Global QCD Analysis of spin PDFs in the proton with high-x data and lattice constraints

Christina Cocuzza

Theory
JAM
Computer Science
Experiment

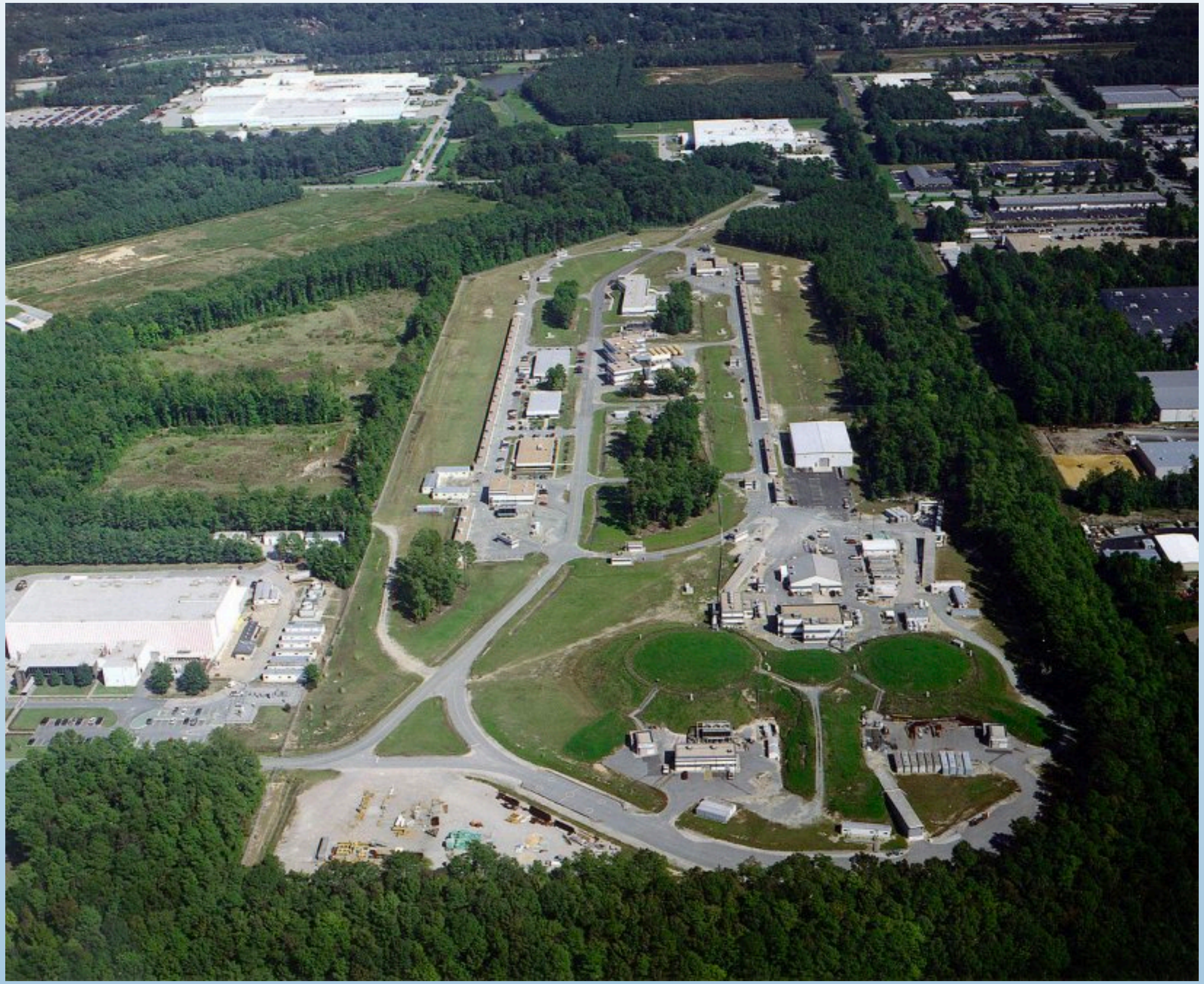
www.jlab.org/theory/jam

Global QCD analysis of spin PDFs in the proton with high-x and lattice constraints

JAM Collaboration (Spin PDF Analysis Group) Collaboration • C. Cocuzza (William-Mary Coll.) Show All(5)

Jun 16, 2025

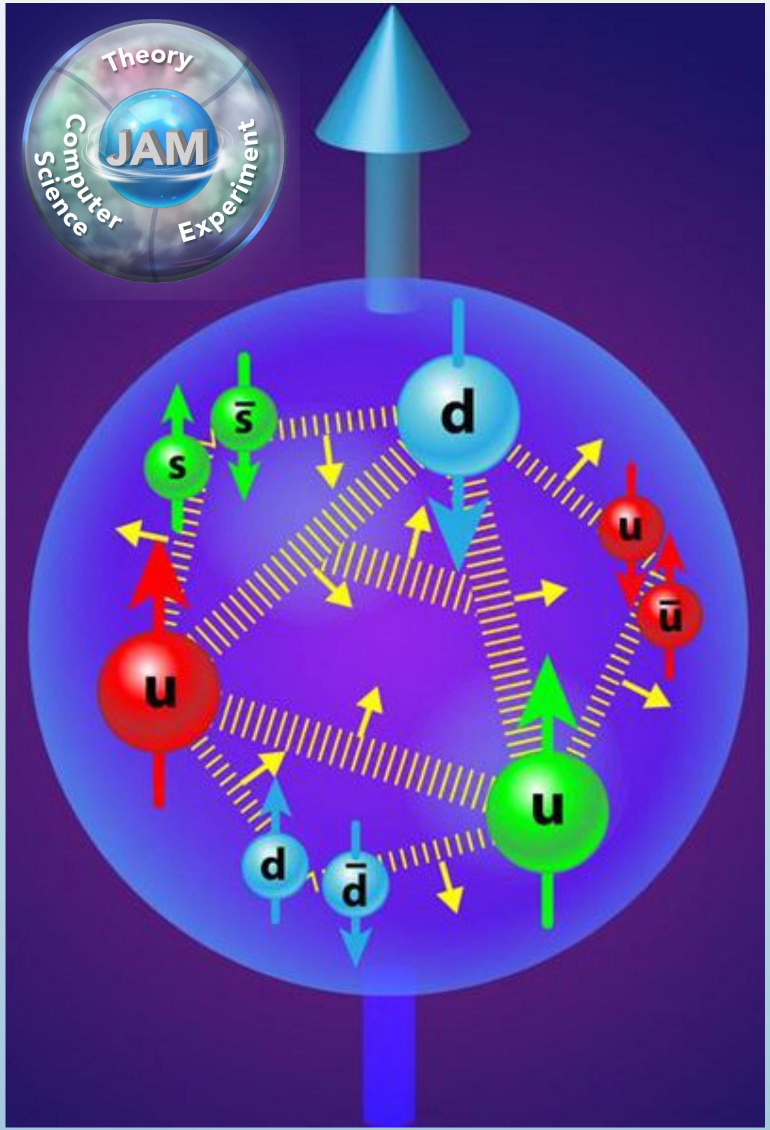
June 24, 2026



JAM Collaboration

3-dimensional structure of nucleons:

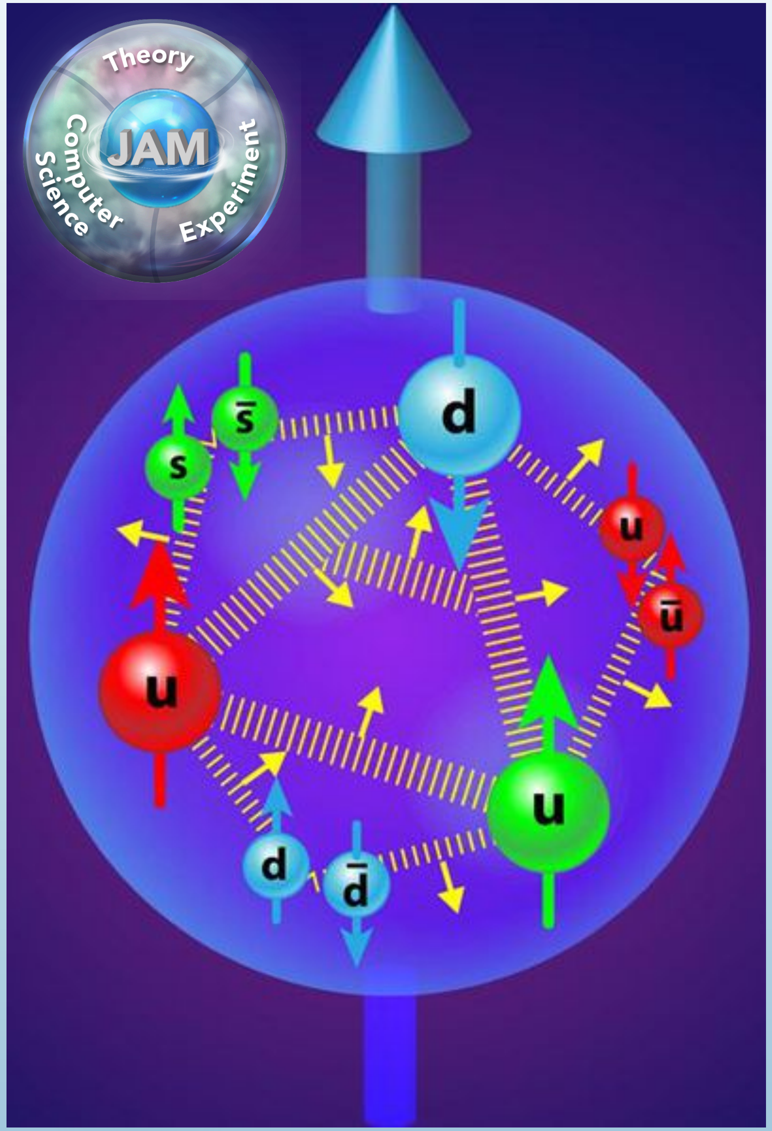
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



JAM Collaboration

3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Hadron Structure

Global QCD Analysis

Hadron Structure

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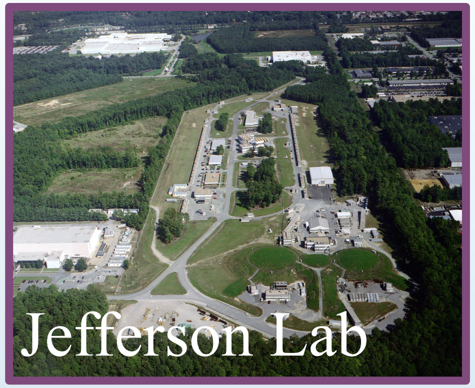
Hadron Structure

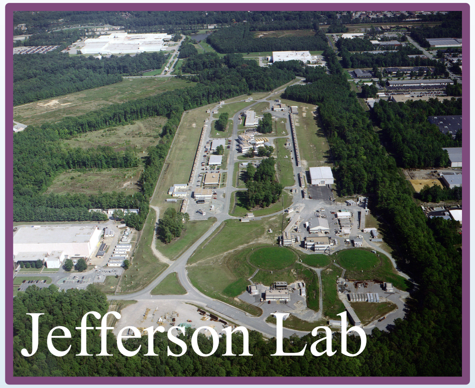
Global QCD Analysis



Hadron Structure

Global QCD Analysis

Introduction



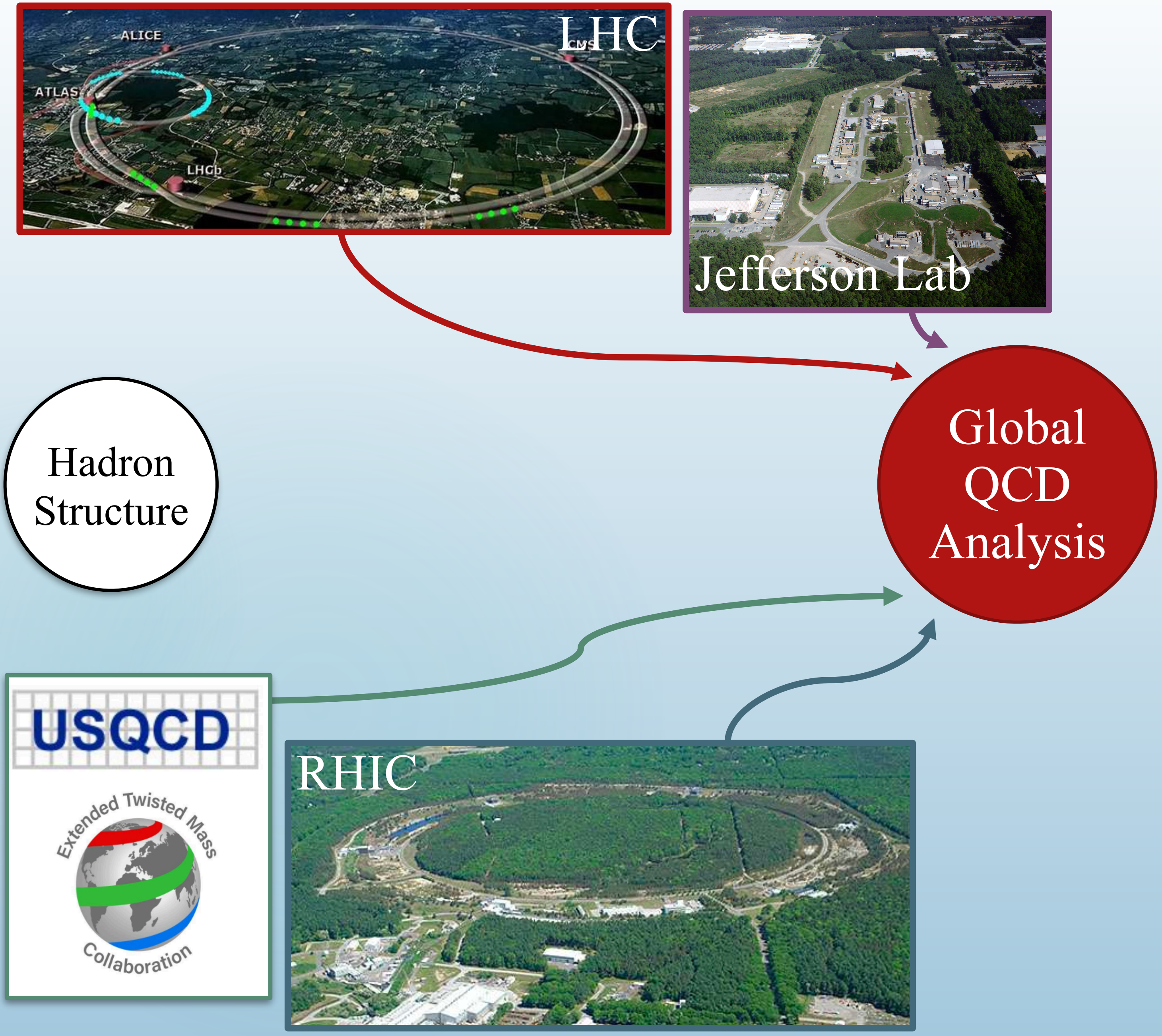
Hadron Structure

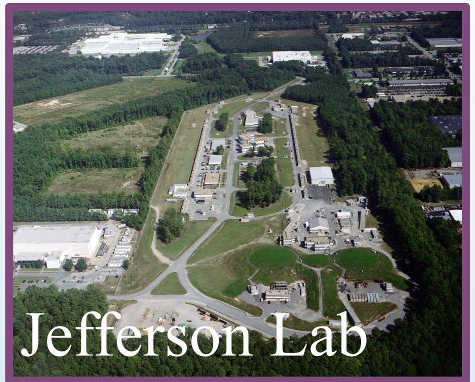
Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

Introduction





Hadron Structure

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j\left(\frac{x}{z},\mu\right)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

USQCD

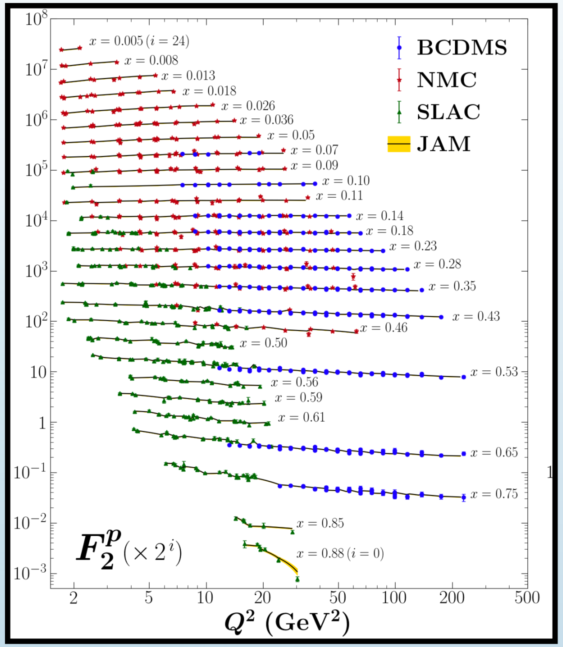
Extended Twisted Mass Collaboration

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e} \right)^2$$

χ^2 Minimization

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data}) \right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



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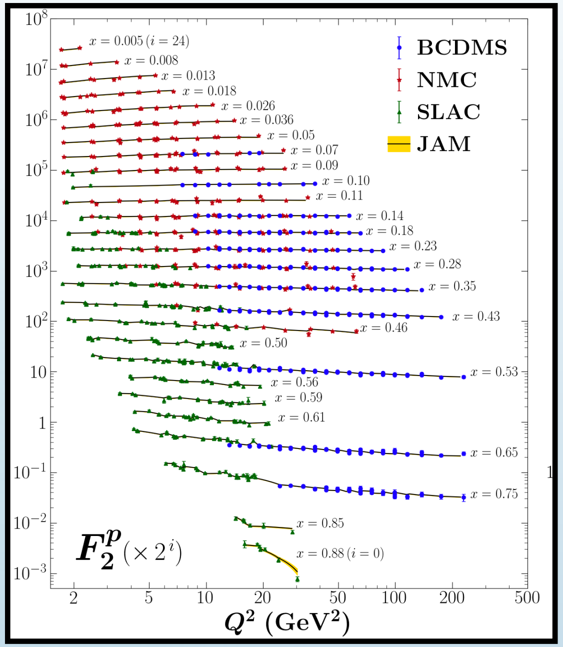

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Global QCD Analysis

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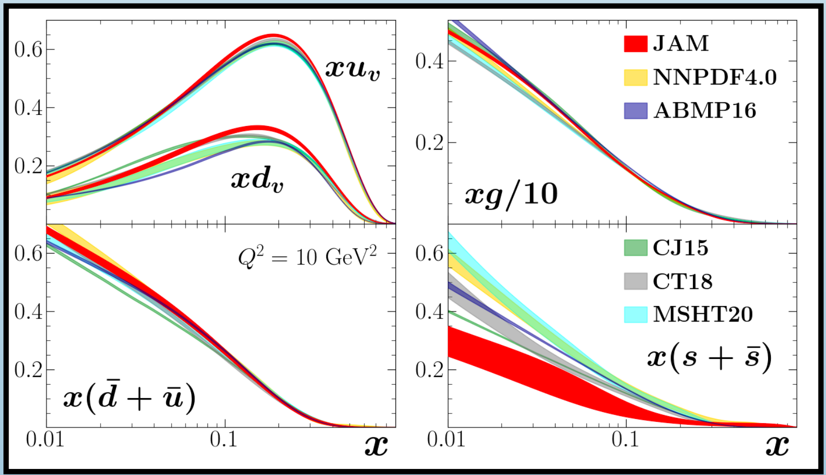


Data Resampling

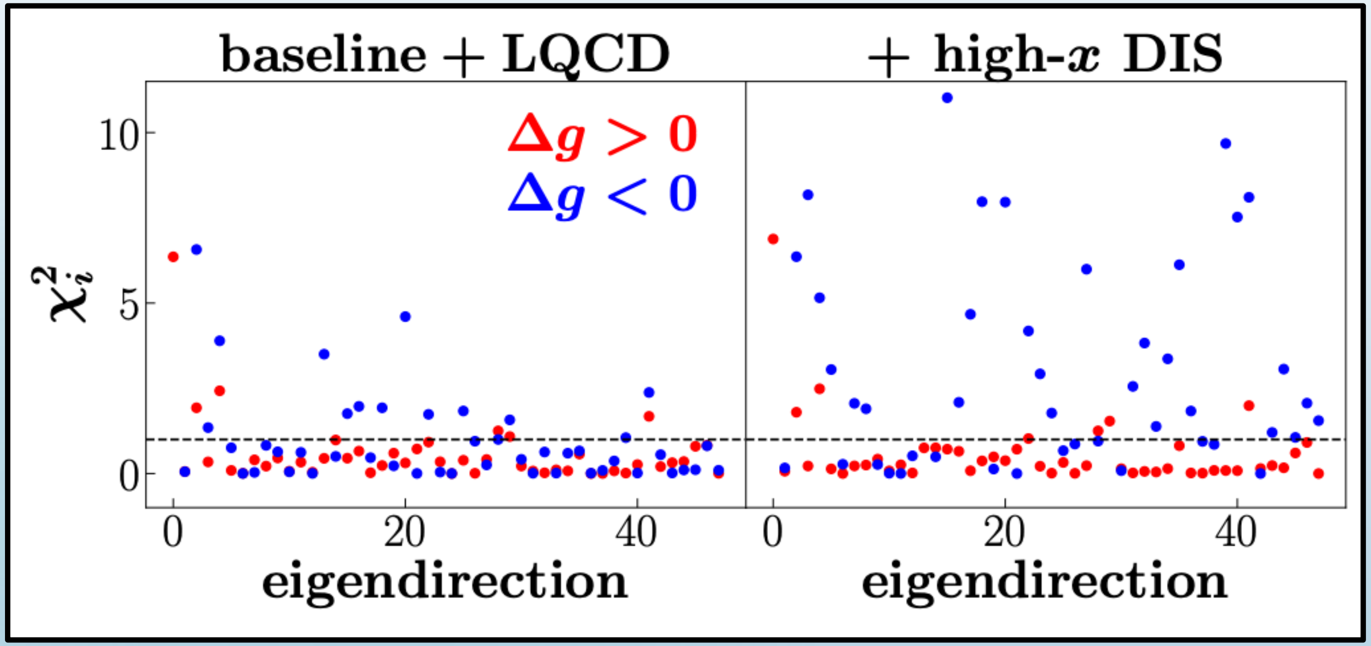
$$\tilde{\sigma} = \sigma + N(0,1)\,\alpha$$

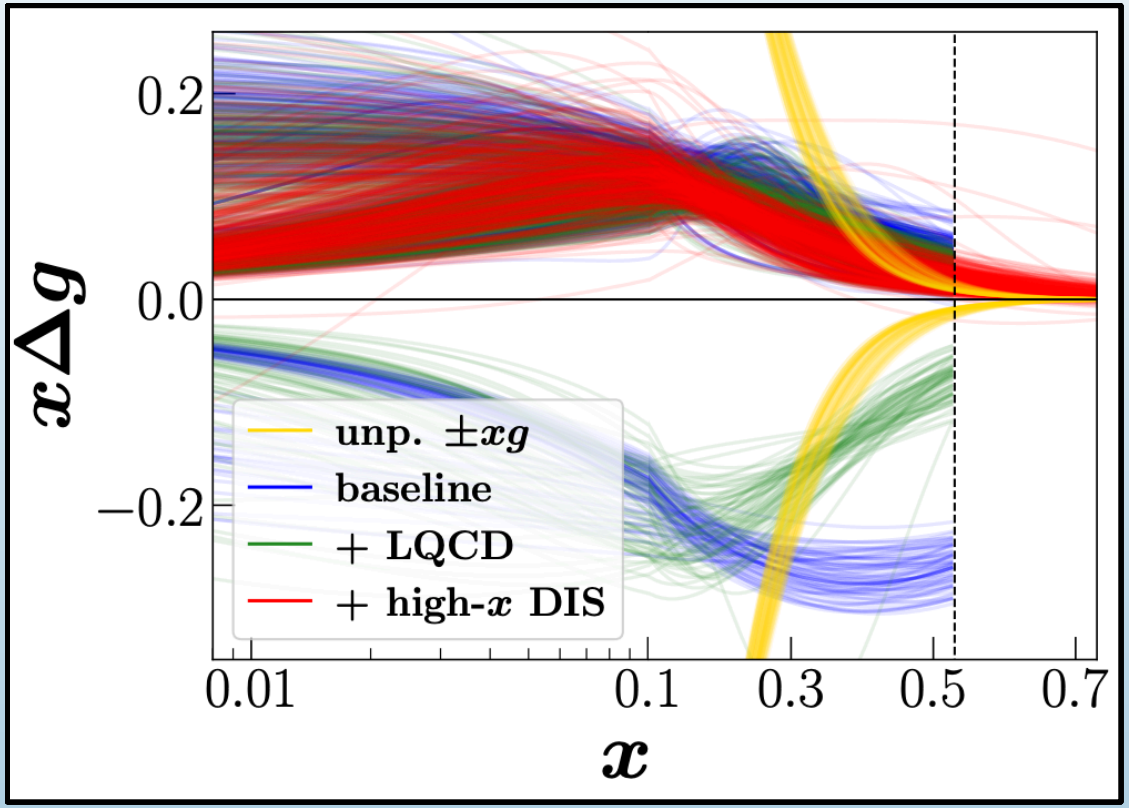
JAM Δg (2024)

New Data-Driven Constraints on the Sign of Gluon Polarization in the Proton

JAM Collaboration • N.T. Hunt-Smith (Adelaide U.) Show All(6)

Mar 12, 2024





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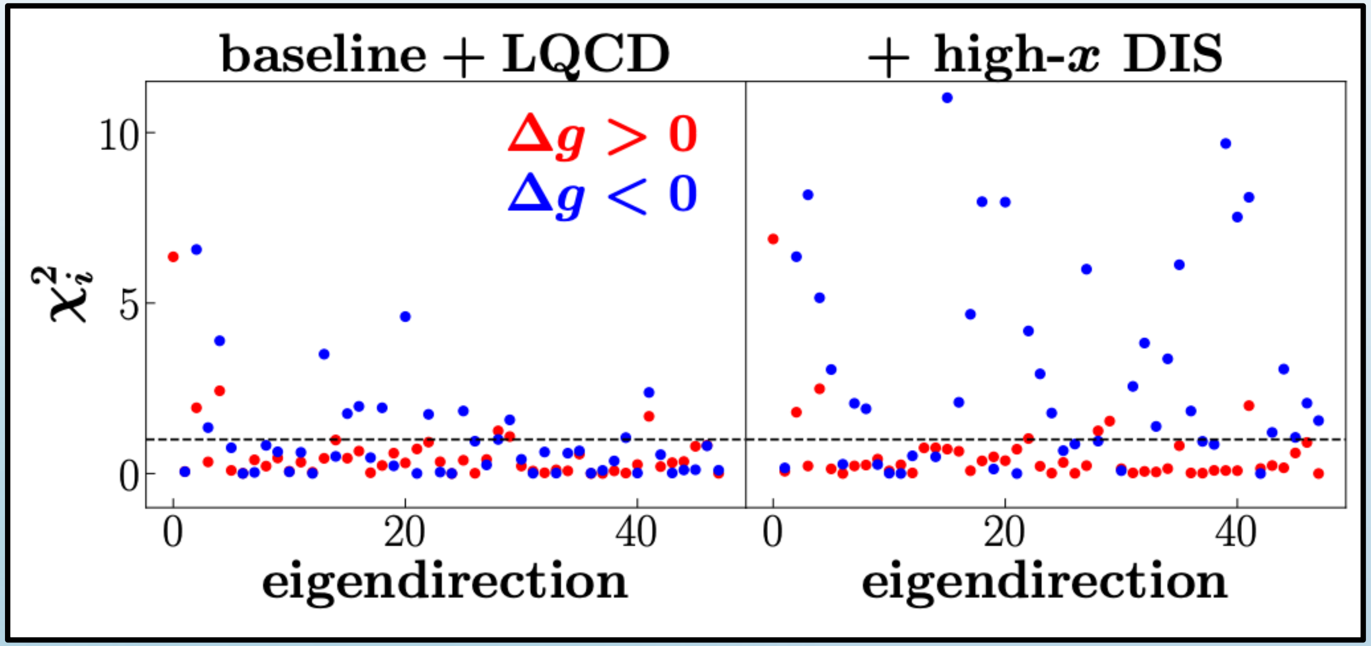
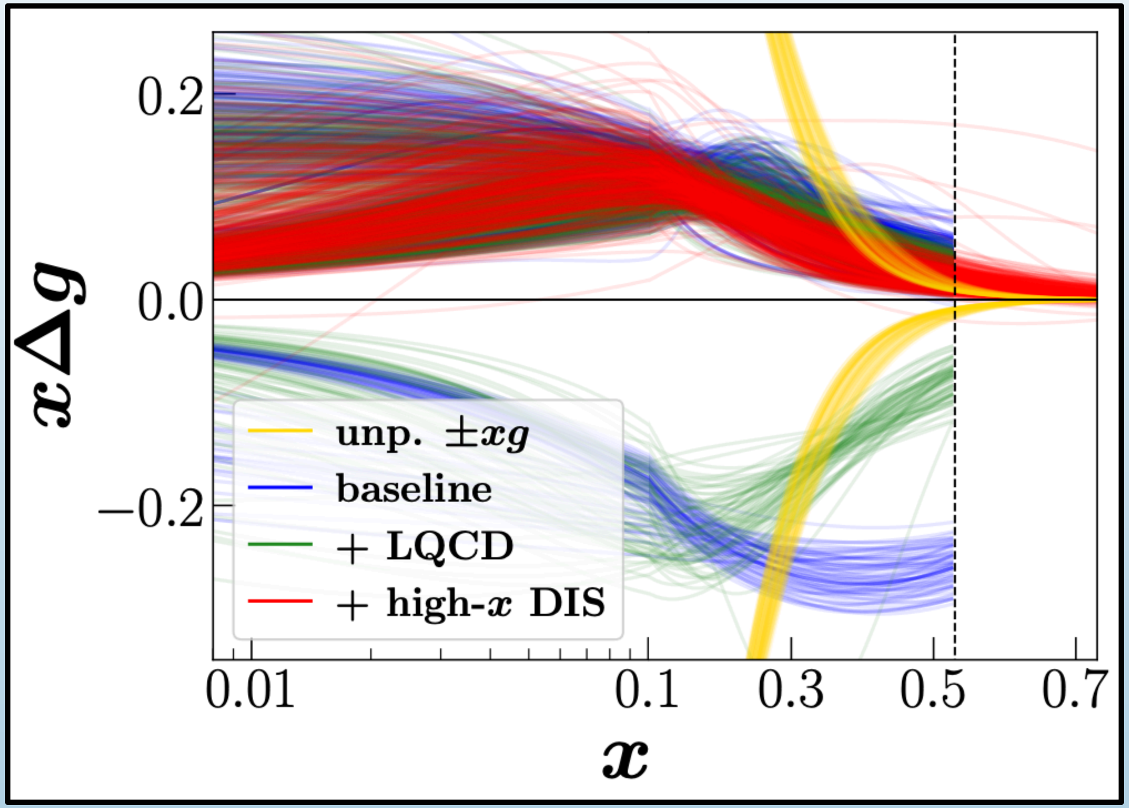
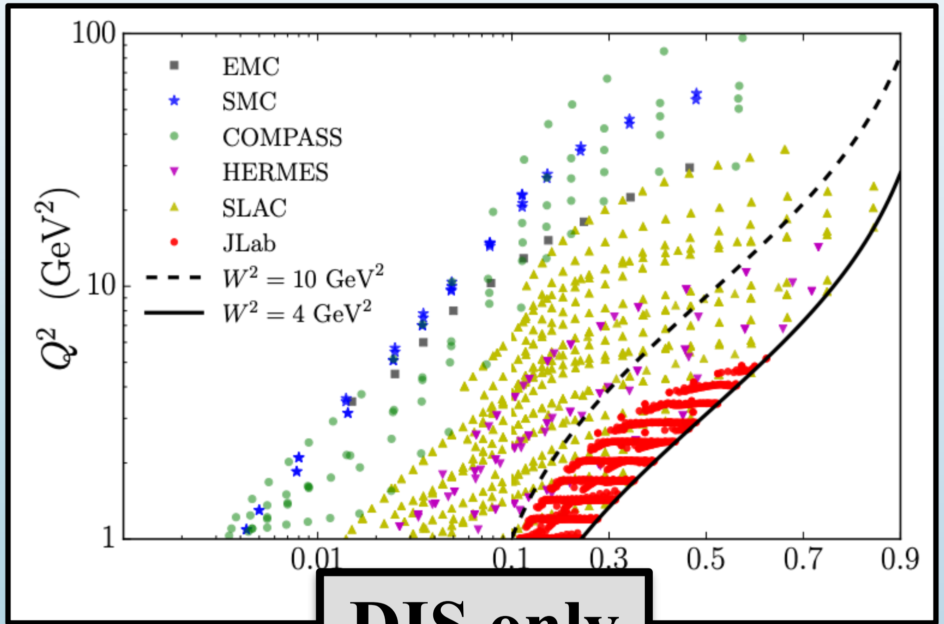




Adding high-x data constrains $\Delta\Sigma$ and leads to a poor description of LQCD for negative gluon

JAM High-x (2015)



DIS only

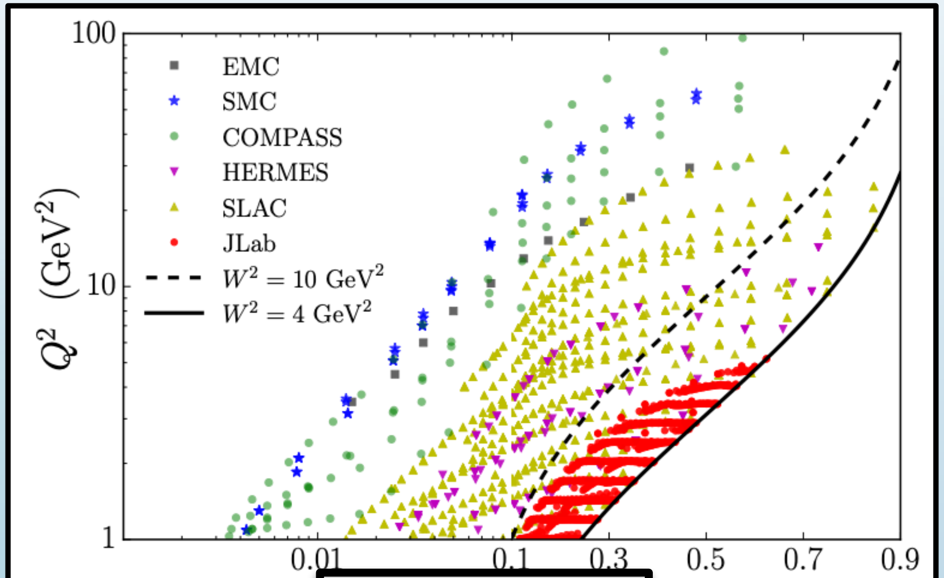
JAM15 was the last JAM polarized high-x analysis with Target Mass Corrections (TMCs) and Higher Twists (HT)

JAM High-x (2015)

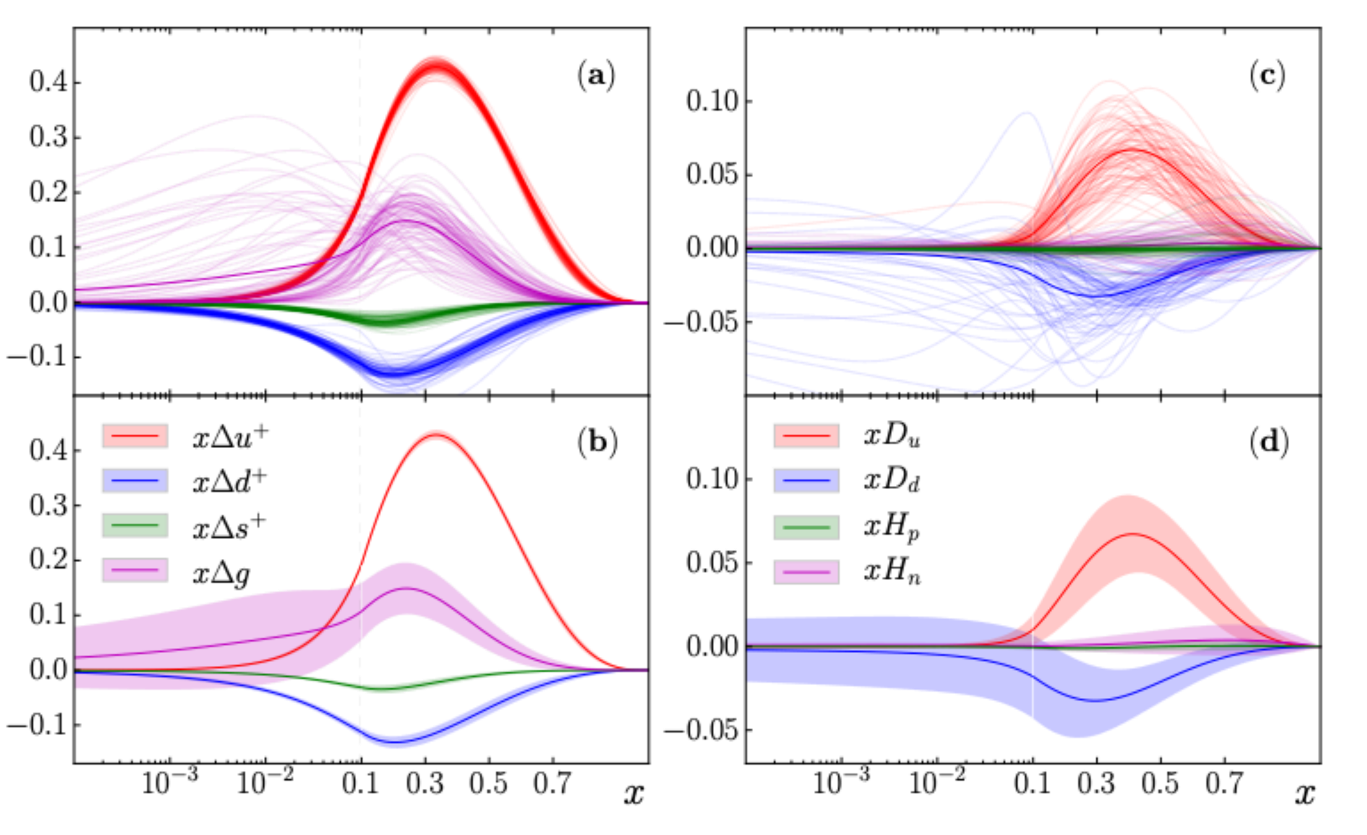
Iterative Monte Carlo analysis of spin-dependent parton distributions

Jefferson Lab Angular Momentum Collaboration • Nobuo Sato (Jefferson Lab) Show All(5)

Jan 28, 2016



DIS only



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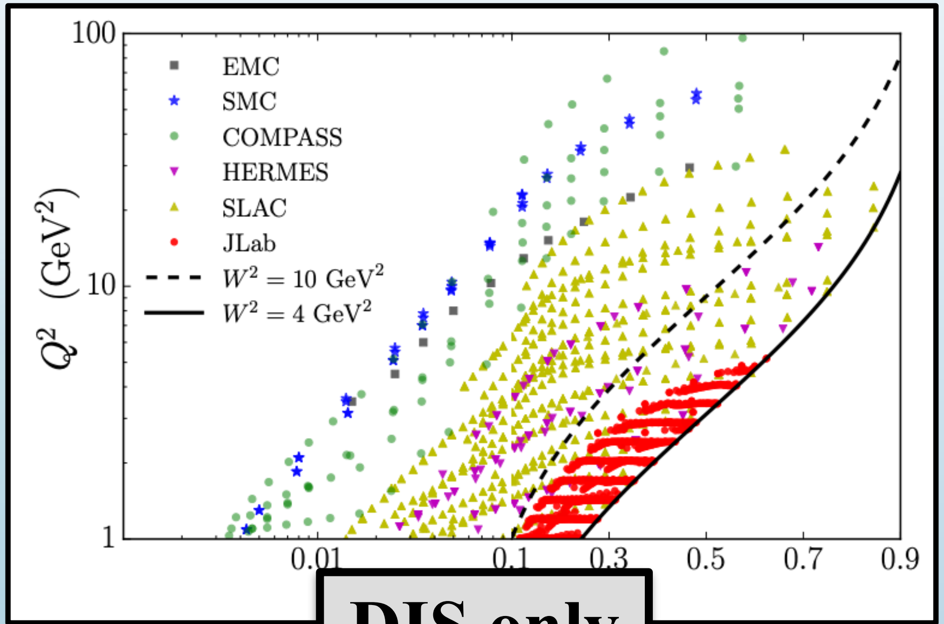
JAM High-x (2015)



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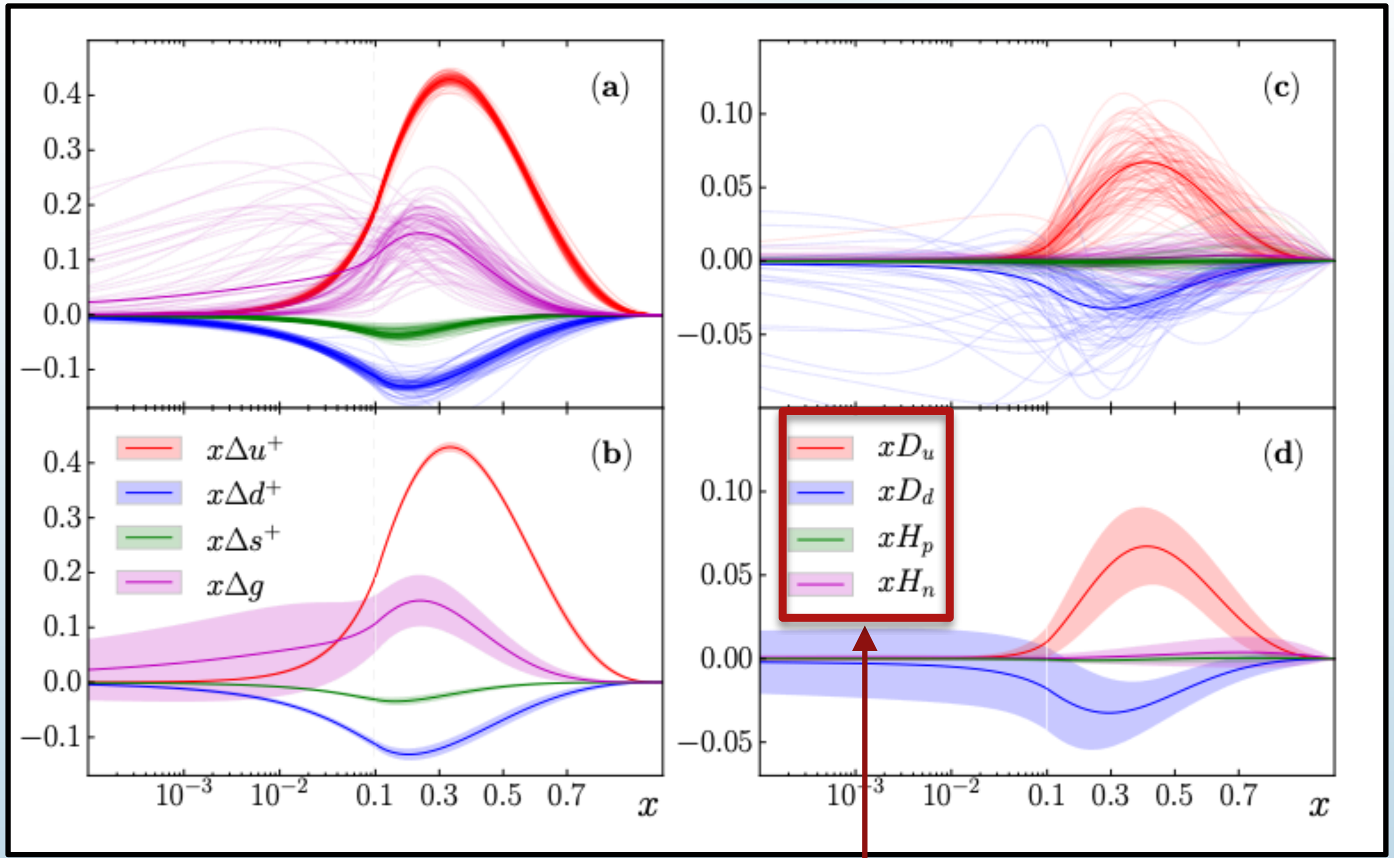
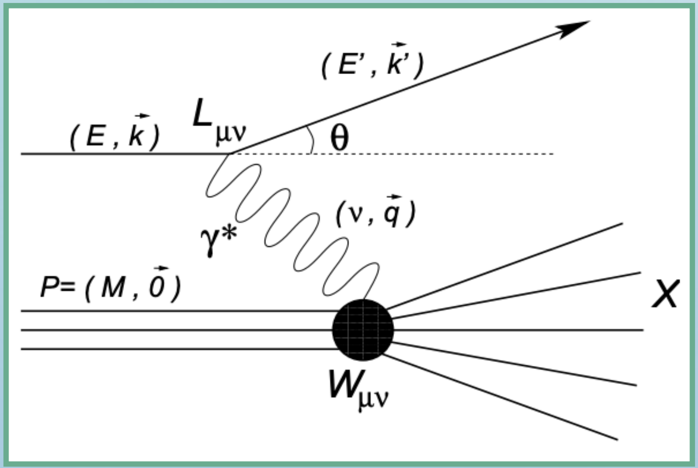


Extracted twist-3 (D_q) and twist-4 (H_N) functions

Ultimate JAM helicity analysis

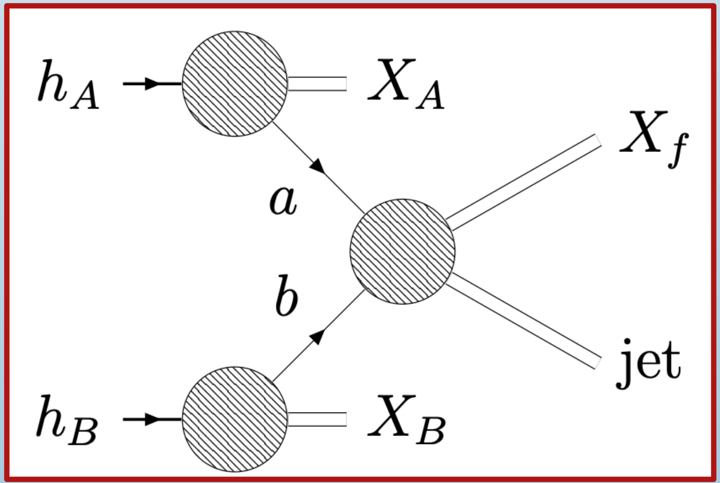
Simultaneous extraction of helicity PDFs (Δq) and single-hadron FFs (D_q^H) ($H = \pi, K, h$) at NLO **with** high-x DIS + TMCs + HTs + nuclear corrections

Polarized DIS



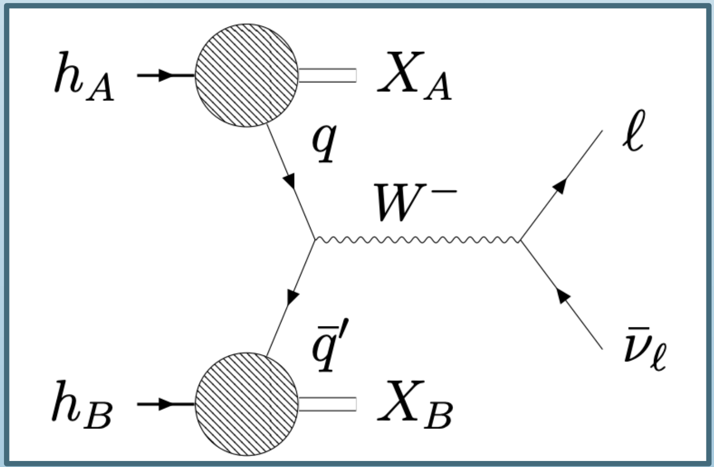
A. Airapetian *et al.*, Phys. Rev. D **75**, 012007 (2007)

Polarized jet production



C. Cocuzza, dissertation (2023)

W/Z production



C. Cocuzza, dissertation (2023)

Polarized DIS Theory

Leading Twist polarized structure functions:

$$g_1^{\mathrm{LT}}(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta C_q^{\mathrm{DIS}} \otimes \Delta q^+ + 2\Delta C_g^{\mathrm{DIS}} \otimes \Delta g](x, Q^2)$$

$$g_2^{\mathrm{LT}}(x, Q^2) = -\, g_1^{\mathrm{LT}}(x, Q^2) + \int_x^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z, Q^2)$$

Wandzura-Wilczek (WW) relation

Polarized DIS Theory

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Wandzura-Wilczek (WW) relation

Structure functions with collinear factorization (CF) TMCs:

$$g_1^{\mathrm{TMC}}(x,Q^2) = \frac{1}{\rho^2} g_1^{\mathrm{LT}}(x_N,Q^2) + \frac{2(\rho-1)}{\rho^2}\int_{x_N}^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z,Q^2)$$

$$g_2^{\mathrm{TMC}}(x,Q^2) = -\frac{1}{\rho^2} g_1^{\mathrm{LT}}(x_N,Q^2) + \frac{2}{(1+\rho)\rho^2}\int_{x_N}^1 \frac{dz}{z} g_1^{\mathrm{LT}}(z,Q^2)$$

[Derivation in Appendix A of our paper]

$$\rho = \sqrt{1+\frac{4M^2x^2}{Q2}},\; x_N = \frac{2x}{1+\rho}$$

Polarized DIS Theory

Higher Twists:

$$g_i^N = g_i^{N,\text{TMC}} + g_i^{N,\text{HT}},\ i = 1,2,\ N = p, n$$

$$g_1^{N,\text{HT}} = \frac{c_1^{N,\text{HT}}}{Q^2}$$

$$g_2^{N,\text{HT}} = c_2^{N,\text{HT}}$$

Polarized DIS Theory

Higher Twists:

$$g_i^N = g_i^{N,\mathrm{TMC}} + g_i^{N,\mathrm{HT}},\ i = 1,2,\ N = p,n$$

$$T(x,\mu^2) = N x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

$$g_1^{N,\mathrm{HT}} = \frac{c_1^{N,\mathrm{HT}}}{Q^2}$$

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Template used for g_1, g_2 proton and neutron HTs

Polarized DIS Theory

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Template used for g_1, g_2 proton and neutron HTs

Nuclear structure functions:

$$g_i^A(x,Q^2) = \sum_N \left[\Delta f_{ij}^{N/A} \otimes g_j^N\right](x,Q^2) \quad (i,j = 1,2)$$

Polarized DIS Theory

Burkhardt-Cottingham (BC) sum rule:

$$\Gamma_i(Q^2) = \int_0^1 dx g_i(x, Q^2), \quad i = 1,2$$

$$\Gamma_2^{\mathrm{LT}}(Q^2) = 0$$

Polarized DIS Theory

Burkhardt-Cottingham (BC) sum rule:

$$\Gamma_i(Q^2) = \int_0^1 dx g_i(x, Q^2), \quad i = 1,2$$

$$\Gamma_2^{\mathrm{LT}}(Q^2) = 0$$

d_2 matrix element:

$$d_2(Q^2) = \int_0^1 dx x^2 \left[2g_1(x, Q^2) + 3g_2(x, Q^2)\right]$$

Related to nucleon's color polarizability or the transverse color force acting on quarks

Zero at LT.
Non-zero with TMCs and/or HTs.

Lattice QCD Data

Singlet Ioffe time helicity distributions:

$$\mathscr{I}_{\Delta g}(\nu,\mu^2) = \int_0^1 dx x \sin(x\nu)\, \Delta g(x,\mu^2)$$

$$\mathscr{I}_{\Delta\Sigma}(\nu,\mu^2) = \int_0^1 dx x \sin(x\nu)\, \Delta\Sigma(x,\mu^2)$$

Lattice QCD Data

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1901 gauge configurations of an ensamble with (2+1)-dynamical clover Wilson fermions with stout-link smearing and tree-level tadpole-improved gauge action with a lattice volume of $32^3 \times 64$

$m_\pi = 358(3)$ MeV, latice spacing $a = 0.096(1)$ fm

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1901 gauge configurations of an ensamble with (2+1)-dynamical clover Wilson fermions with stout-link smearing and tree-level tadpole-improved gauge action with a lattice volume of $32^3 \times 64$

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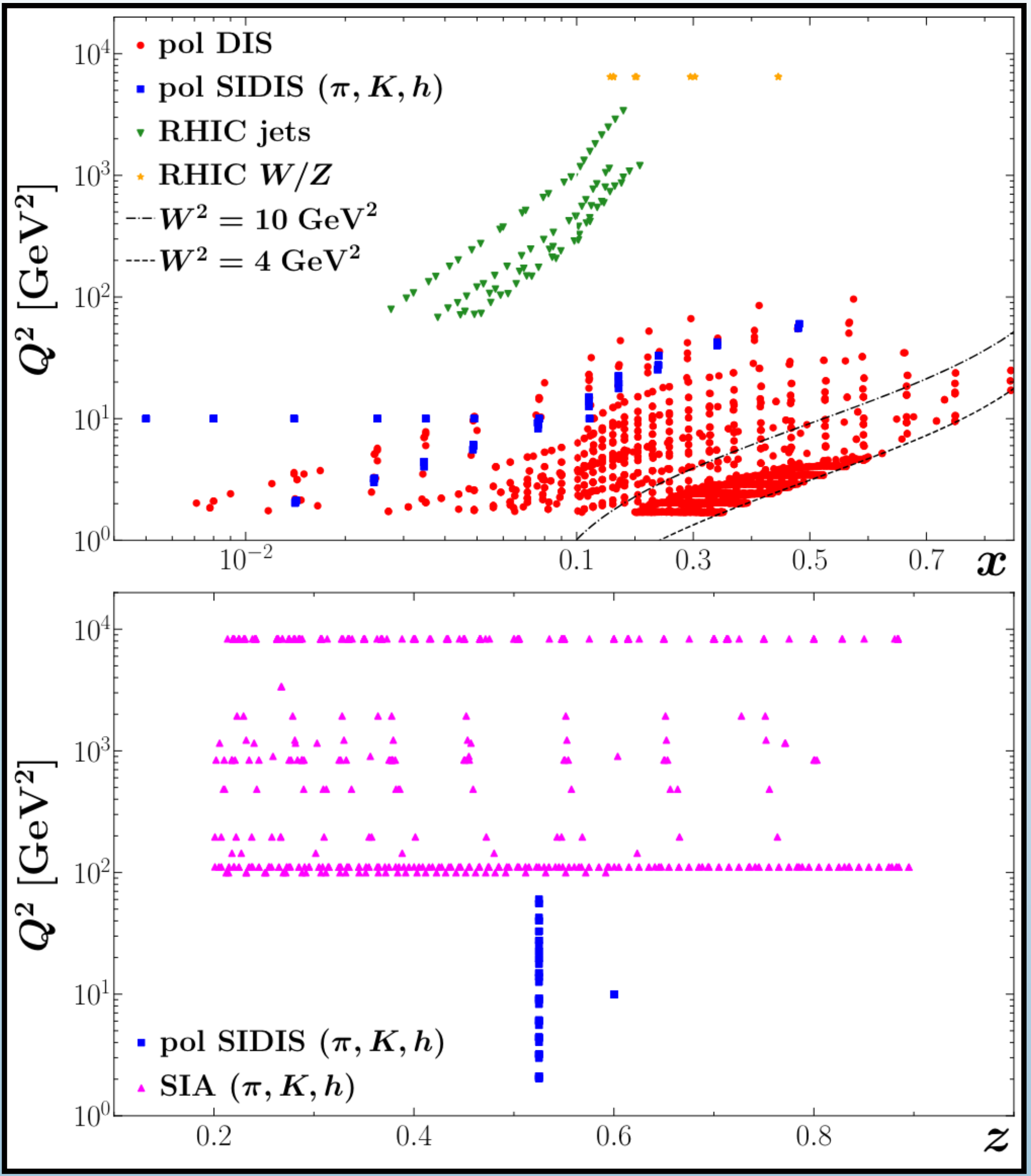
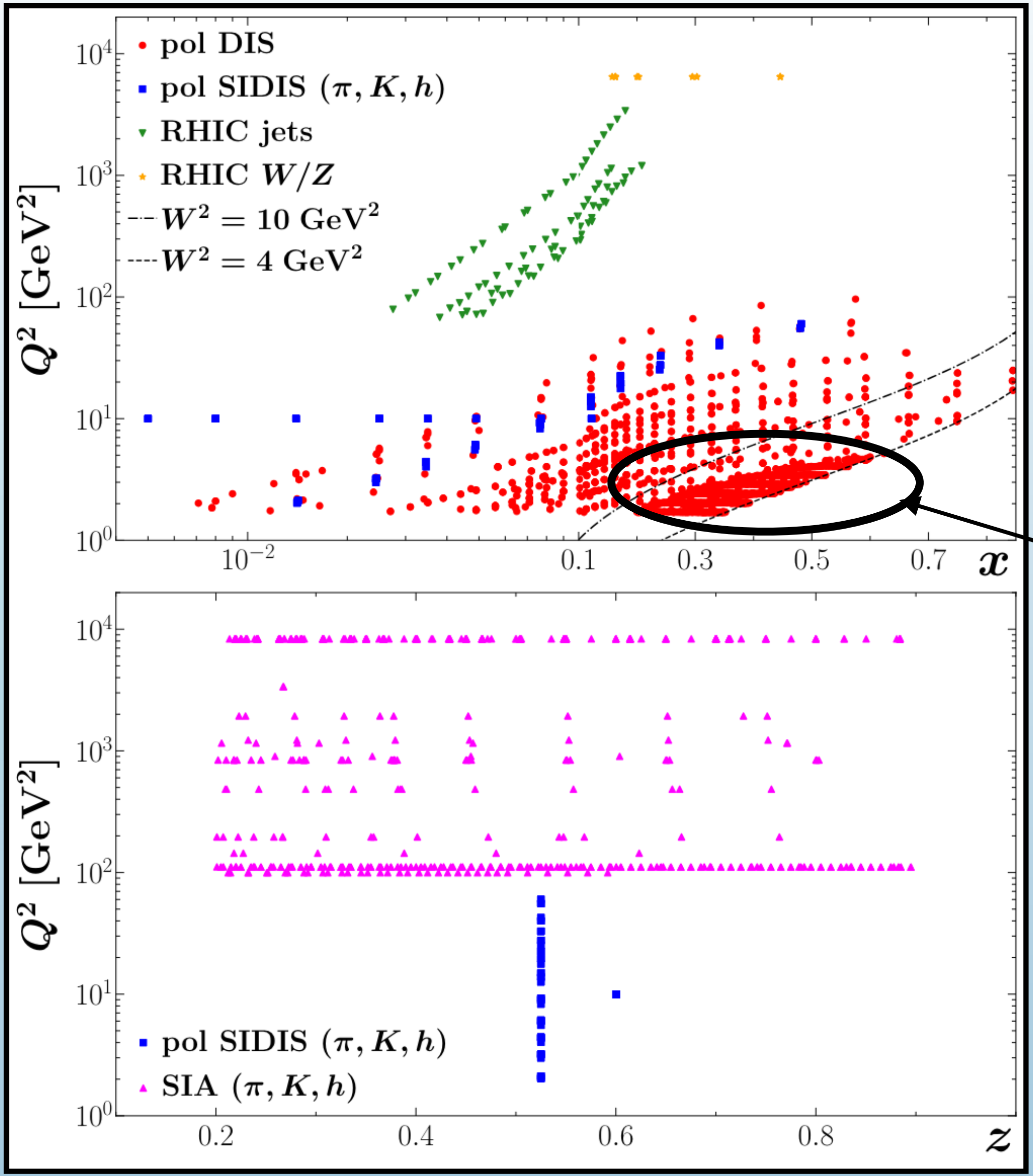
Included on same footing as experimental data

Data Summary



Polarized DIS	EMC, SMC, COMPASS, SLAC, HERMES, JLab	1735 points
Polarized SIDIS	SMC, COMPASS, HERMES	124 points
Polarized Jets	STAR, PHENIX	83 points
Polarized W/Z	STAR, PHENIX	18 points
LQCD	Radyushkin	48 points

Data Summary



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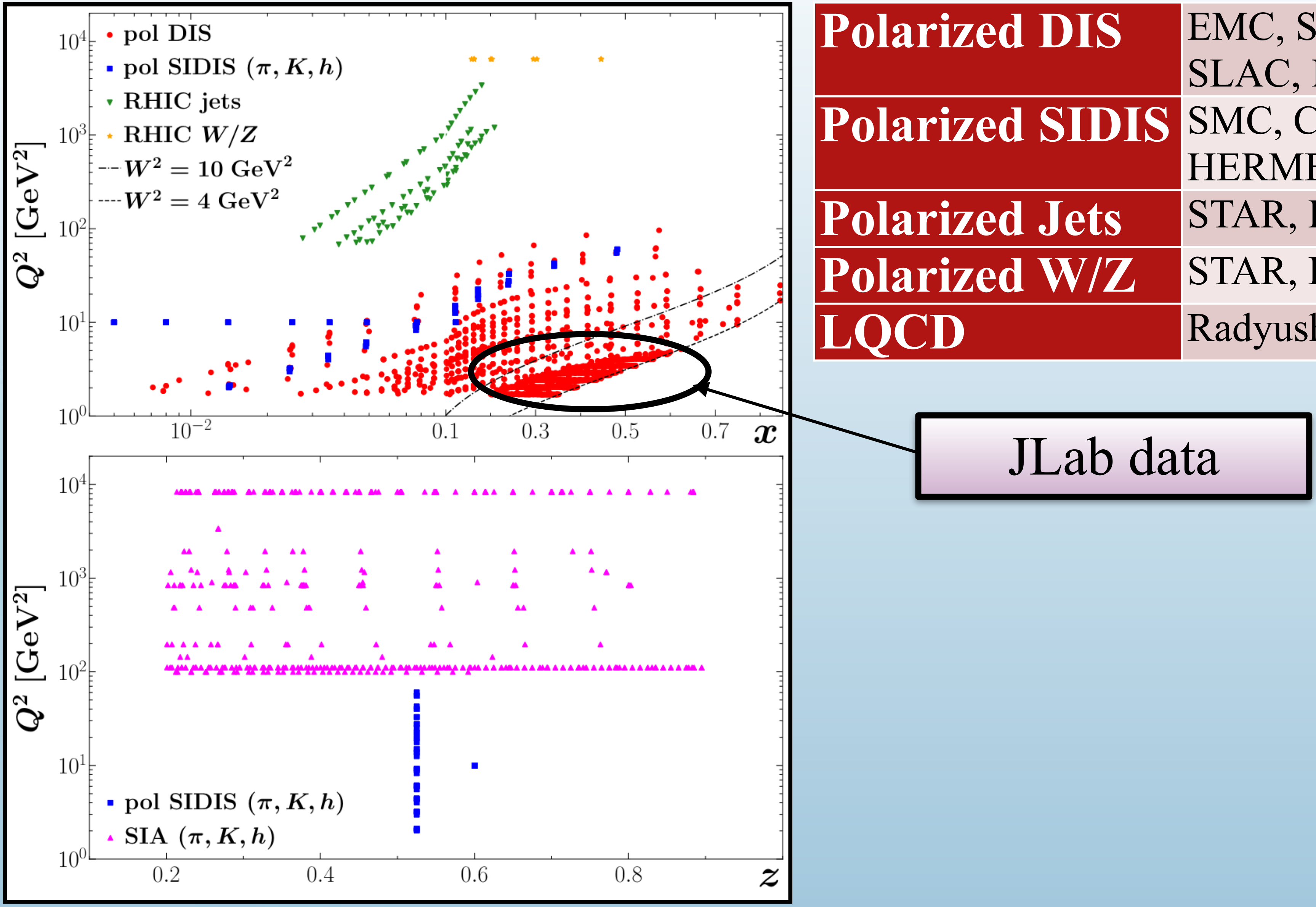
JLab data

Data Summary



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Helicity PDFs
(no pos. constraints)

$$\Delta u = \Delta u_v + \Delta \bar{u},$$
$$\Delta d = \Delta d_v + \Delta \bar{d},$$
$$\Delta \bar{u} = \Delta S + \delta \bar{u},$$
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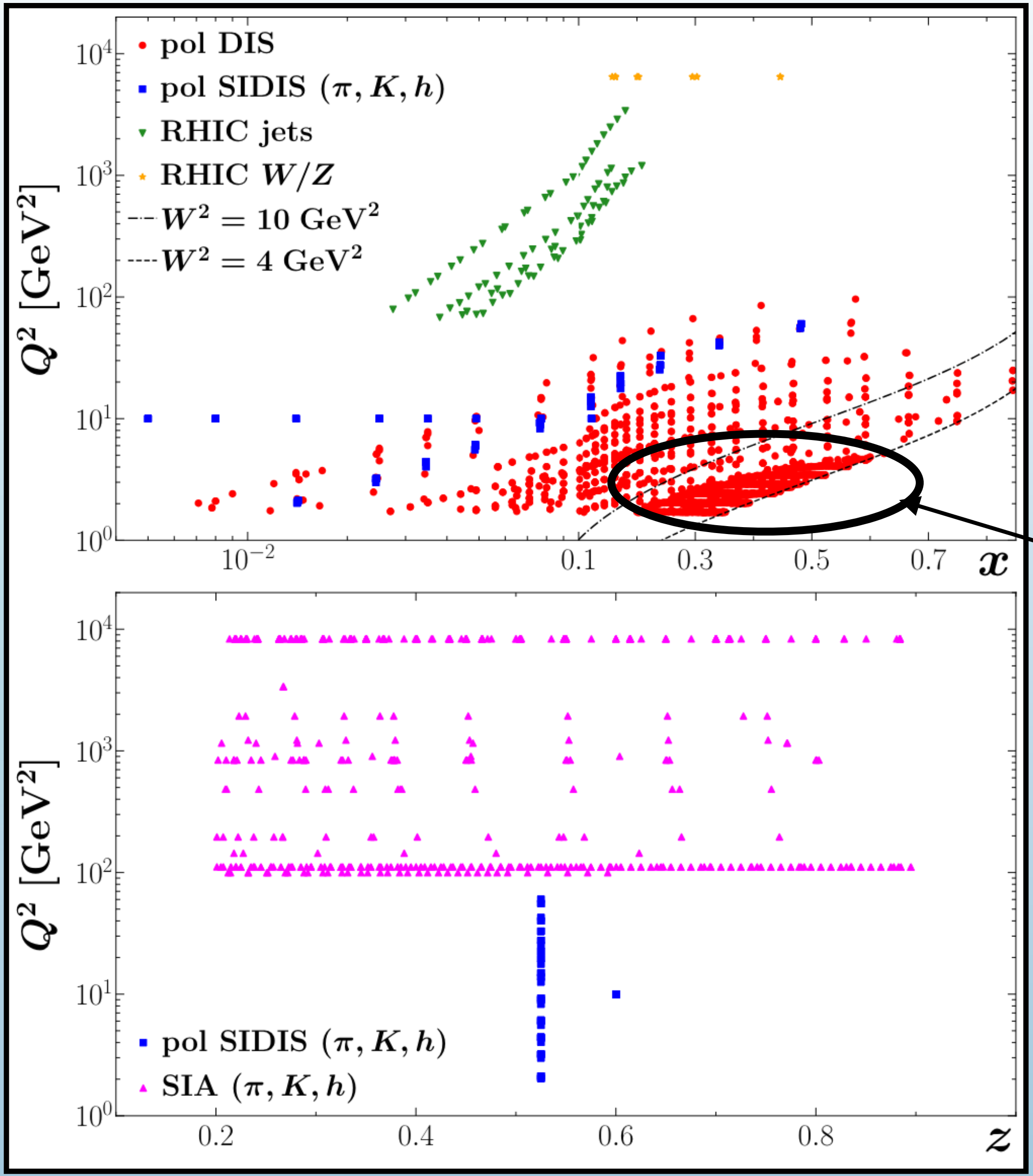
$$\Delta s = \Delta \bar{s} = \Delta S\ .$$

Data Summary



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Enforce $g_A = 1.269(3)$

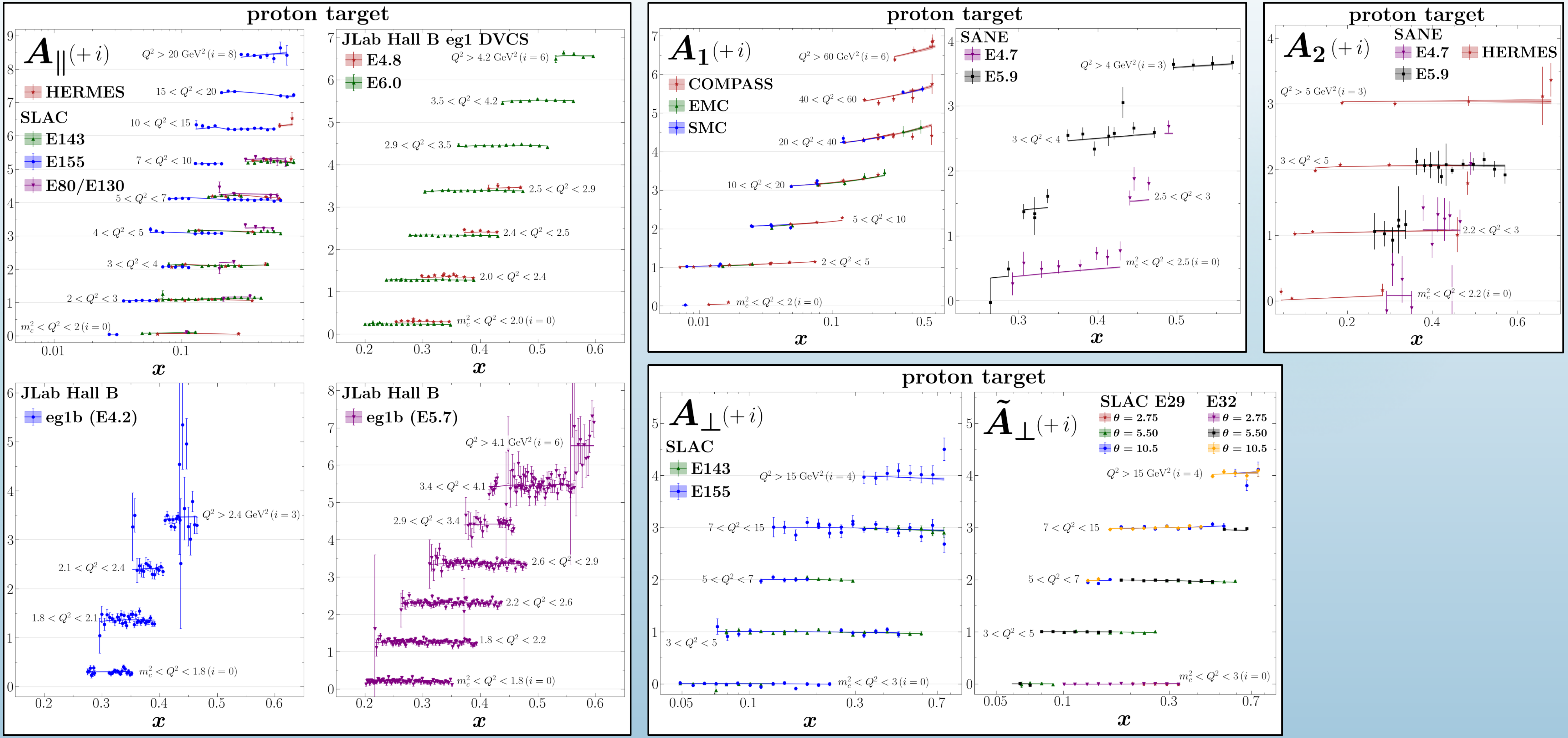
$$g_A = \int_0^1 dx\, [\Delta u^+ - \Delta d^+](x, Q^2),$$

χ^2 Summary

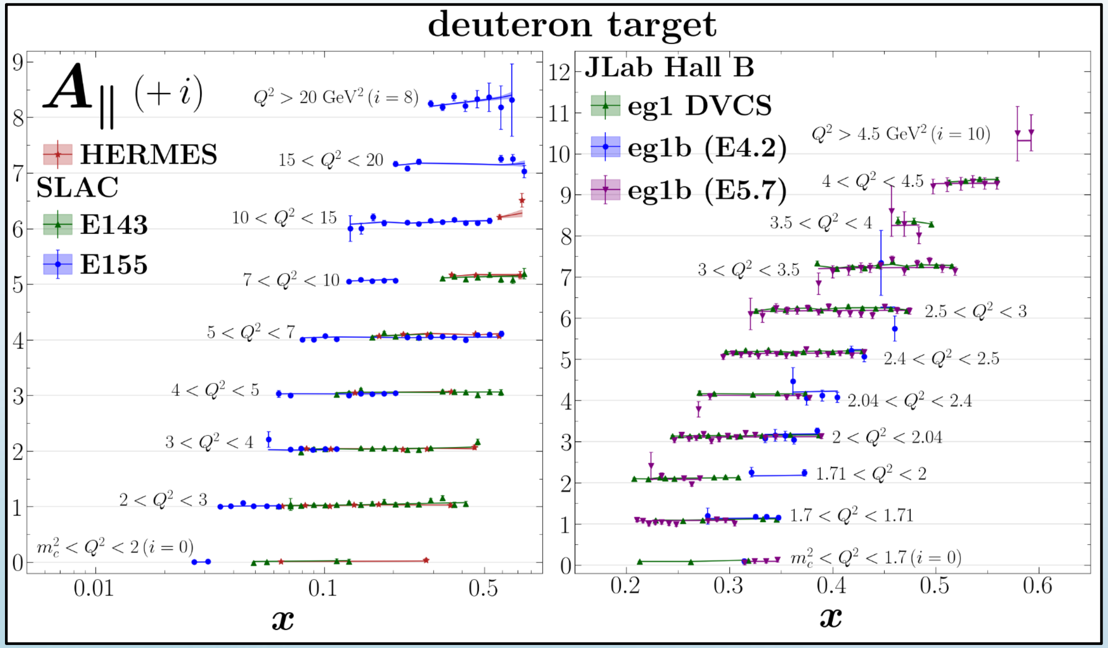
Data	N_{dat}	χ^2_{red} (Z-score) $W^2 > 4$ GeV^2 (with TMC & HT)
Polarized	1960†	0.99 (−0.30)
— DIS	1735†	1.02 (+0.61)
— SIDIS	124	0.76 (−1.99)
— jets	83	0.81 (−1.29)
— W/Z boson	18	0.75 (−0.71)
Lattice QCD	48	0.58 (−2.35)
Total	**2008†**	**0.99 (−0.22)**

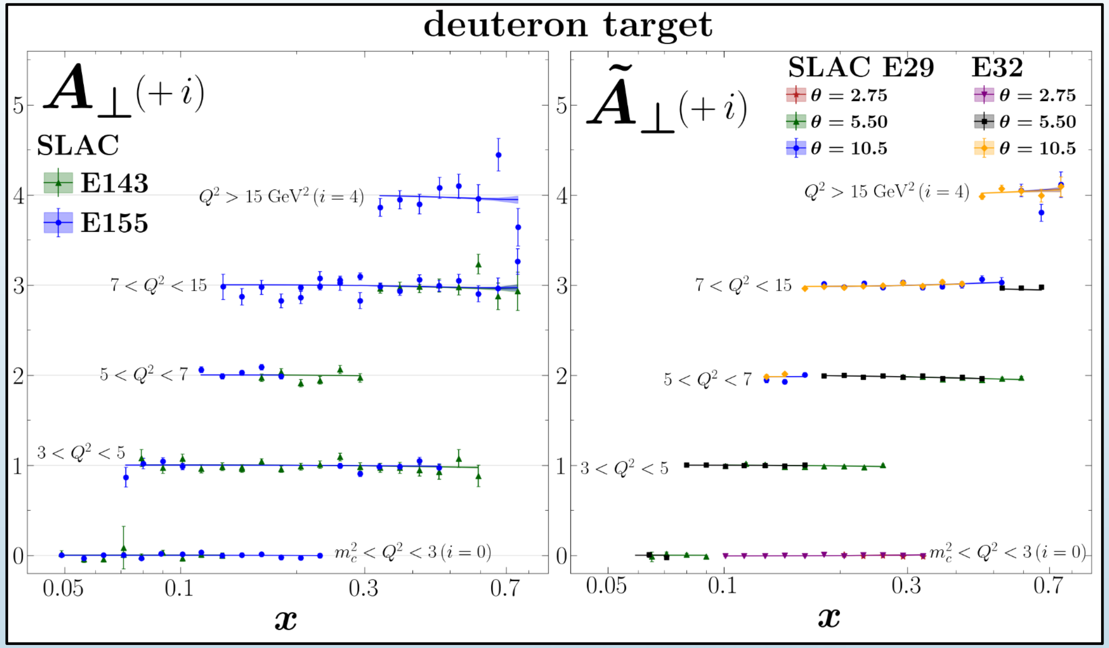
Good description of all experimental data and LQCD calculations

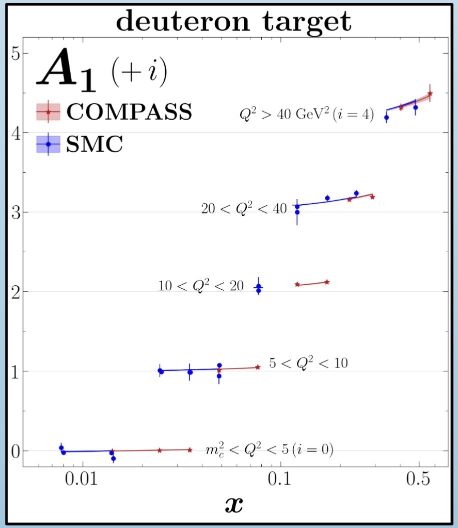
Data vs. Theory (DIS) (proton)



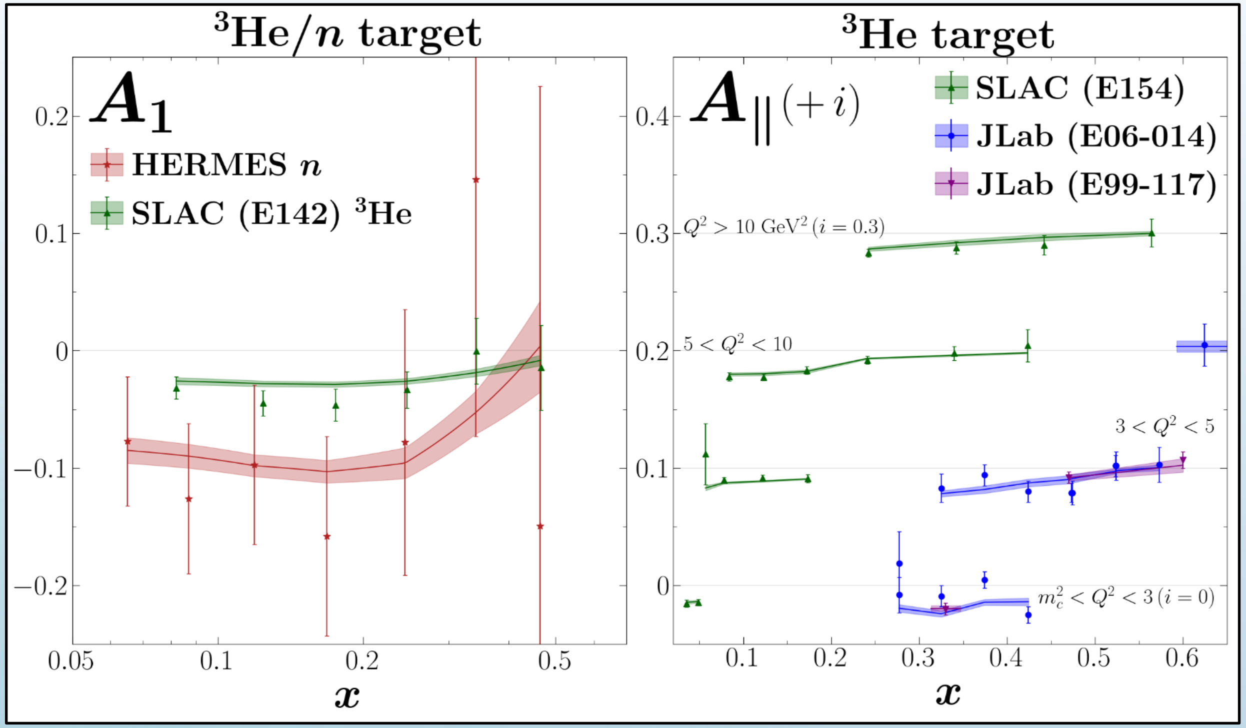
Data vs. Theory (DIS) (deuteron)

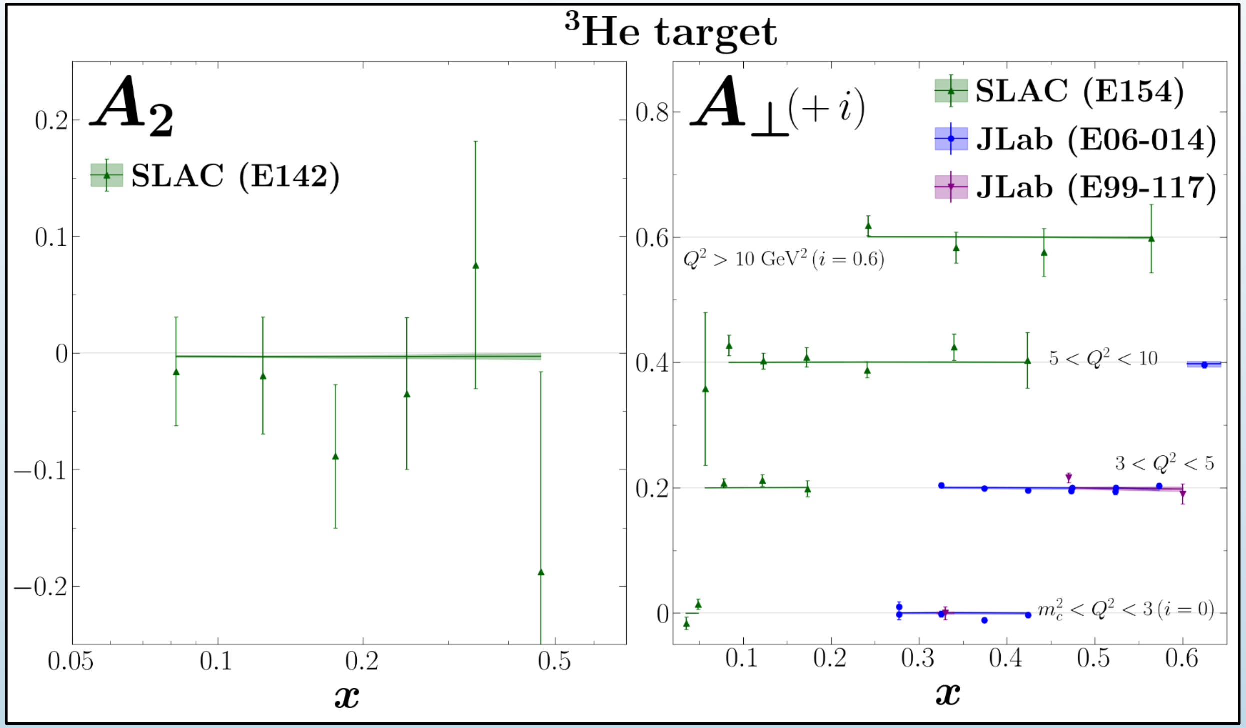




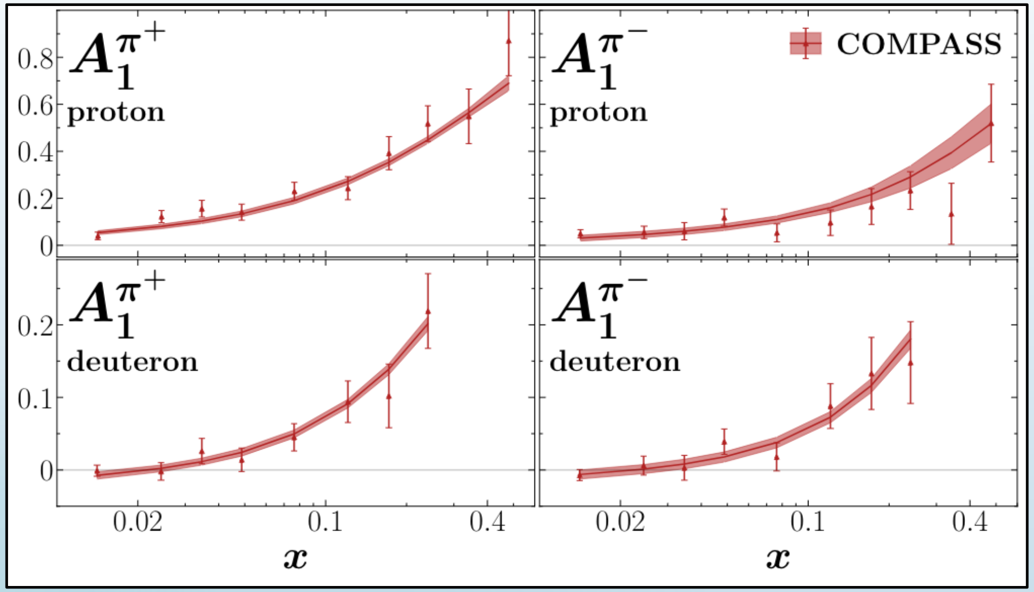


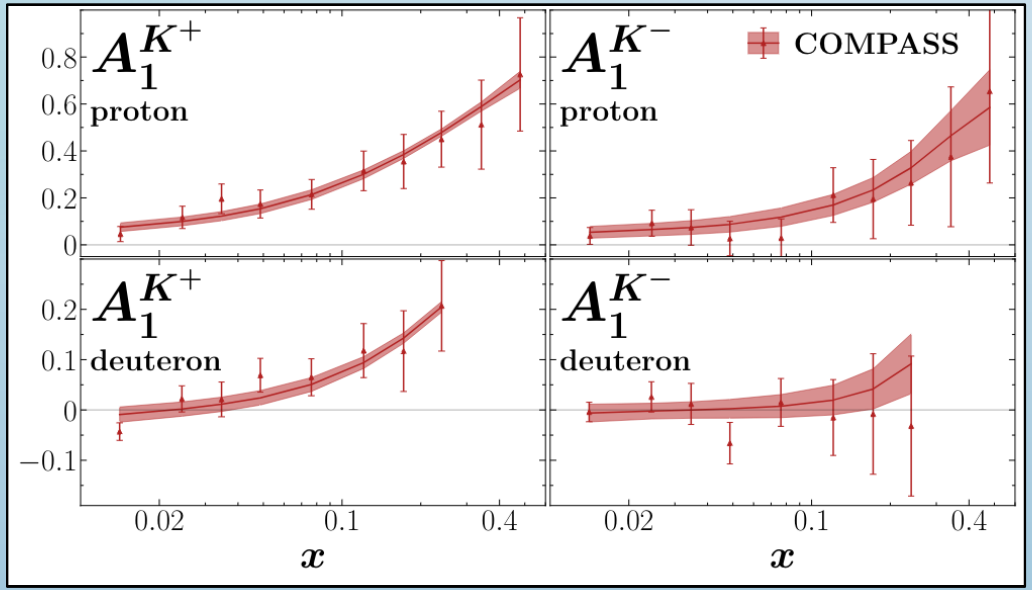
Data vs. Theory (DIS) (helium/neutron)

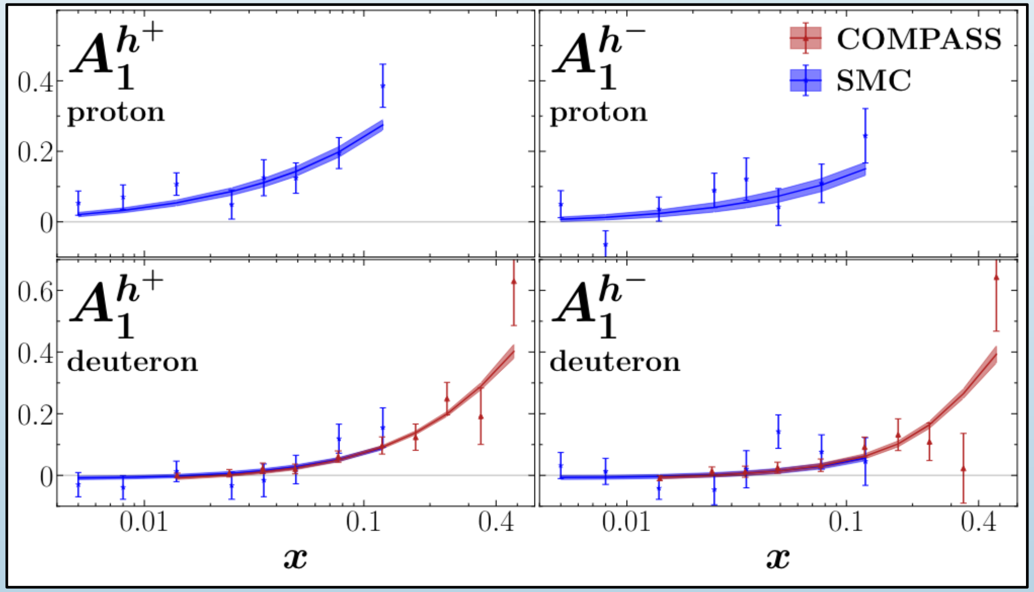




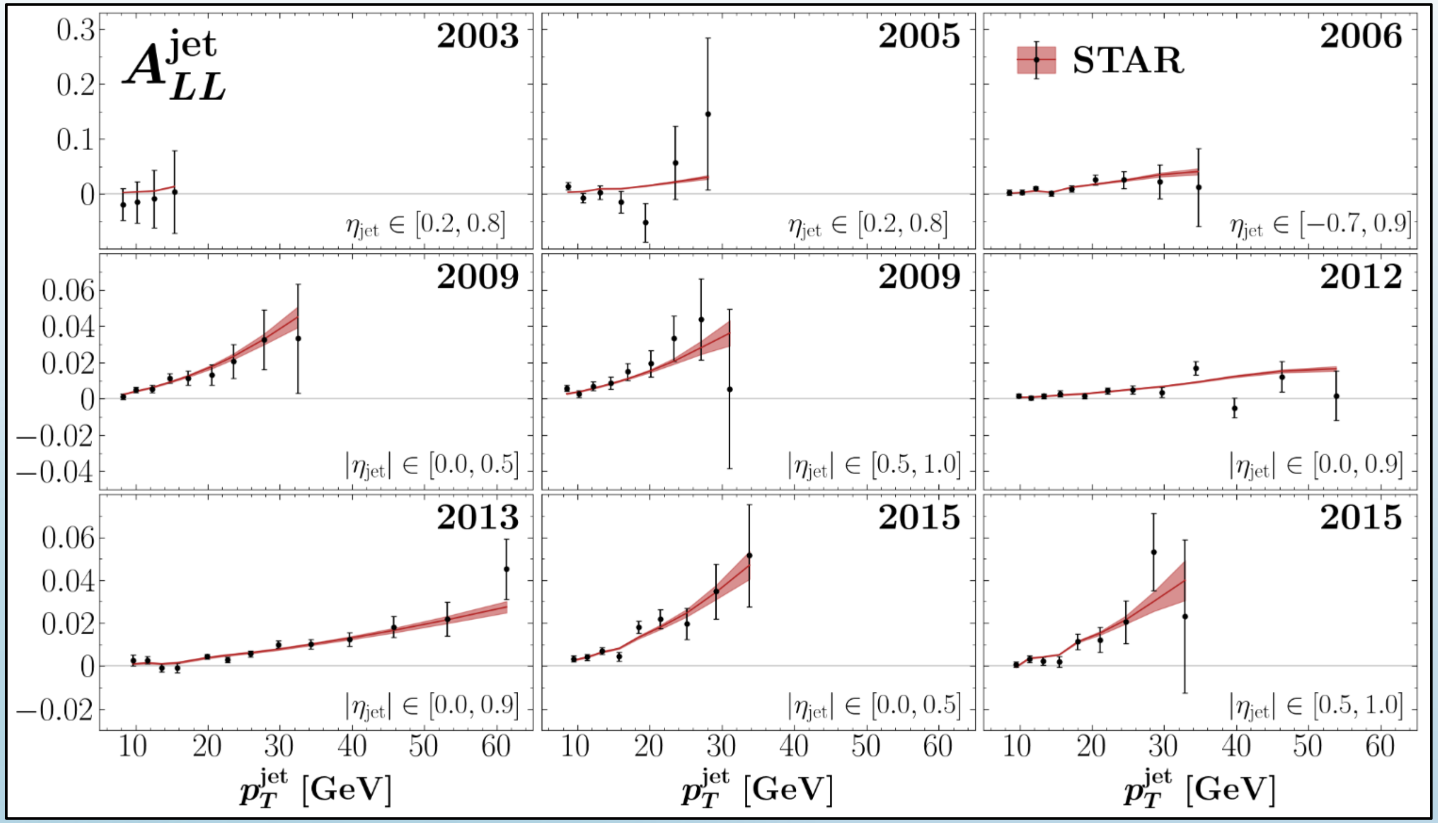
Data vs. Theory (SIDIS)

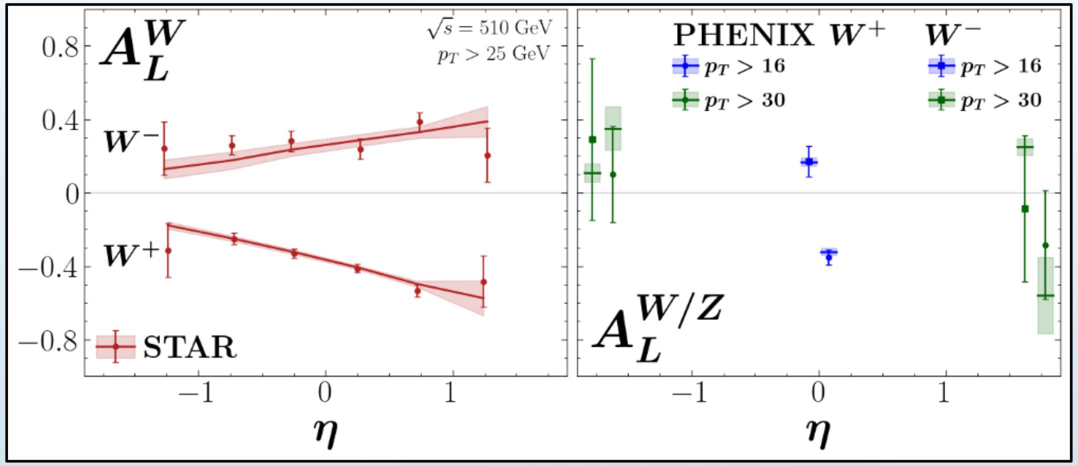




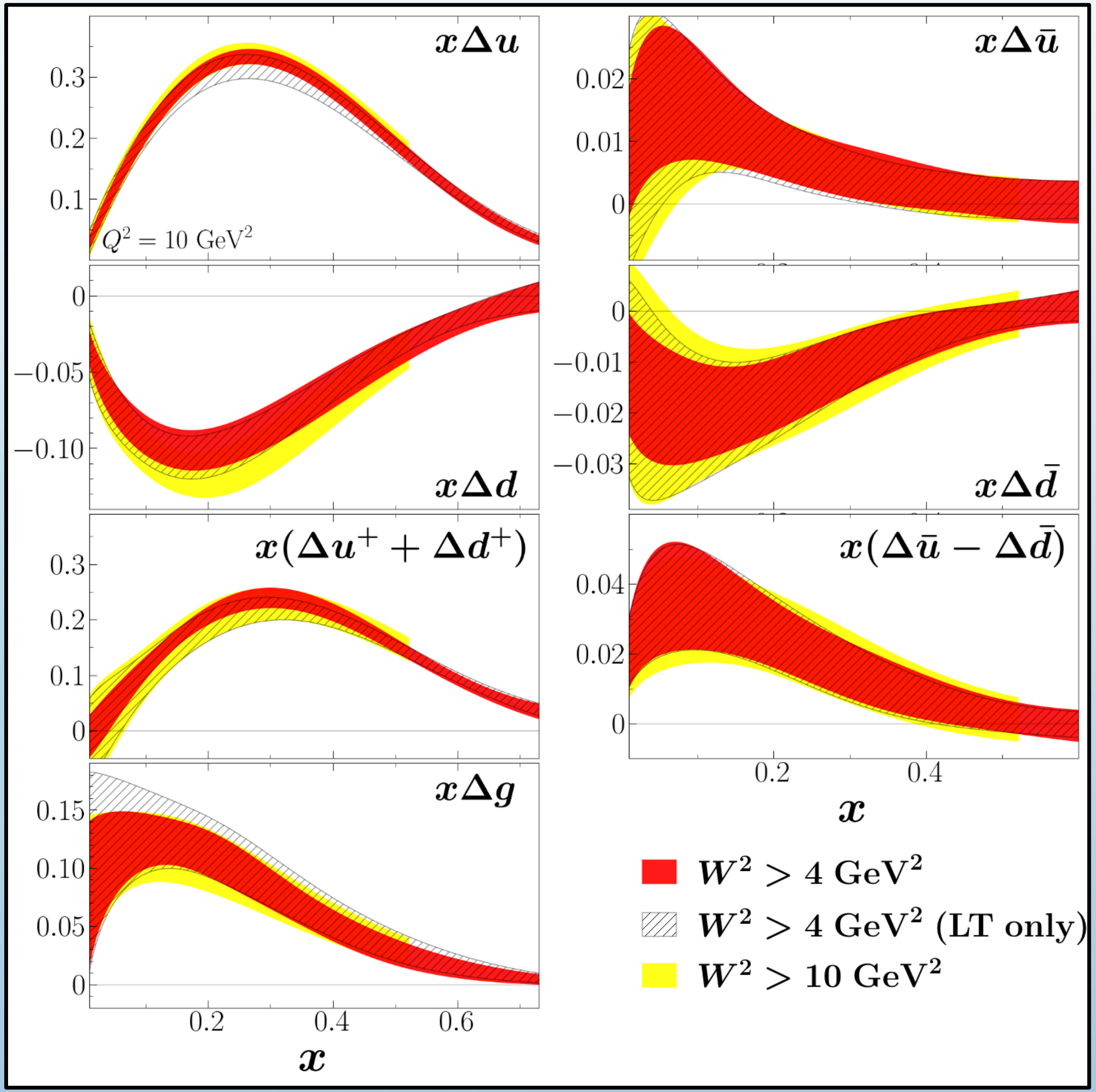


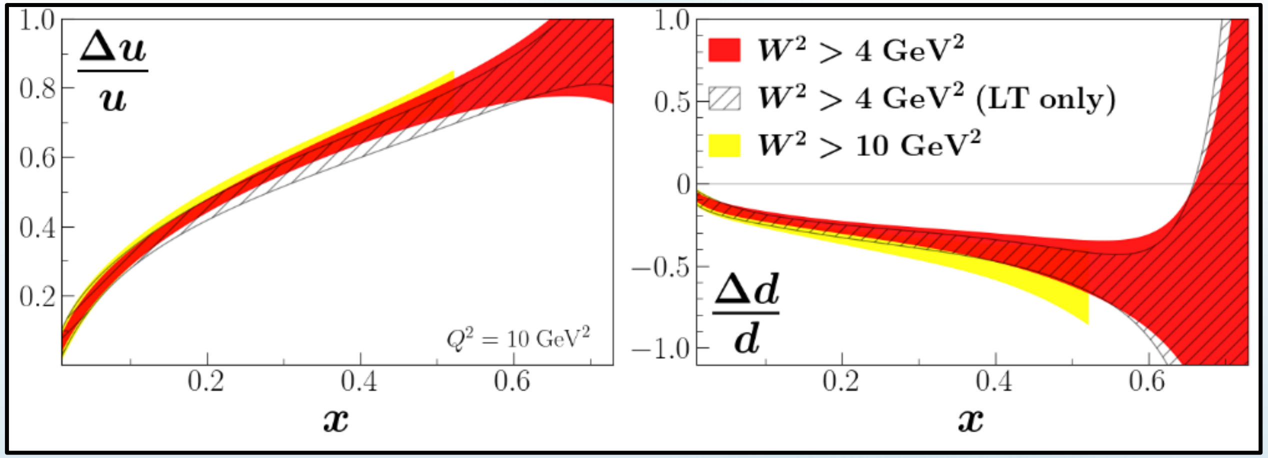
Data vs. Theory (*pp*)





Helicity PDFs





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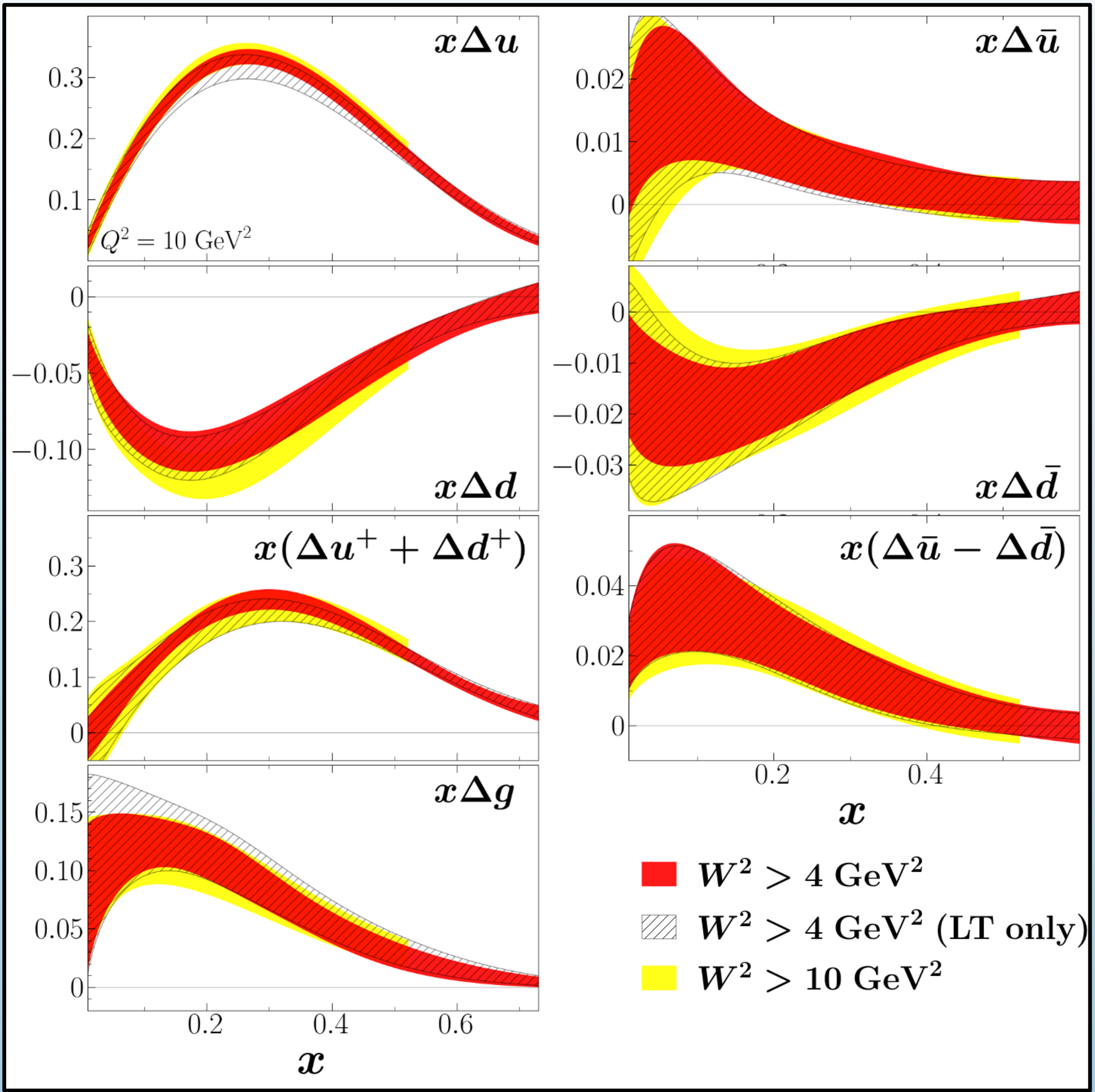

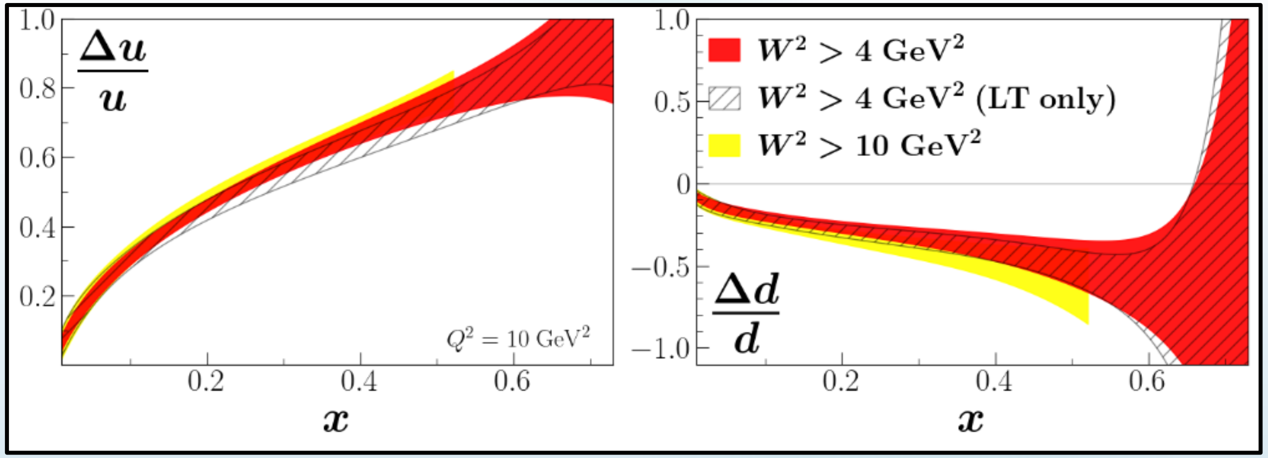

Sizable reduction in uncertainties going from $W^2 > 10 \rightarrow W^2 > 4$ GeV^2, particularly on valence distributions and moments

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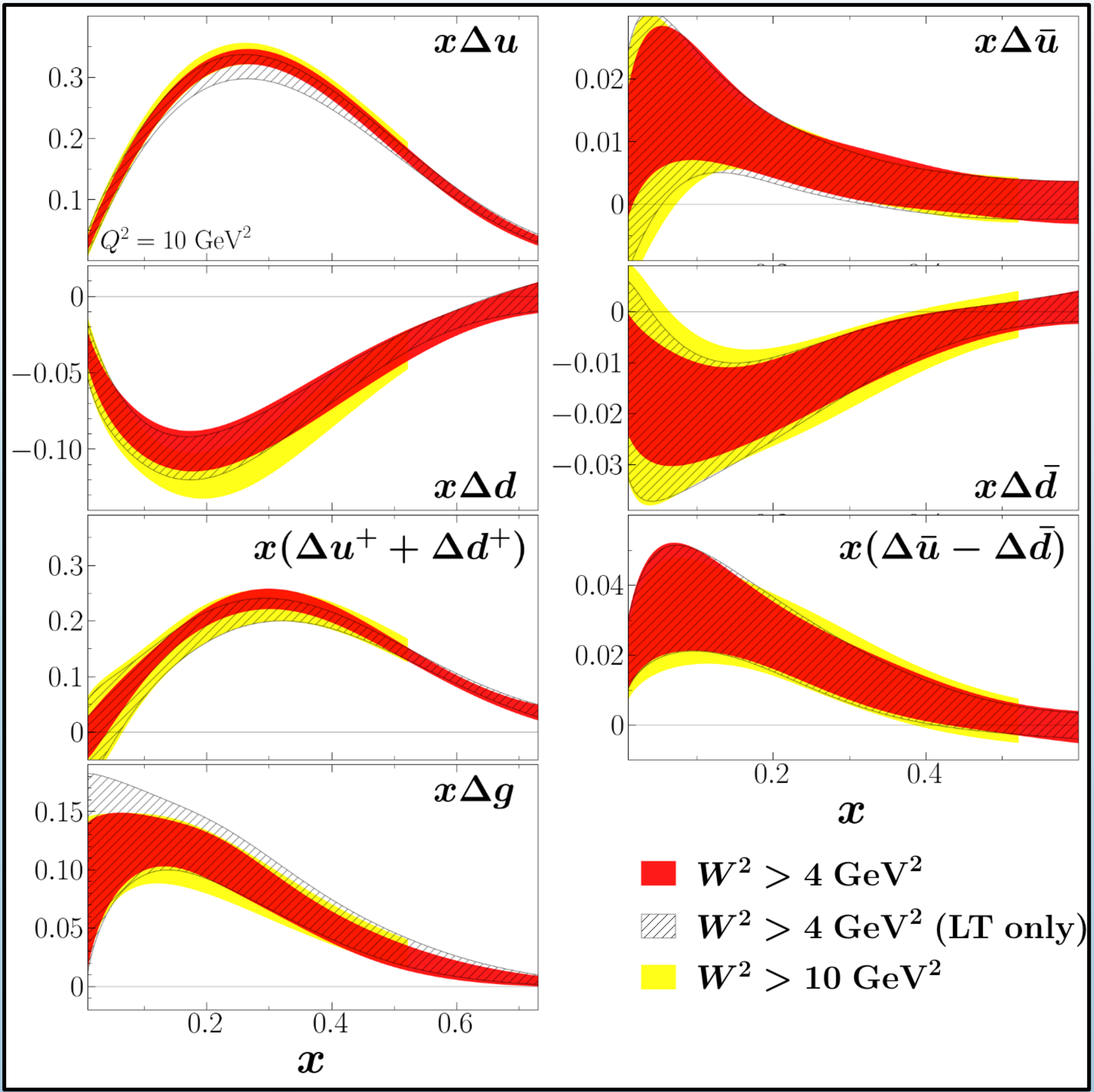
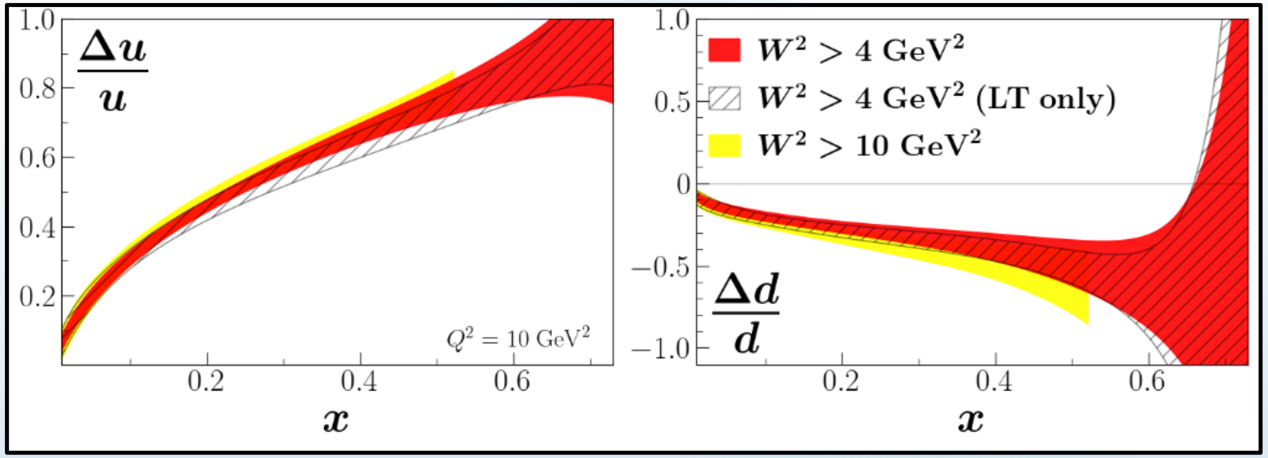
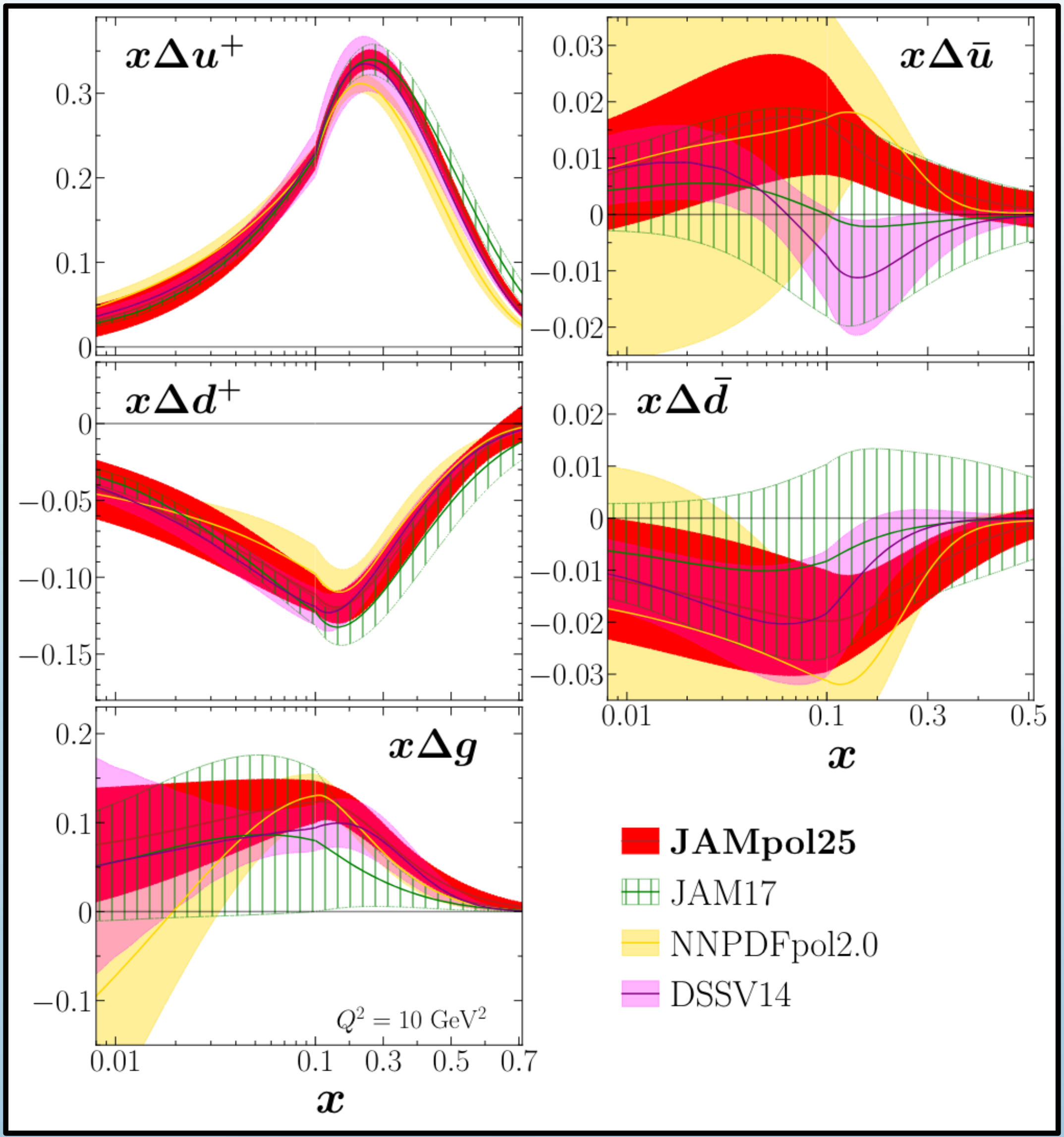




Sizable reduction in uncertainties going from $W^2 > 10 \rightarrow W^2 > 4$ GeV^2, particularly on valence distributions and moments

	$[x_{\min}, x_{\max}; W^2_{\min}$ (GeV^2)]		
Truncated Moment	[0.005, 0.52; 10]	[0.005, 0.52; 4]	**[0.005, 0.76; 4]**
Δu^+	0.75(12)	0.75(6)	**0.78(6)**
Δd^+	−0.39(12)	−0.36(5)	**−0.37(5)**
Δg	0.44(15)	0.44(14)	**0.44(14)**
$\Delta u^+ + \Delta d^+$	0.41(37)	0.42(17)	**0.42(17)**
$\Delta u^+ - \Delta d^+$	1.14(3)	1.11(4)	**1.15(4)**

Helicity PDFs



JAM17: Included SIDIS and FFs in fit, but not high-x data

Δu^+ and Δd^+ have smaller uncertainties at $x > 0.1$ due to low W^2, high-x data.

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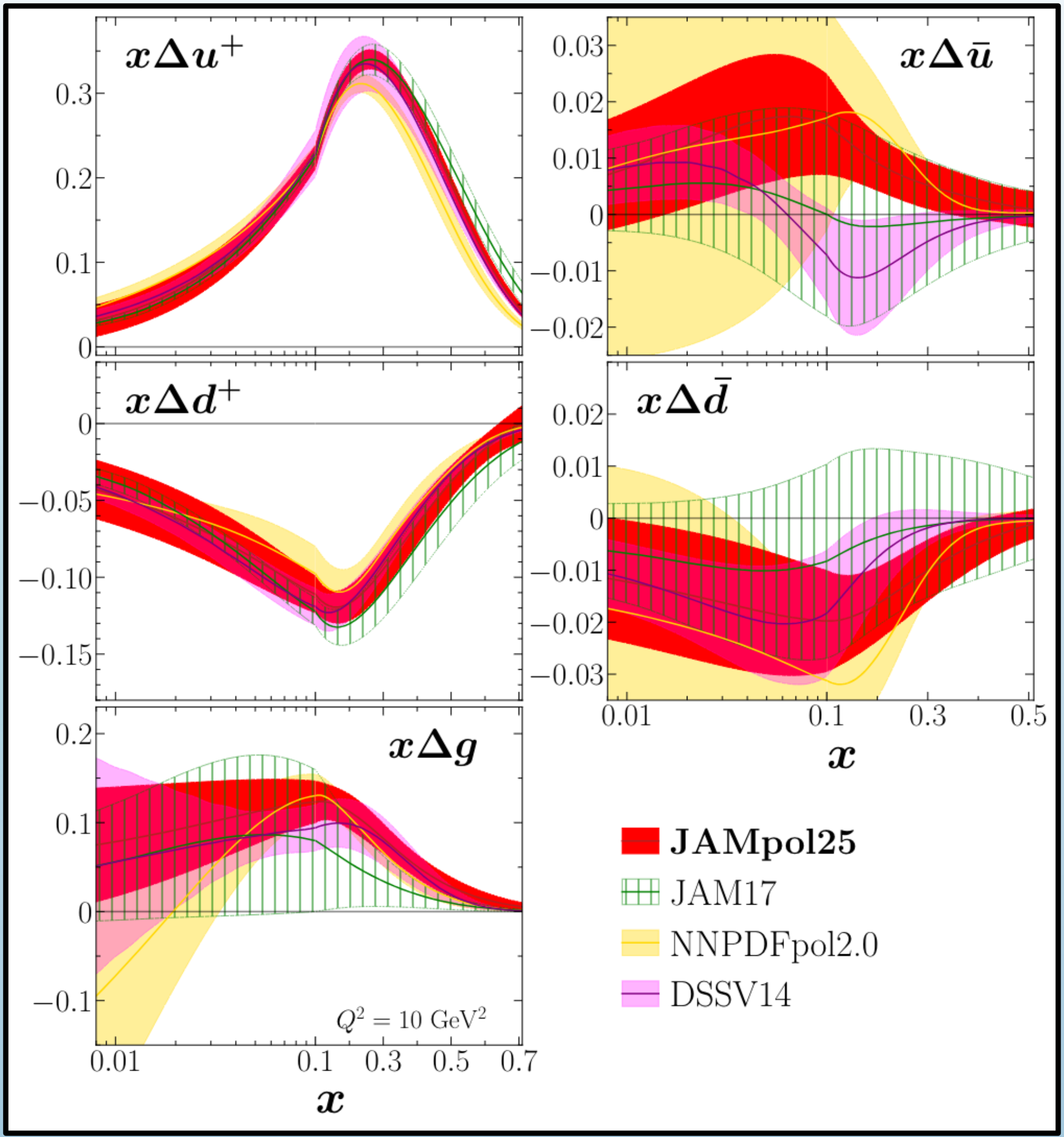

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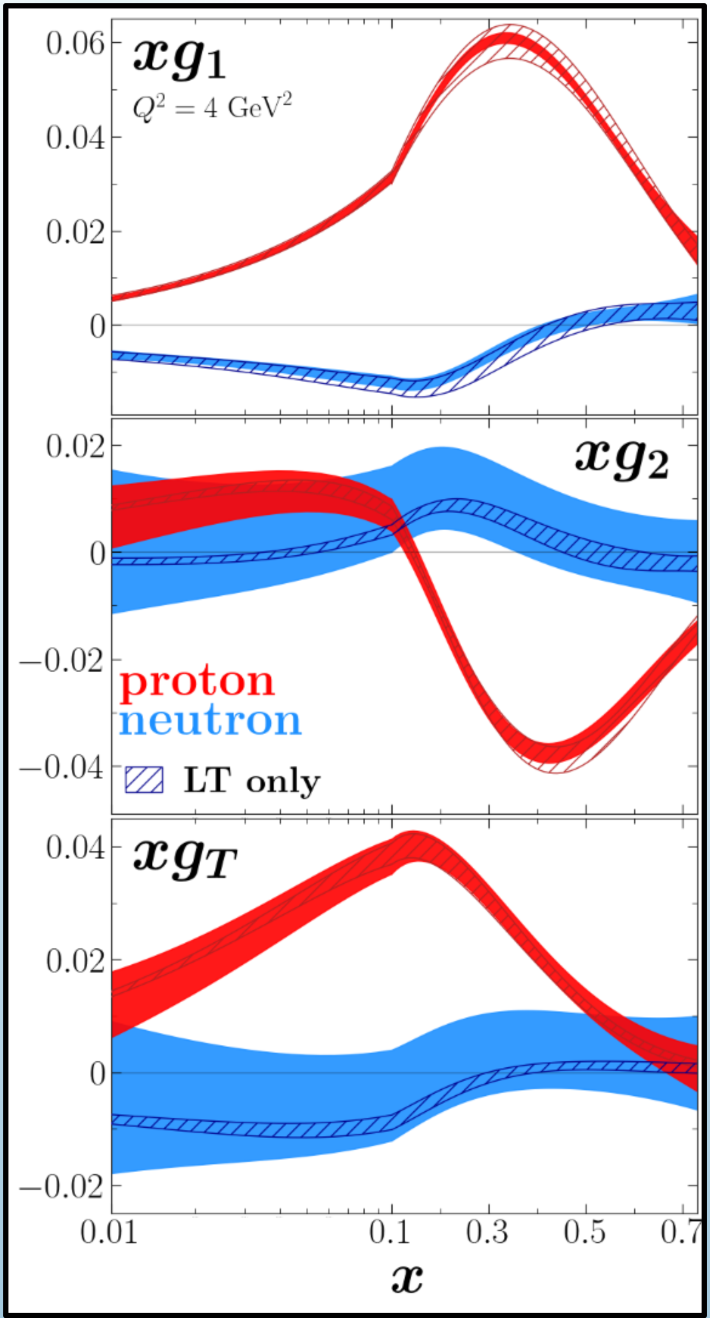
Small errors on Δg, due to jet and LQCD data

Structure Functions

g_1^p, g_1^n and g_2^p stable with regard to HTs; sufficient data to constrain them

$$g_T = g_1 + g_2$$



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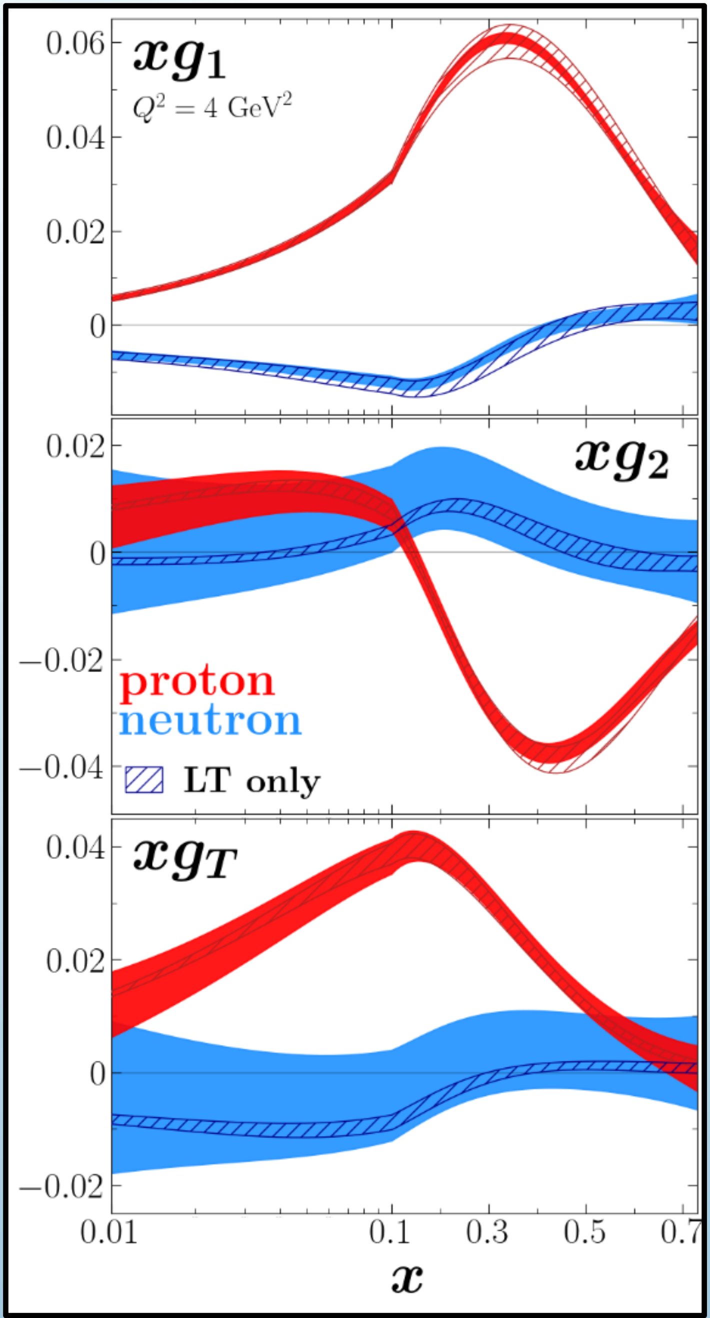


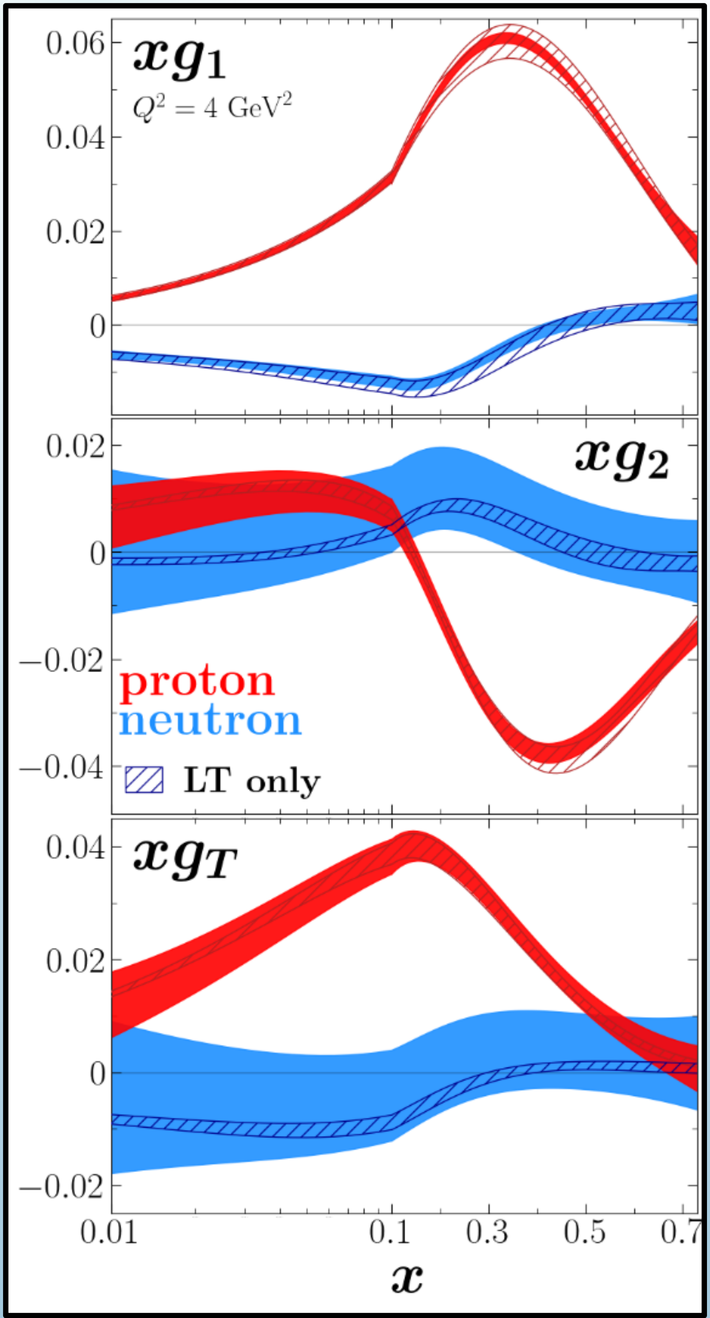
Structure Functions

g_1^p, g_1^n and g_2^p stable with regard to HTs; sufficient data to constrain them

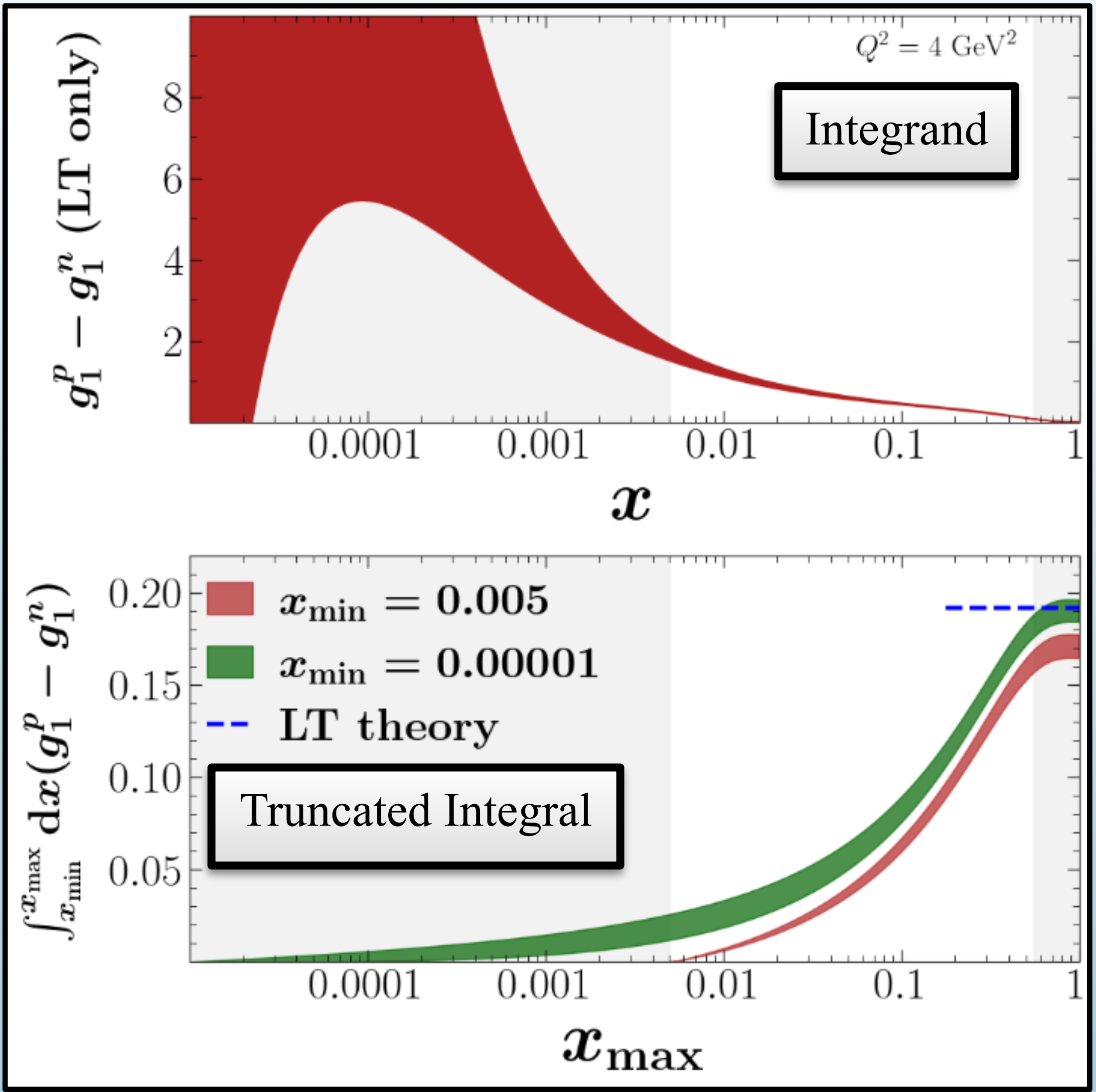
For g_2^n, errors get much larger due to very sparse $A_\perp$ data for deuteron/helium

All HTs are consistent with zero, but give a more accurate reflection of the errors.
I.e. for g_T^n, qualitative change in conclusion from nonzero to consistent with zero

$$g_T = g_1 + g_2$$



Bjorken Sum Rule



Bjorken sum rule at NLO and LT:

$$\int_0^1 dx[g_1^p(x,Q^2) - g_1^n(x,Q^2)] = \frac{1}{6} g_a(1 - \alpha_s/\pi)$$

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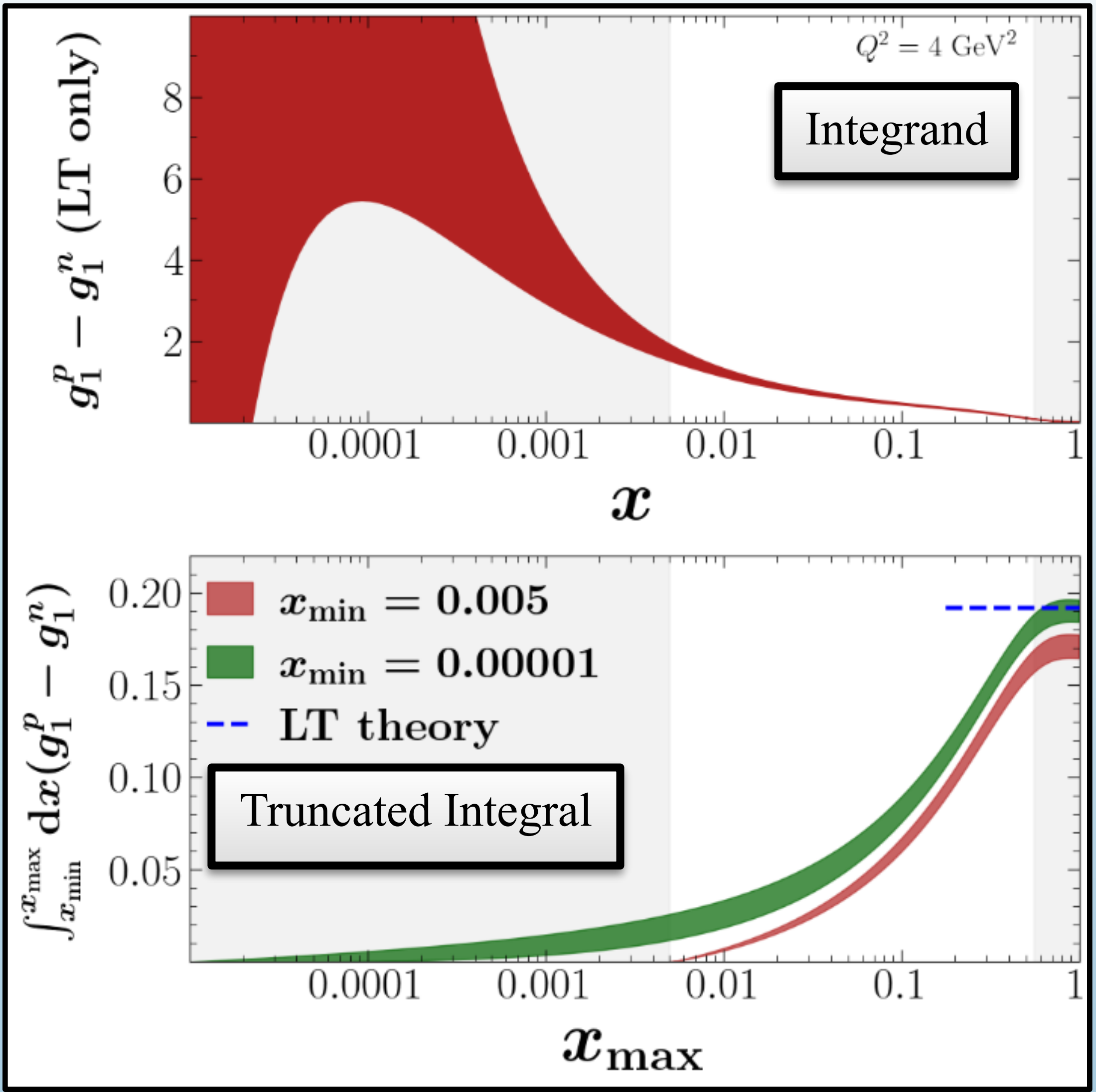

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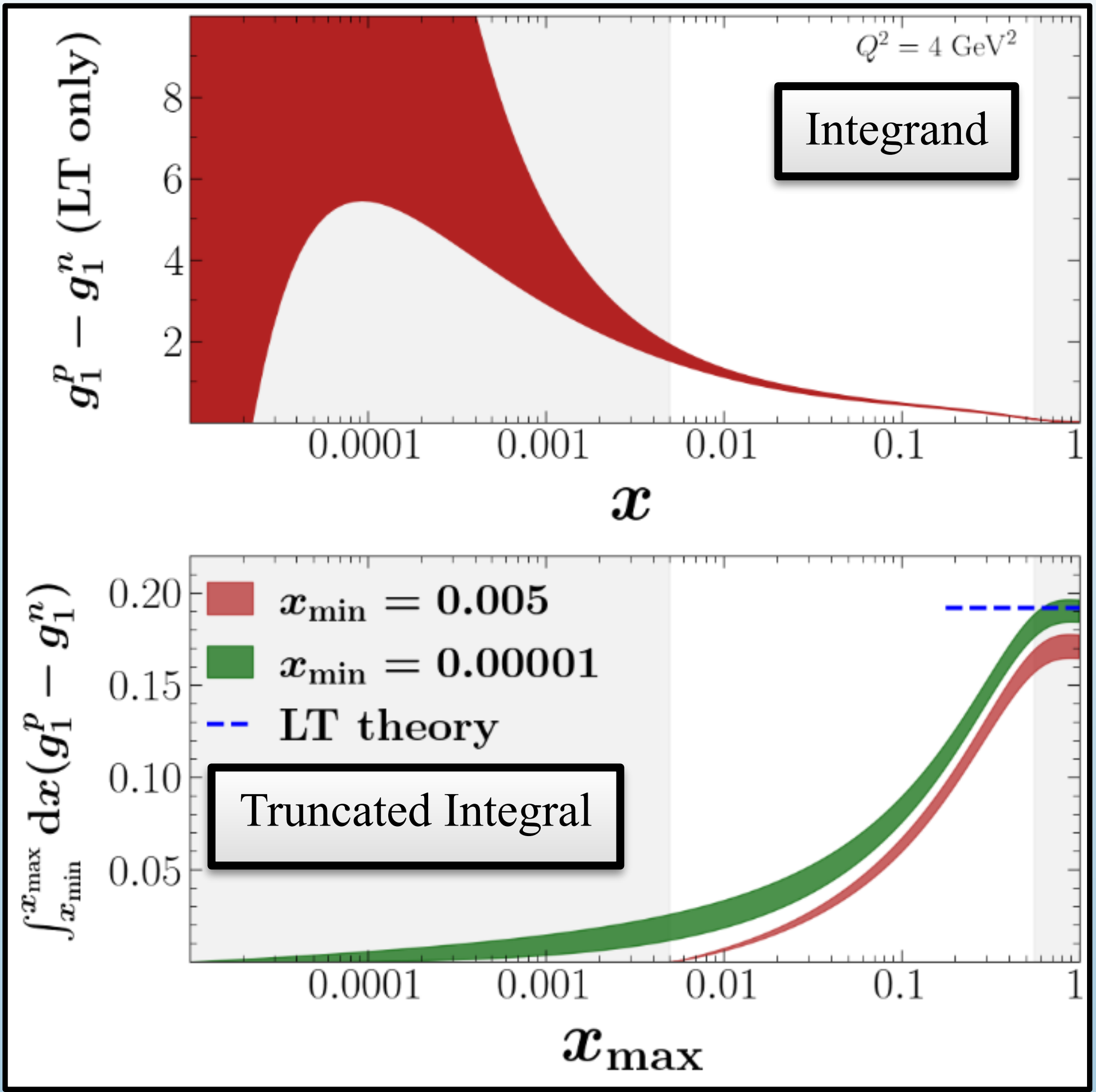

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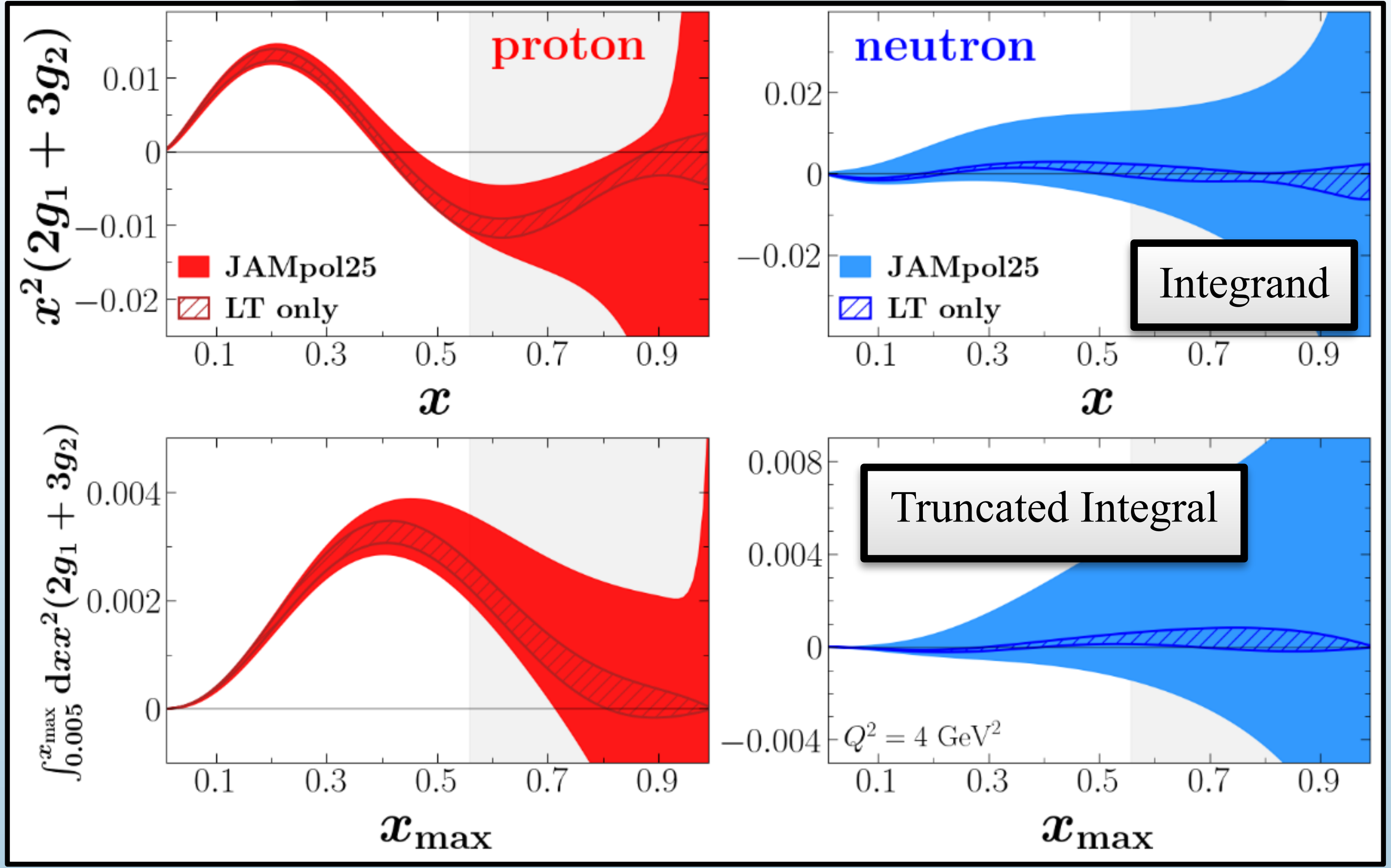
Integrand begins to explode in unmeasured low-x region (shaded area)

$x_{\rm min} = 0.005$ result slightly undershoots theory. $x_{\rm min} = 0.00001$ result matches theory very well.

d_2 Matrix Element

$$d_2(Q^2) = \int_0^1 dx x^2 \left[2g_1(x,Q^2) + 3g_2(x,Q^2)\right]$$

Factor of x^2 means contribution below 0.005 is negligible



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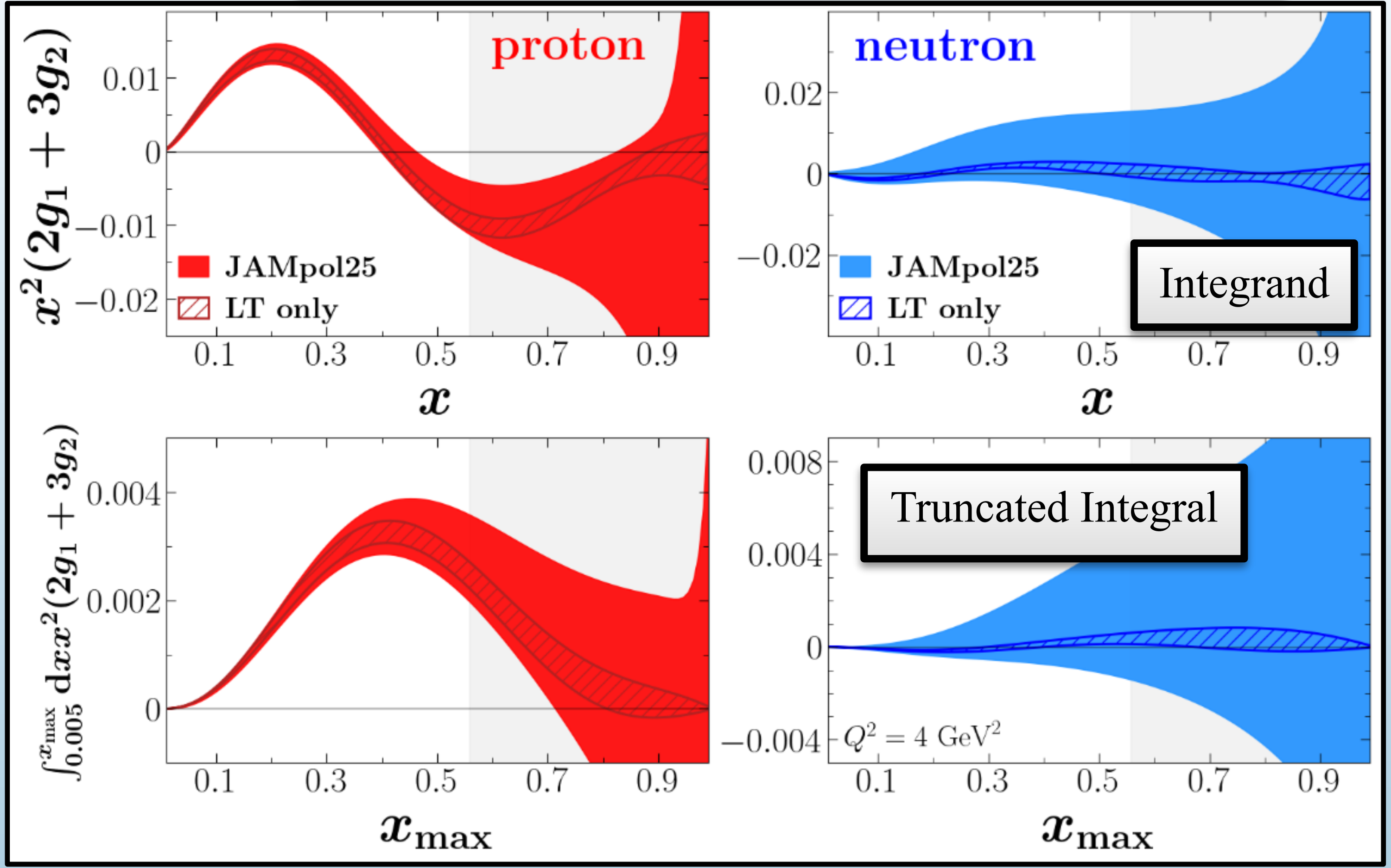

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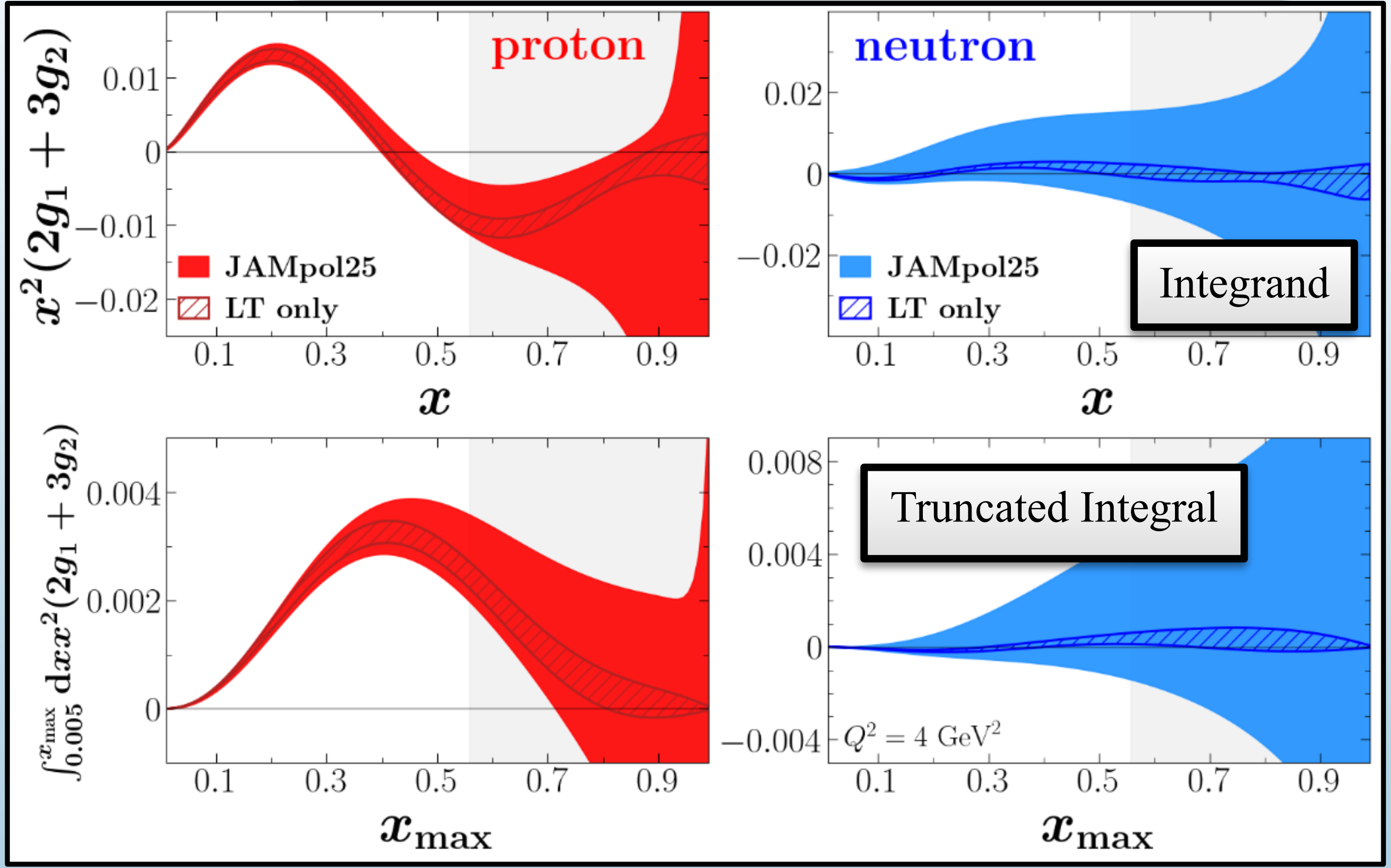

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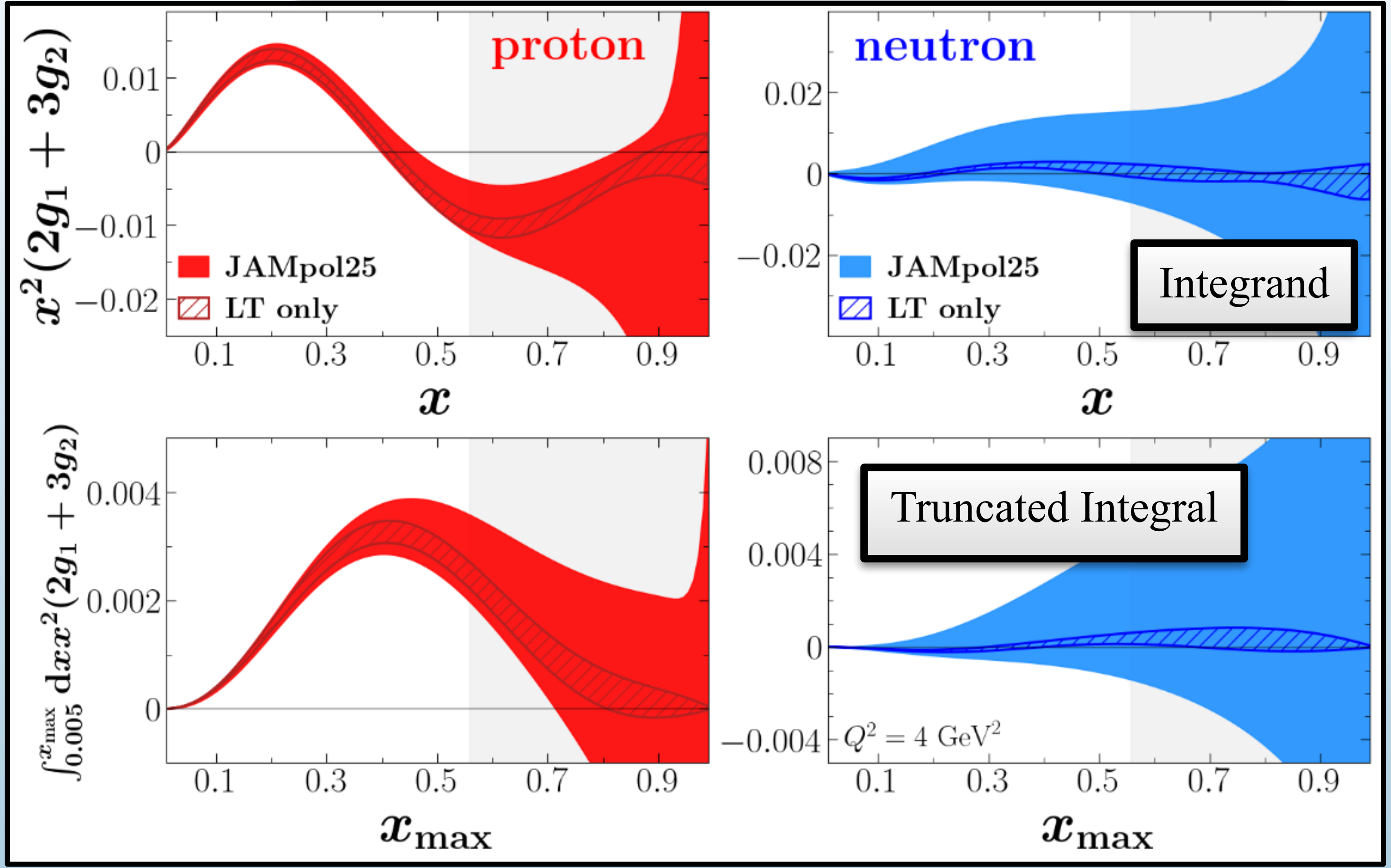
With HTs, errors get larger at large x

Neutron is always consistent with zero. Integrand for proton is non-zero up to $x \approx 0.8$



Conclusions

Simultaneous extraction of PDFs and FFs, with low $W^2 > 4\ \mathrm{GeV}^2$ cut, with TMCs and HTs

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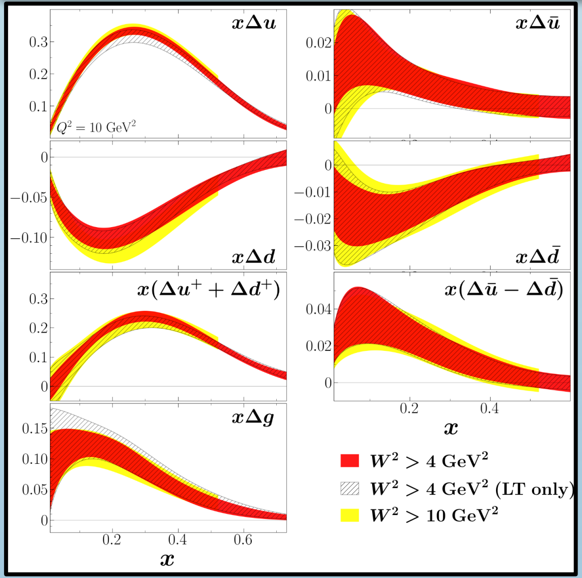

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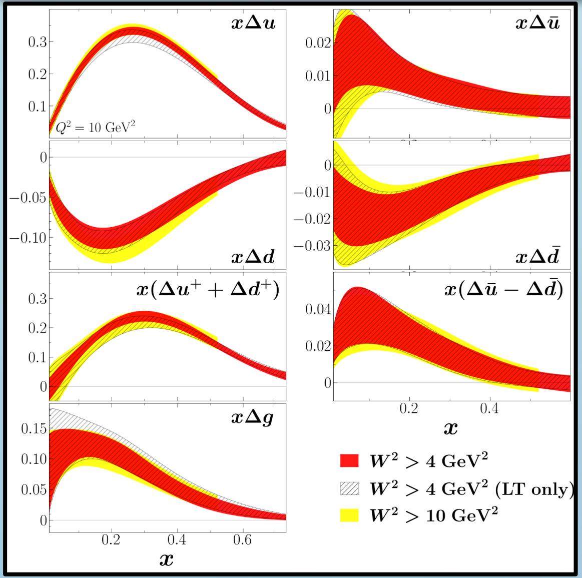
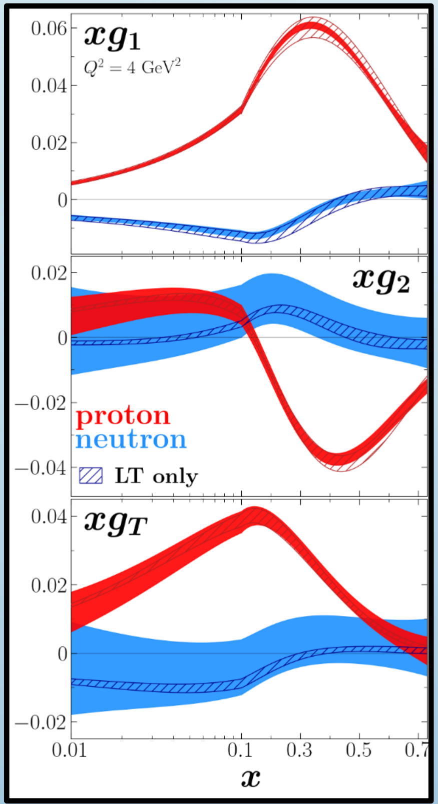


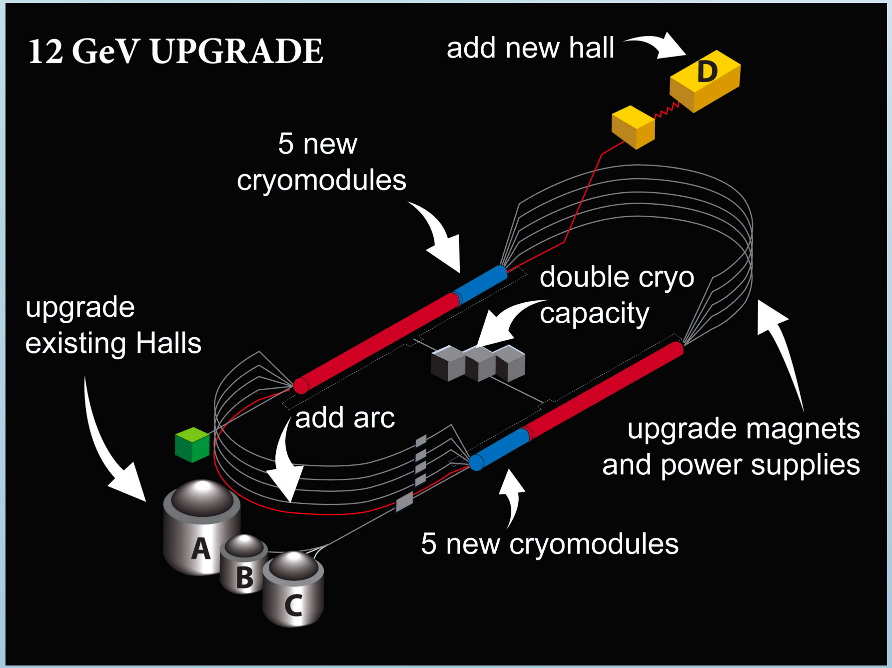
No signal found for HT effects, but they are necessary to get accurate error estimates, especially on g_2^n

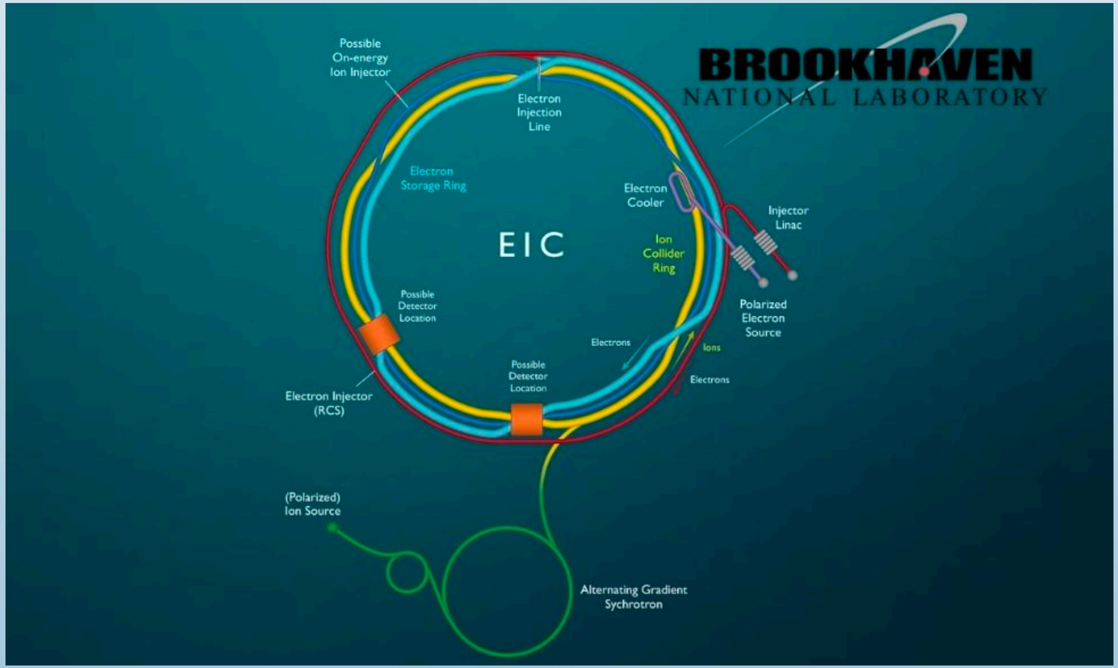


Outlook

JLab 12 GeV data, particularly transversely polarized asymmetries on helium, will help further constrain the d_2 matrix element

EIC can provide constraints at small x, and reduce uncertainties on the gluon and singlet distributions in the extrapolation region





Collaboration

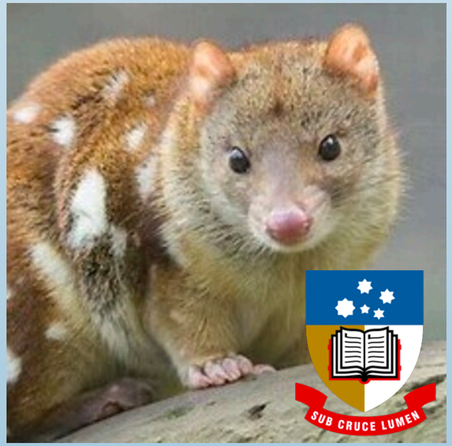
Christina Cocuzza

Nobuo Sato

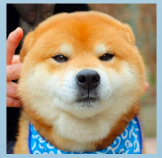
Nick Hunt-Smith

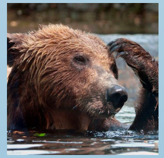
Wally Melnitchouk

Anthony Thomas



Thank you to Yiyu Zhou and
Patrick Barry for helpful discussions





Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

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Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

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Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M \, \tilde{f}_j(N,\mu_0) \tilde{f}_l(M,\mu_0) \otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0) \right]$$

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

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Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

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Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

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Data

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Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

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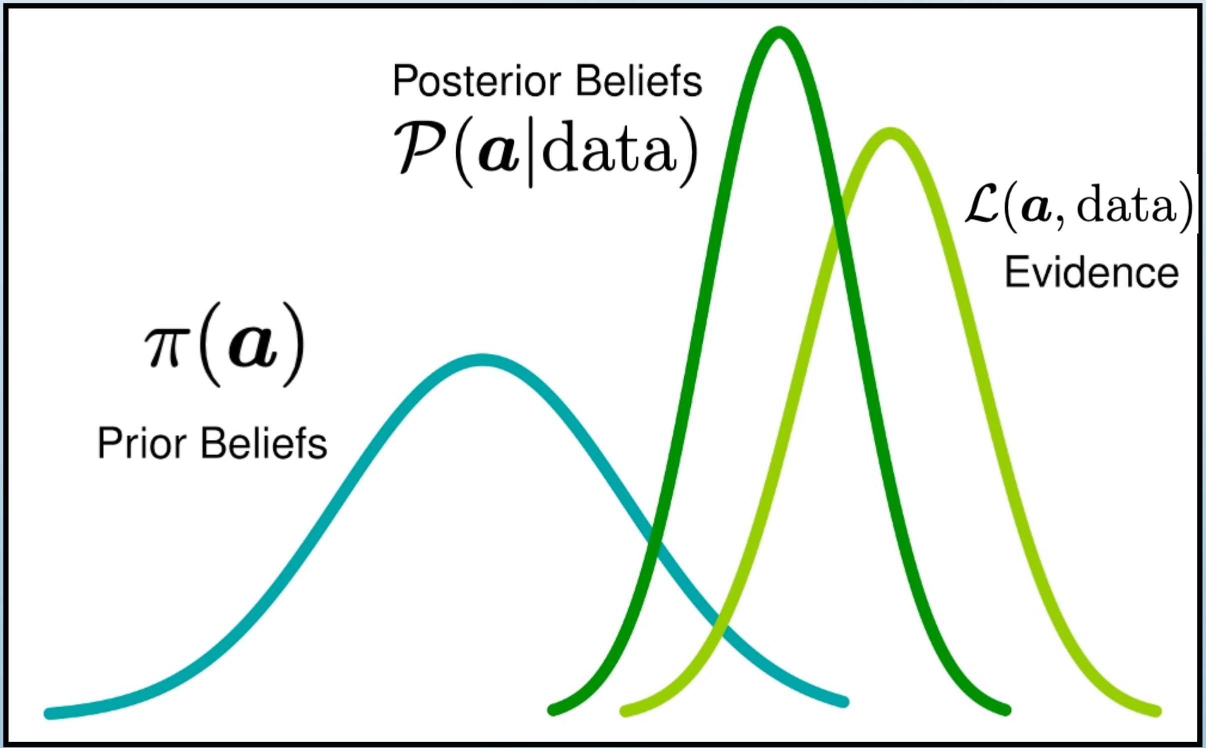
Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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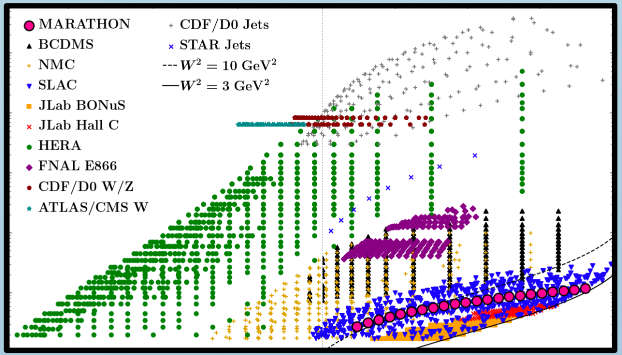
Data

Original Data



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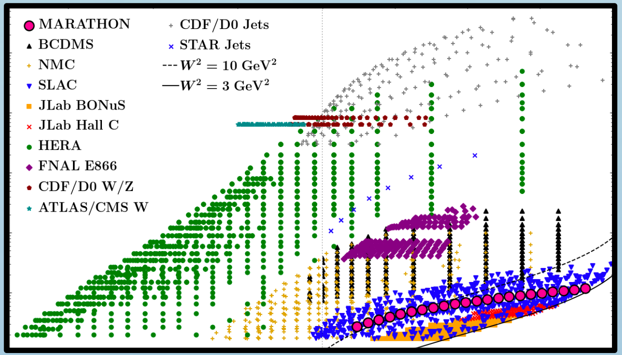


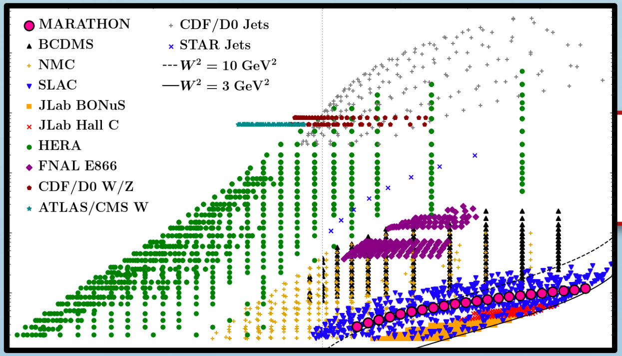
$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Pseudo-Data → $\tilde{\sigma}$

Data → σ

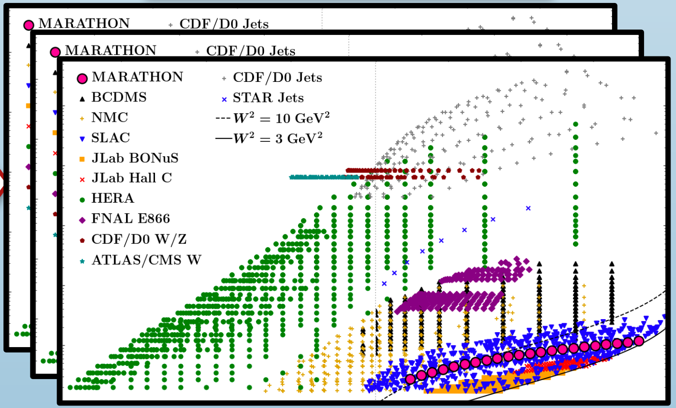
Uncorrelated Uncertainties → α

Original Data



DR

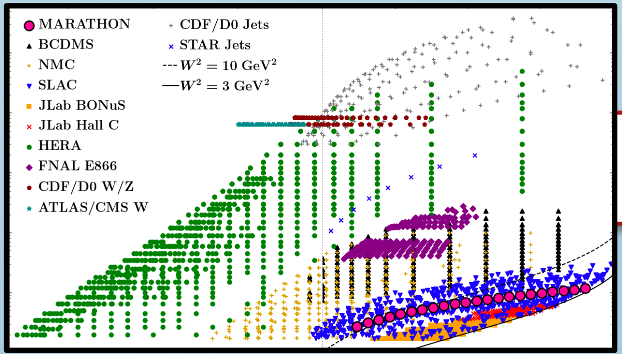
Replica Data



Pseudo-Data → $\tilde{\sigma} = \sigma + N(0,1)\ \alpha$ ← Uncorrelated Uncertainties

↑ Data

Original Data



DR →

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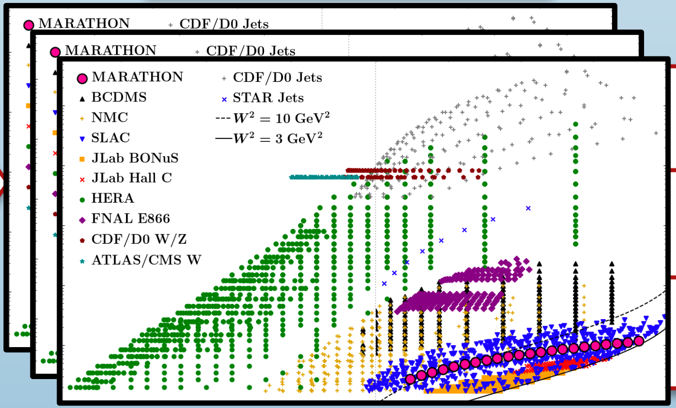
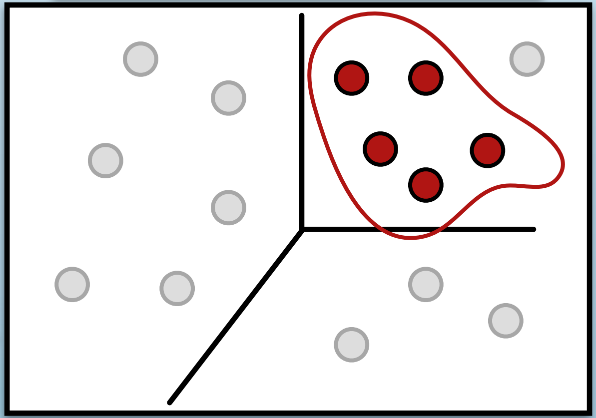


Maximum Likelihood →

Maximum Likelihood →

Maximum Likelihood →

Parameter Space



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \ \rho(\boldsymbol{a} \,|\, data) \ O(\boldsymbol{a})$$

$$V[O] = \int d^n a \ \rho(\boldsymbol{a} \,|\, data) \ \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but
$n = \mathcal{O}(100)$!

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$$V[O] = \int d^n a \ \rho(\boldsymbol{a} \,|\, data) \ \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but

$n = \mathcal{O}(100)$!

Build an MC ensemble

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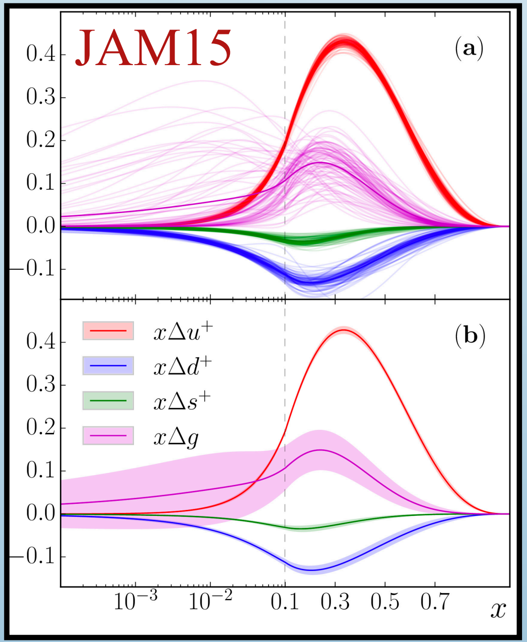

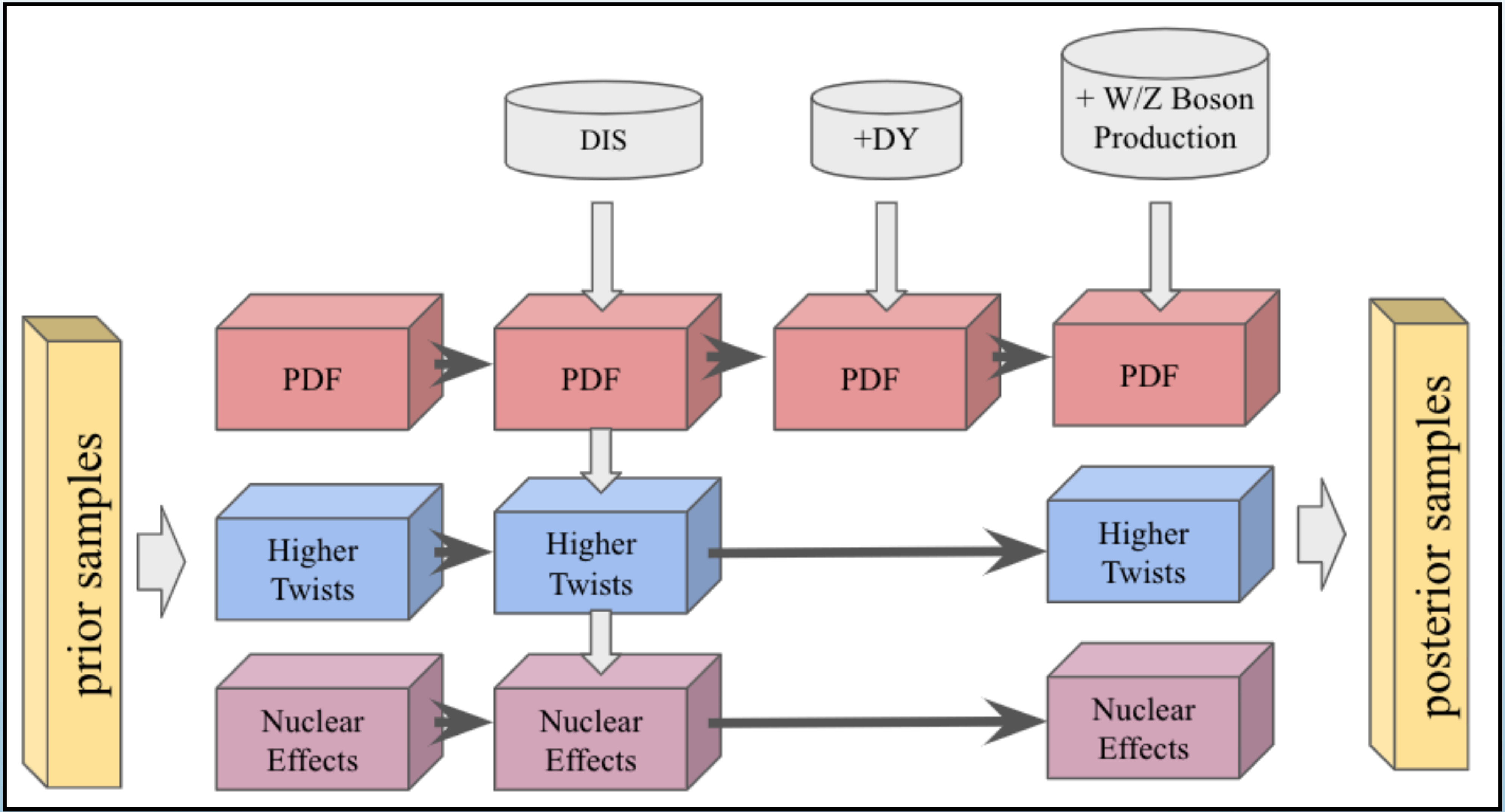
DIS
+DY
+ W/Z Boson Production
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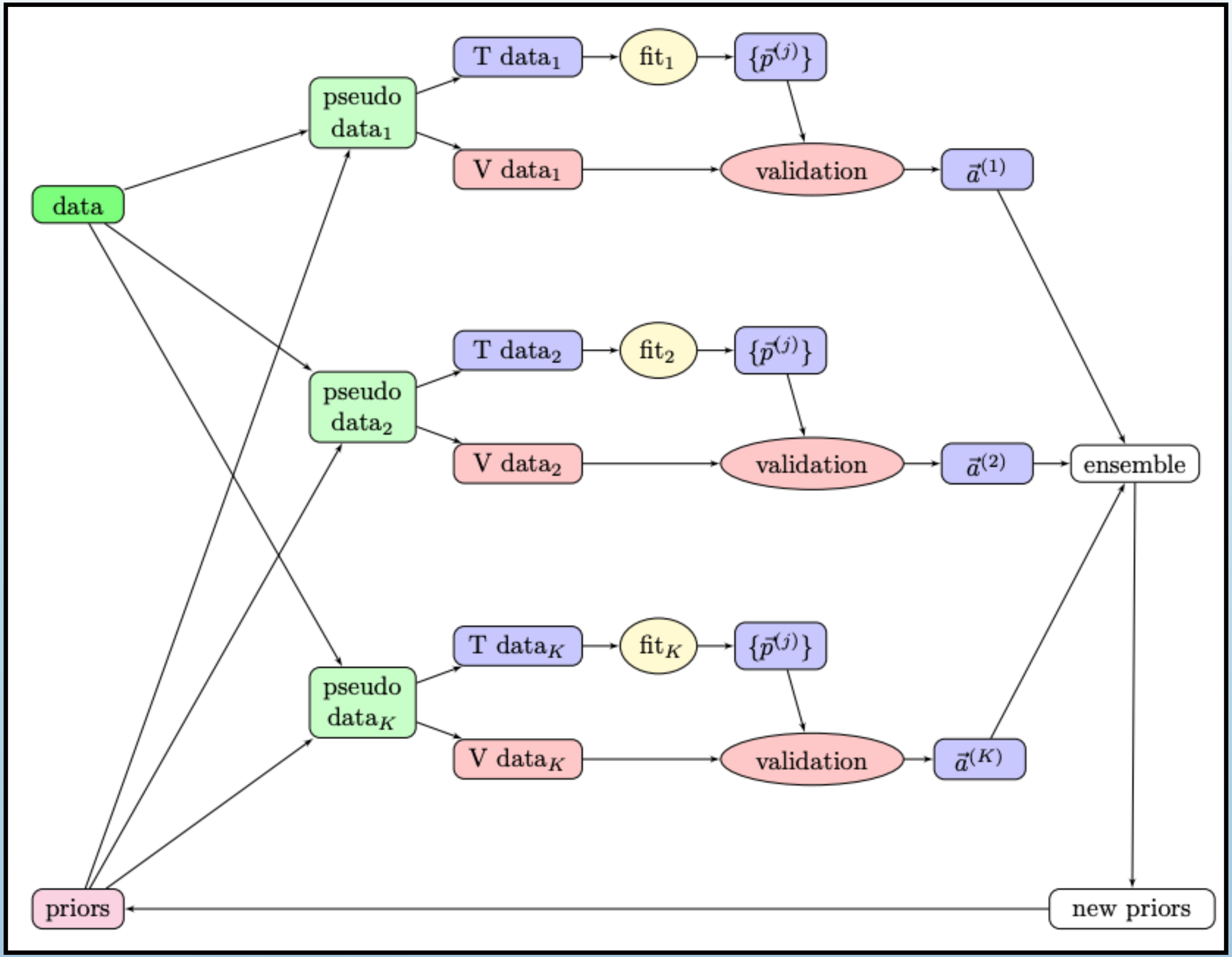
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples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new priors
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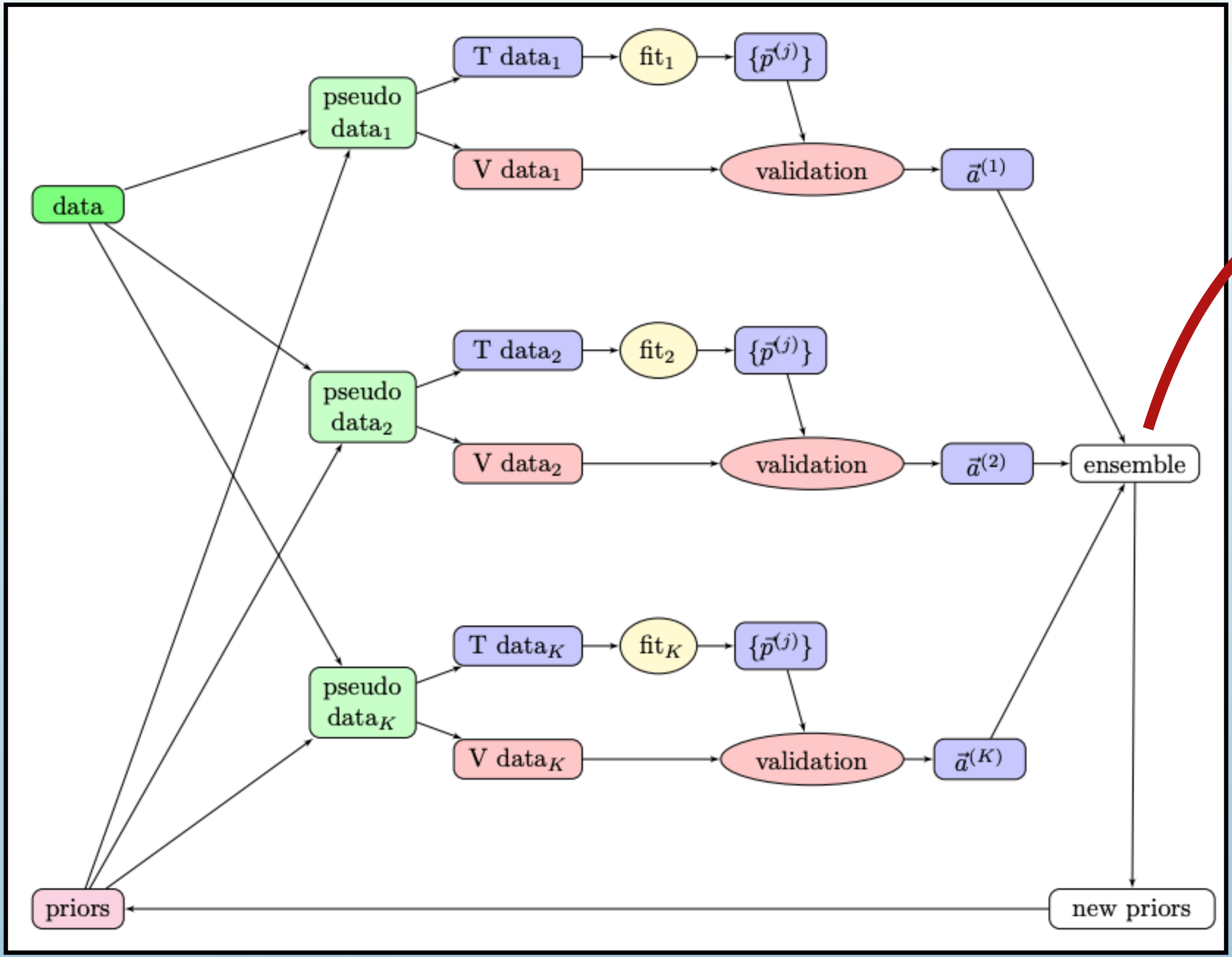
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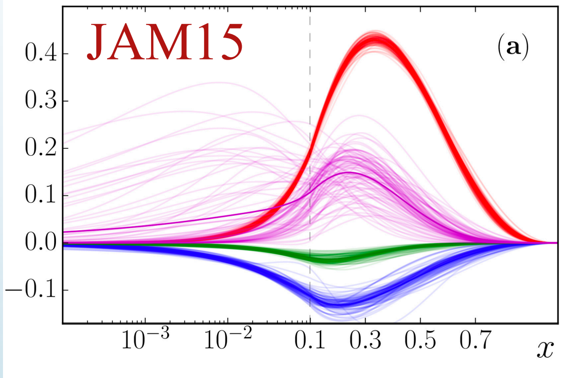
JAM15
(a)
0.4
0.3
0.2
0.1
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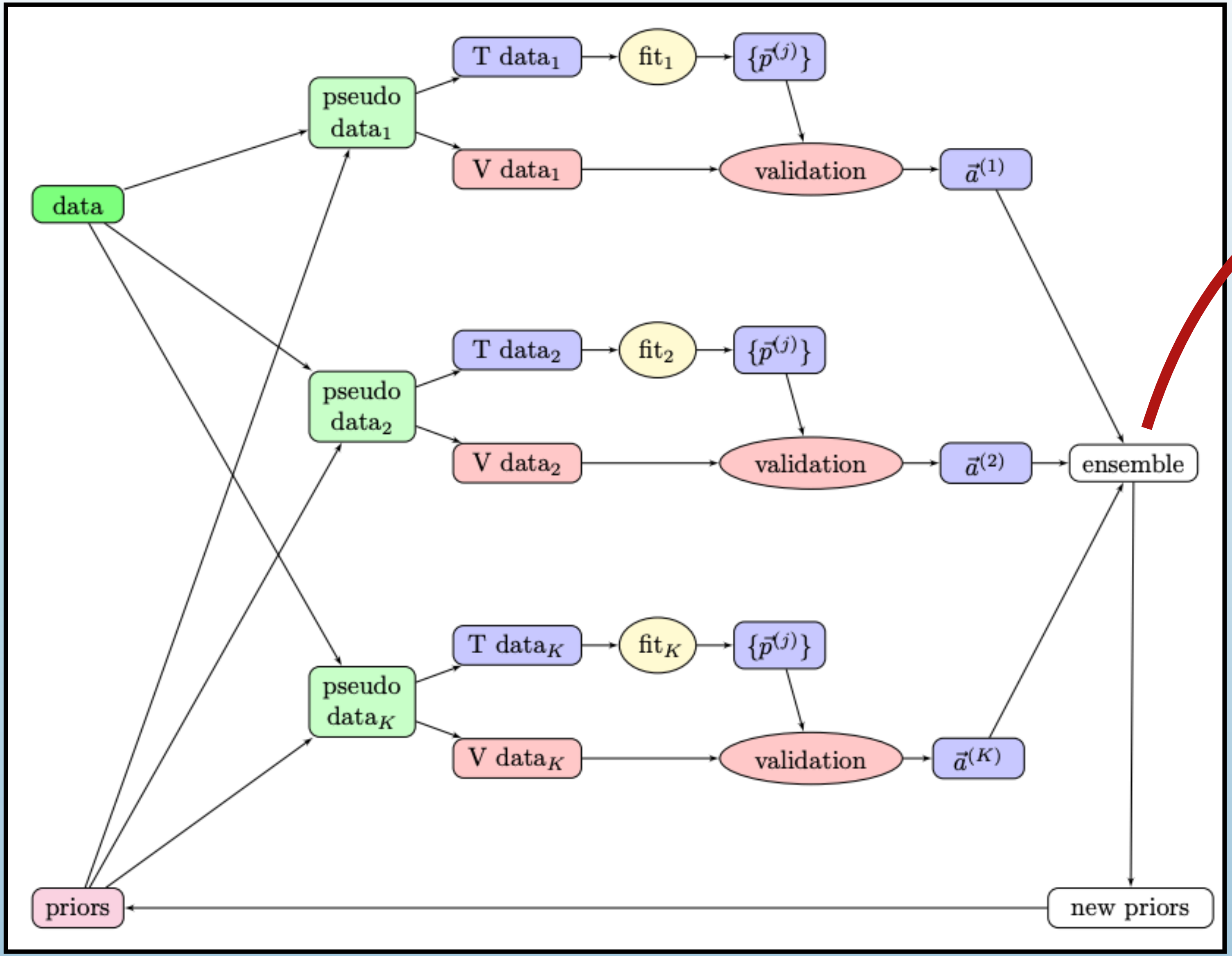
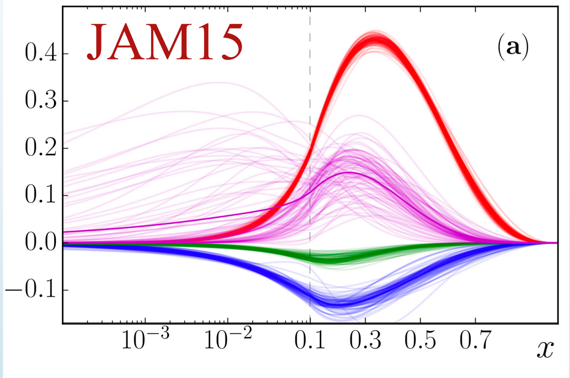
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x





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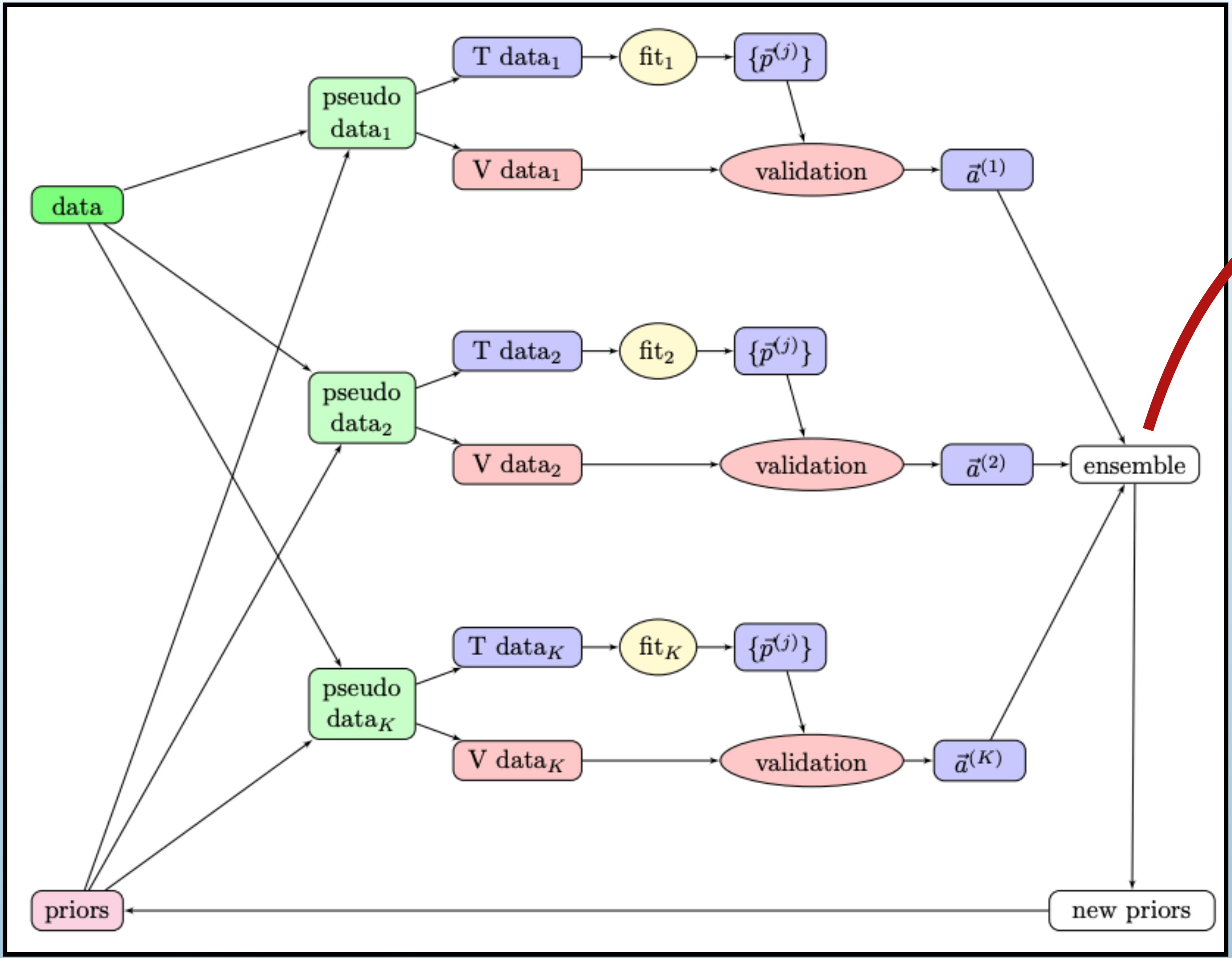

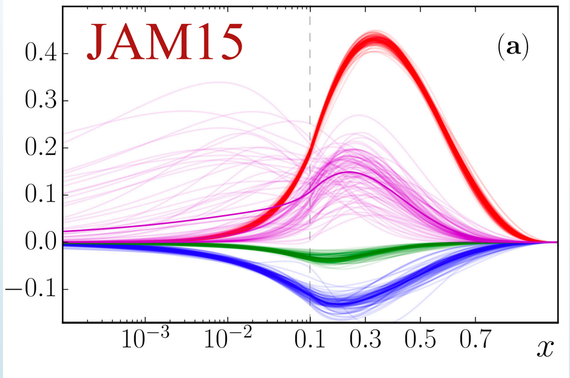

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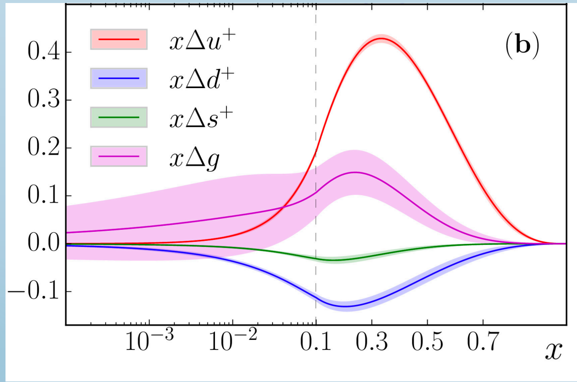


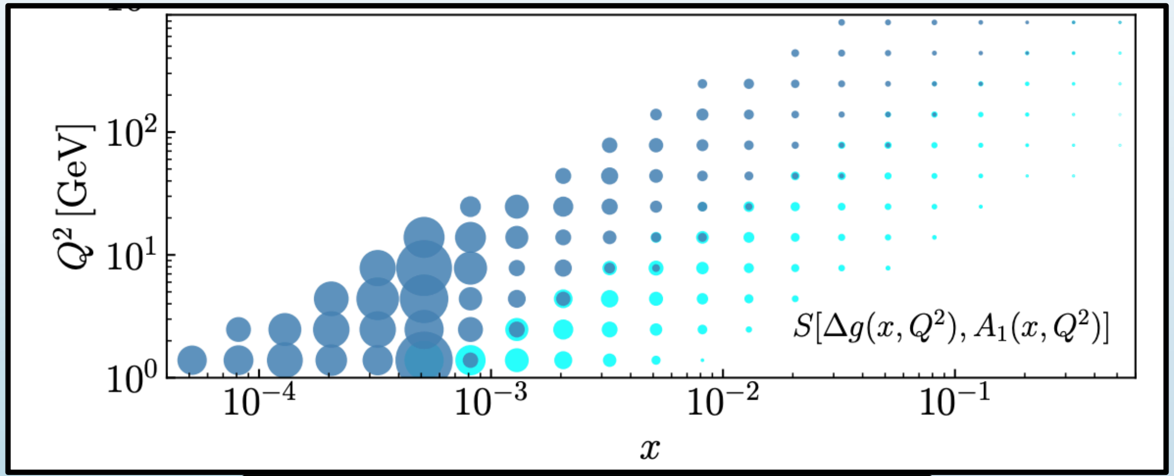
Future Experiments

Revisiting helicity parton distributions at a future electron-ion collider #2

Ignacio Borsa (U. Buenos Aires), Gonzalo Lucero (U. Buenos Aires), Rodolfo Sassot (U. Buenos Aires), Elke C. Aschenauer (Brookhaven Natl. Lab.), Ana S. Nunes (Brookhaven Natl. Lab.) (Jul 16, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 094018 • e-Print: 2007.08300 [hep-ph]

$$\vec{l} + \vec{N} \rightarrow l' + X$$



Sensitivity of A_1 to Δg

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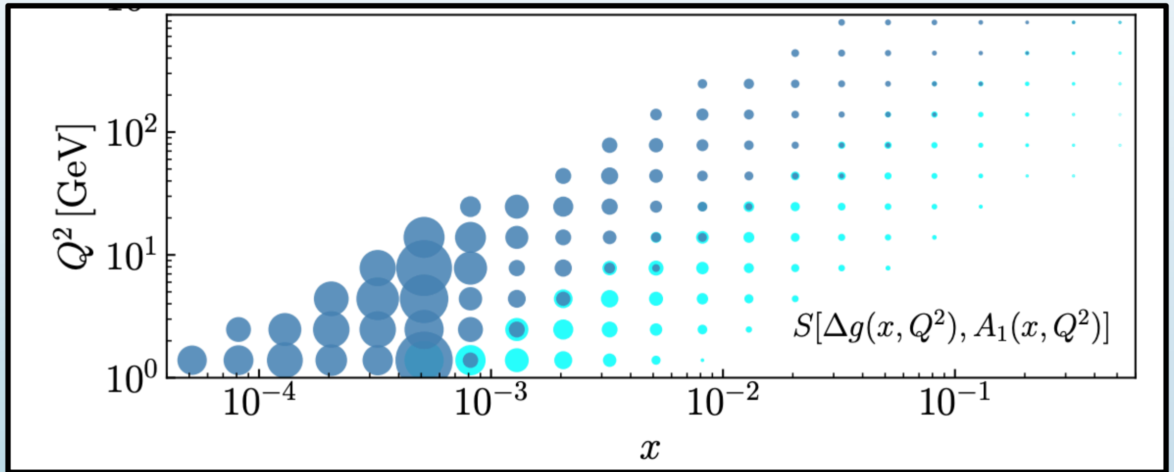
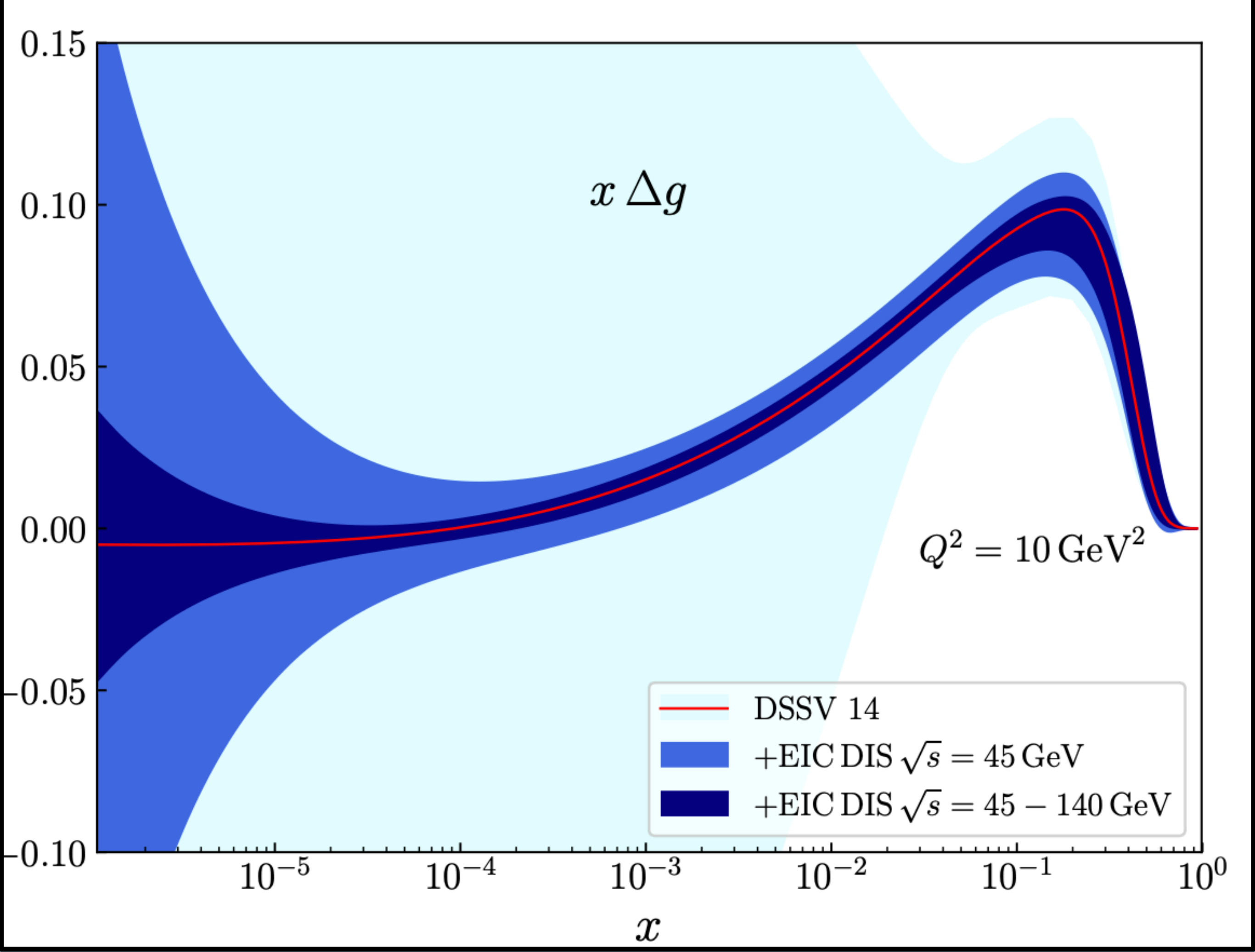


Sensitivity of A_1 to Δg

Large impact on Δg predicted, especially below $x \approx 0.01$



Positivity and renormalization of parton densities #1

John Collins (Penn State U.), Ted C. Rogers (Old Dominion U. and Jefferson Lab), Nobuo Sato (Jefferson Lab) (Nov 1, 2021)

Published in: *Phys.Rev.D* 105 (2022) 7, 076010 • e-Print: 2111.01170 [hep-ph]

As regards the positivity issue itself, there are several points to make. First, we emphasize that we have not argued that $\overline{\text{MS}}$ pdfs *must* be negative for any particular choice of scales or $\mu_{\overline{\text{MS}}}$. Rather we proved that nothing in the definition of pdfs or in the factorization theorems themselves excludes negativity as a possibility, especially at low or moderate input scales. But we did show arguments that indicate that certain generic situations do tend to lead to negative pdfs of partons with small pdfs, notably for non-valence quarks. Giving a full theoretical answer to the question of whether a particular pdf turns negative depends on its large distance/low energy non-perturbative properties, as the sensitivity to mass scales in the example of Sec. VIII illustrates. Also, the failure of

Current State of Helicity PDFs

Proton spin puzzle:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

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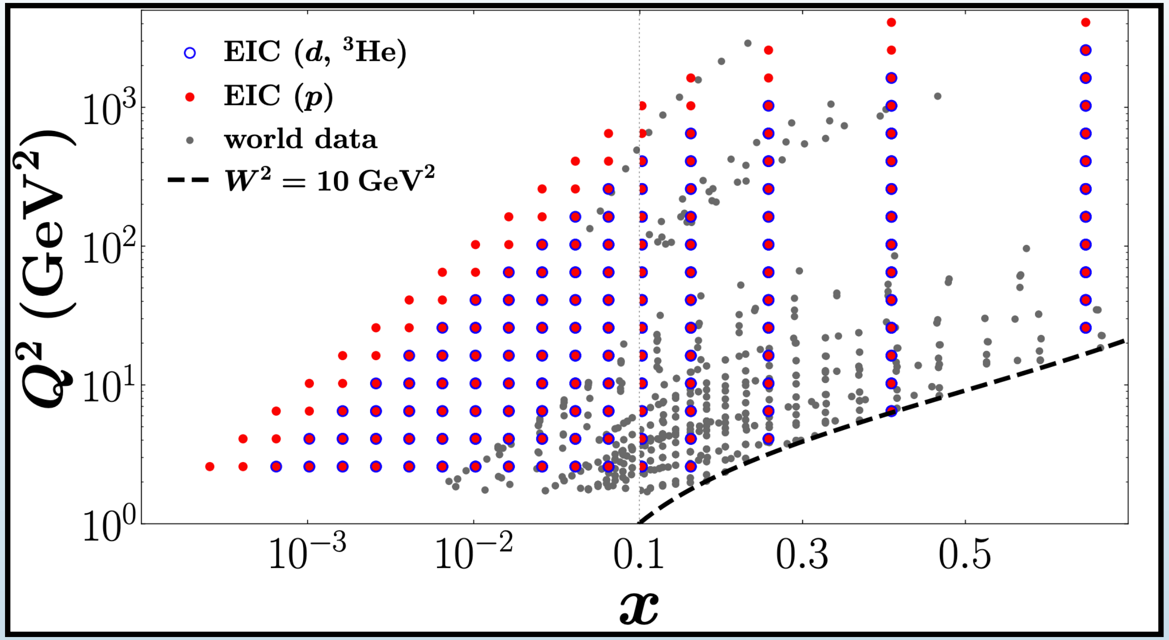

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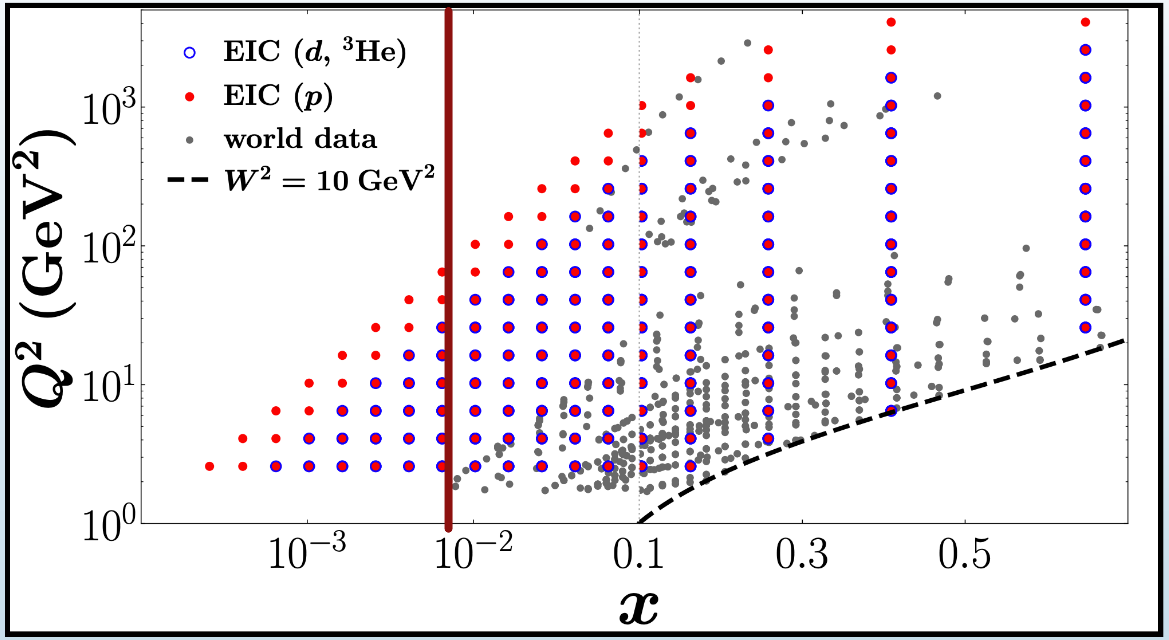

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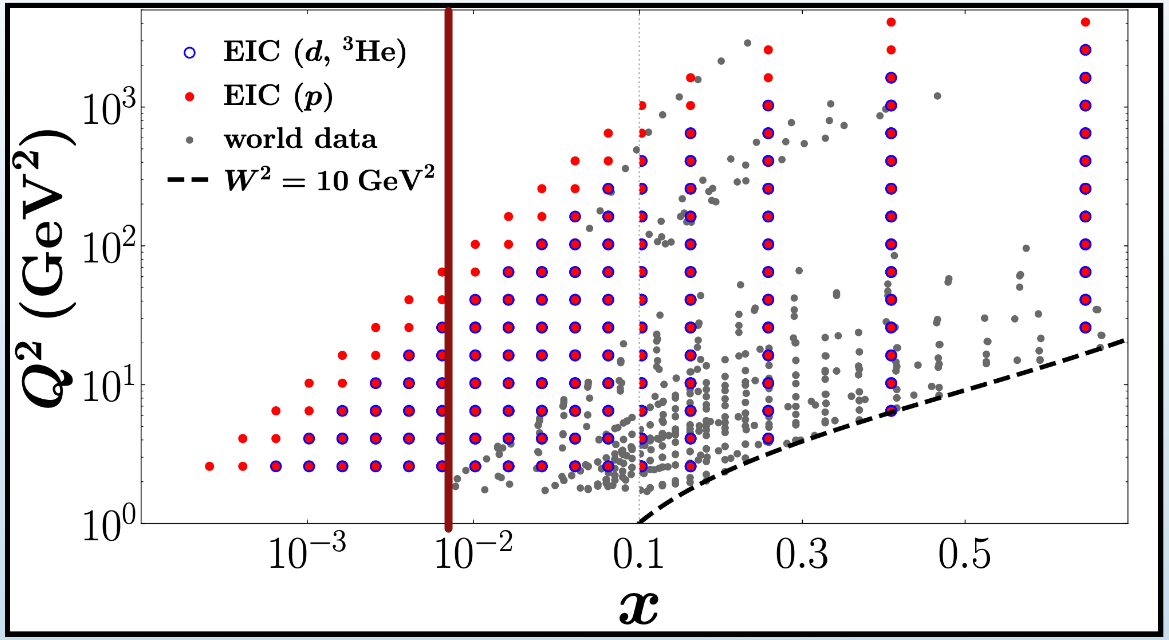

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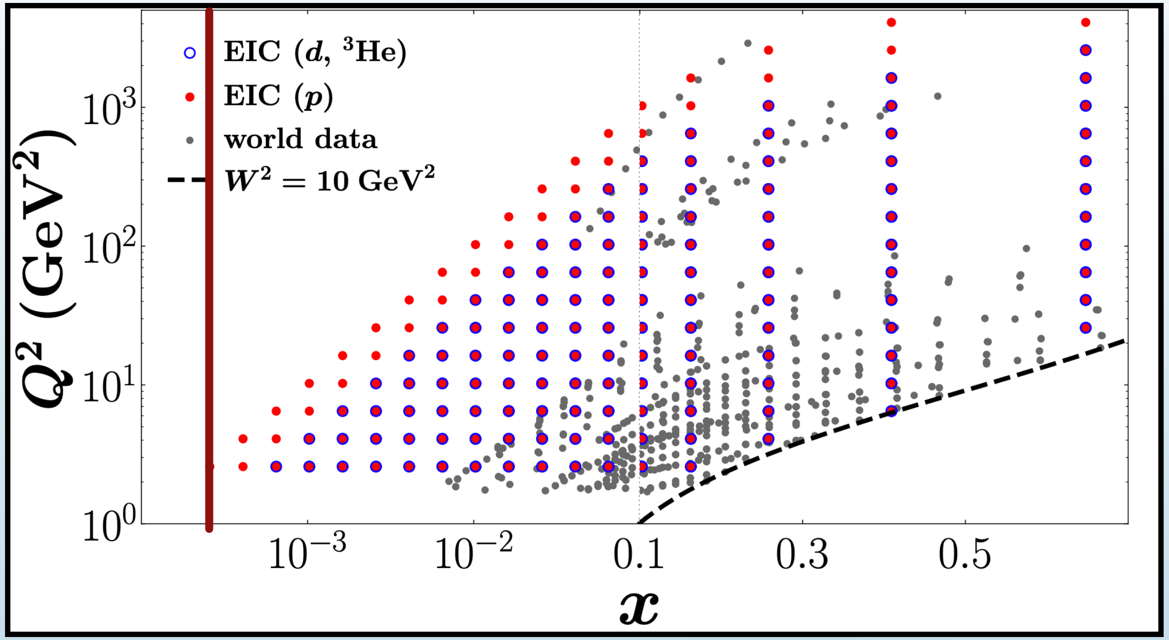

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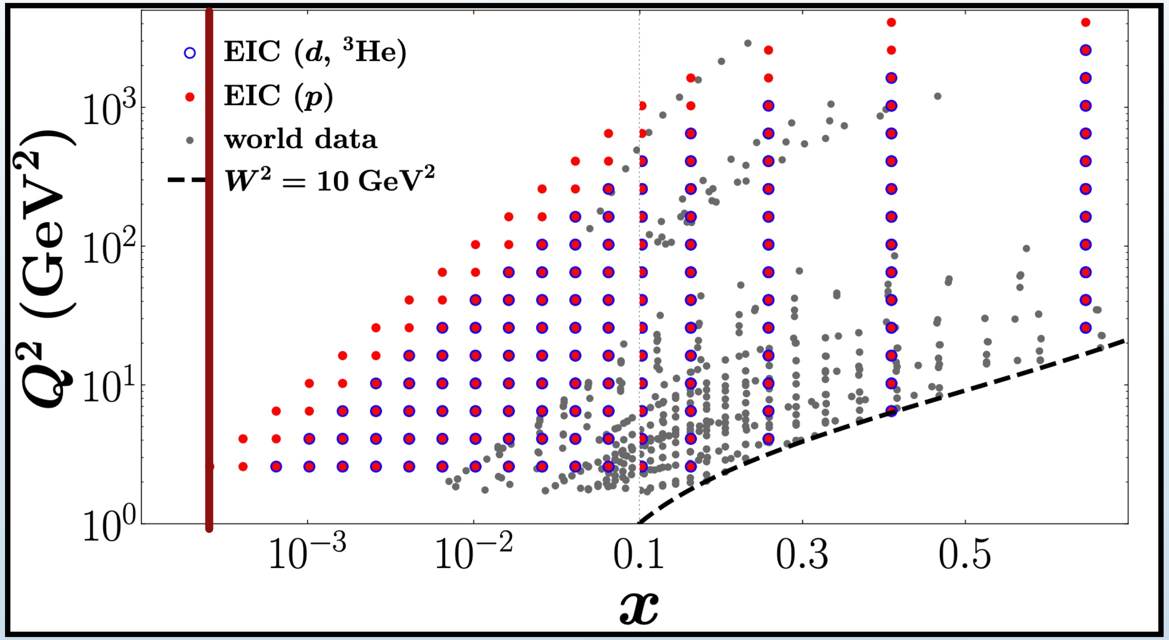

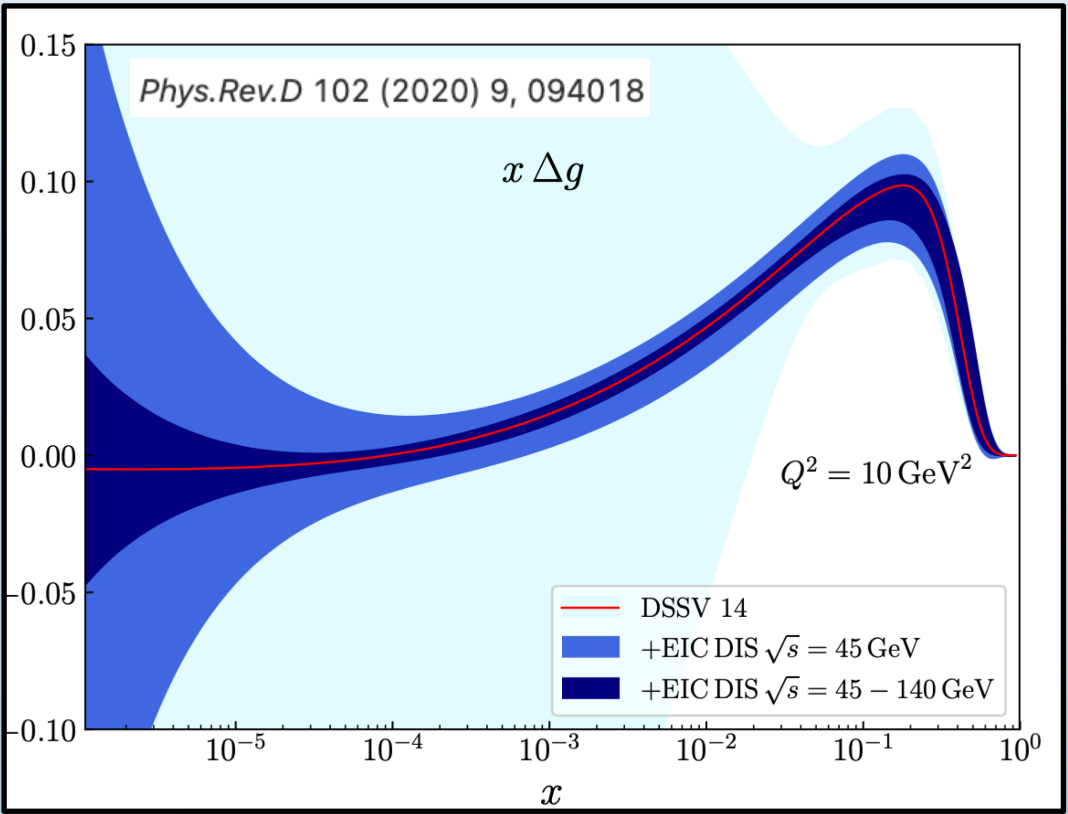

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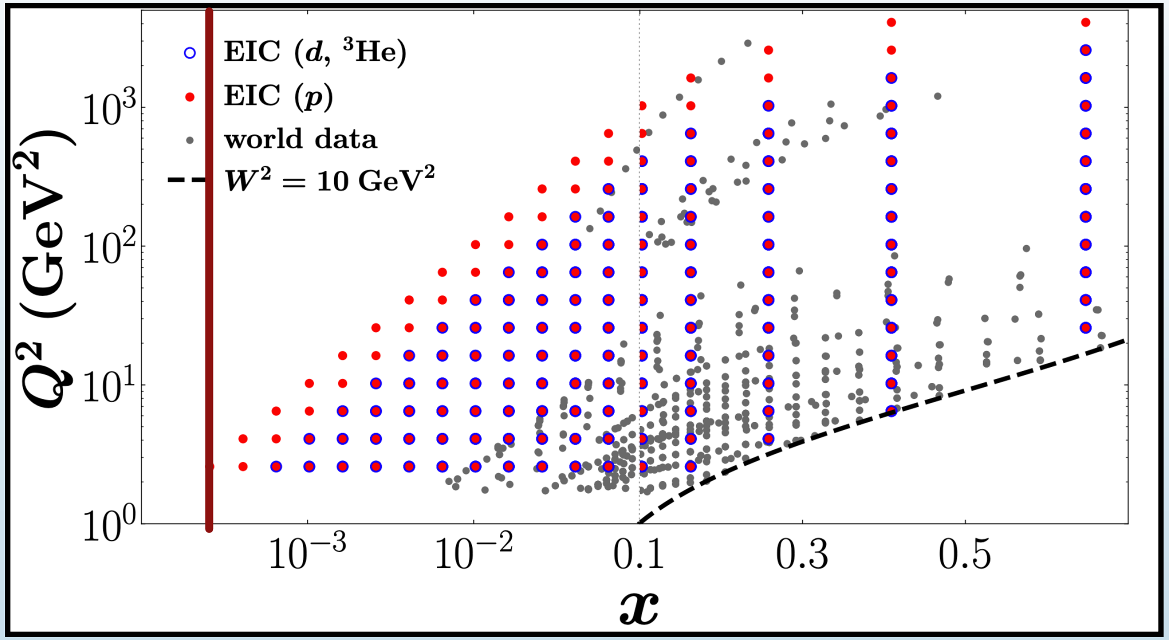

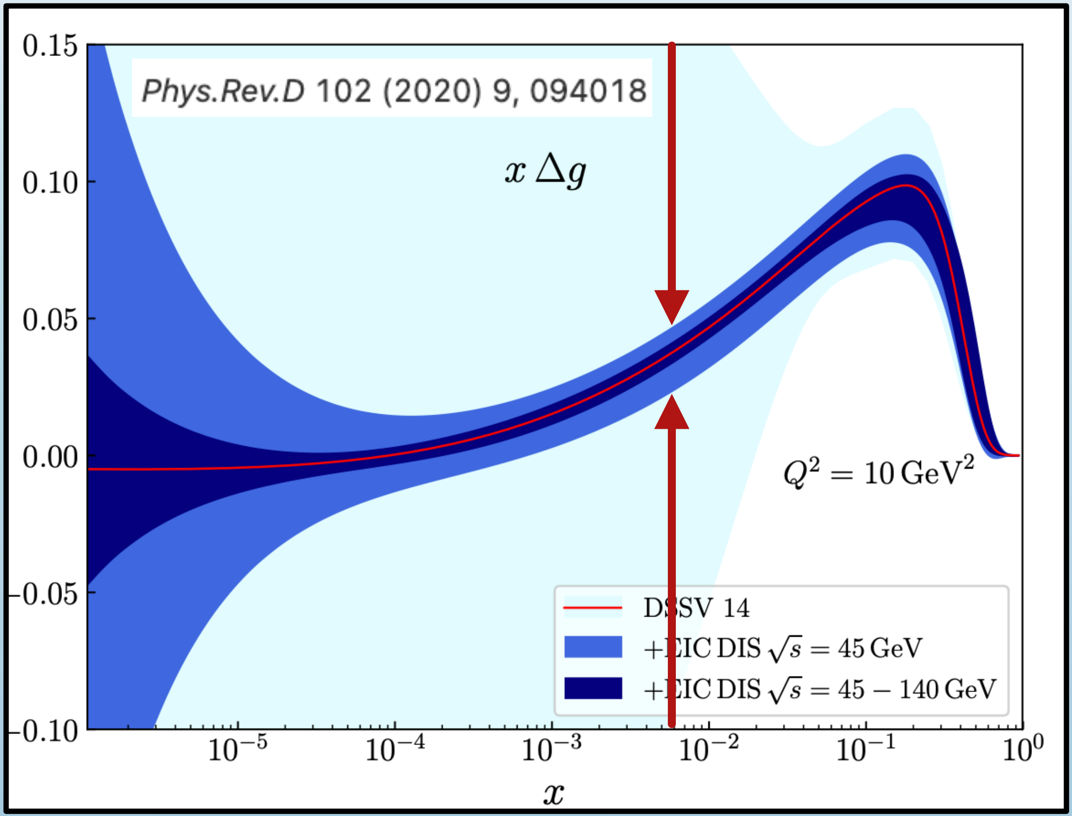

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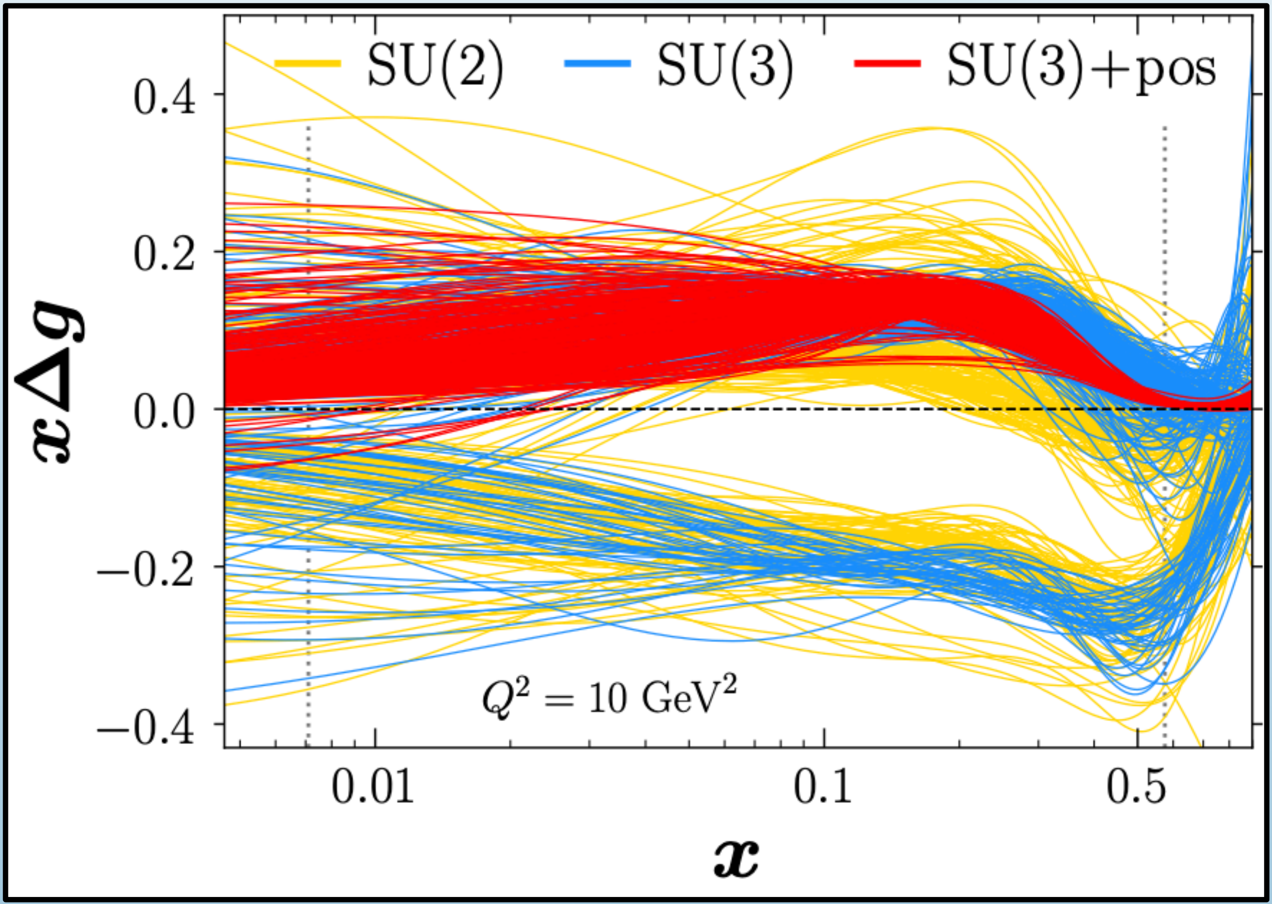
Published in: *Phys.Rev.D* 105 (2022) 7, 074022 • e-Print: 2201.02075 [hep-ph]



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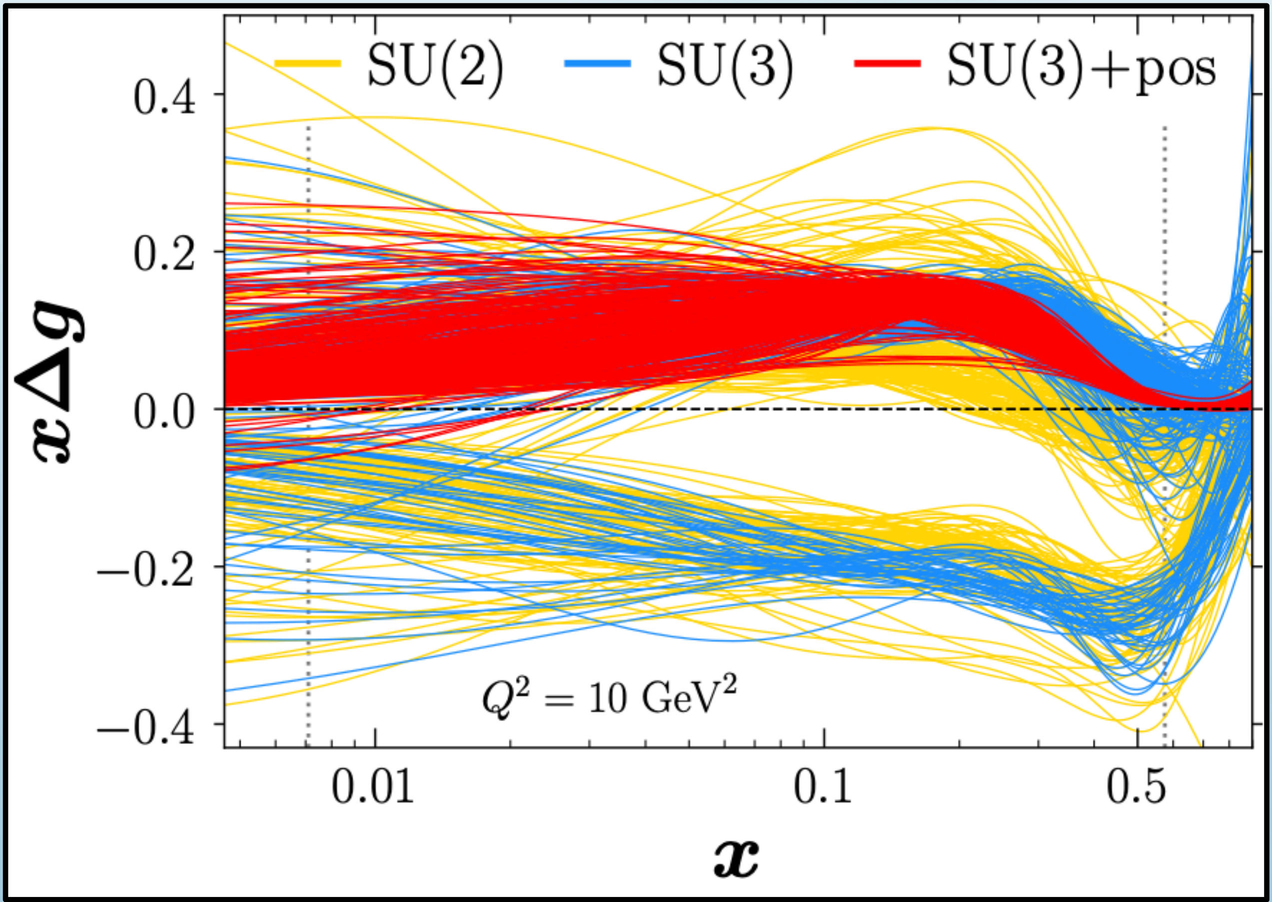

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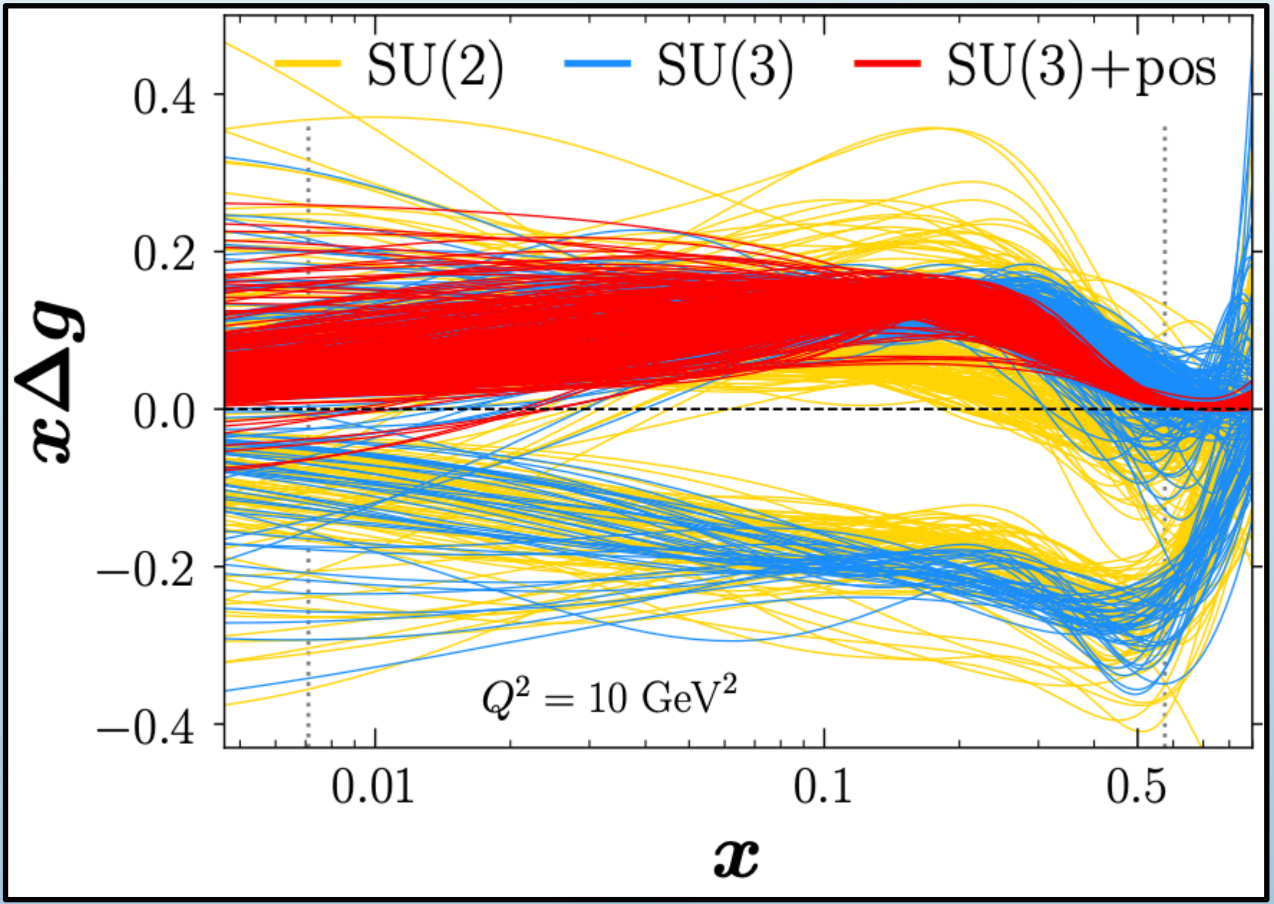

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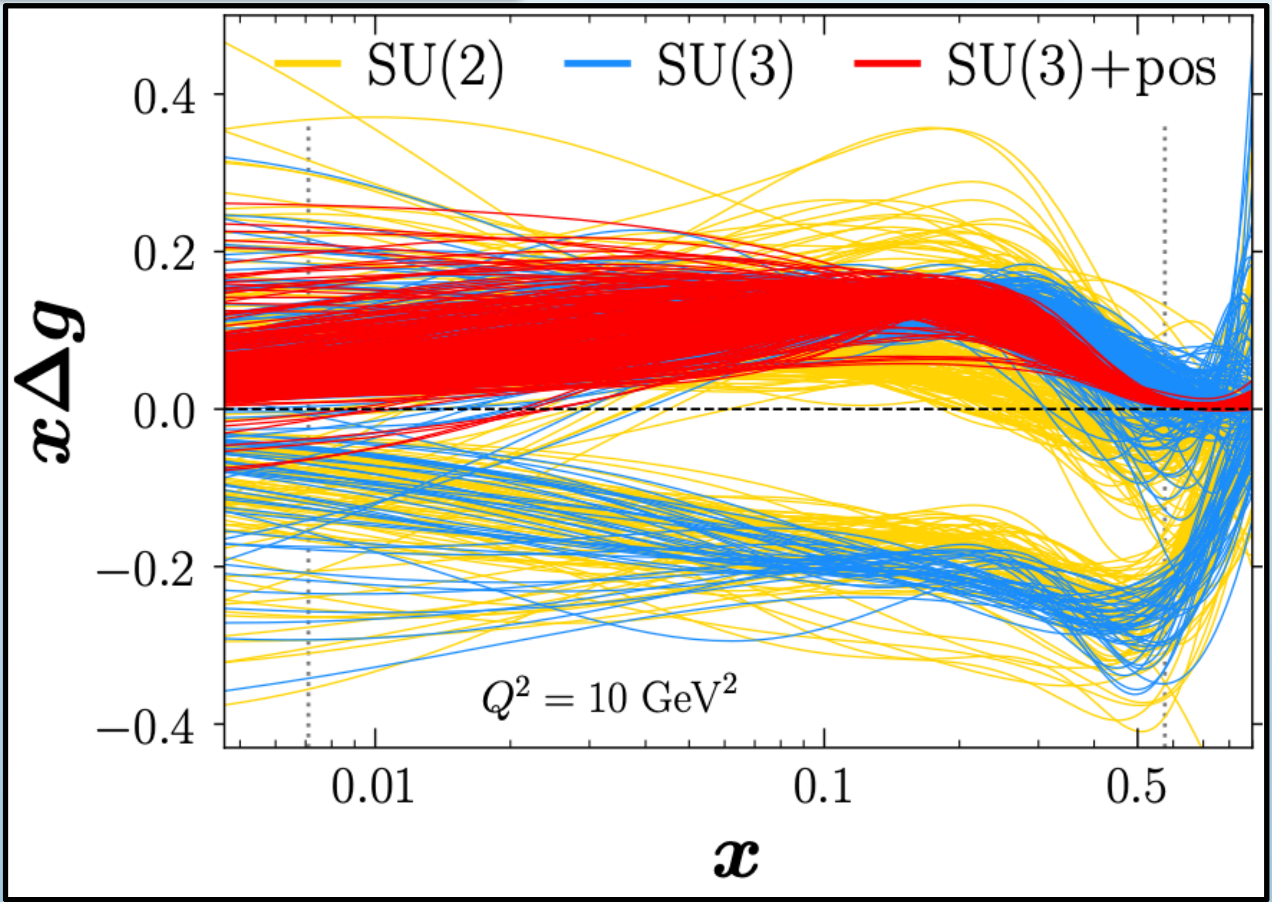

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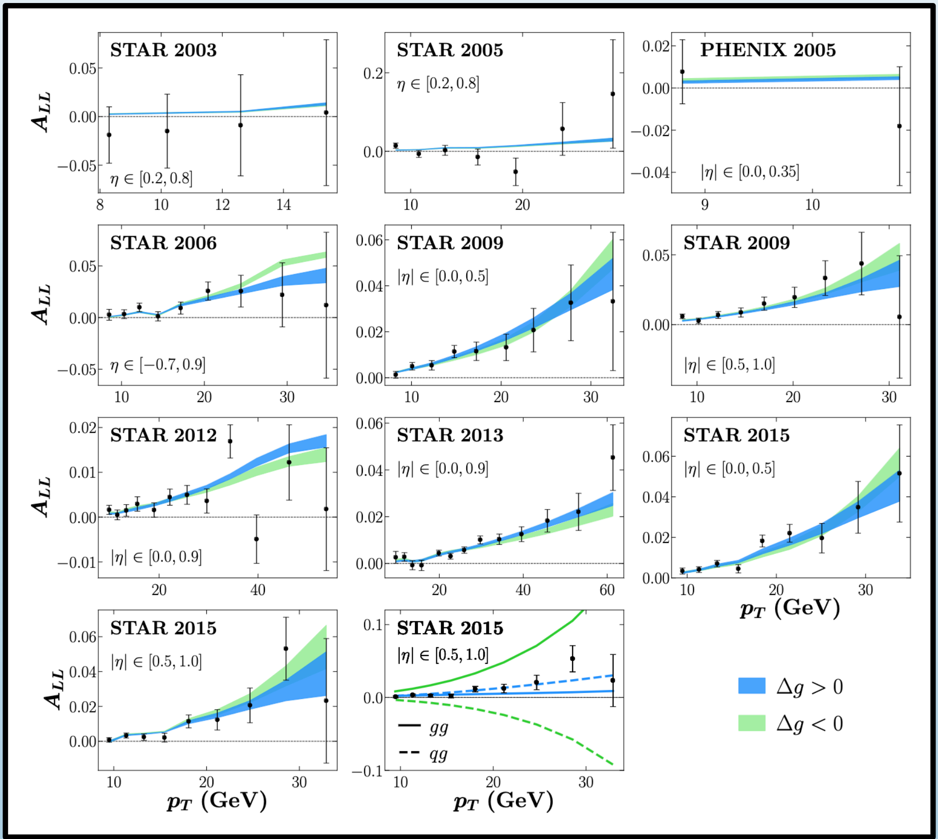

$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \ldots$

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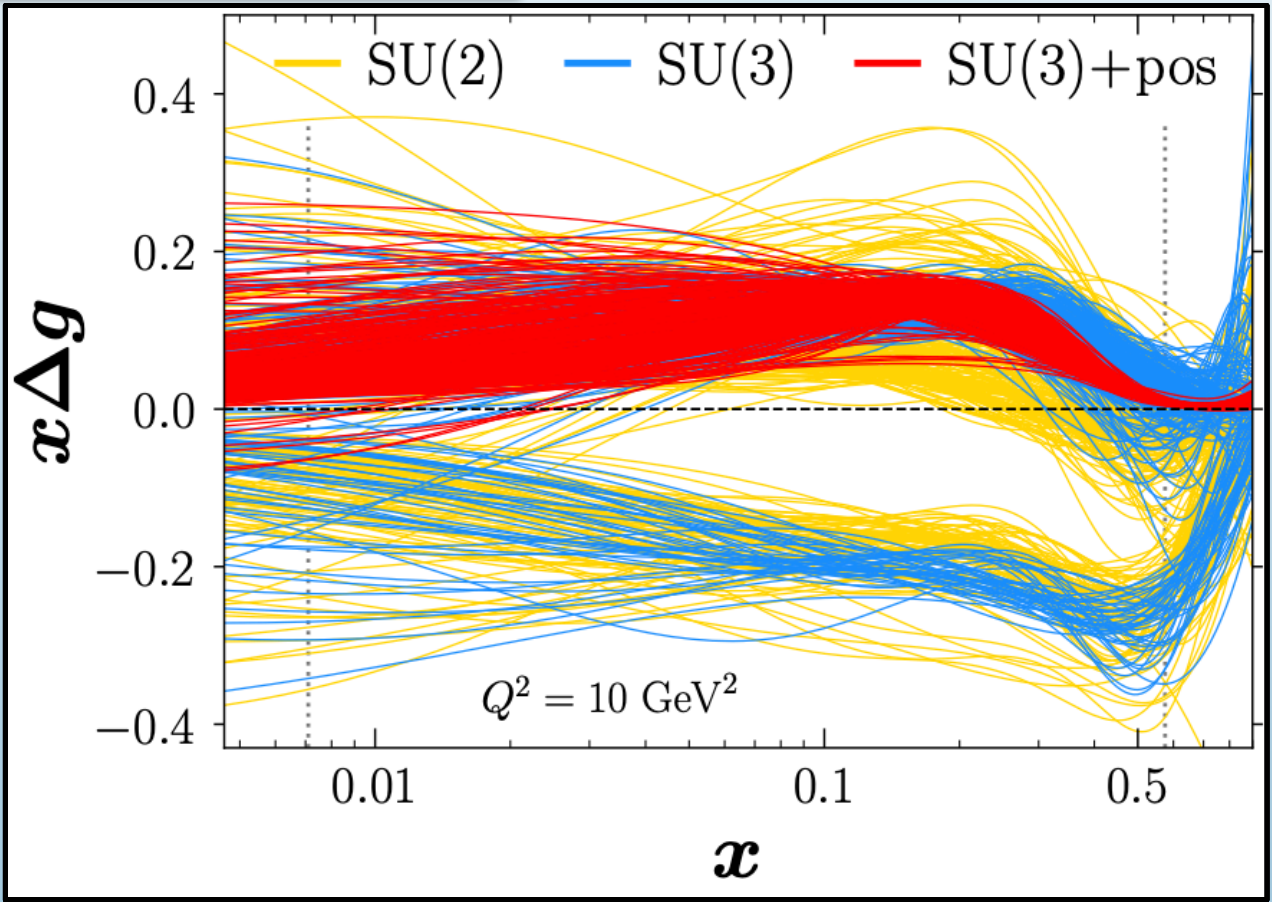

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Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions

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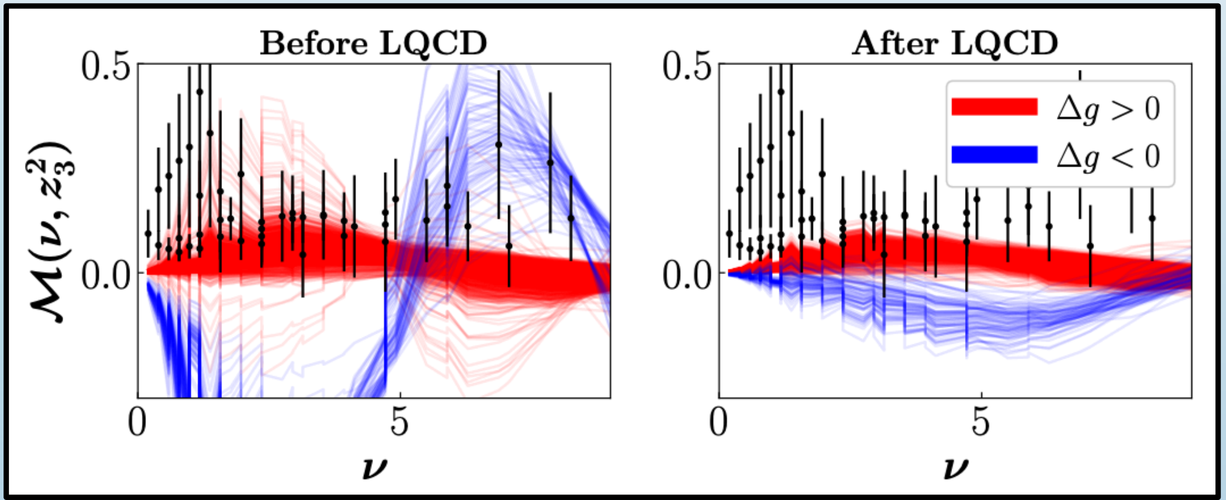
Oct 27, 2023

$$\mathcal{M}(\nu, z_3^2) = \int_0^1 dx\ x\ sin(x\nu)\Delta g(x) \quad \text{(LO)}$$



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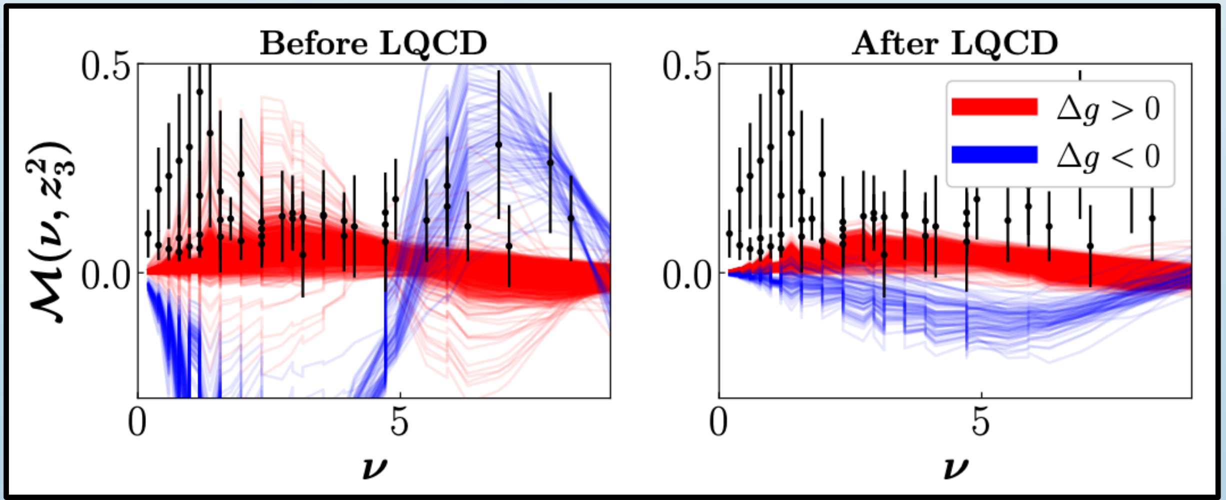

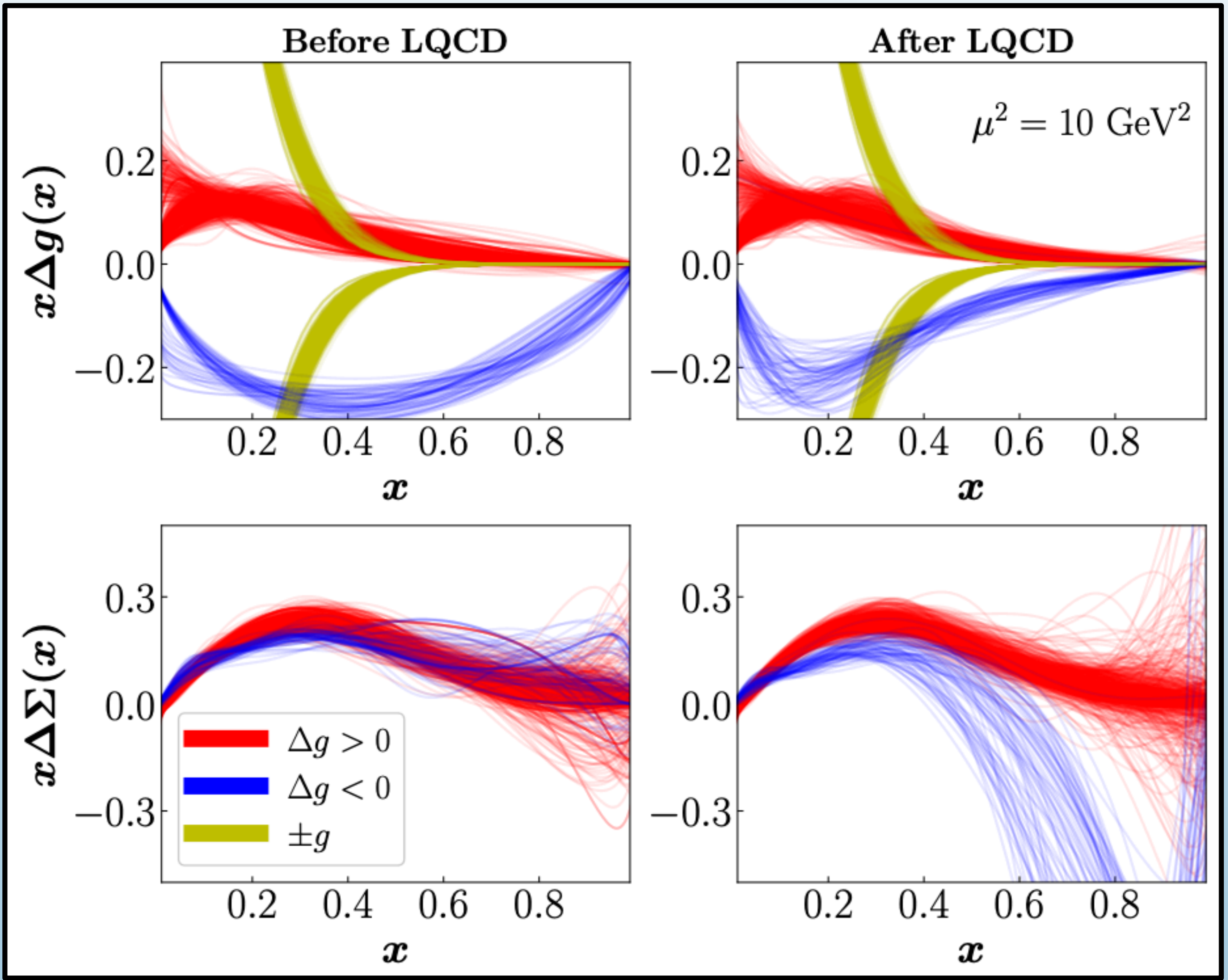

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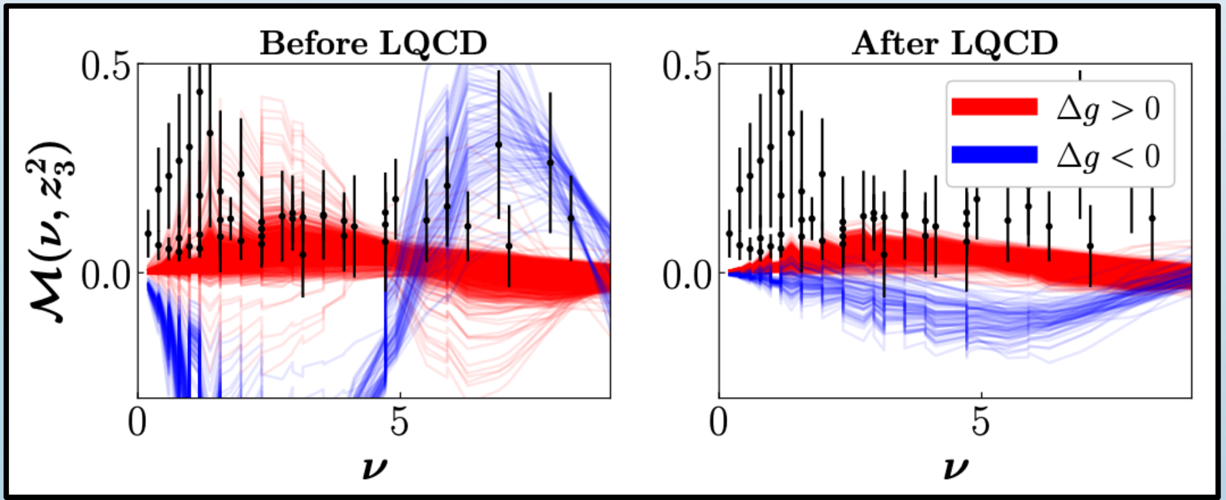
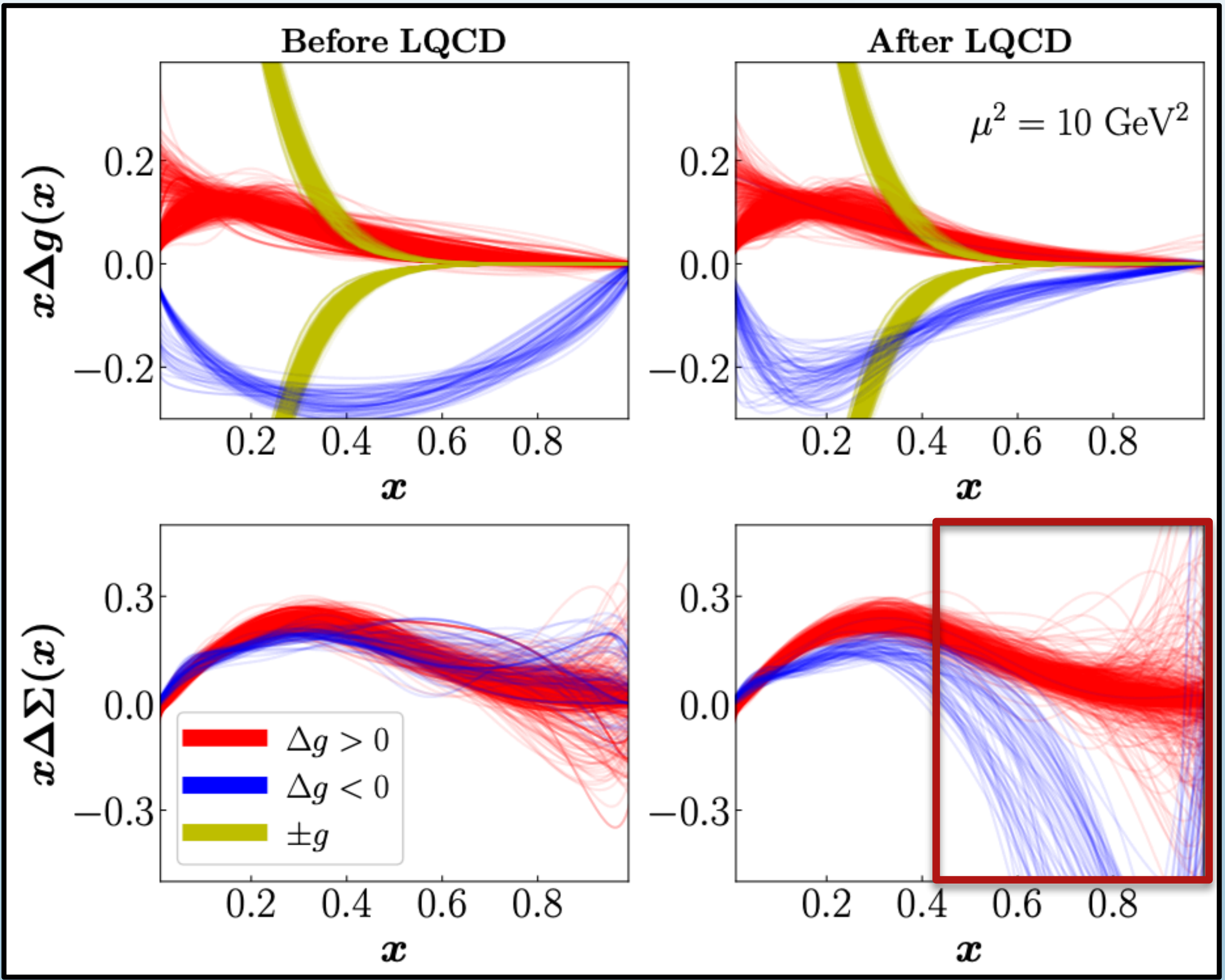


LQCD data does not rule out negative gluon, but leads to wild behavior for $\Delta\Sigma$ at large x



W^2_{min} Analysis and Stability

W^2_{min} (GeV^2)	3.5	**4**	5	6	10
N_{dat}	2002	**1735**	1287	1008	689
χ^2_{red}	1.11	**1.02**	0.97	1.00	1.01
Z-score	+3.23	**+0.61**	−0.83	+0.03	+0.22

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Data	N_{dat}	χ^2_{red} (Z-score) $W^2 > 10$ GeV^2 (LT only)	$W^2 > 4$ GeV^2 (LT only)	$W^2 > 4$ GeV^2 (with TMC & HT)
Polarized	1960†	0.96 (−0.85)	1.13 (+3.92)	0.99 (−0.30)
— DIS	1735†	1.01 (+0.24)	1.17 (+4.86)	1.02 (+0.61)
— SIDIS	124	0.80 (−1.66)	0.82 (−1.47)	0.76 (−1.99)
— jets	83	0.81 (−1.22)	0.79 (−1.38)	0.81 (−1.29)
— W/Z boson	18	0.83 (−0.43)	0.92 (−0.14)	0.75 (−0.71)
Lattice QCD	48	0.59 (−2.29)	0.59 (−2.34)	0.58 (−2.35)
Total	**2008**†	**0.94** (−1.33)	**1.13** (+4.00)	**0.99** (−0.22)

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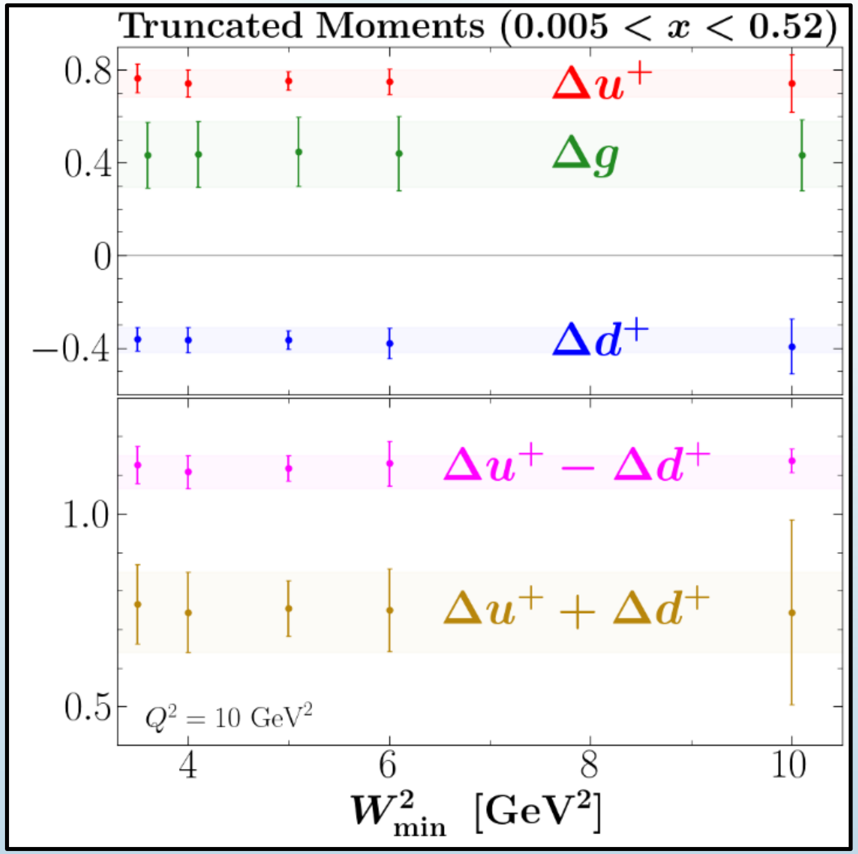
— W/Z boson	18	0.83 (−0.43)	0.92 (−0.14)	0.75 (−0.71)
Lattice QCD	48	0.59 (−2.29)	0.59 (−2.34)	0.58 (−2.35)
Total	**2008†**	**0.94** (−1.33)	**1.13** (+4.00)	**0.99** (−0.22)

TMCs necessary for description of high-x DIS data
(1.03 → 1.02 when HT is added)

$W^2_{\rm min}$ Analysis and Stability

$W^2_{\rm min}$ (GeV^2)	3.5	**4**	5	6	10
$N_{\rm dat}$	2002	**1735**	1287	1008	689
$\chi^2_{\rm red}$	1.11	**1.02**	0.97	1.00	1.01
Z-score	+3.23	**+0.61**	−0.83	+0.03	+0.22



Data	$N_{\rm dat}$	$\chi^2_{\rm red}$ (Z-score) $W^2 > 10$ GeV^2 (LT only)	$W^2 > 4$ GeV^2 (LT only)	$W^2 > 4$ GeV^2 (with TMC & HT)
Polarized	1960†	0.96 (−0.85)	1.13 (+3.92)	0.99 (−0.30)
— DIS	1735†	1.01 (+0.24)	1.17 (+4.86)	1.02 (+0.61)
— SIDIS	124	0.80 (−1.66)	0.82 (−1.47)	0.76 (−1.99)
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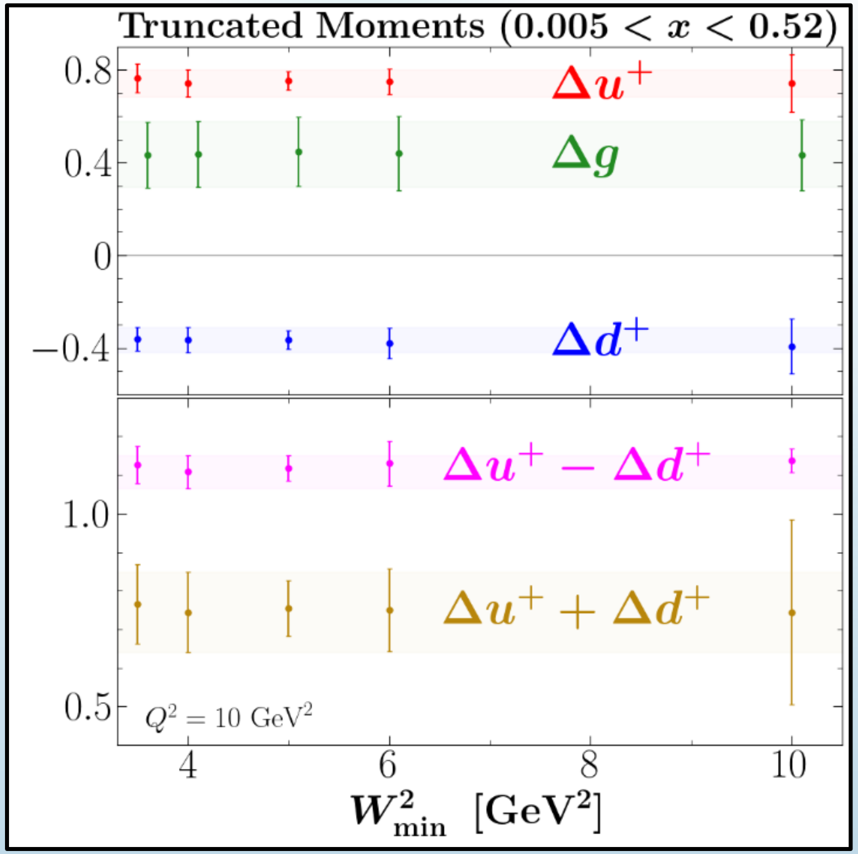

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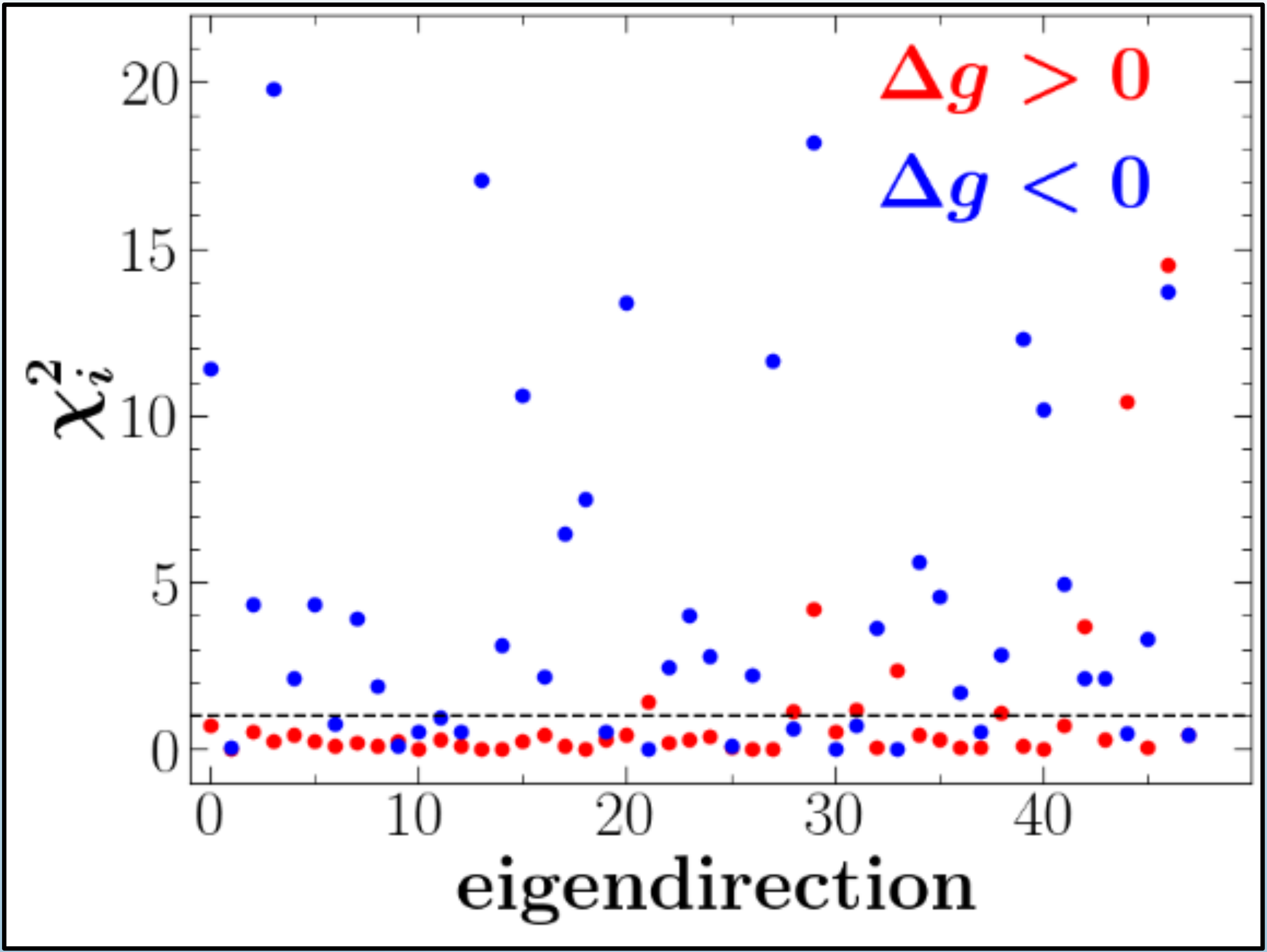
Moments stable for $3.5 < W^2_{\rm min} < 6$ GeV^2

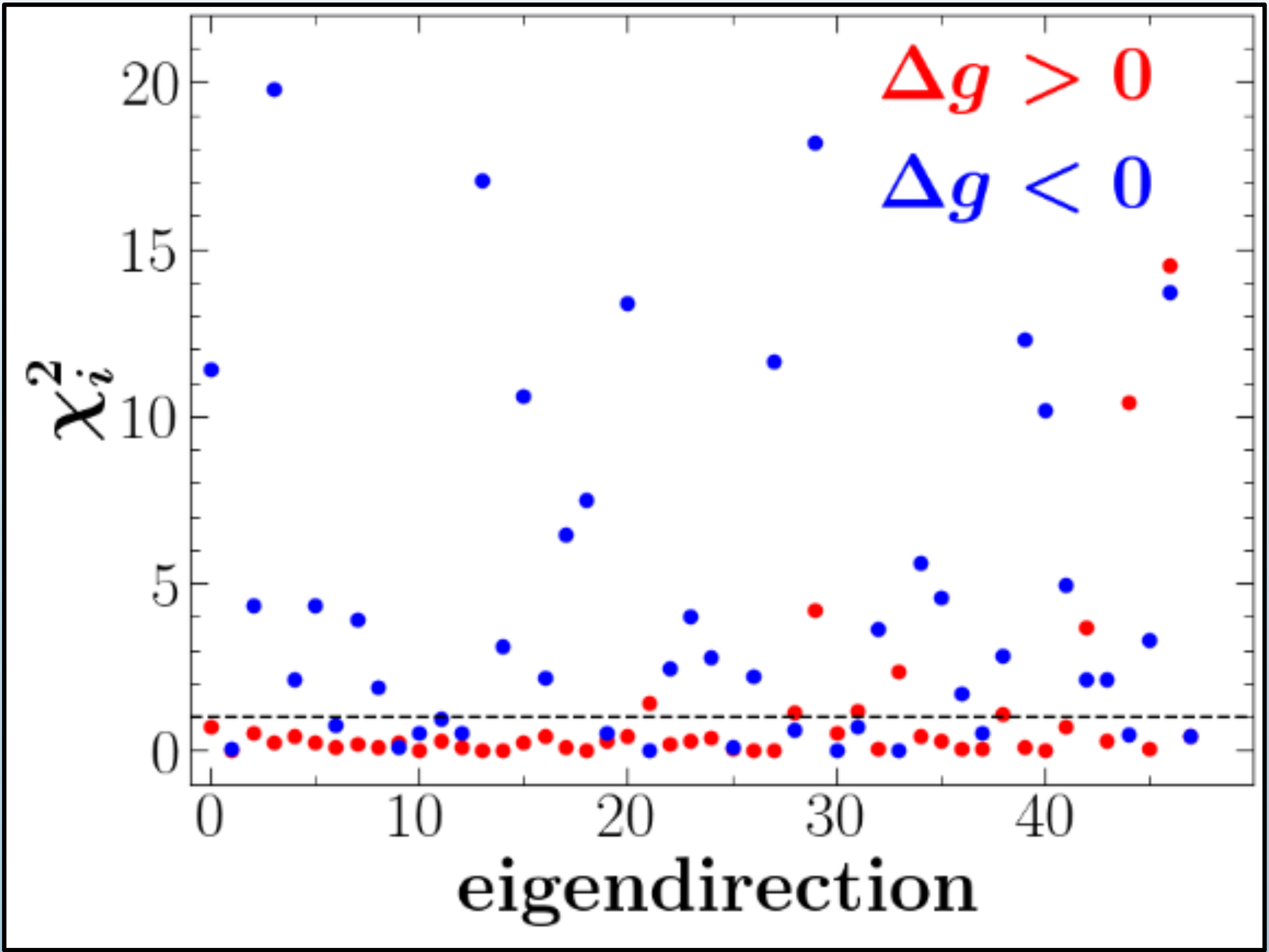
TMCs necessary for description of high-x DIS data
(1.03 → 1.02 when HT is added)

Gluon Solutions



For LQCD calculation, we show the residuals as a function of the 48 eigendirections

Gluon Solutions

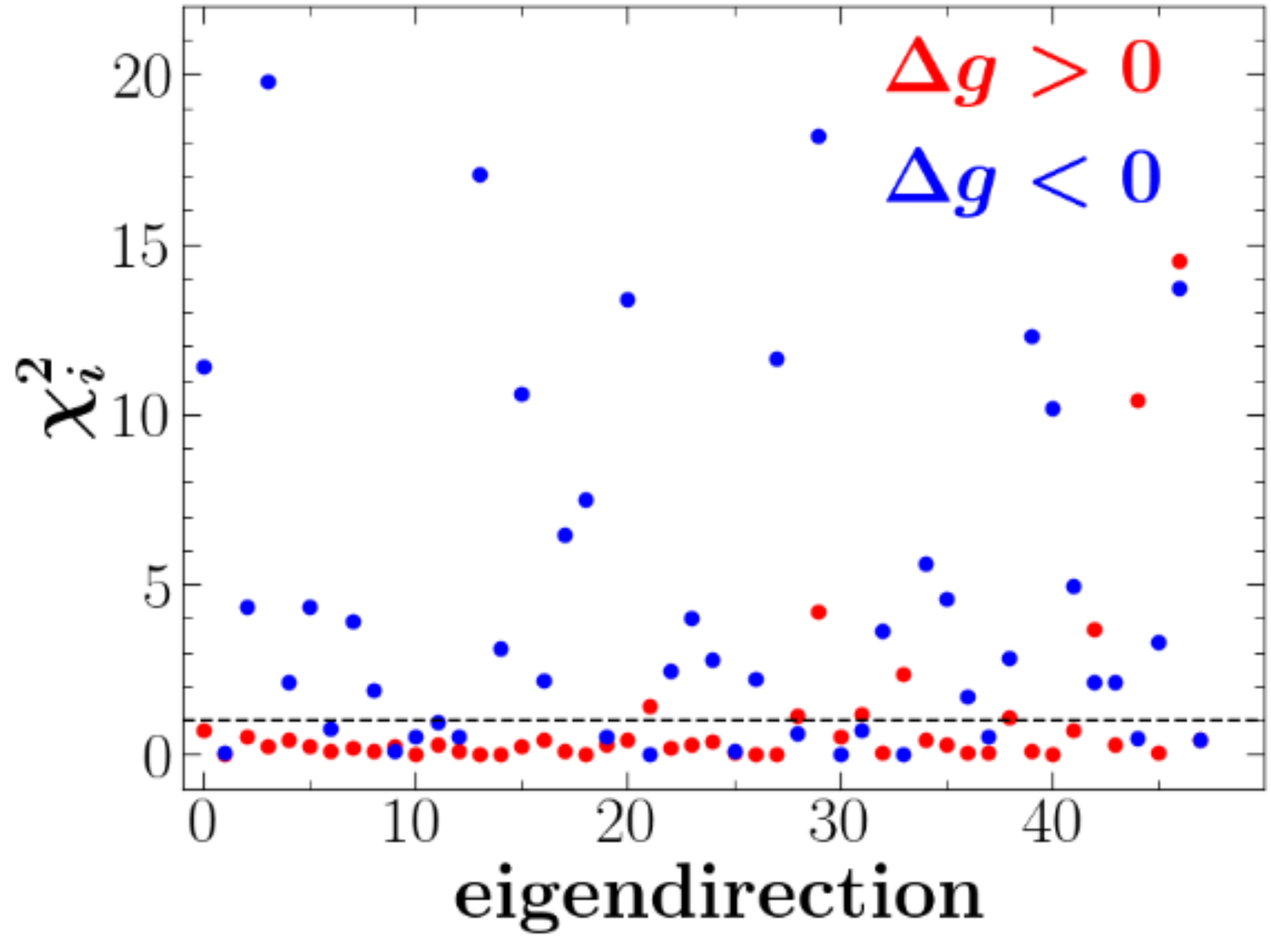


For LQCD calculation, we show the residuals as a function of the 48 eigendirections

When combined with high-x DIS data and polarized jet data, the negative Δg solution gives a far worse description of LQCD

N. T. Hunt-Smith *et al.*, Phys. Rev. Lett. **133**, 161901 (2024)

Gluon Solutions



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In future results, we ignore the negative Δg solution