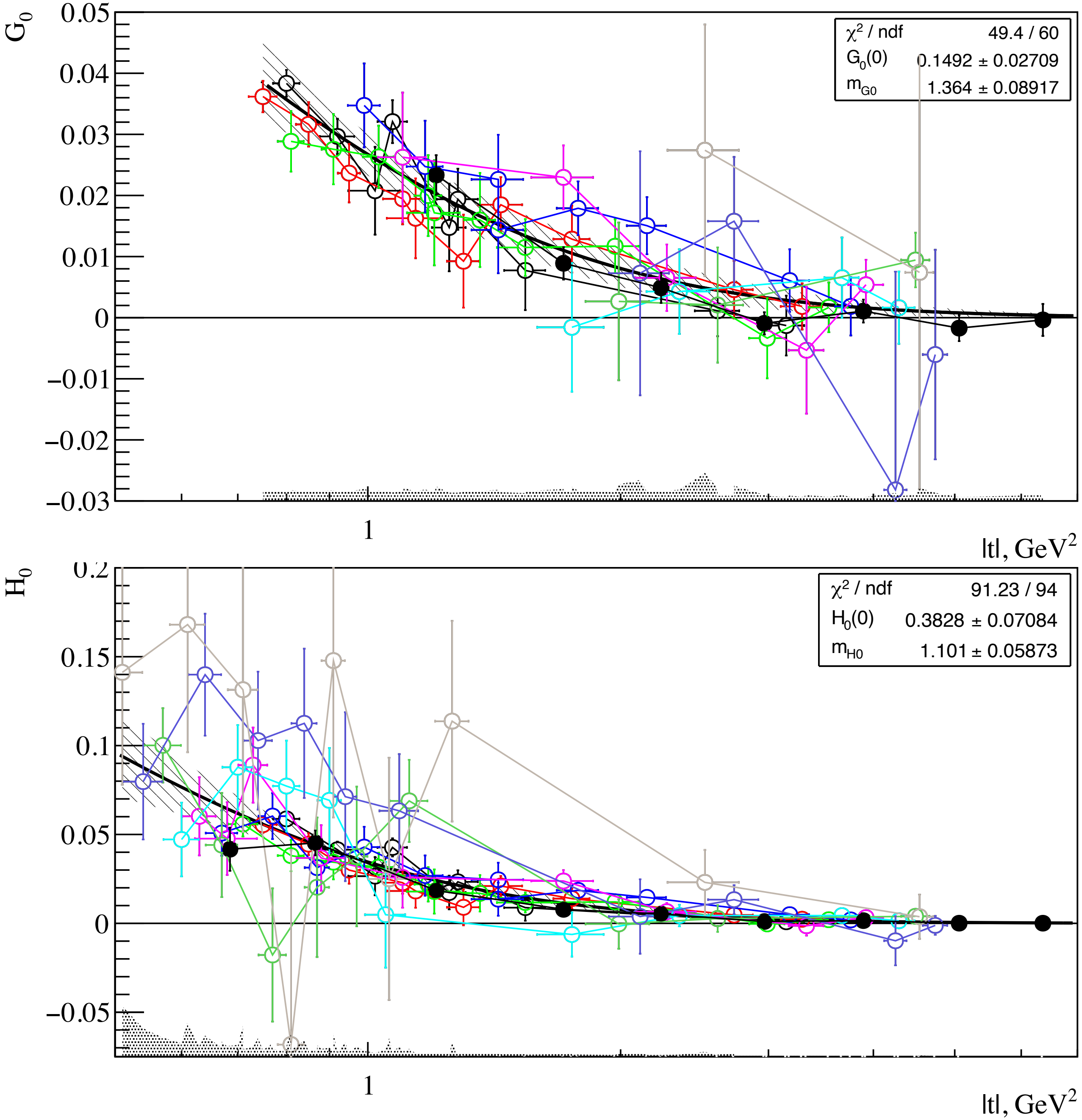
$$\sigma_{H0} = \frac{d\sigma}{dt}\frac{\eta^{-2}}{H(E_\gamma)} = \eta^{-2}H_0(t) + 4H_2(t)$$

$$H_0(t) = \left[\sigma_{R0}(E_i,t) - \sigma_{R0}(E_j,t)\right] / \left[\eta^{-2}(E_i,t) - \eta^{-2}(E_j,t)\right]$$



GlueX data - Phys.Rev.C 108 (2023)

Gluon Form Factors - energy independence



Energy independence of the $G(t)/H(t)$ functions (within errors)

Fits with:

$$\frac{G_0(0)}{(1 - t/m_{G0}^2)^4}$$

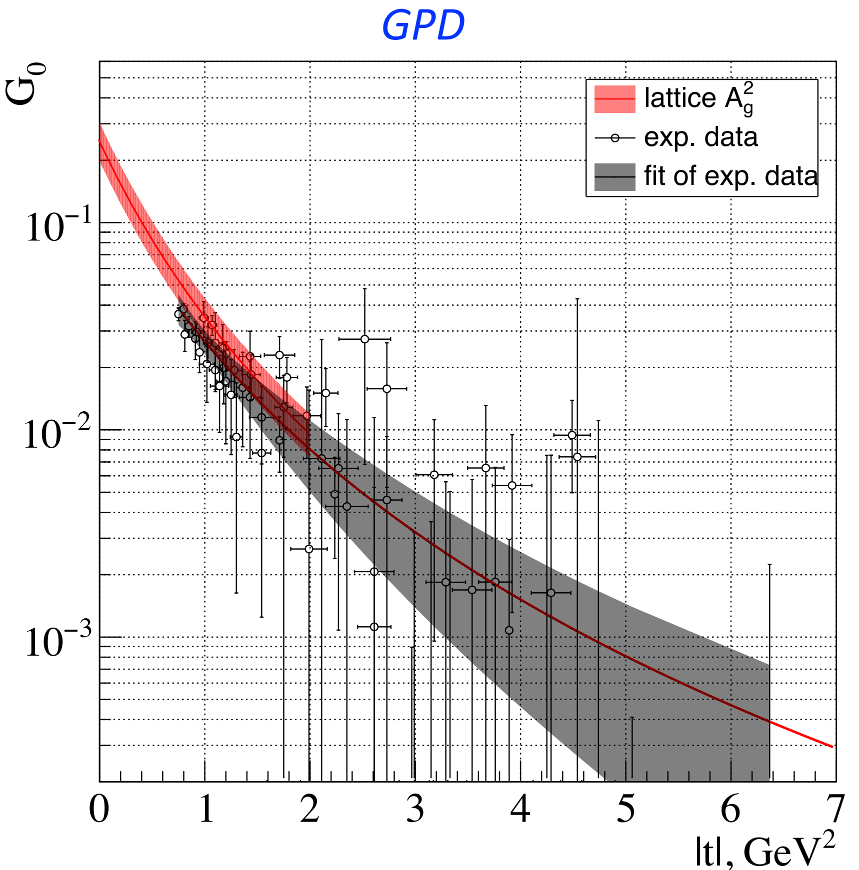
$$\frac{H_0(0)}{(1 - t/m_{H0}^2)^4}$$

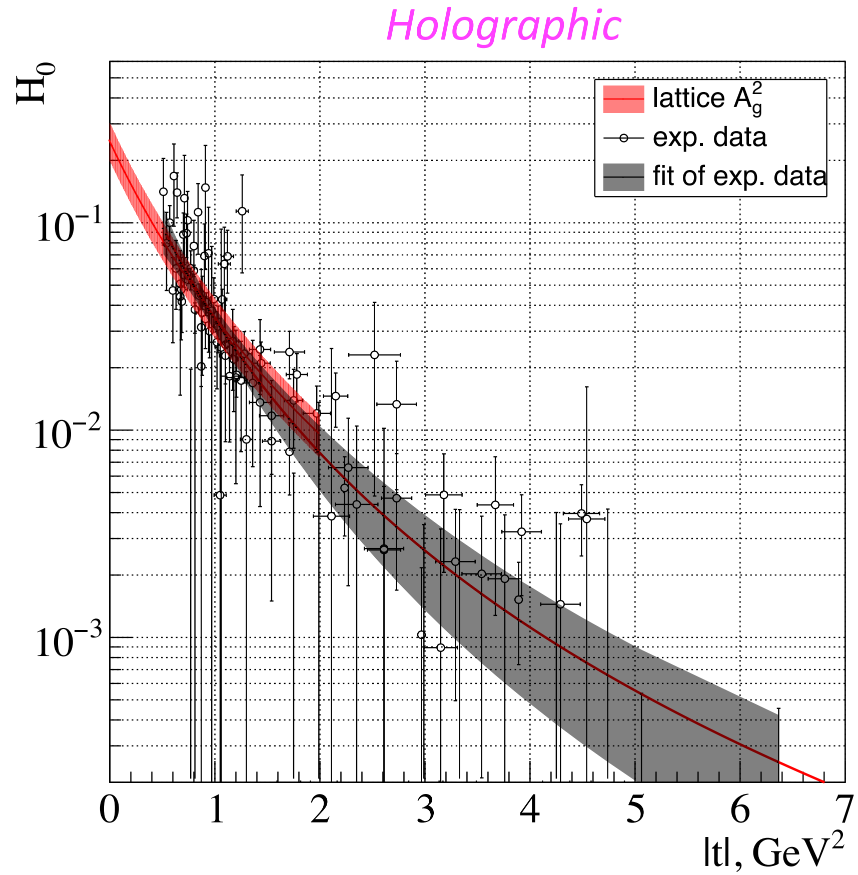
Using GlueX and J/ψ-007 data - different colors - different energies

No theory/lattice constraints used!

LP and E.Chudakov
Phys. Rev. D **112**, 052009 (2025)

Assuming leading-term approximation - data vs lattice





$G_0(t) = \left(\mathscr{A}_g^{(2)}(t)\right)^2 - \frac{t}{4m^2}\left(\mathscr{B}_g^{(2)}(t)\right)^2$ *for high ξ values*

$\mathscr{A}_g^{(2)}(t) = A_g(t)$

$\mathscr{B}_g^{(2)}(t) = B_g(t)$

leading-moment approximation

neglecting $B_g(t)$

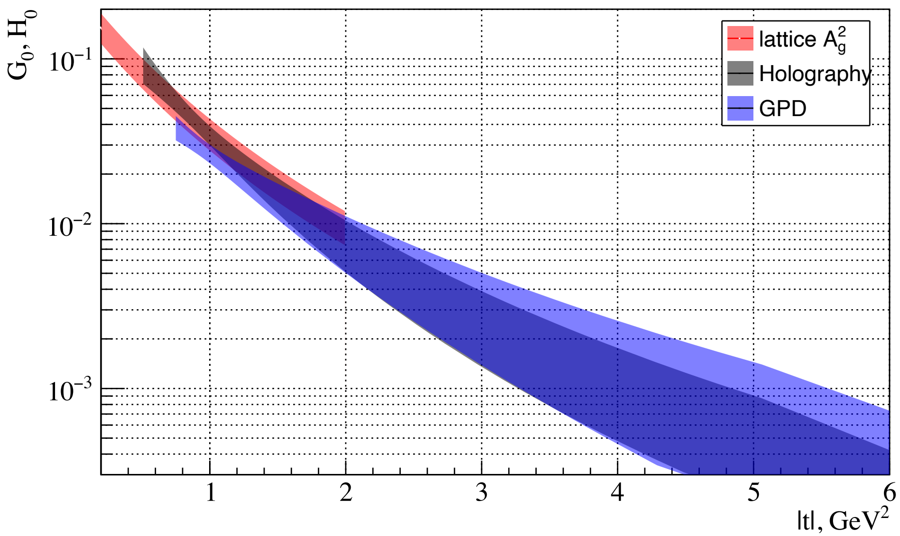
$G_0(t) = A_g^2(t)$

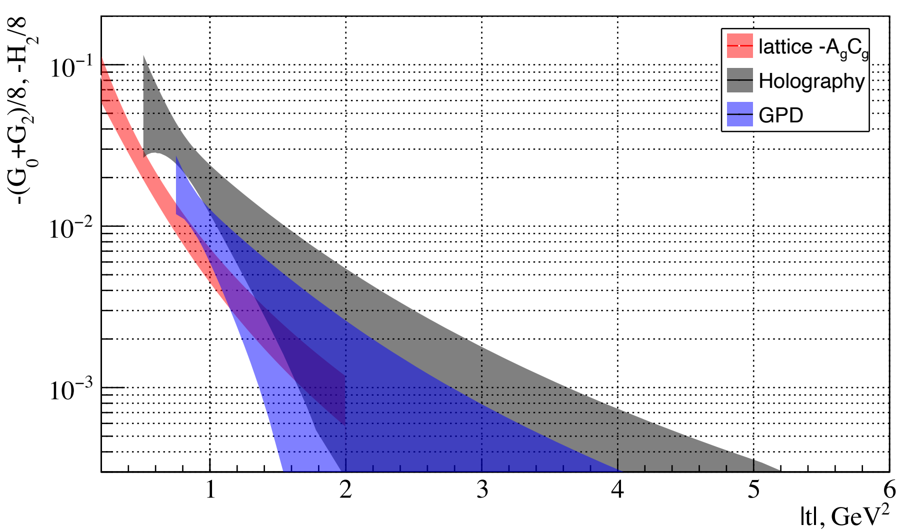
$B_g(t)$=0

$H_0(t) = A_g^2(t)$

for large N_c and strong α_s

Gluon Gravitational Form Factors - summary





Features in data consistent with the GFF models:

- $d\sigma/dt(E_\gamma, t \to fixed)$ increases with energy
- $G(t)$ and $H(t)$ form factor functions are energy independent (within the experimental errors)

- In leading-term approximation (and neglecting B_g):

$G_0(t) = H_0(t) = A_g^2(t)$ and

$G_0(t) + G_2(t) = H_2(t) = 8C_g(t)A_g(t)$

General agreement b/n extracted FFs using two diametric theories

- agreement with lattice
- possible conclusion: the model corrections are not dominant

GPD & FF definitions

GPDs, $H^{q,g}(x,\xi,t)$ and $E^{q,g}(x,\xi,t)$

- Electromagnetic FFs (EFFs) of the proton

$$\int_{-1}^{1} dx H^{q,g}(x,\xi=0,t) = A_{10}^{q,g}(t) \equiv H_{00}^{q,g}(t) \equiv F_1^{q,g}(t)$$

$$\int_{-1}^{1} dx E^{q,g}(x,\xi=0,t) = B_{10}^{q,g}(t) \equiv E_{00}^{q,g}(t) \equiv F_2^{q,q}(t)$$

- Mass/mechanical/gravitational FFs (GFFs) of the proton

$$\int_{-1}^{1} dx\, x H^{q,g}(x,\xi=0,t) = A_{20}^{q,g}(t) \equiv A^{q,g}(t)$$

$$\int_{-1}^{1} dx\, x E^{q,g}(x,\xi=0,t) = B_{20}^{q,g}(t) \equiv B^{q,g}(t)$$

$J^{q,g} = \frac{1}{2}\left(A^{q,g} + B^{q,g}\right)$ is q, g contribution to proton angular momentum

Proton angular momentum decomposition

Global (lattice + data) GPD extraction (ignoring strangeness)
$J^u = 0.33, J^d = -0.11$, and $J^g = 0.26$, with $J_{tot} = 0.484(17)$

"First Global Extraction of Generalized Parton Distributions from Experiment and Lattice Data with Next-to-Leading-Order Accuracy" Y.Guo, F.Aslan, X.Ji, M.Santiago, Phys. Rev. Lett. ***135****, 261903 (2025)*

Lattice simulations of the GFFs that include strangeness ($N_f = 2 + 1$):
$J_u = 0.22, J_d = 0.02, J_s = 0.01$, and $J_g = 0.26$, with $J_{tot} = 0.506(25)$

"Gravitational Form Factors of the Proton from Lattice QCD",D. C. Hackett, D. A. Pefkou, and P. E. Shanahan, Phys. Rev. Lett. ***132****, 251904 (2024).*

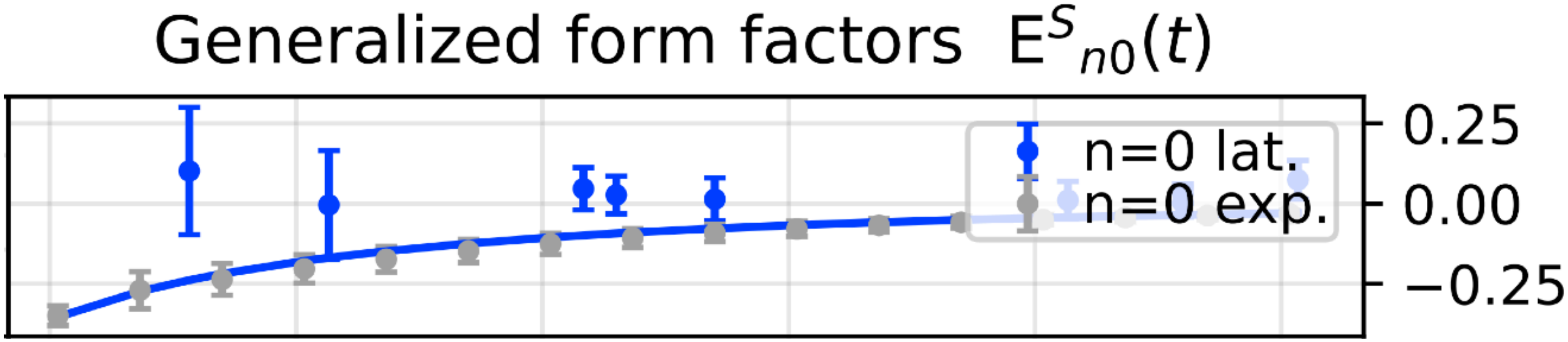
The disagreement of the gravitational FFs when including data (lattice+data) comes from disagreement b/n lattice and data of the electro-magnetic FFs !

Proton angular momentum decomposition

Iso-scalar electromagnetic form factor from experiment:

$F_2^p + F_2^n = (F_2^u + F_2^d)/3 = B_{10}^S \equiv E_{00}^S < 0,$

while from lattice tends to be positive:



Phys. Rev. Lett. ***135****, 261903 (2025)*

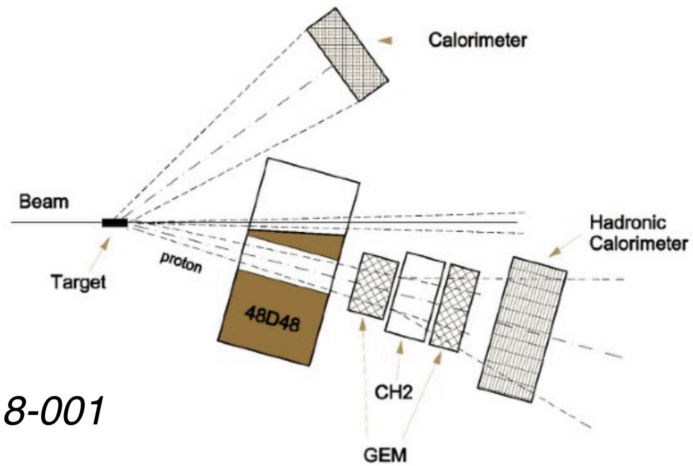
$B_{10}^S < 0$ from experiment implies that $B_{20}^S < 0$:

$$\int_{-1}^{1} dx E(x, \xi = 0, t) = B_{10}^S \rightarrow \int_{-1}^{1} dx\, x E(x, \xi = 0, t) = B_{20}^S$$

While from lattice $B_{20}^S > 0$ or ~ 0

Phys. Rev. Lett. ***125****, 262001 (2023)*

SBS for J/ψ production



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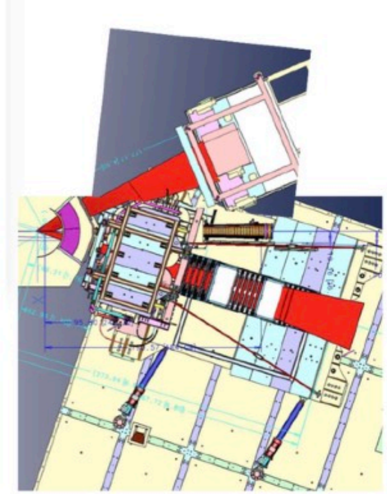

$$\gamma p \rightarrow J/\psi p \rightarrow e^{+} p (e^{-})$$

- Using 6-10% radiator, 5 μA, thousands J/ψ per day
- Proton and positron detected in SBS - best resolution achieved for J/ψ mass
- Electron identified in ECAL (at 22°) and energy used for trigger and background suppression
- K_{LL} with proton polarimeter and A_{LL} with NH_3 long. polarized target (at 0.01 lower rates)

$$ep \rightarrow e' \gamma^{*} p \rightarrow e' J/\psi p \rightarrow e' e^{+} p (e^{-})$$

  - Photon virtuality equivalent of charmonium mass increase: $M^2_{J/\psi} + Q^2$
  - Important for testing GPD approach - factorization validated at high masses

Summary

- "Rosenbluth" (kinematic) separation of the J/ψ near-threshold photo-production demonstrates FFs are independent of energy - no theoretical assumption
- Extracting gluon GFFs from the data only, without any additional theoretical/lattice constraints - using two theoretical approaches
- Extracted gluon GFFs on the same scale with lattice results

- GFFs and E.M. FFs as moments of GPD - tension b/n lattice and data for $F_2^p + F_2^n$, results in tension for angular momentum decomposition - could be explained by strangeness contribution (?) or lattice uncertainties

- What can be done with SBS - thousands J/ψ's per day in photo/electro-production - need much more work to become realistic proposal