# Unraveling QCD with TMD hadronic structures

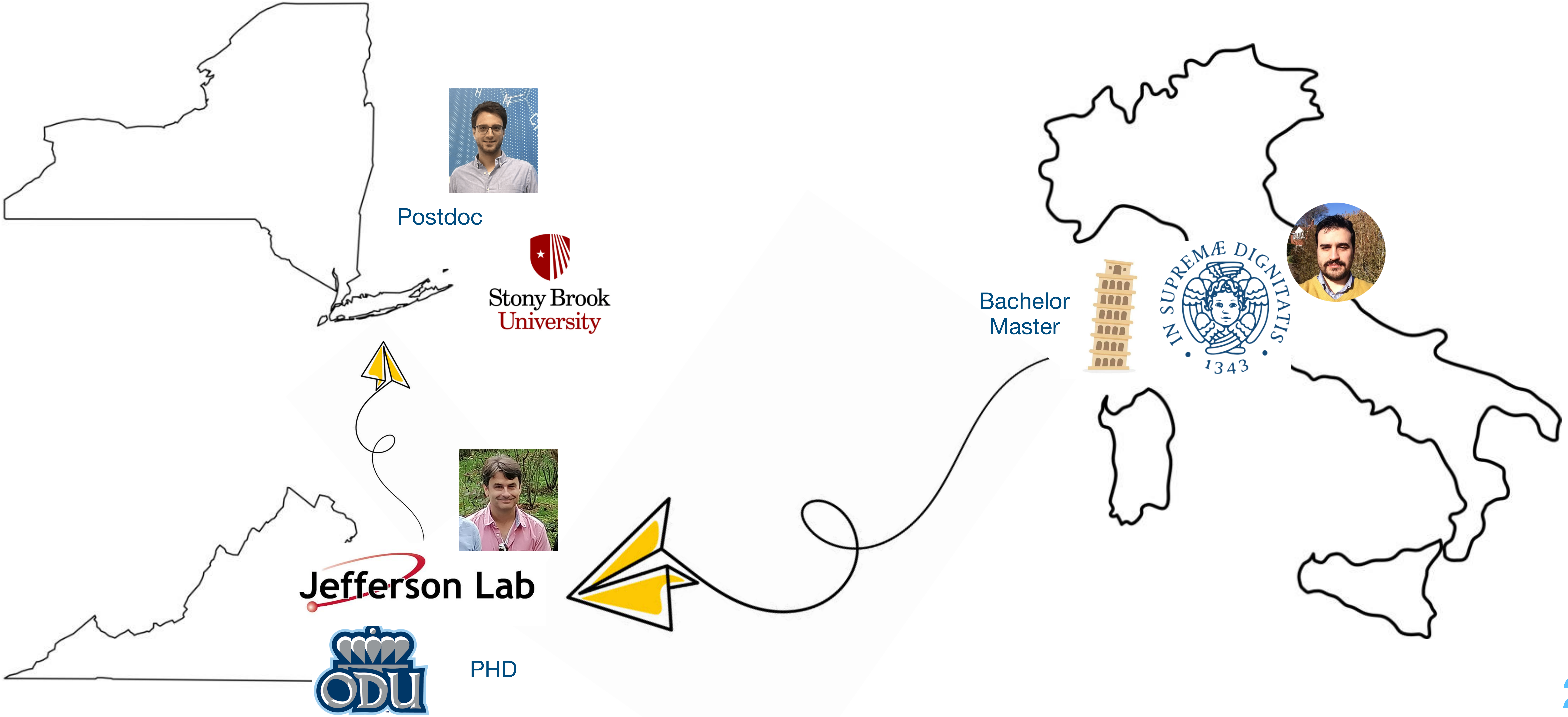
## JLUO Annual Meeting
## JSA PhD Thesis Prize

Tommaso Rainaldi - June 24, 2026

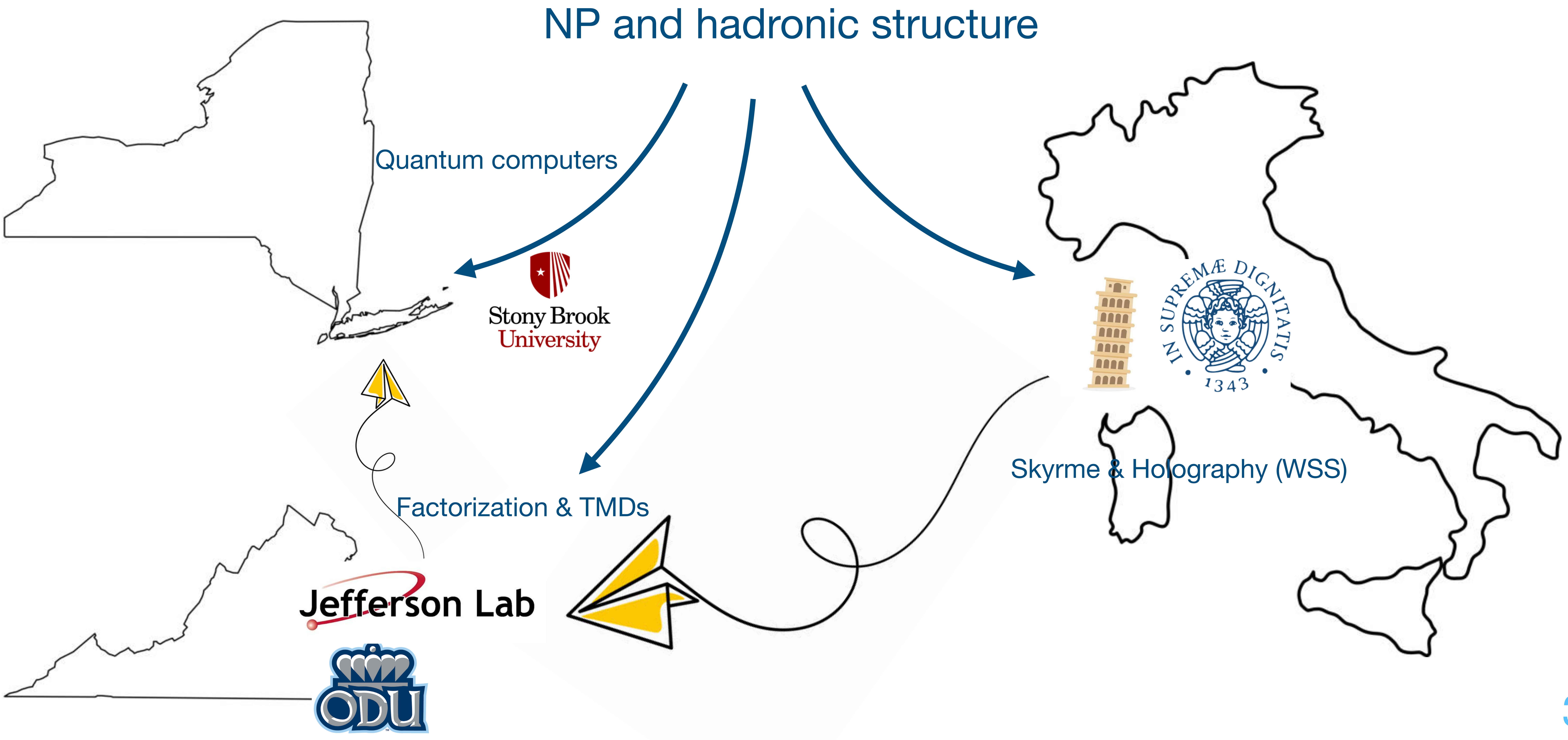
Old Dominion University

Stony Brook University

# About me


Postdoc
Stony Brook
University
Jefferson Lab
ODU
PHD
Bachelor
Master
IN SUPREMÆ DIGNITATIS
1343

# About me


NP and hadronic structure
Quantum computers
Stony Brook University
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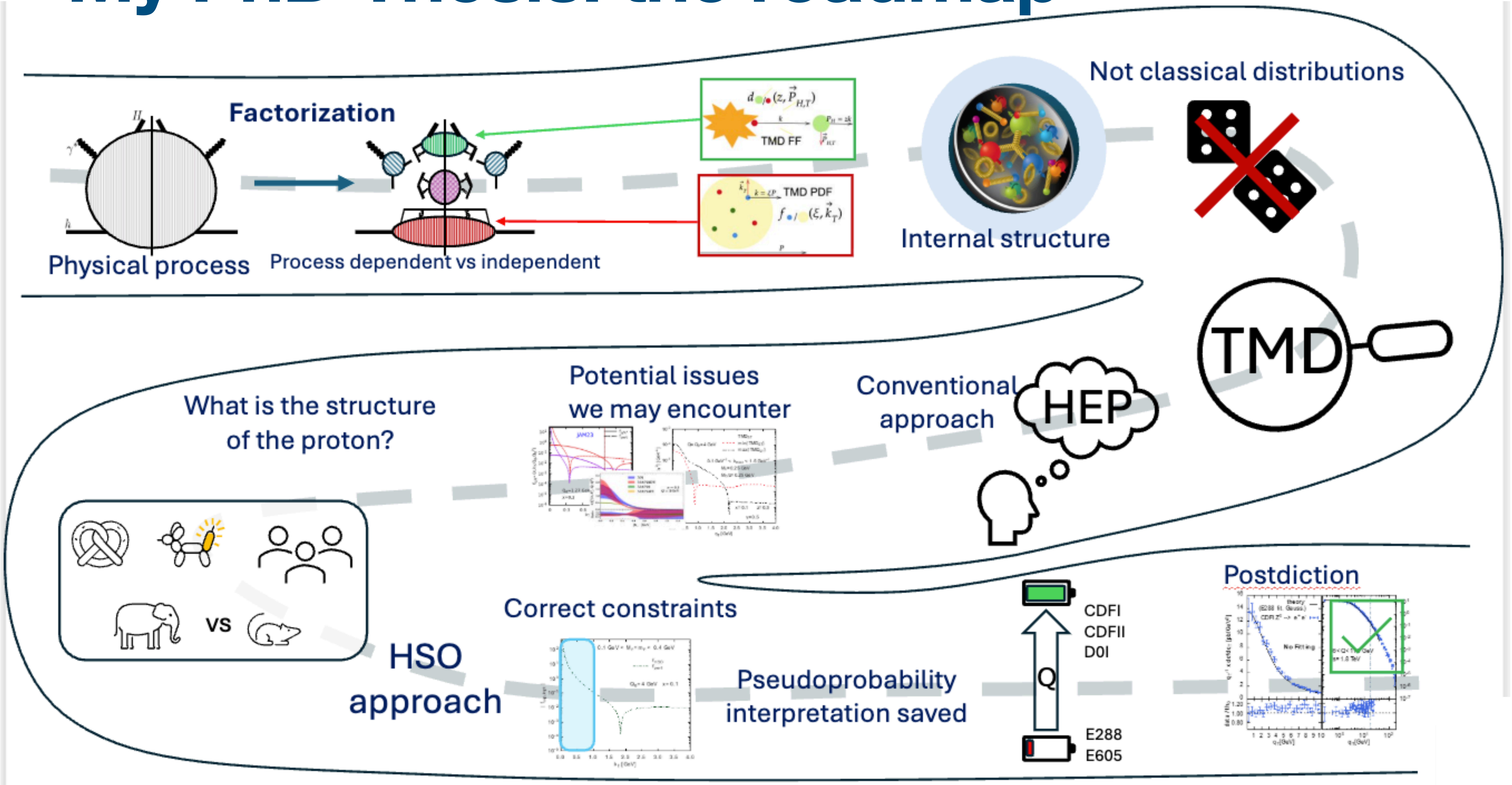
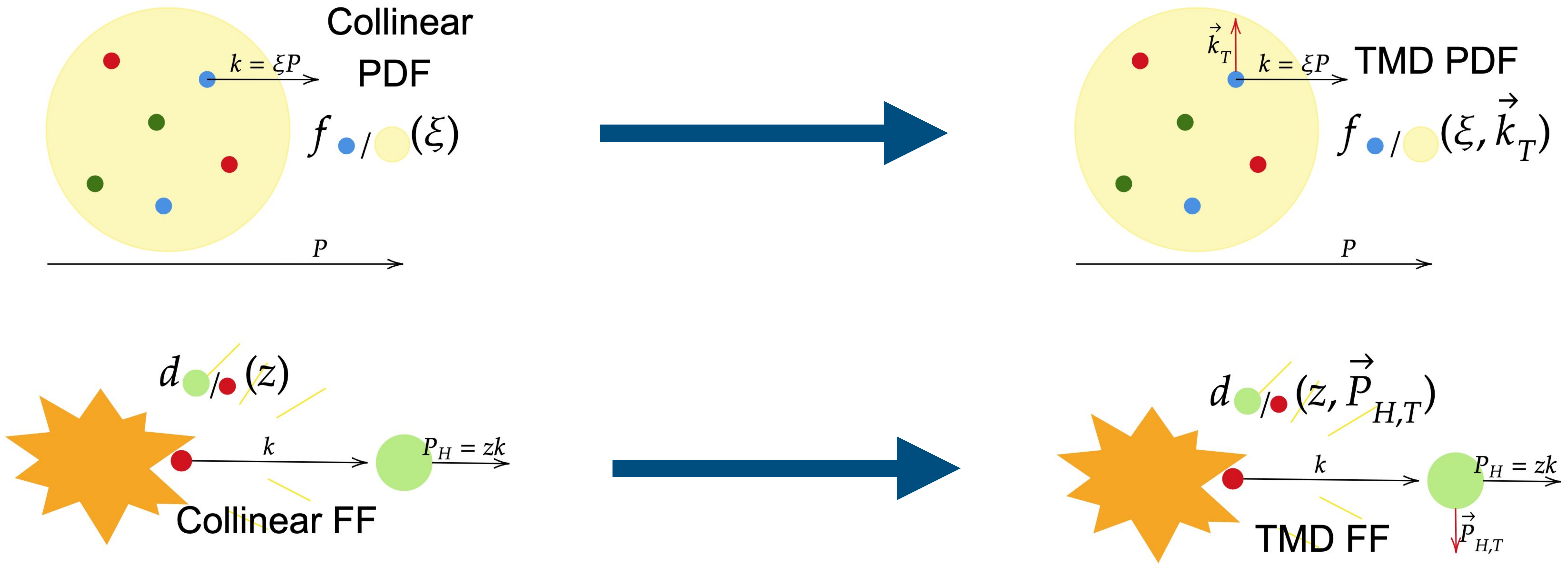
Skyrme & Holography (WSS)

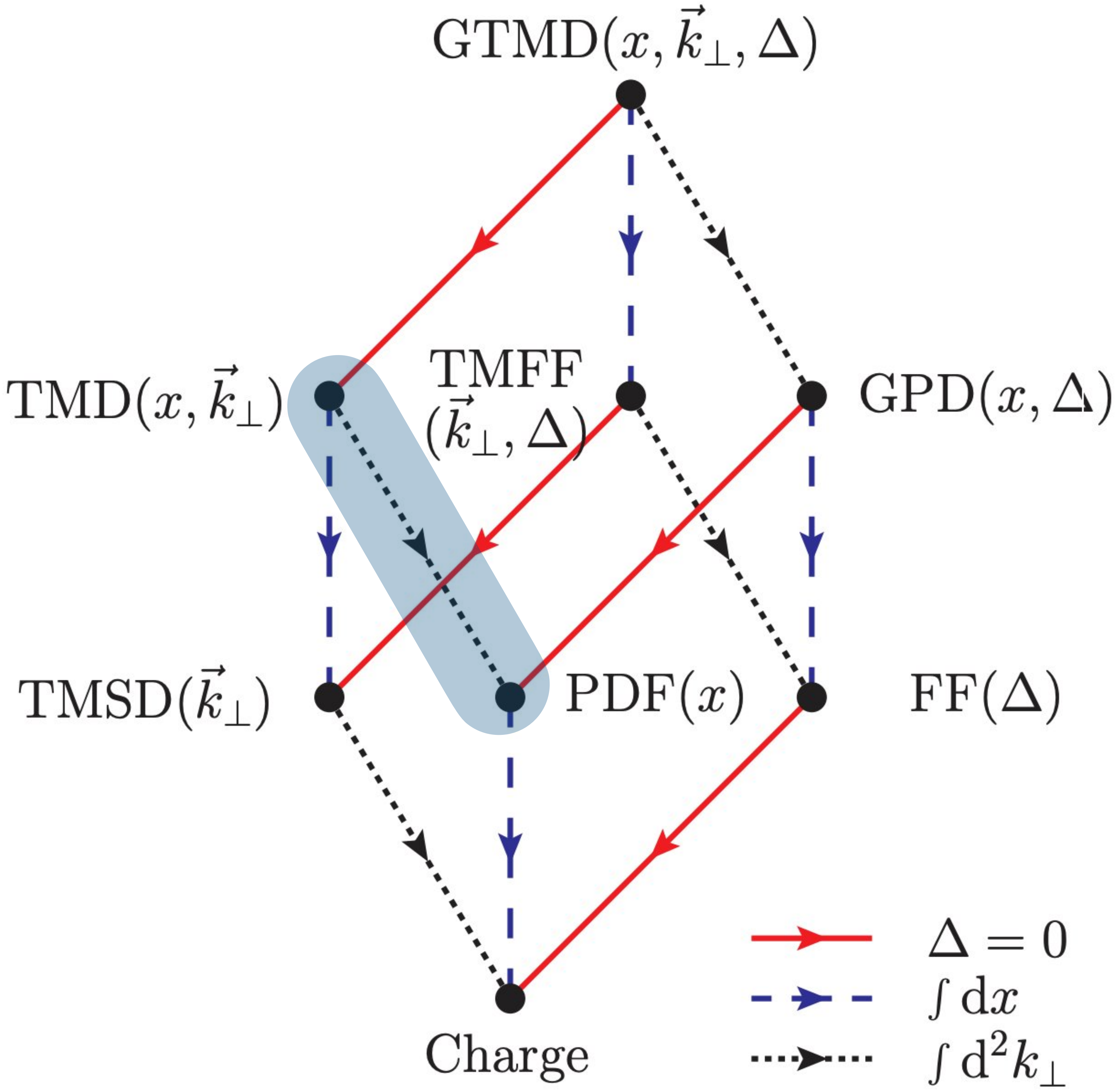
# My PhD Thesis: the roadmap



# Hadronic structure picture



# Useful relations with caveats




Lorcé, Pasquini et al `11

**Hadron structure interpretation but not true probabilities:**

Can become negative sometimes

$$\text{PDF}(x) < 0\,, \quad \text{TMD}(x, k_T) < 0$$

Naive relation doesn't hold

$$\text{PDF}(x) \neq \int d^2\boldsymbol{k}_T \text{TMD}(x, k_T)$$

They run with the energy scale(s)

$$\text{PDF}(x) = \text{PDF}(x; \mu) \quad (\text{DGLAP})$$

$$\text{TMD}(x, k_T) = \text{TMD}(x, k_T; \mu, \zeta) \quad (\text{CSS})$$

# Convenient $b_T$ space

## Just a Fourier transform away

$$\tilde{f}_{i,H}(x; \boldsymbol{b}_T; \mu, \zeta) = \int \mathrm{d}^2\boldsymbol{k}_T \, e^{-i\boldsymbol{k}_T \cdot \boldsymbol{b}_T} f_{i/H}(x, \boldsymbol{k}_T; \mu, \zeta)$$

# Renormalization Group equations

$$\frac{\partial \ln \tilde{f}_{j/p}(x, \boldsymbol{b}_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(\boldsymbol{b}_T; \mu)$$

$$\frac{\partial \ln \tilde{f}_{j/p}(x, \boldsymbol{b}_T; \mu, \zeta)}{\partial \ln \mu} = \gamma\left(\alpha_S(\mu); \mu/\sqrt{\zeta}\right)$$

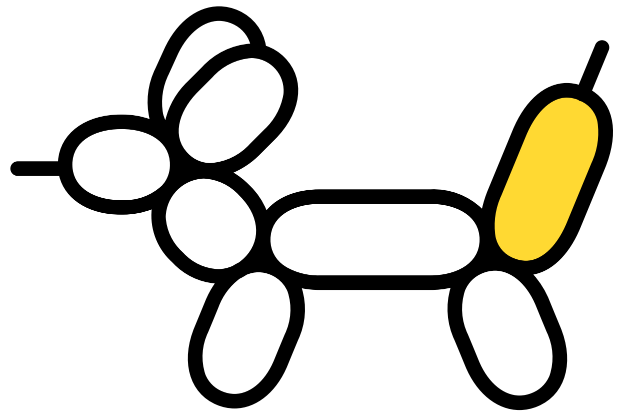
Perturbative at small $\boldsymbol{b}_T$
Nonperturbative elsewhere

$$\frac{d\tilde{K}(\boldsymbol{b}_T; \mu)}{d \ln \mu} = -\gamma_K(\alpha_S(\mu))$$

Perturbative

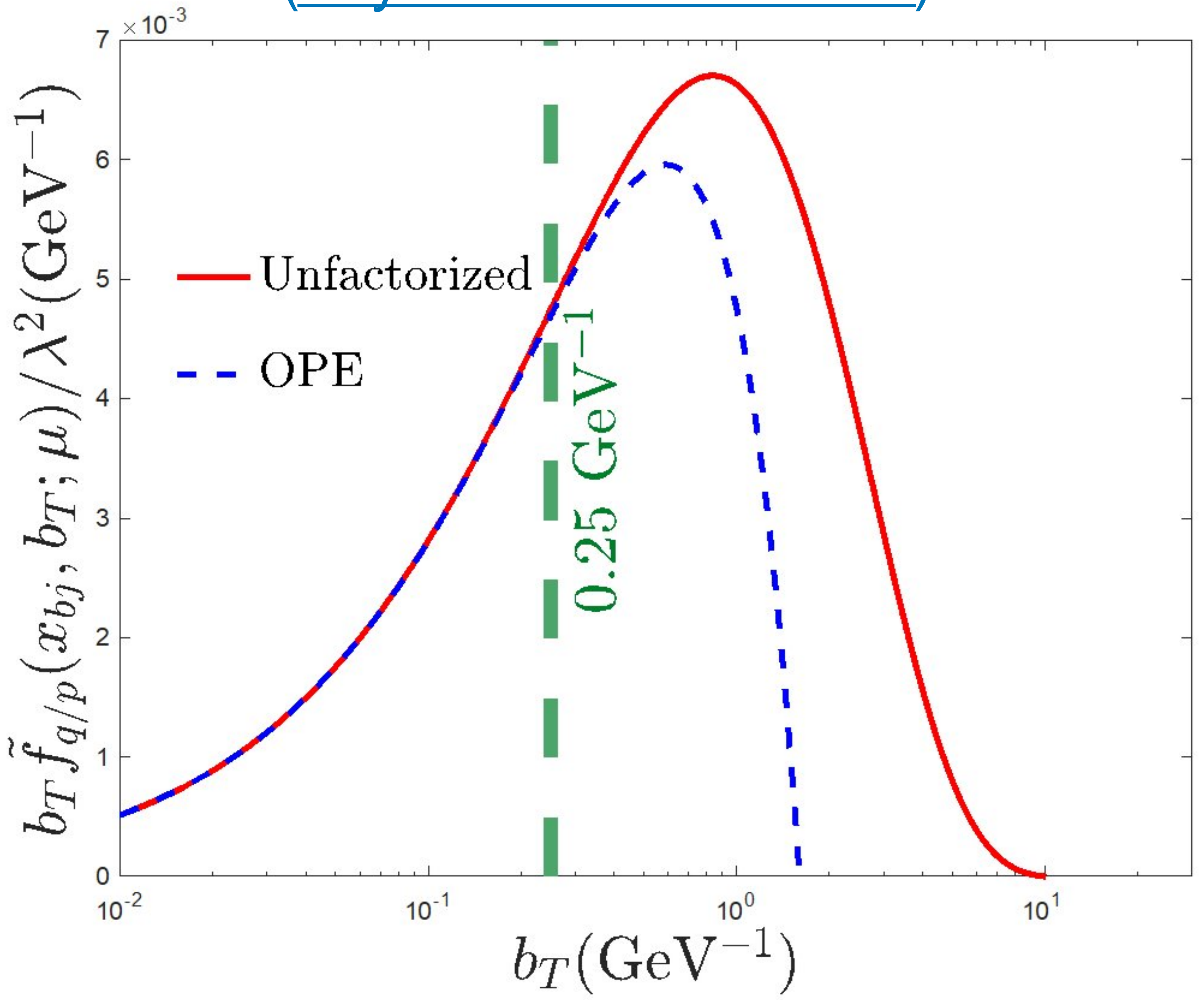
$$\tilde{f}_{i,H}(x; \boldsymbol{b}_T; \mu, \zeta) = \tilde{f}_{i,H}(x; \boldsymbol{b}_T; \mu_0, \zeta_0)\ \exp\left\{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_S(\mu'); 1) - \ln\left(\frac{\sqrt{\zeta}}{\mu'}\right)\gamma_K(\alpha_S(\mu'))\right] + \ln\left(\sqrt{\frac{\zeta}{\zeta}}\tilde{K}(\boldsymbol{b}_T; \mu)\right)\right\}$$

# Tail of the TMDs from OPE



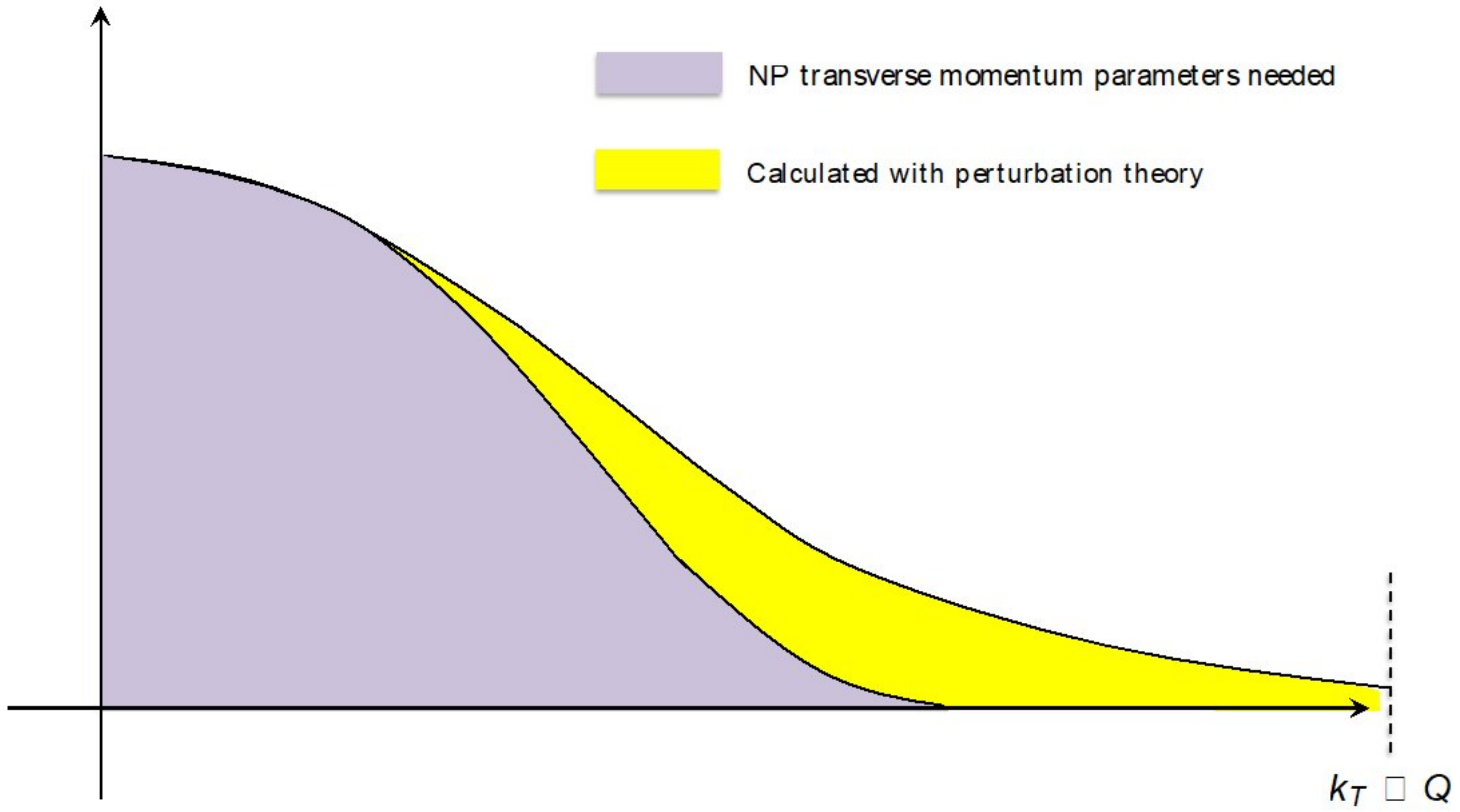
$$\tilde{f}_{i,H}(x;\boldsymbol{b}_T;\mu,\zeta) = \tilde{C}_{ij}(x,\boldsymbol{b}_T;\mu,\zeta) \otimes f_{j/H}(x;\mu) + \mathcal{O}(mb_T)$$

An explicit verification in the Yukawa model (PhysRevD.107.074031)




This means that the TMD must match the OPE for $k_T/\mu \gtrsim 1$ ($\mu b_T \lesssim 1$)

$$\boldsymbol{b}_T \longleftrightarrow \boldsymbol{k}_T$$



# Integral relation

DGLAP with “new” kernels

(3D info) (1D info) “violation”

$$\int_0^{\mu_c} d^2\boldsymbol{k}_T\, f(x, k_T; \mu, \mu^2) = f(x;\mu) + \Delta(x;\mu,\mu_c) + \mathrm{p.s.}$$

$\overline{\mathrm{MS}}$ PDF Scheme change terms

See also: Rio, Prokudin et al (PhysRevD.110.016003)

$$\Delta(x;\mu,\mu_c) = C_\Delta(x;\mu,\mu_c) \otimes f(x;\mu)$$

OPE coefficients

# Large logs

$$\tilde{f}_{i,H}(x;\boldsymbol{b}_T;\mu,\zeta) = \tilde{C}_{ij}(x,\boldsymbol{b}_T;\mu,\zeta) \otimes f_{j/H}(x;\mu) + \mathcal{O}(mb_T)$$

$$b_*(b_T) = \begin{cases} b_T, & mb_T \ll 1 \\ b_{\max}, & \text{otherwise} \end{cases}$$

Auxiliary parameter (not physical)

$$\text{powers of } \ln\left(\frac{\mu b_T}{2e^{-\gamma_E}}\right) \xrightarrow{b_T\to+\infty} +\infty \qquad \xrightarrow{b_T\to-\infty} -\infty$$

Here the OPE just beaks down

$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$

Natural scale to RG resum the logs
(“tainted” by $b_*$)

Here the OPE is valid but we need RG

# CSS parametrization

## Conventional approach

$$\tilde{f}_{j/p}\left(x;\boldsymbol{b}_{\mathrm{T}};\mu_Q,Q\right)=\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x;\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\times$$
$$\times\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu');1\right)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right]+\ln\left(\frac{Q}{\mu_{b_*}}\right)\tilde{K}\left(\boldsymbol{b}_*;\mu_{b_*}\right)\right\}$$
$$\times\exp\left\{-g_{j/p}\left(x,\boldsymbol{b}_{\mathrm{T}}\right)-g_K\left(\boldsymbol{b}_{\mathrm{T}}\right)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

OPE is recovered at small $b_T$ (but…)

$$\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)=\widetilde{C}_{j/j'}\left(x/\xi,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\otimes\tilde{f}_{j'/p}\left(\xi;\mu_{b_*}\right)+\mathcal{O}\left(m^2b^2_{\mathrm{max}}\right)$$

# CSS parametrization

## Conventional approach

$$\tilde{f}_{j/p}(x; \boldsymbol{b}_\mathrm{T}; \mu_Q, Q) =$$ OPE at optimal scale $\times$

$\times$ Evolution from optimal scale to the desired one

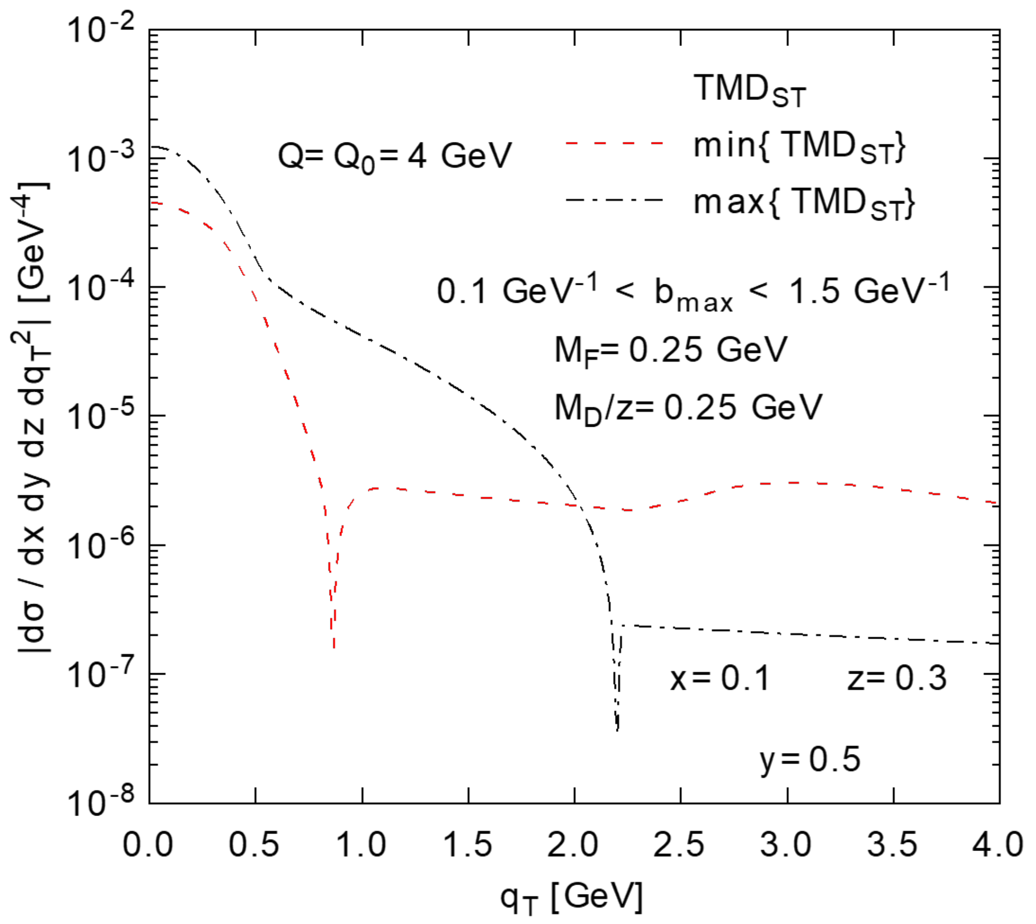
$\times$ Nonperturbative contribution

OPE is recovered at small $b_T$ (but…)

$$\tilde{f}^{\mathrm{OPE}}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}) = \widetilde{C}_{j/j'}(x/\xi, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}) \otimes \tilde{f}_{j'/p}(\xi; \mu_{b_*}) + \mathcal{O}\left(m^2 b^2_{\mathrm{max}}\right)$$

# What could go wrong?

$b_{\max}$ dependence propagating in both TMDs and cross sections




Risks: prediction and interpretation

$$\frac{\mathrm{d}\tilde{f}(x_{bj},\mathrm{b_T})}{\mathrm{db_{max}}} = \mathcal{O}(m\mathrm{b_{max}})$$

Tail mismatching at $k_T/\mu \approx 1$ and beyond

Risks: misrepresenting perturbative/nonperturbative contributions

Integrate relation not satisfied

Risks: pseudoprobability is lost

# HSO parametrization at the input scale

## (Fixed scale additive choice)

OPE $\mathcal{O}(\alpha_S)$

$$f_{\mathrm{inpt,i/p}}(x, \boldsymbol{k}_T; \mu_{Q_0}; Q_0^2) = \frac{1}{2\pi}\frac{1}{k_T^2 + m^2}\left[A^f_{i/p}(x; \mu_{Q_0}) + B^f_{i/p}(x; \mu_{Q_0}) \ln \frac{Q_0^2}{k_T^2 + m^2}\right] + \frac{1}{2\pi}\frac{1}{k_T^2 + m^2} A^{f,g}_{i/p}(x; \mu_{Q_0})$$
$$+ C^f_{i/p} f_{\mathrm{core,i/p}}(x, \boldsymbol{k_T}; Q_0^2)$$

Any nonperturbative model
(Gaussian, lattice, EFT…)
Easily swappable

Pseudoprobability interpretation is saved by construction

$$\int_0^\mu d^2\boldsymbol{k}_T\, f(x, k_T; \mu, \mu^2) = f(x;\mu) + C_\Delta \otimes f(x;\mu) \equiv f^c(x;\mu,\mu)$$

# All orders fixed-order additive HSO

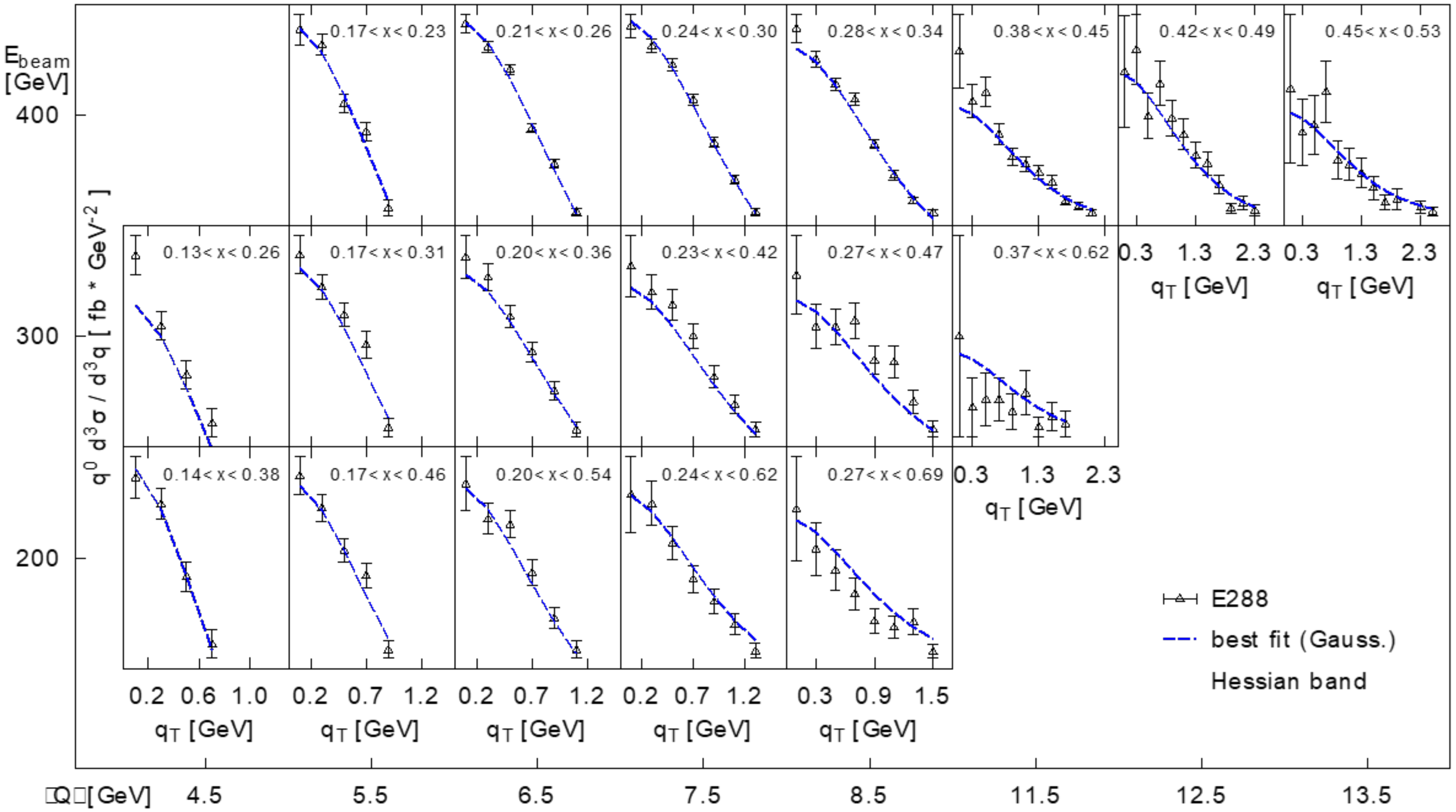
$$f_{i/h}(x; k_T; \mu, \zeta; m, M_{\text{small}})$$
$$= \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \sum_{l=0}^{k-1} a_S^n(\mu) \mathcal{M}_{kl} \frac{\ln^l \left(\frac{\mu^2}{k_T^2 + m^2}\right)}{2\pi(k_T^2 + m^2)} \tilde{C}_{ij}^{(n,k)}(x; \mu, \zeta) \otimes f_{j/h}(x; \mu) + C f_{1,i/h}^{\text{core}}(x, k_T, M_{\text{small}}).$$

Any nonperturbative model (Gaussian, lattice, EFT…) Easily swappable

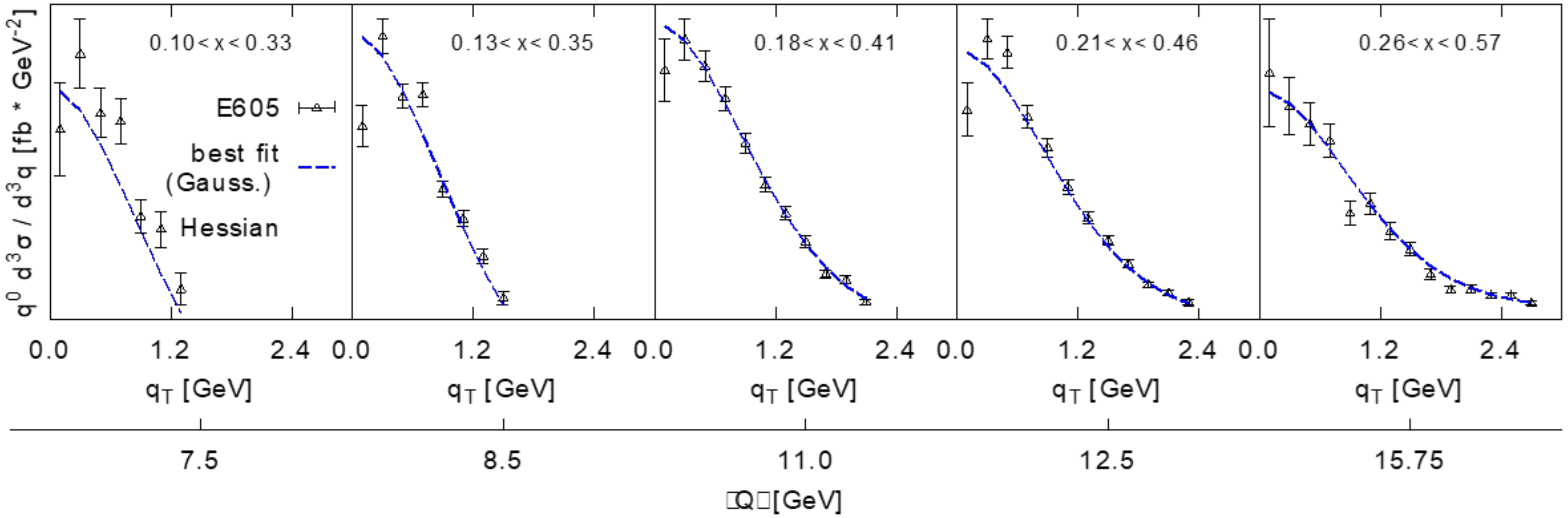
$$C = \frac{1}{\tilde{f}_{i/h}^{\text{core}}(x, 0, M_{\text{small}})} \times$$
$$\left[ \sum_{n=0}^{\infty} a_S^n \tilde{C}_{ij}^{(n,0)}(x; \mu, \zeta) \otimes f_{j/h}(x; \mu) \right.$$
$$\left. - \sum_{n=0}^{\infty} \sum_{k=1}^{2n} \sum_{l=0}^{k-1} a_S^n \mathcal{M}_{kl} \left( \frac{2^l}{1+l} \ln^{1+l} \left(\frac{\mu}{m}\right) + \overline{\mathcal{L}}_0^{(l)} \right) \tilde{C}_{ij}^{(n,k)}(x; \mu, \zeta) \otimes f_{j/h}(x; \mu) \right],$$

# Low-energy Fits

Just 3+1 parameters

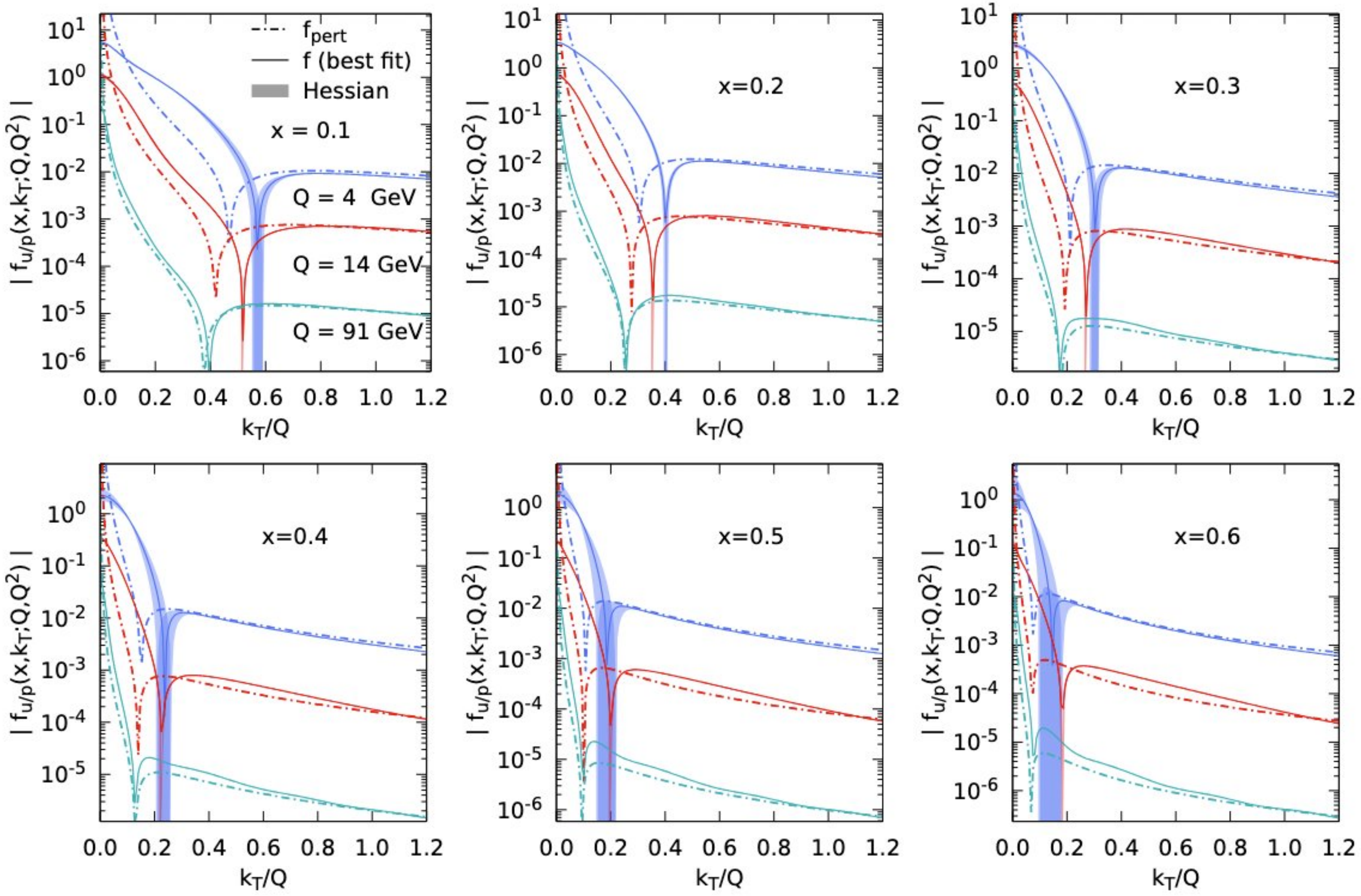



| Gaussian fits | | |
|---|---|---|
| | E288 (130 pts.) | E605 (52 pts.) |
| $\chi^2_{\rm dof}$ | 1.04 | 1.68 |




| Spectator model fit | |
|---|---|
| | E288 (130 pts.) |
| $\chi^2_{\rm dof}$ | 1.04 |

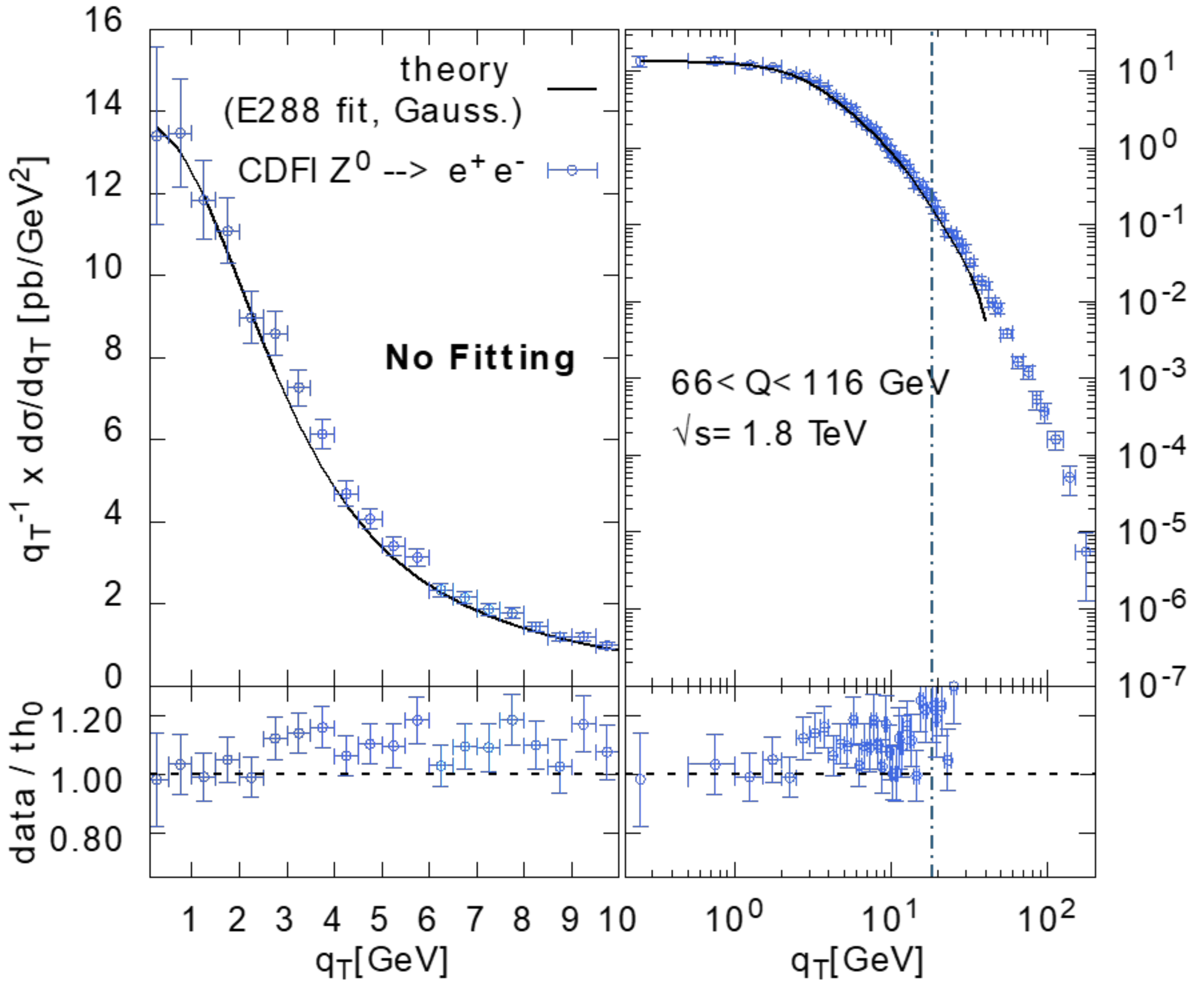
# Results

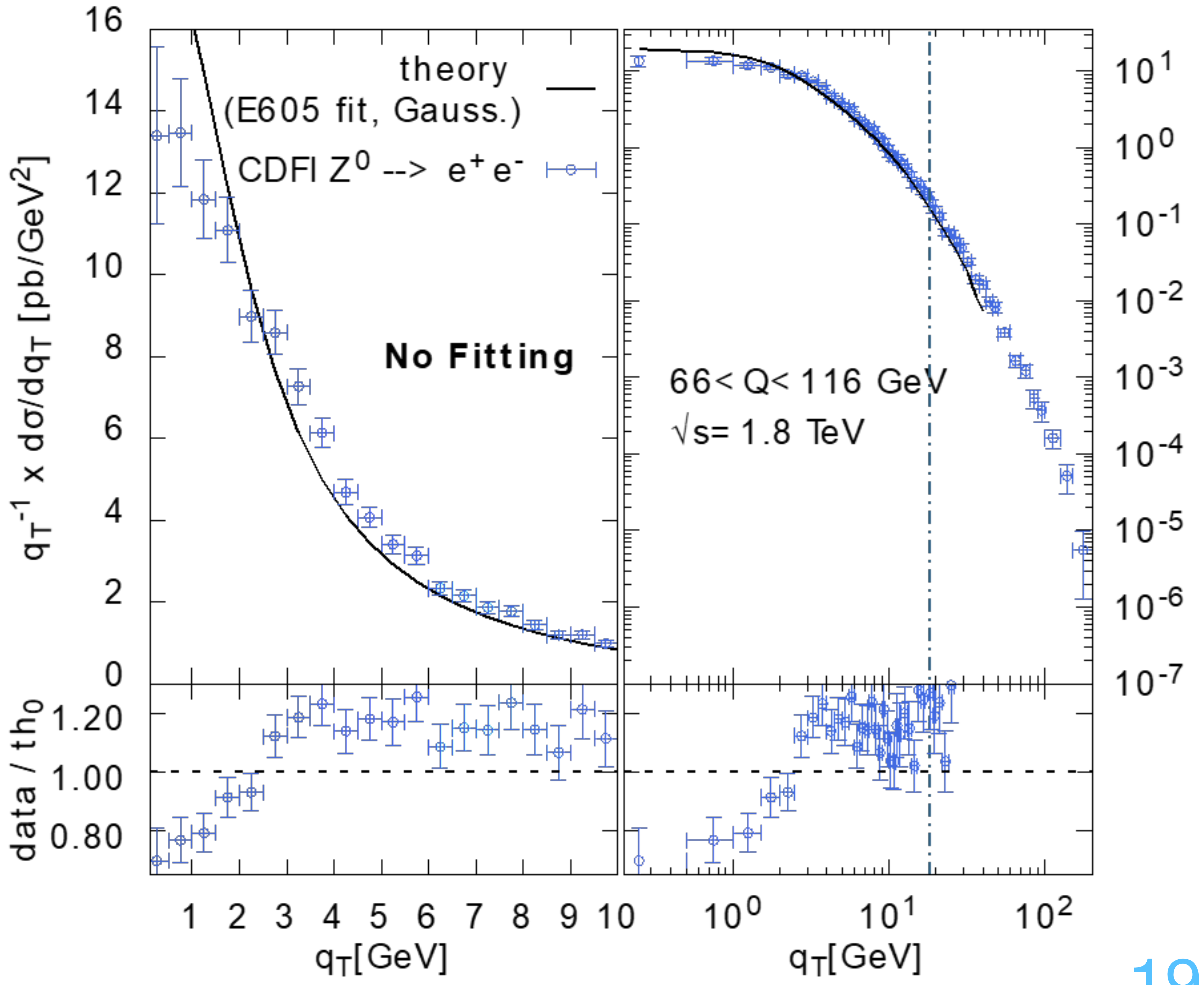


fpert
f (best fit)
Hessian
x = 0.1
Q = 4 GeV
Q = 14 GeV
Q = 91 GeV
x=0.2
x=0.3
x=0.4
x=0.5
x=0.6
| f_u/p(x,k_T;Q,Q^2) |
k_T/Q

# Testing predictive power

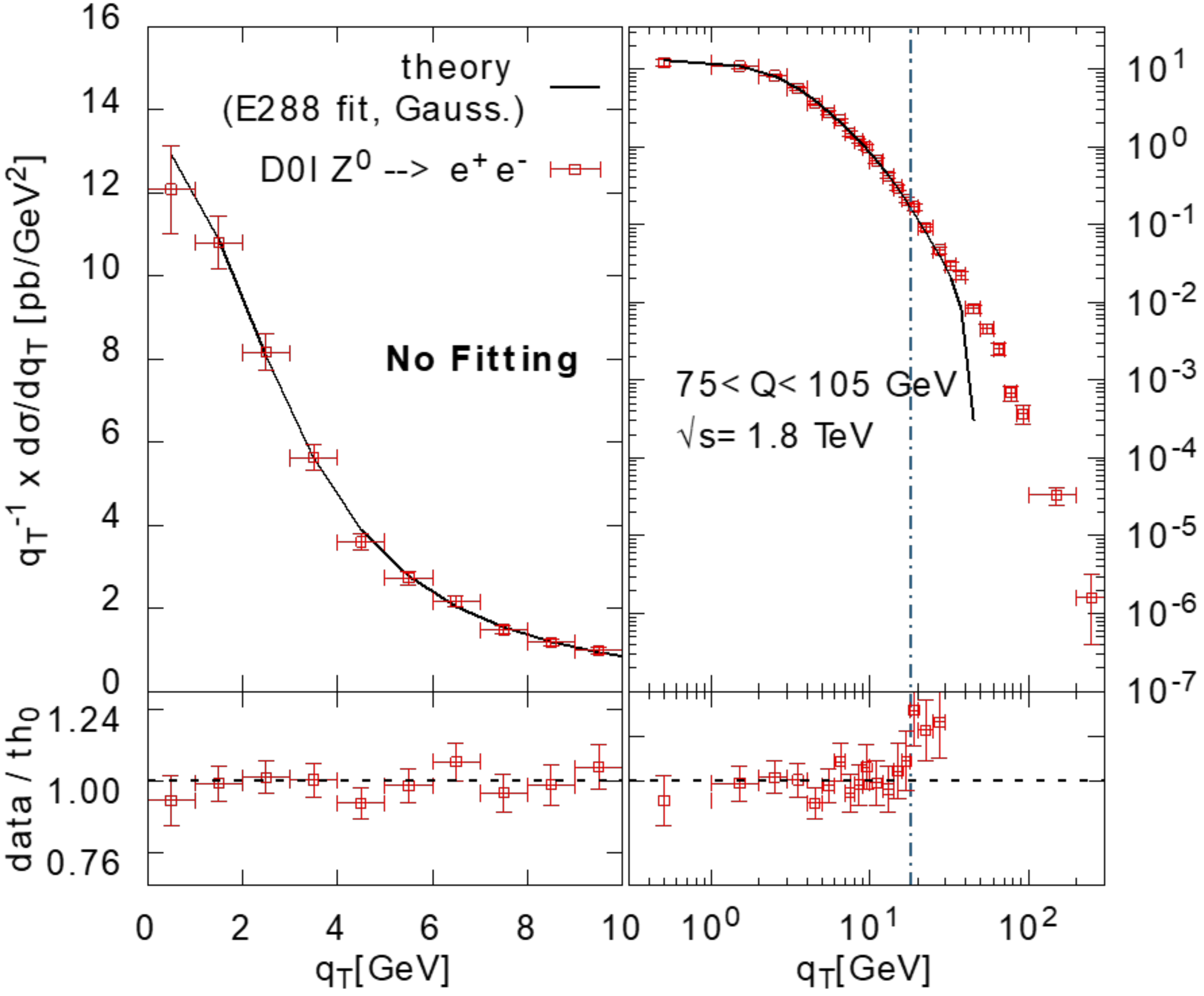
## Just RG evolve low-energy fits

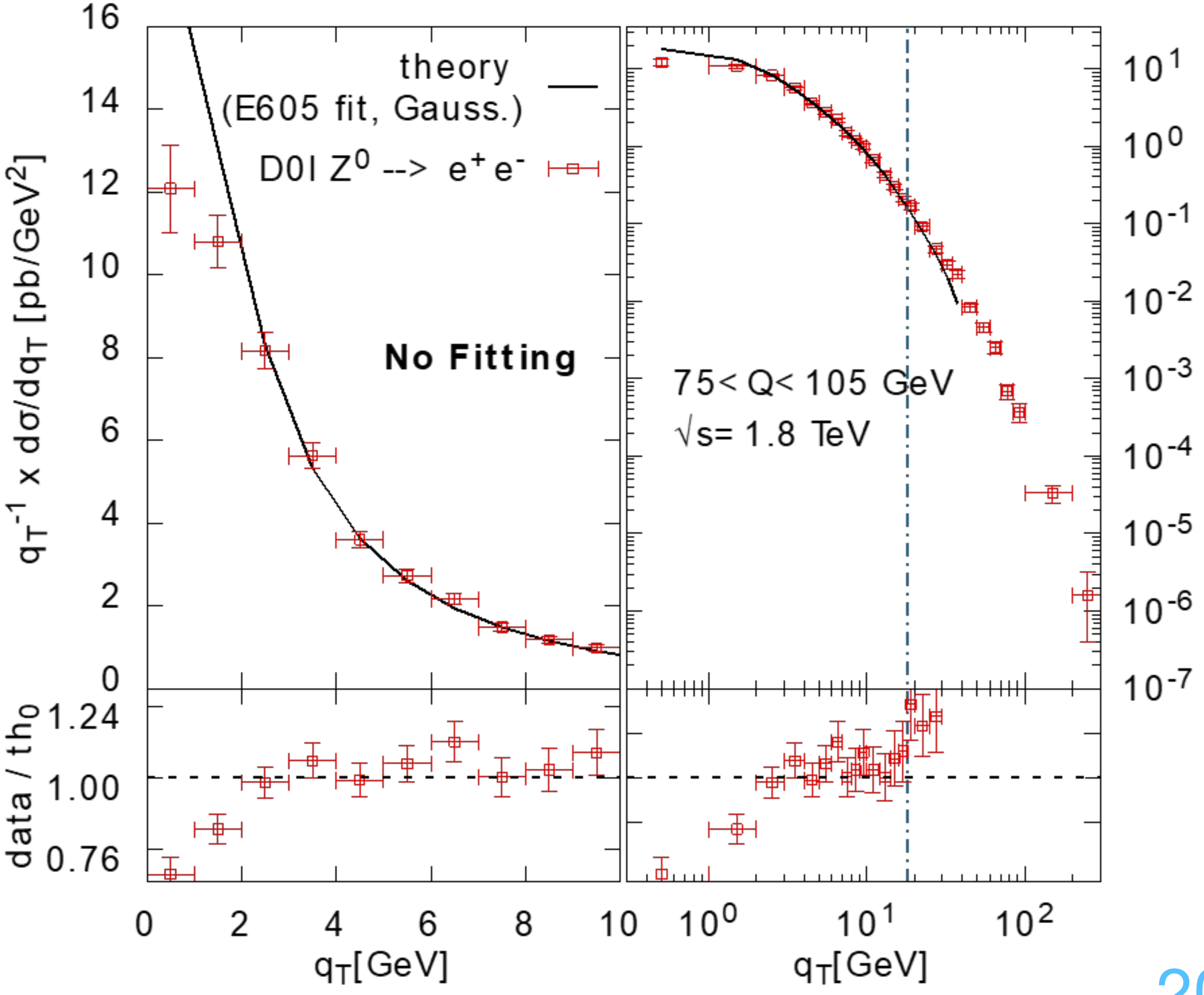


# Testing predictive power

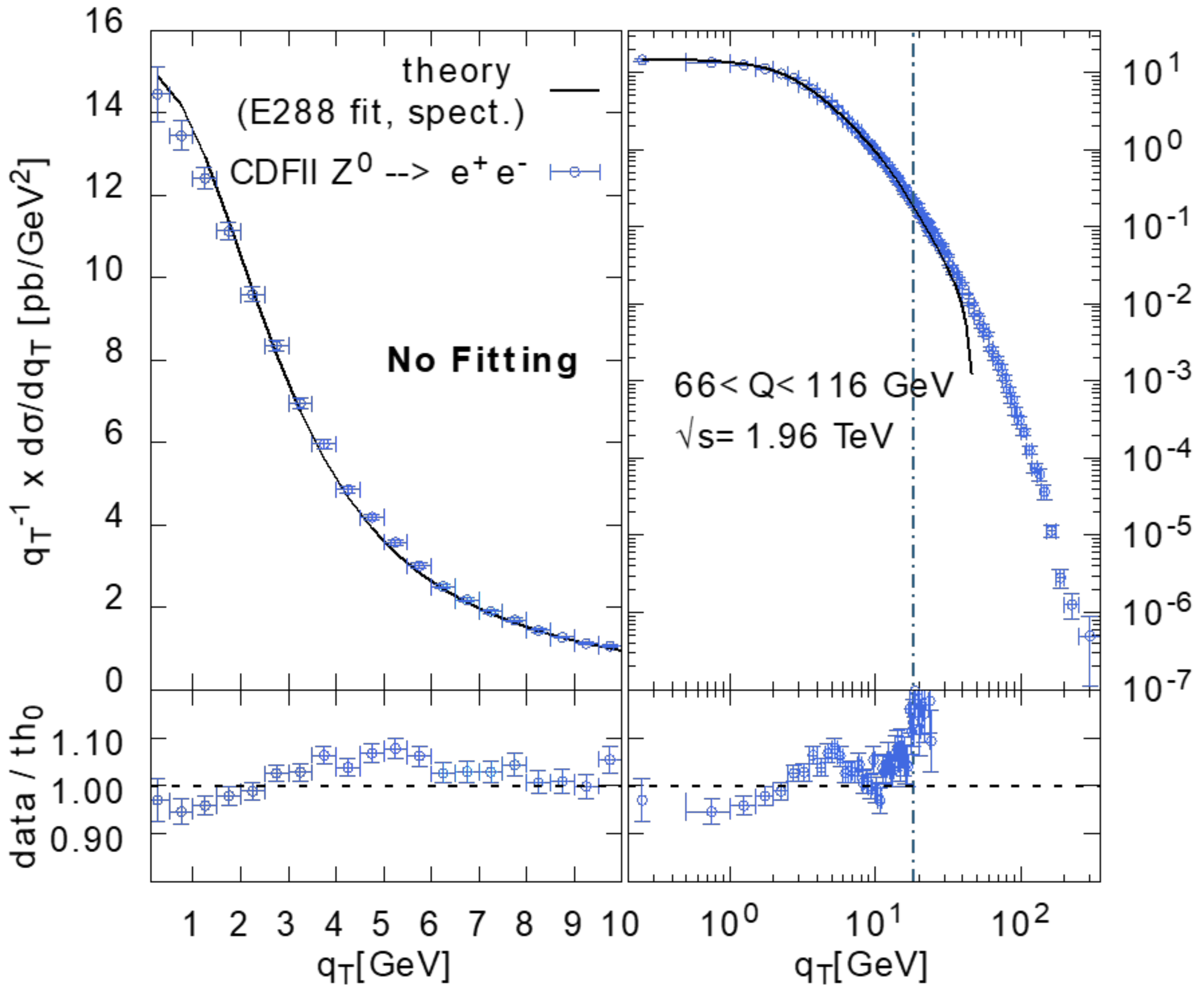
## Different fits on the same experiment

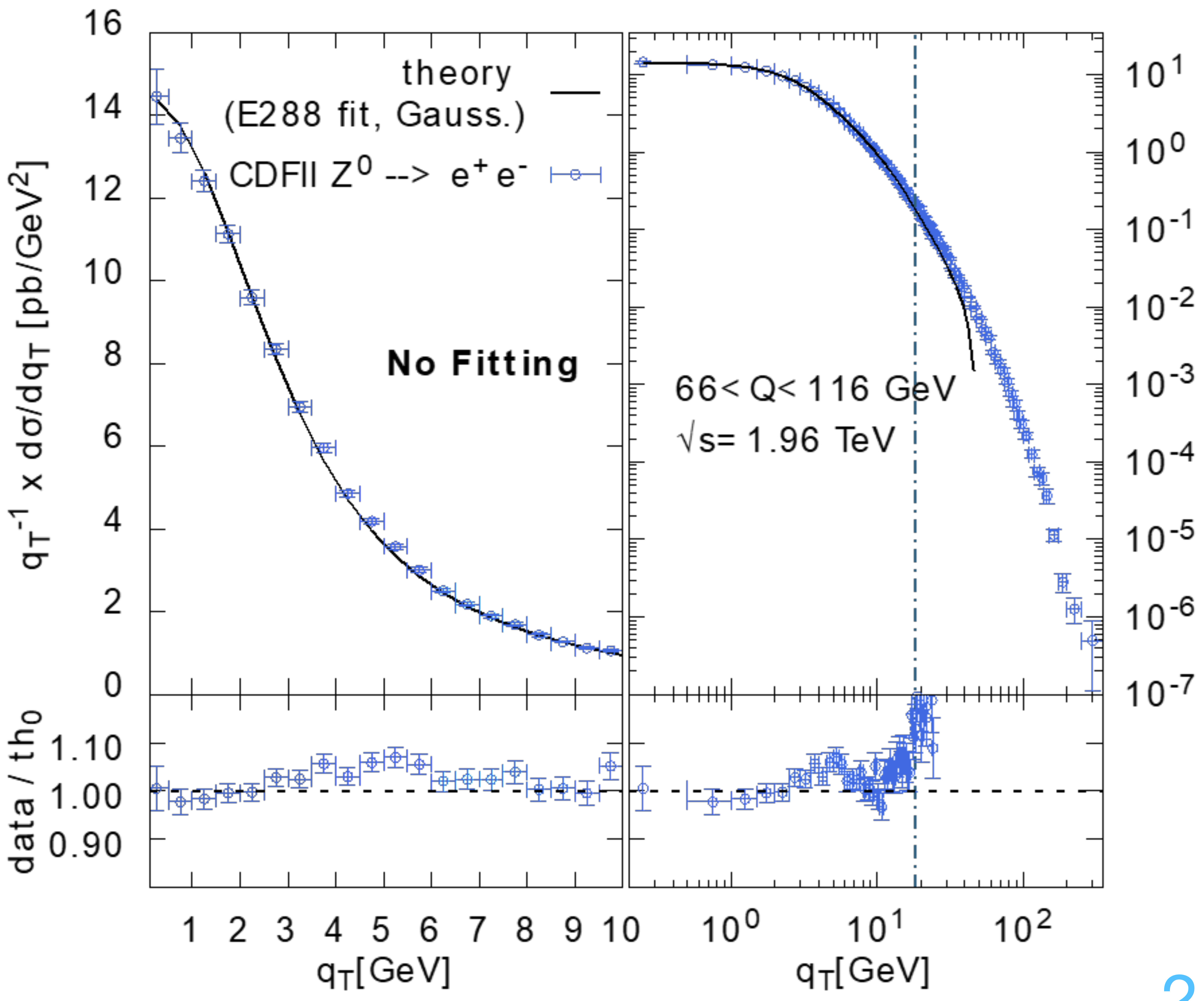


# Testing predictive power

## Different models on the same experiment



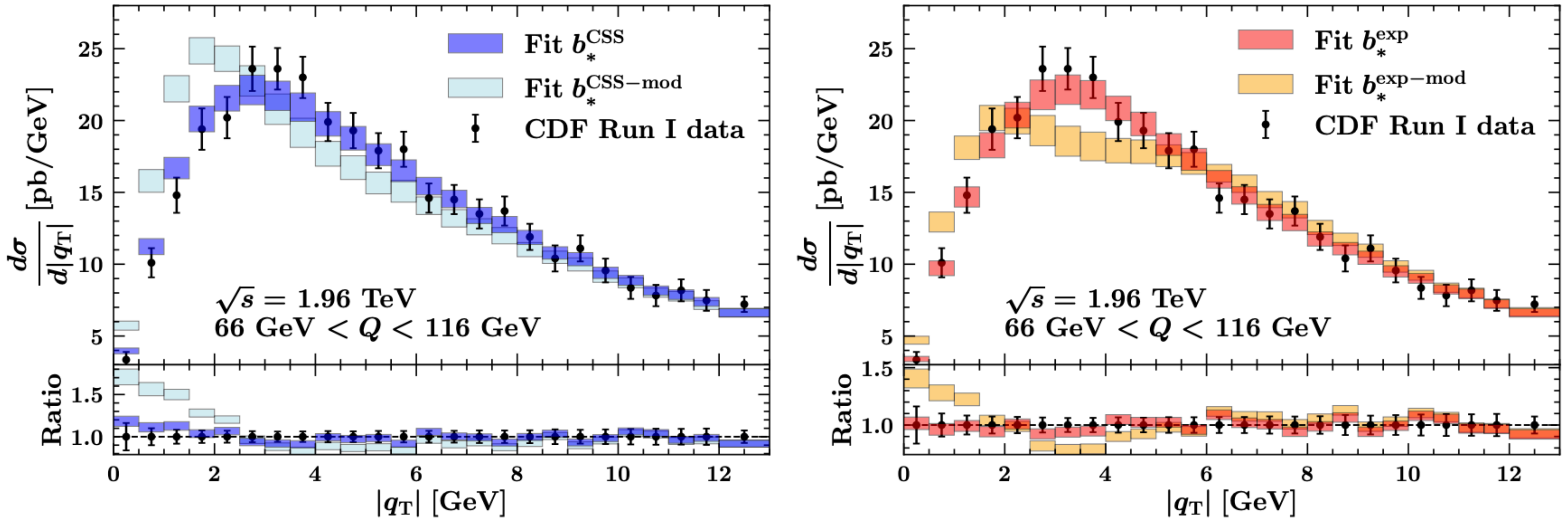



# Possible extension

## Resummed in $k_T$ space

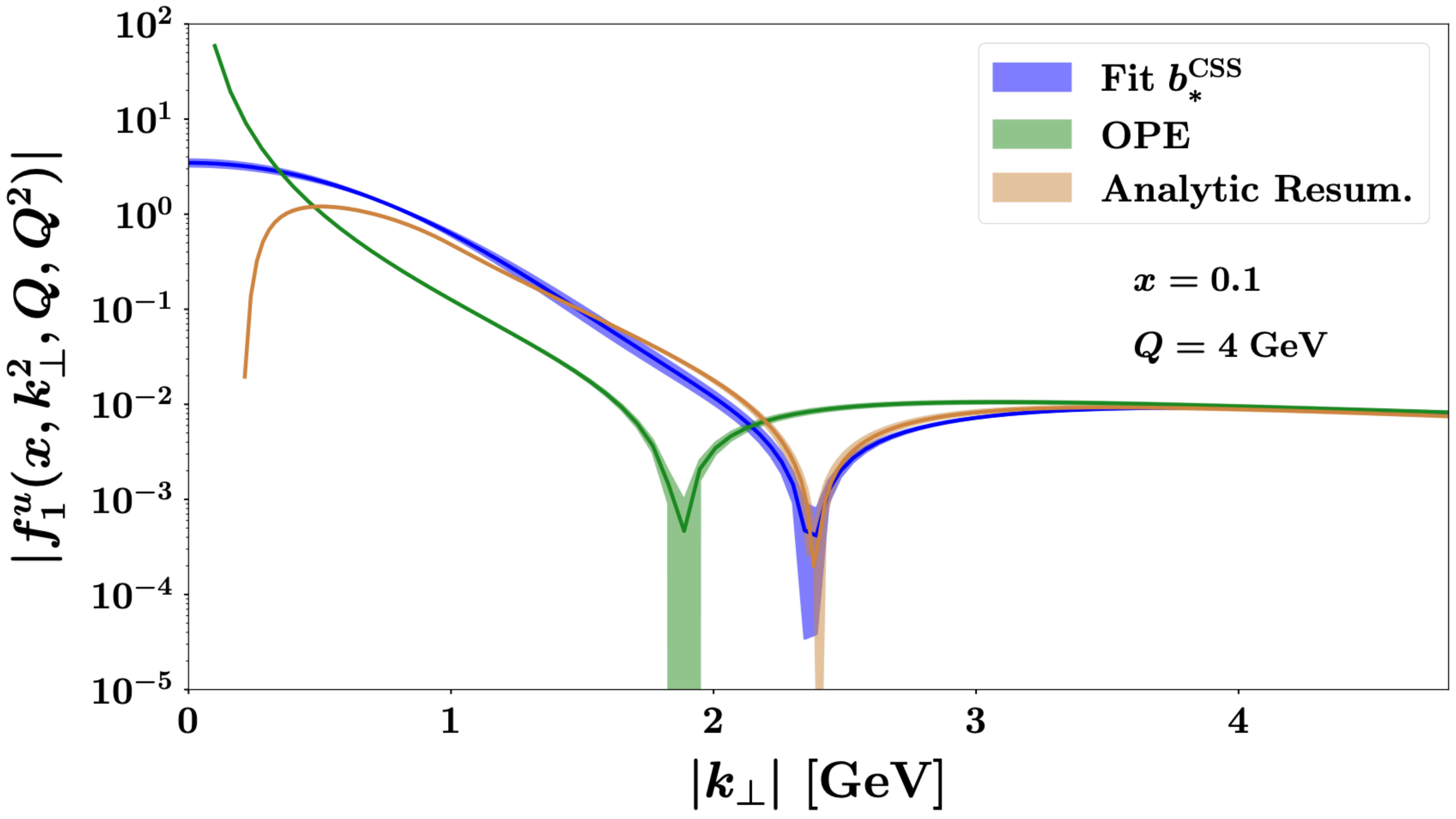
**The impact of prescriptions in phenomenological extractions of Transverse Momentum Dependent distributions**

Matteo Cerutti[1,*] and Andrea Simonelli[2,†]




Transverse momentum Resummation and Analytic continuation into the Deep Infrared

Andrea Simonelli[*1]

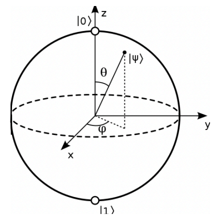


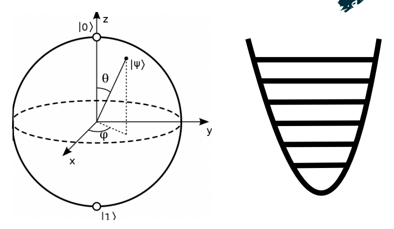
# What now?

# QCD with hybrid platforms

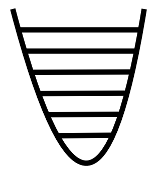
Simulate quarks and gluons with different quantum resources

QCD Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s\bar{\psi}\gamma^{\mu}T_a\psi A^a_{\mu} - \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}$

Qubits 

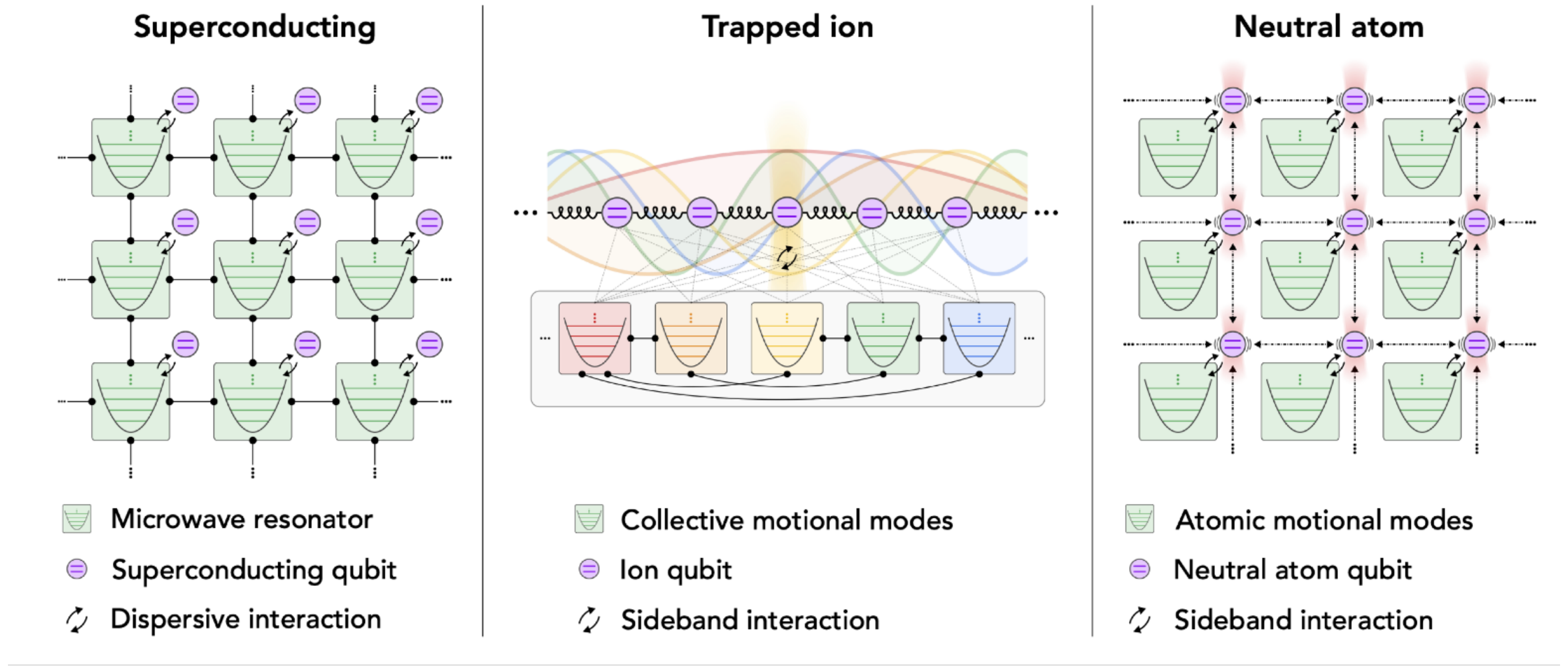


Qubit-qumode interaction



Qumodes

$$\mathcal{H}^m_{\mathrm{qumode}} \otimes \mathcal{H}^n_{\mathrm{qubit}}$$




Hybrid qubit/qumode approach

# Hadron structure on a quantum computer

Building on

**Quasiparton distributions in massive QED2: Toward quantum computation**

Sebastian Grieninger[1,2,*] Kazuki Ikeda[1,2,†] and Ismail Zahed[1,‡]

**Tensor network simulations of quasi-GPDs in the massive Schwinger model**

Sebastian Grieninger[1,2,3,4,*] Jake Montgomery[4,†] Felix Ringer[4,‡] and Ismail Zahed[4,§]

Actual hardware implementation
in collaboration with companies

quasi-PDFs and GPDs
(boost excited state)

QED2
(Schwinger model)

quasi-FFs
(real time dynamics needed)

QCD2
(non-Abelian)

TMDs? (Higher dimensions)

# Summary

- Consistent TMD parametrization
- pQCD and operator constrained are satisfied
- Predictive power increases: tested
- Model independent
- W+Y matching (not here)
- Improvable with better models and resummed versions

- Hadronic structure with quantum computers

# Special thanks to

Ted

Andrea

Fatma

Osvaldo

Elena

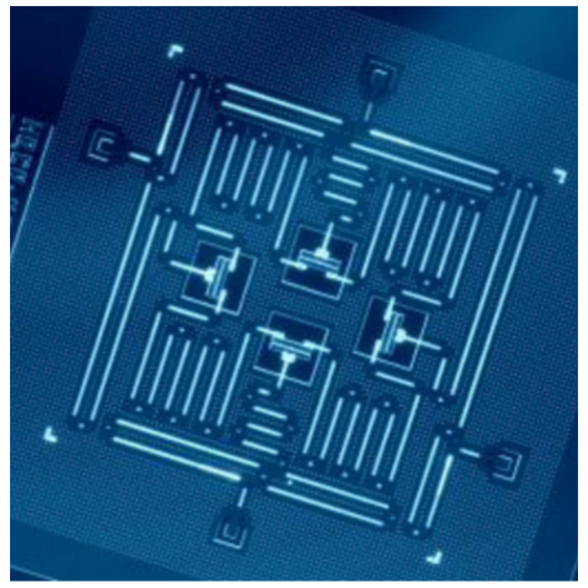
&

Jefferson Lab

# Thanks

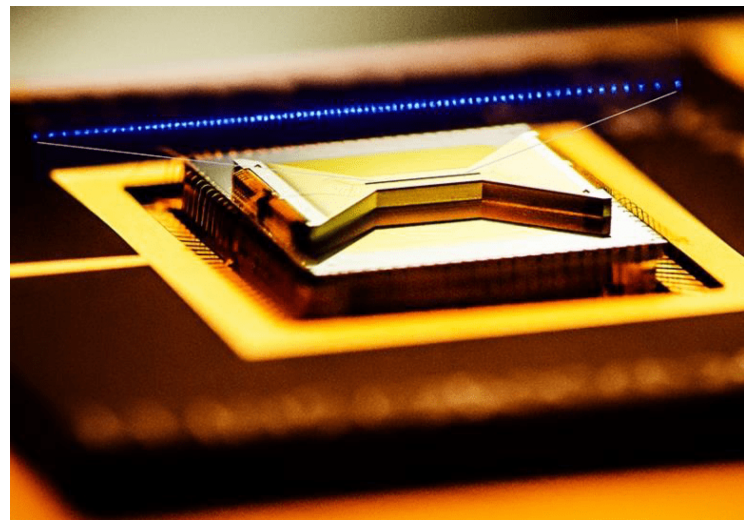
# Industry level effort (some examples)

### Superconducting qubits



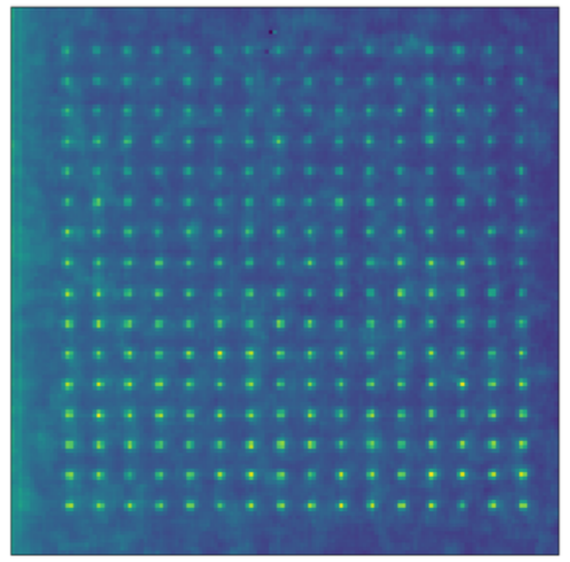
IBM
IQM
rigetti
Google AI Quantum
ALICE & BOB

### Trapped ion qubits



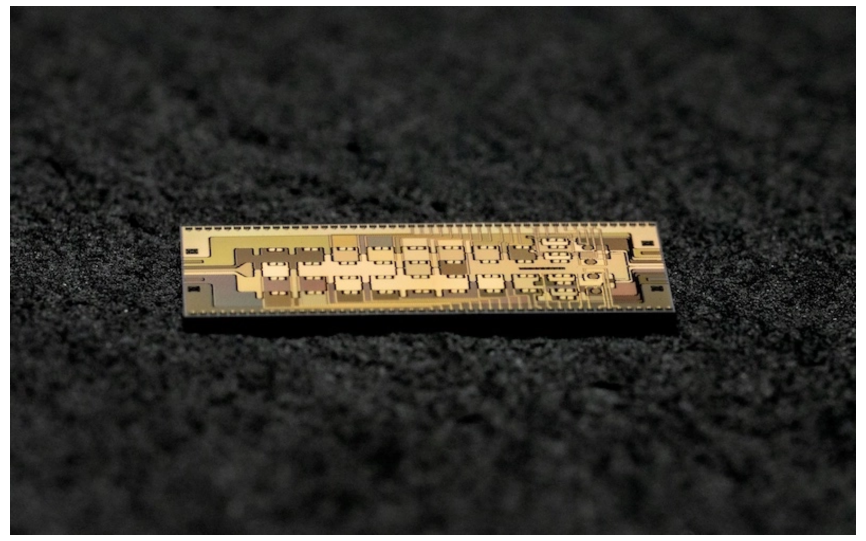
IONQ
oxford ionics
QUANTINUUM
AQT

### Neutral atoms qubits



|QuEra>
Computing Inc.
Infleqtion
atom computing
Pasqal

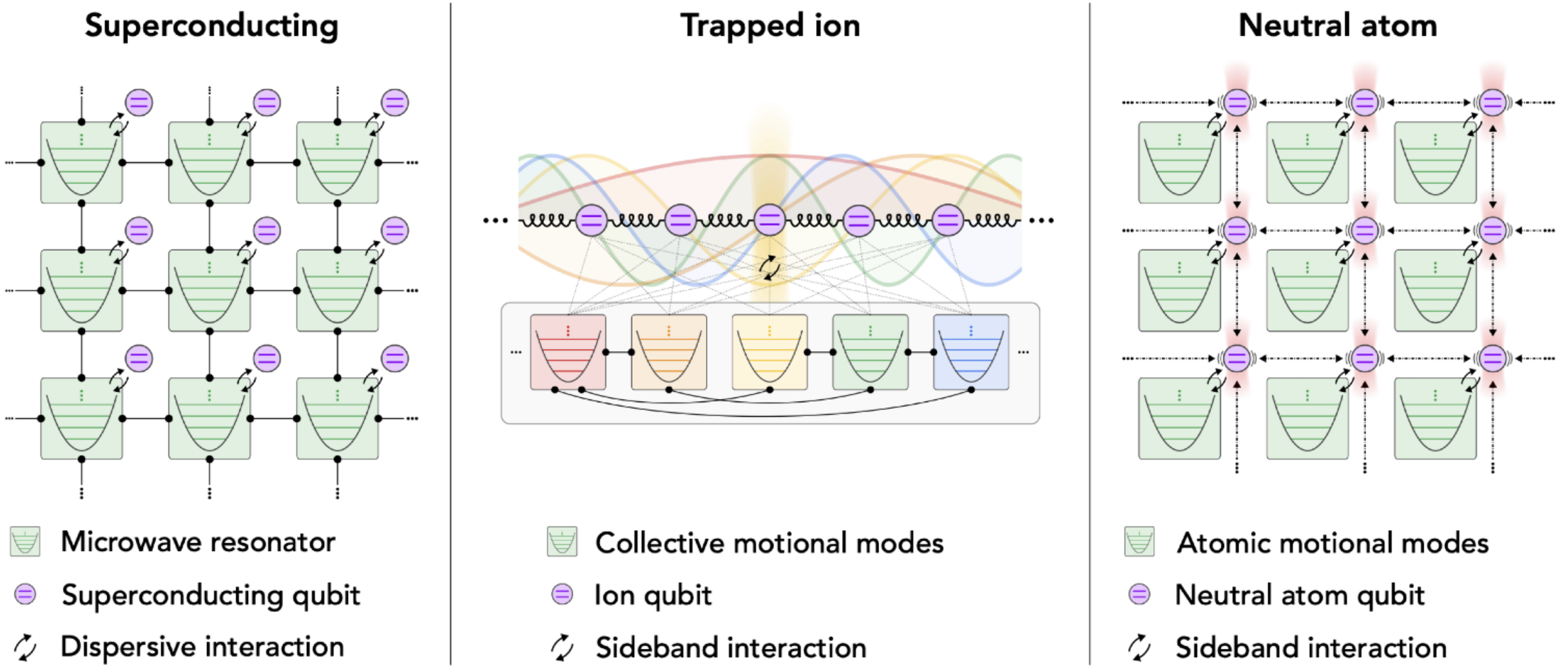
### Photonics qubits



XANADU
PsiQuantum
QUIX QUANTUM

# New paradigm: Hybrid architectures

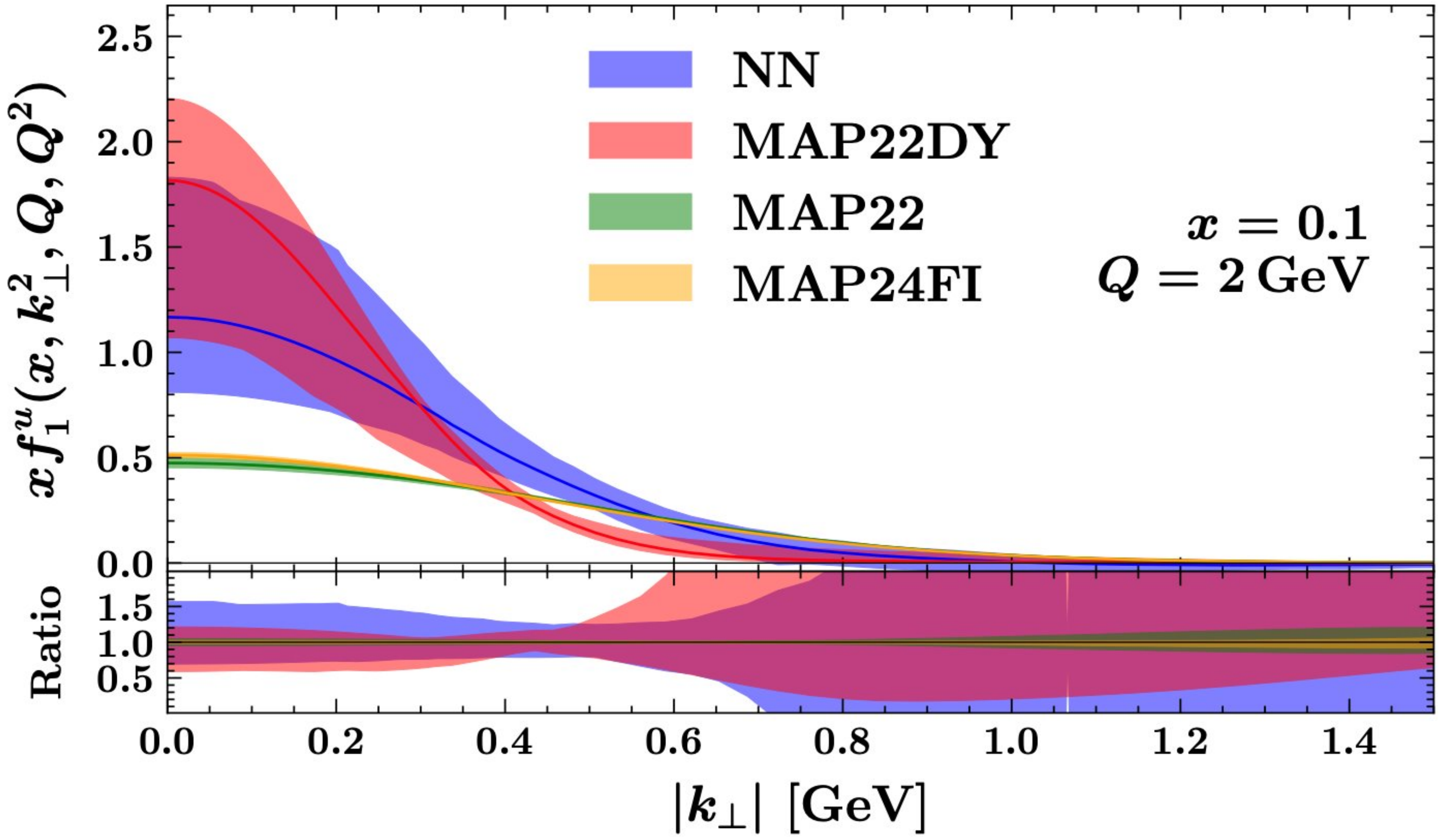
Girvin, Wiebe et al `24




Platform dependence: Native gates, Coherence time, Connectivity, …

# Fits comparison

Credits: Chiara Bissolotti (QCD Evolution 2025)




Different extractions from good fits ($\chi^2 \approx 1$) do not agree with each other

Bollweg, Gao et al (2504.04625)