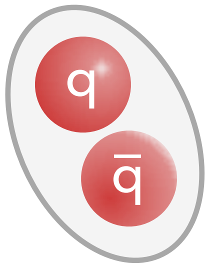
η and η' meson production in J/ψ radiative decays from lattice QCD

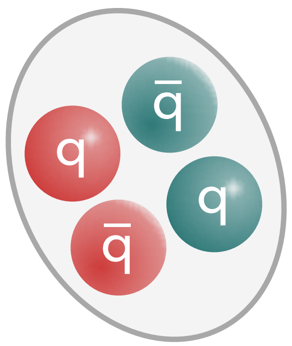
Jozef Dudek

William & Mary
CHARTERED 1693

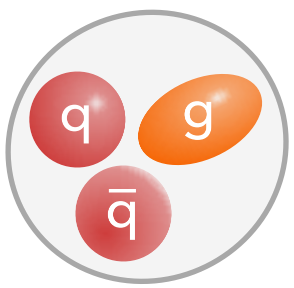
"pictures"



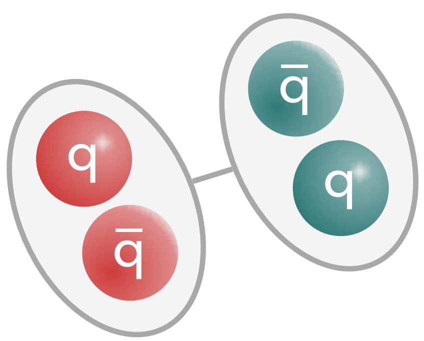
conventional meson



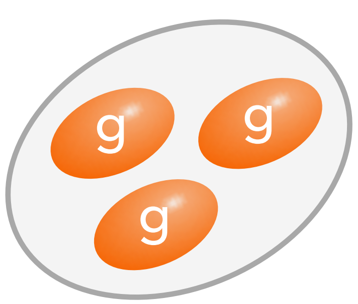
tetraquark



hybrid meson



meson-meson molecule



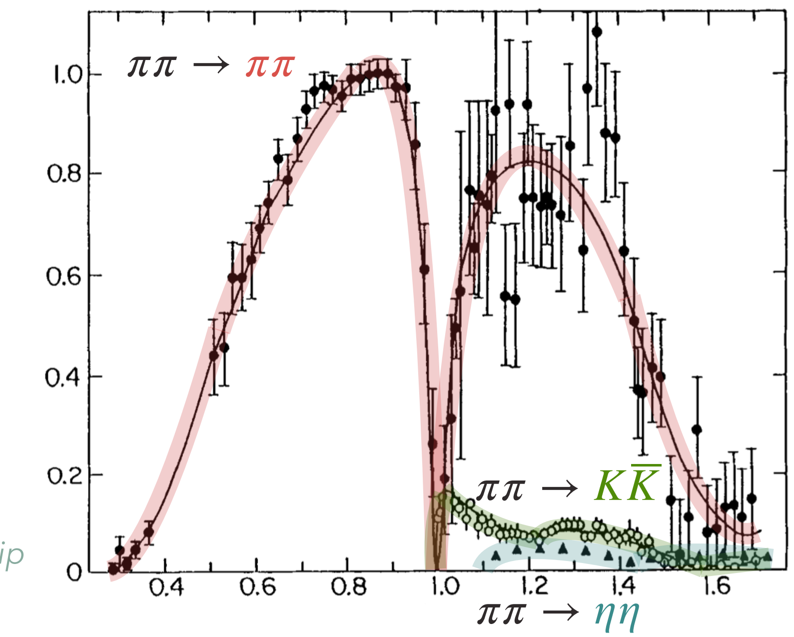
glueball

⋮

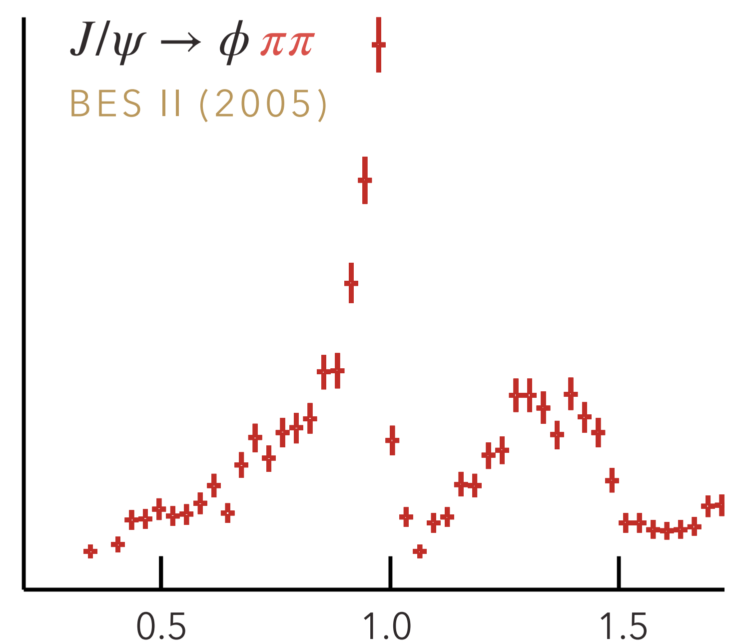
"reality"

resonances in complicated production/decay

e.g. $f_0(980)$



sometimes a dip



sometimes a peak

meson spectroscopy

"pictures"

conventional meson

tetraquark

hybrid meson

meson-meson molecule

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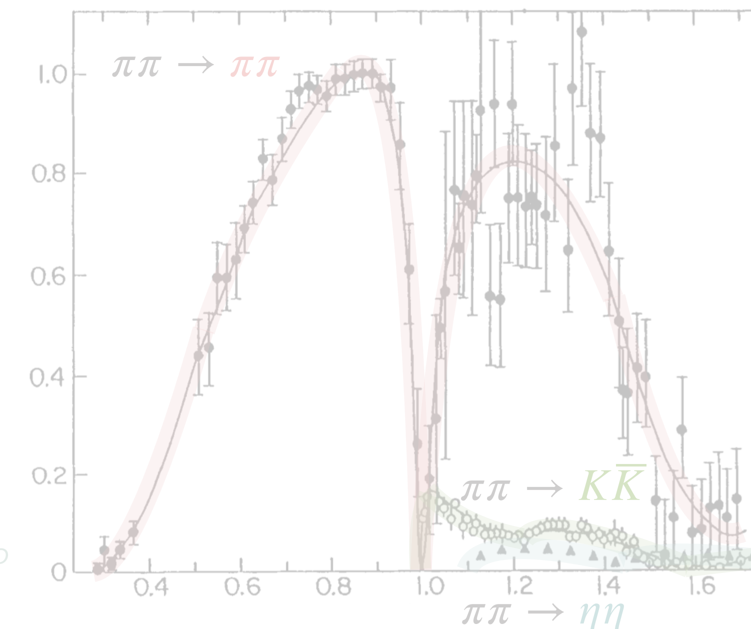
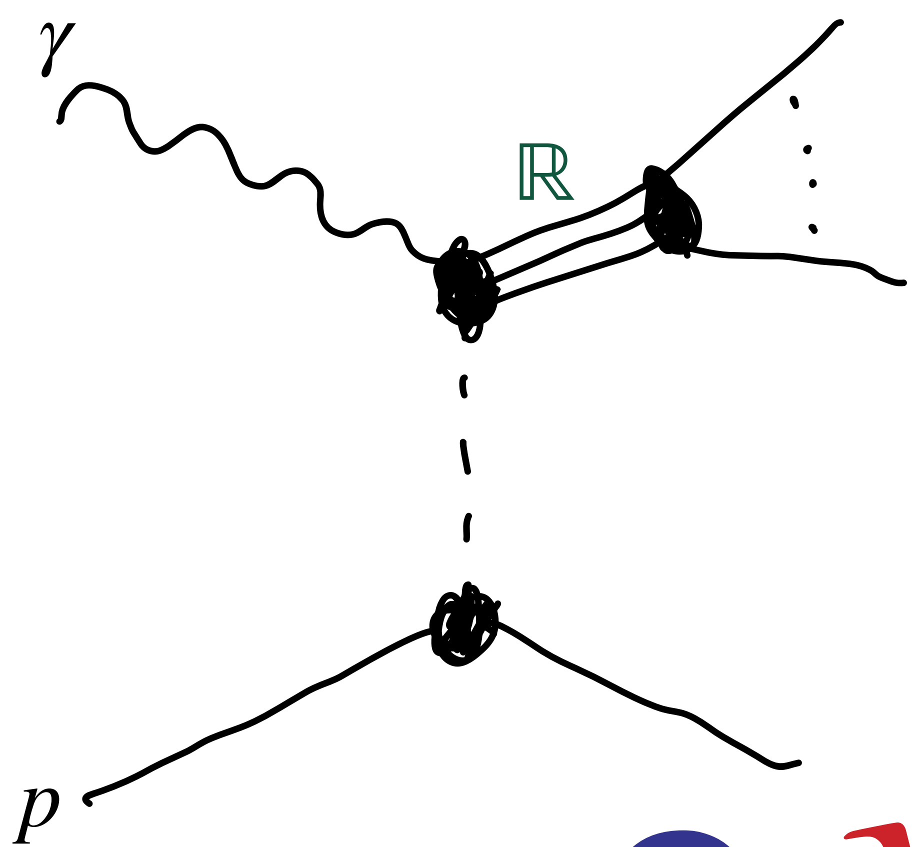


sometimes a peak

first principles QCD will need to deal with **amplitudes**

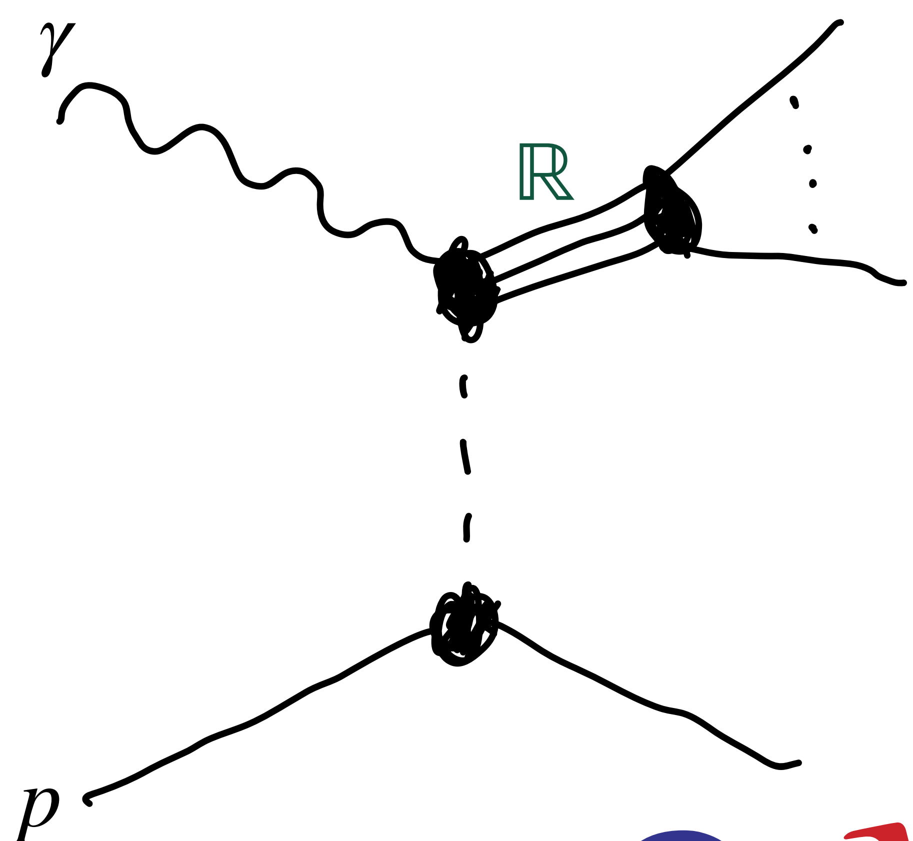
producing meson resonances

the local meson resonance factory

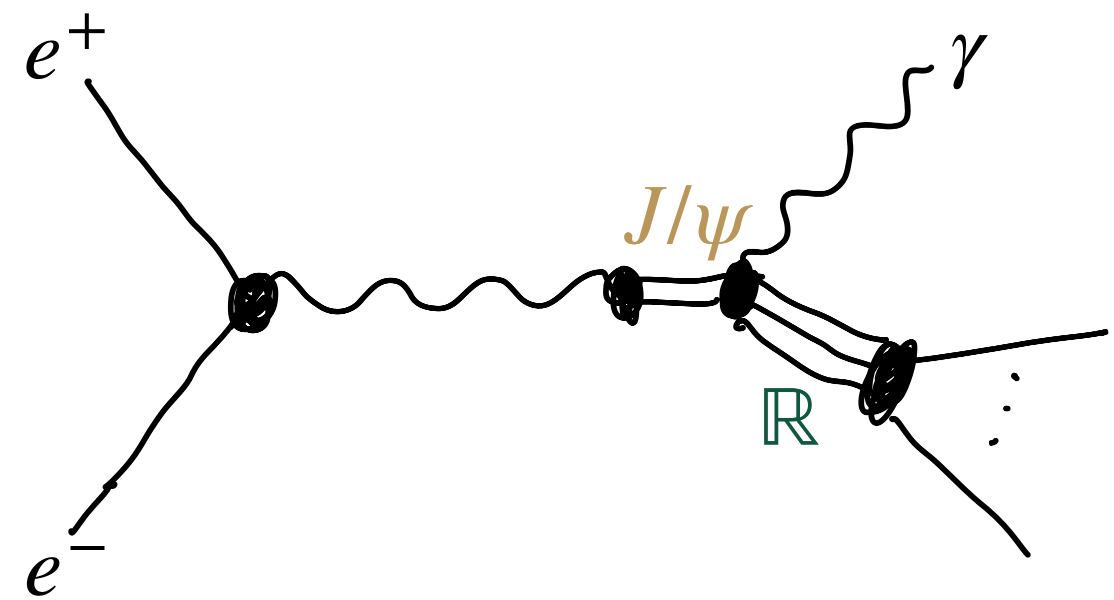


producing meson resonances

the local meson resonance factory



a complimentary approach

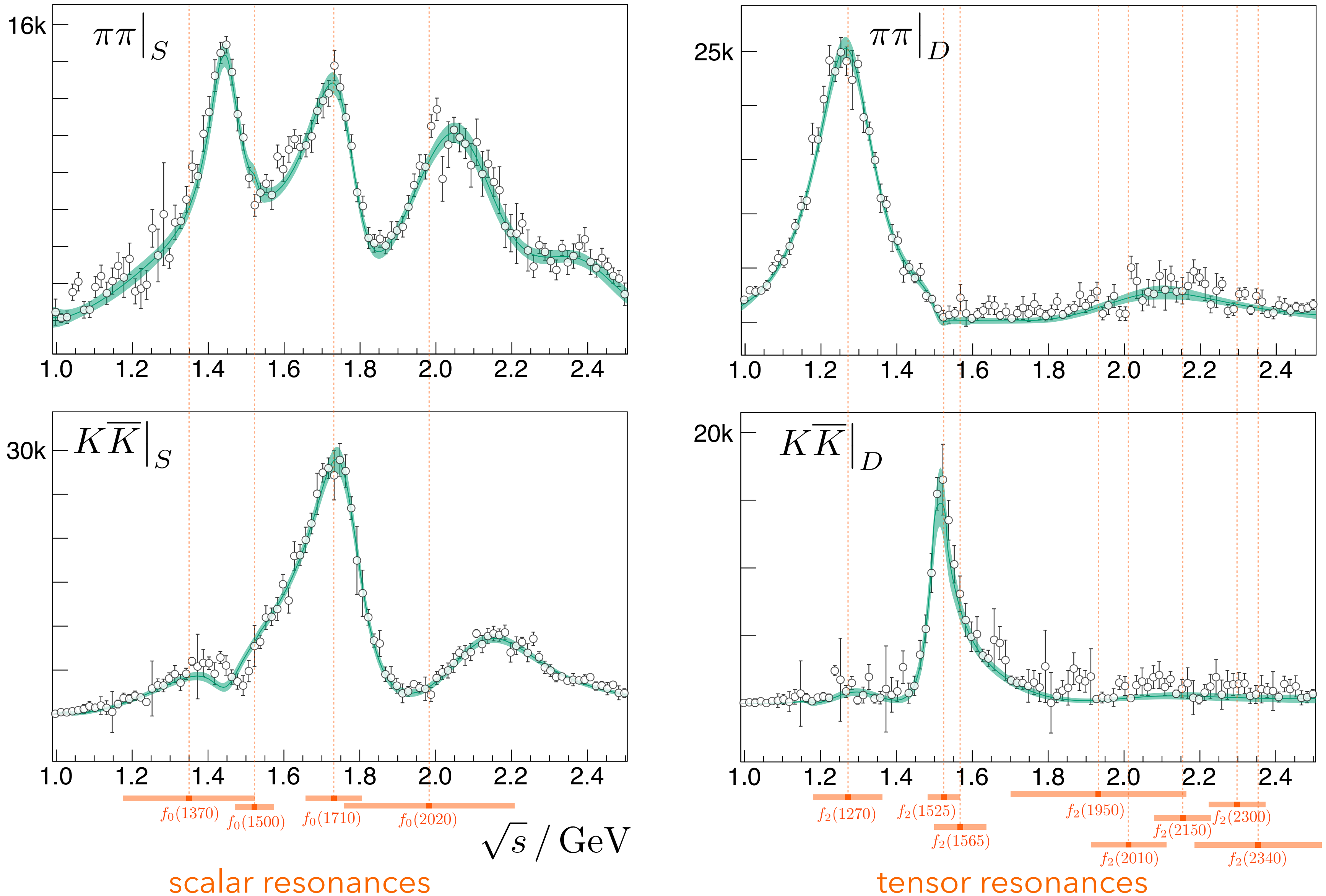


10 billion *J/ψ* decays at BESIII

charmonium radiative decays as light meson factories

10 billion J/ψ decays at BESIII

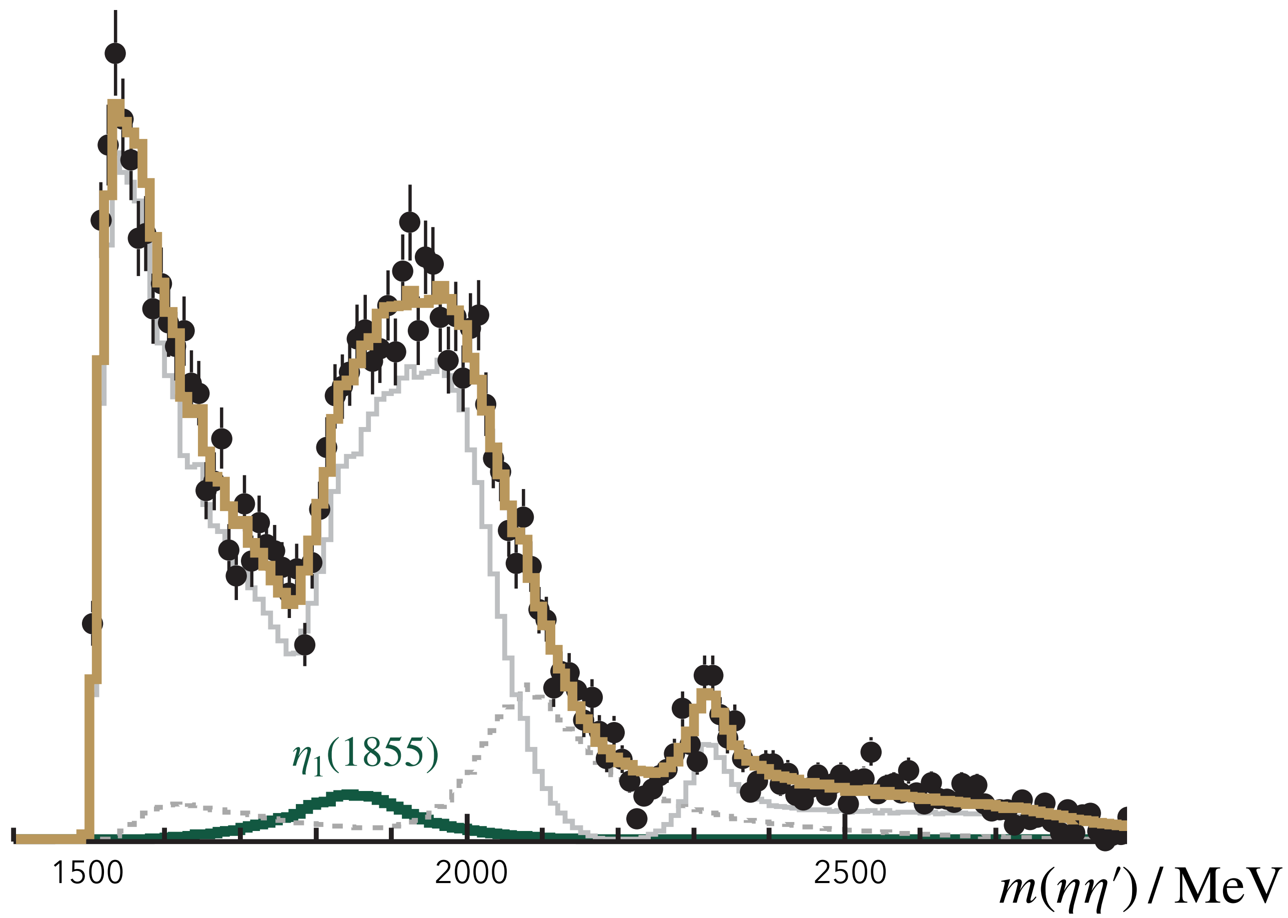
$$J/\psi \to \gamma\,(\pi\pi, K\overline{K}, \ldots)$$



amplitude description by **JPAC**

10 billion J/ψ decays at BESIII

$J/\psi \to \gamma\, \eta\eta'$

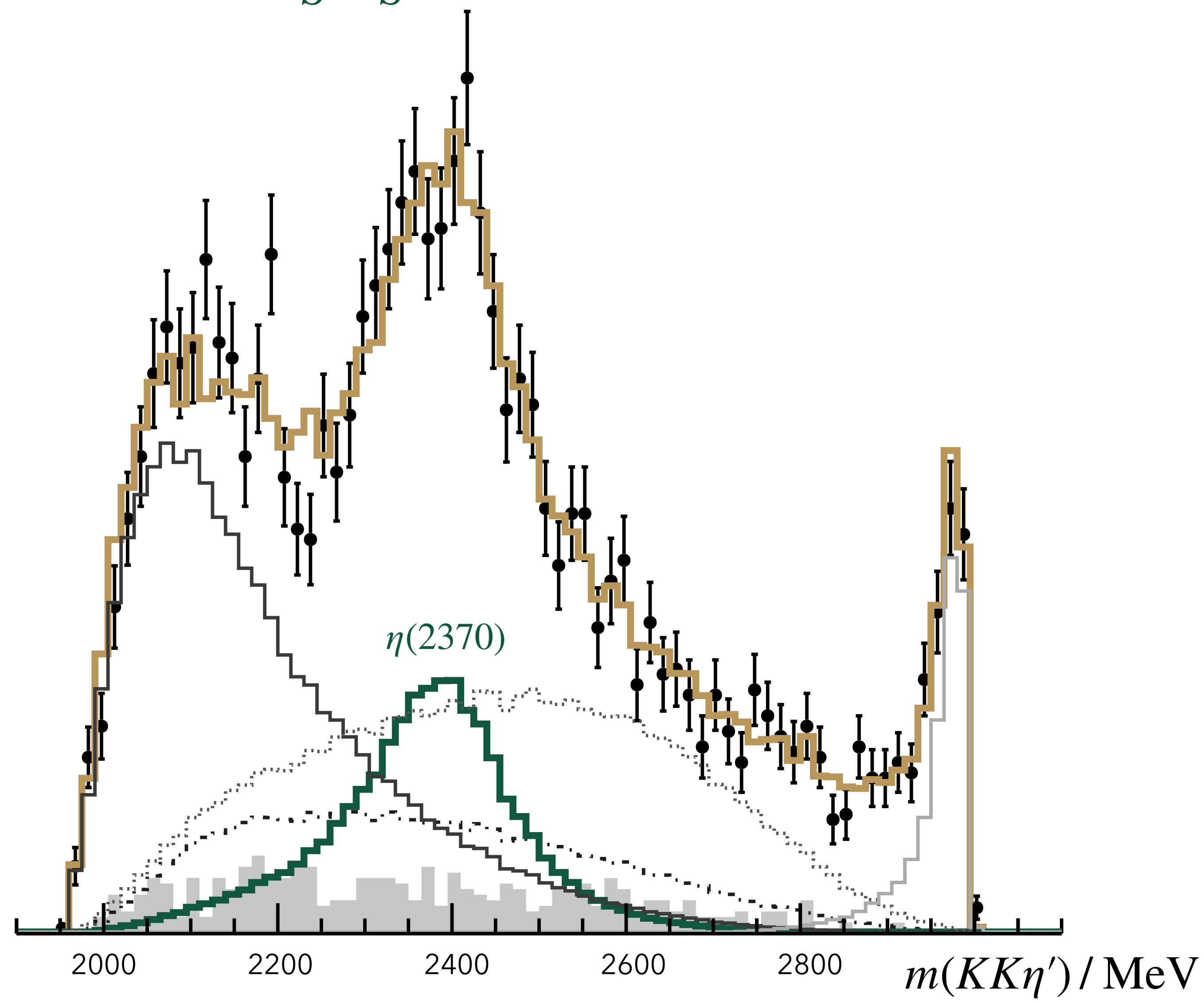


isospin-0 1^{-+} **exotic hybrid meson** candidate

charmonium radiative decays as light meson factories

10 billion J/ψ decays at BESIII

$J/\psi \to \gamma K_S^0 K_S^0 \eta'$



isospin-0 0^{-+} **glueball** candidate

charmonium radiative decays as light meson factories

10 billion J/ψ decays at BESIII

'simplest' cases: $J/\psi \to \gamma\,\eta$ 2.2 million

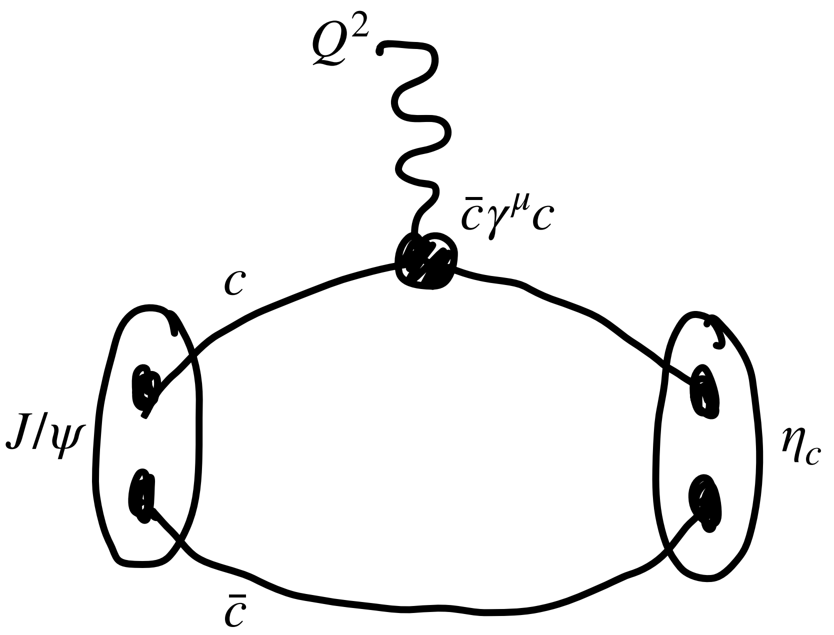
$J/\psi \to \gamma\,\eta'$ 0.6 million

γP	$N^{\text{obs}}_{J/\psi\to\gamma P}$	ε (%)	$\mathcal{B}(J/\psi \to \gamma P)(\times 10^{-4})$ This work	BESIII	PDG [31]
$\gamma\pi^0$	175893 ± 839	52.90 ± 0.03	$0.334 \pm 0.002 \pm 0.009$	$0.361 \pm 0.012 \pm 0.016$ [16]	0.356 ± 0.017
$\gamma\eta$	2209063 ± 2592	50.78 ± 0.03	$10.96 \pm 0.01 \pm 0.19$	$10.67 \pm 0.05 \pm 0.23$ [17]	10.85 ± 0.18
$\gamma\eta'$	638206 ± 1061	50.77 ± 0.03	$54.0 \pm 0.1 \pm 1.1$	$52.7 \pm 0.3 \pm 0.5$ [18]	52.5 ± 0.7

can we describe these decays in first-principles QCD ?

charmonium **radiative transitions** in lattice QCD

state of the art circa 2006



heavy quarks +
no annihilation
= clean signals

PHYSICAL REVIEW D **73,** 074507 (2006)

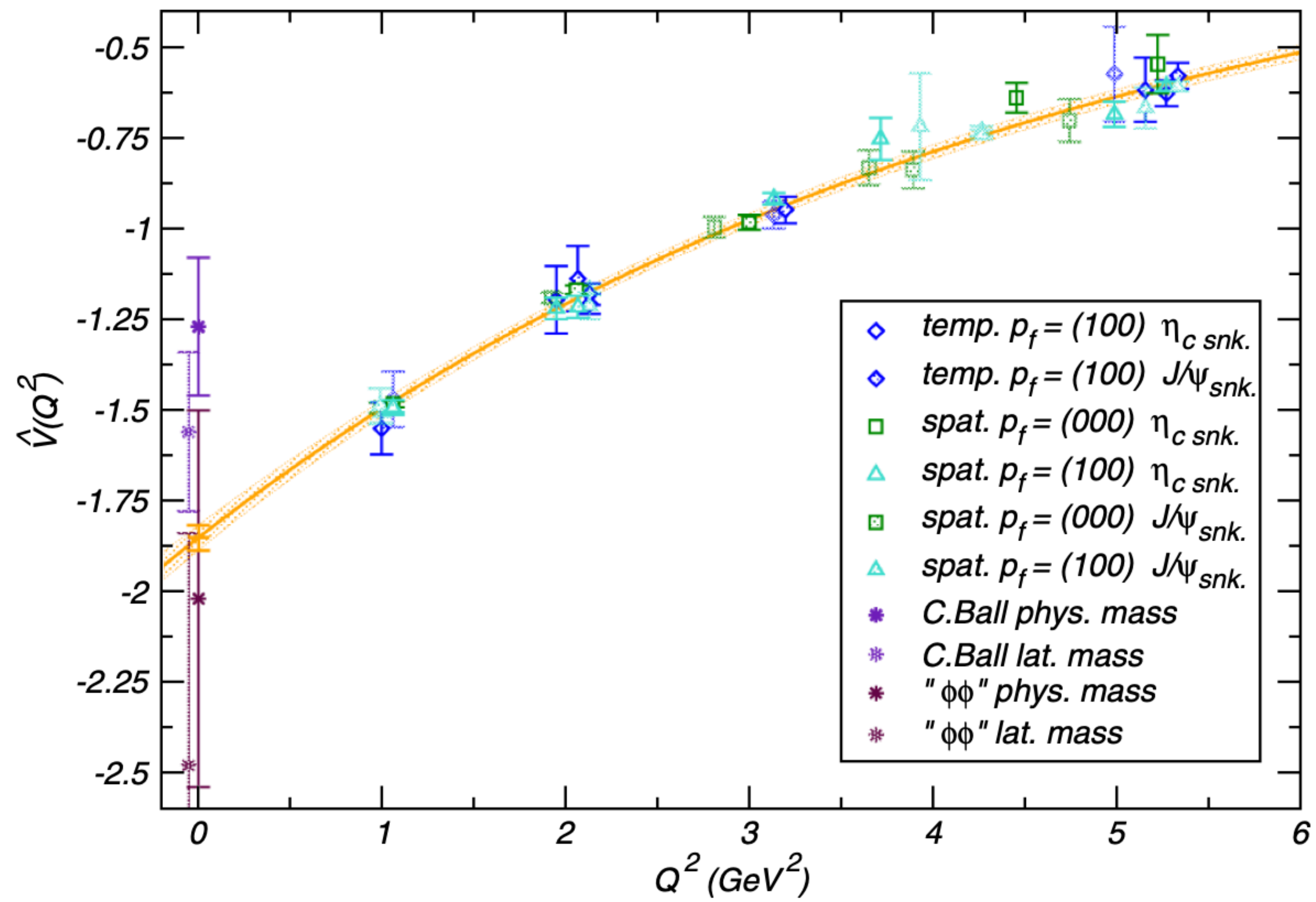
Radiative transitions in charmonium from lattice QCD

Jozef J. Dudek,* Robert G. Edwards, and David G. Richards

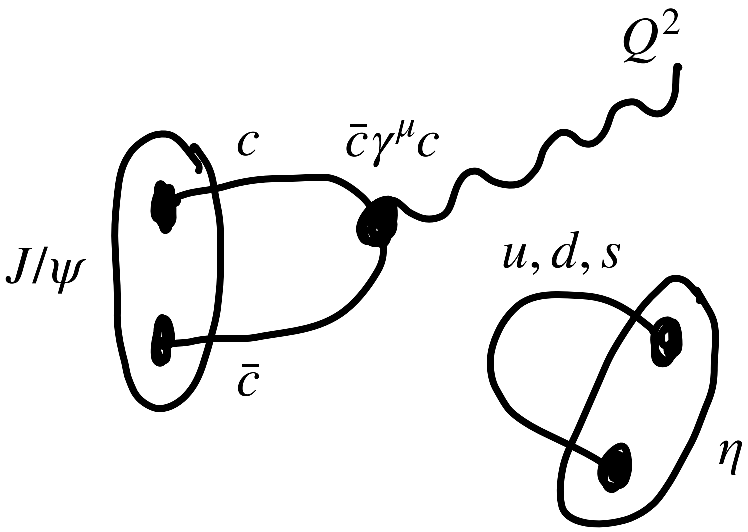
Jefferson Laboratory Mail-Stop 12H2, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

(Received 17 January 2006; published 20 April 2006)

Radiative transitions between charmonium states offer an insight into the internal structure of heavy-quark bound states within QCD. We compute, for the first time within lattice QCD, the transition form factors of various multipolarities between the lightest few charmonium states. In addition, we compute the



remain challenging …



annihilation
= noisy signals

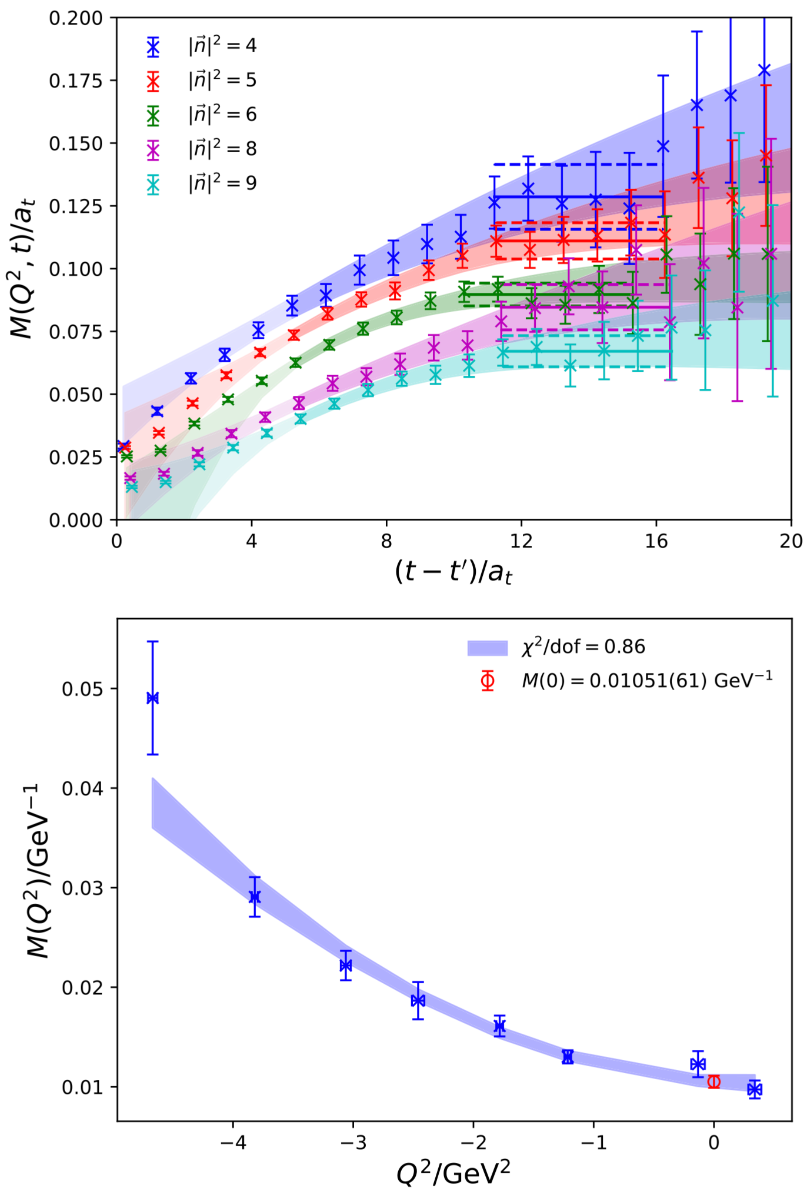
previous state-of-the-art

PHYSICAL REVIEW LETTERS **130,** 061901 (2023)

Radiative Decay Width of $J/\psi \to \gamma\eta_{(2)}$ from $N_f = 2$ Lattice QCD

Xiangyu Jiang,[1,2,*] Feiyu Chen,[1,2] Ying Chen,[1,2,†] Ming Gong,[1,2] Ning Li,[3] Zhaofeng Liu,[1,2,4] Wei Sun,[1] and Renqiang Zhang[1,2]

$m_\pi \sim 350\,\mathrm{MeV}$



two-flavor: ~~η, η'~~

simple operators

limited kinematics

previous state-of-the-art

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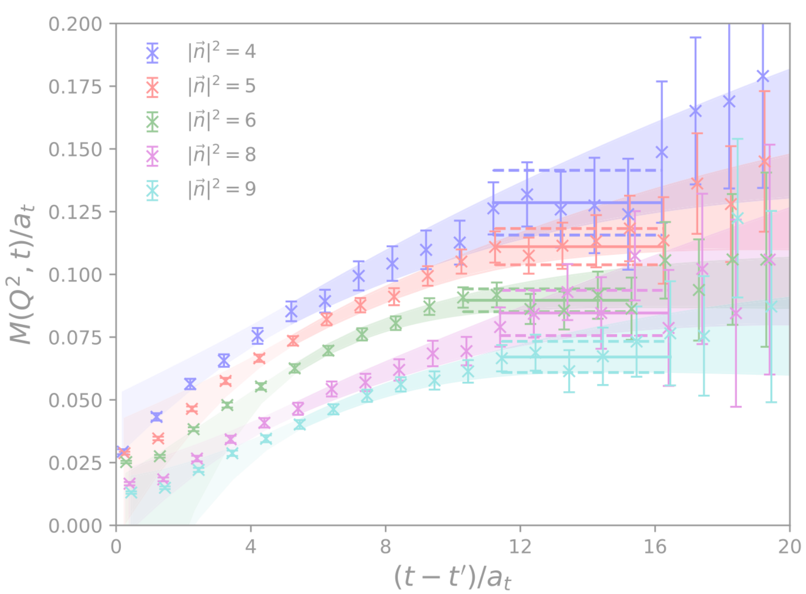
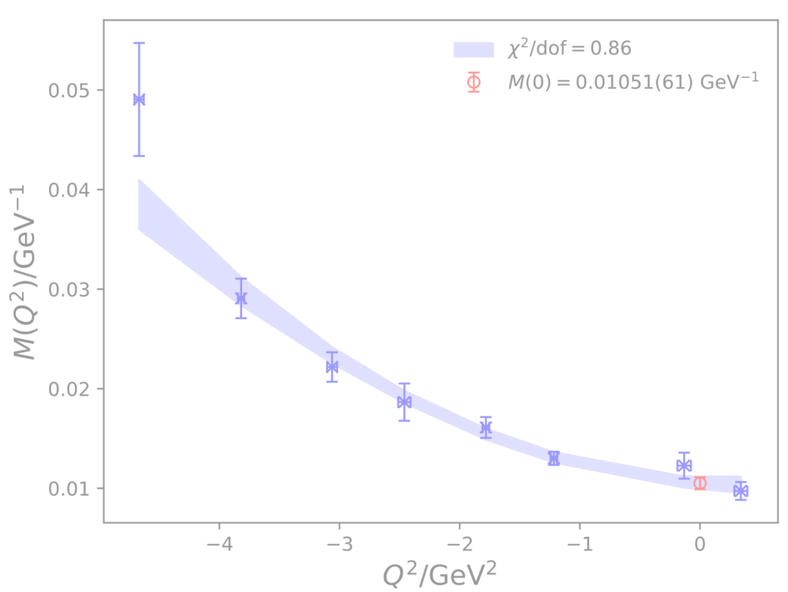




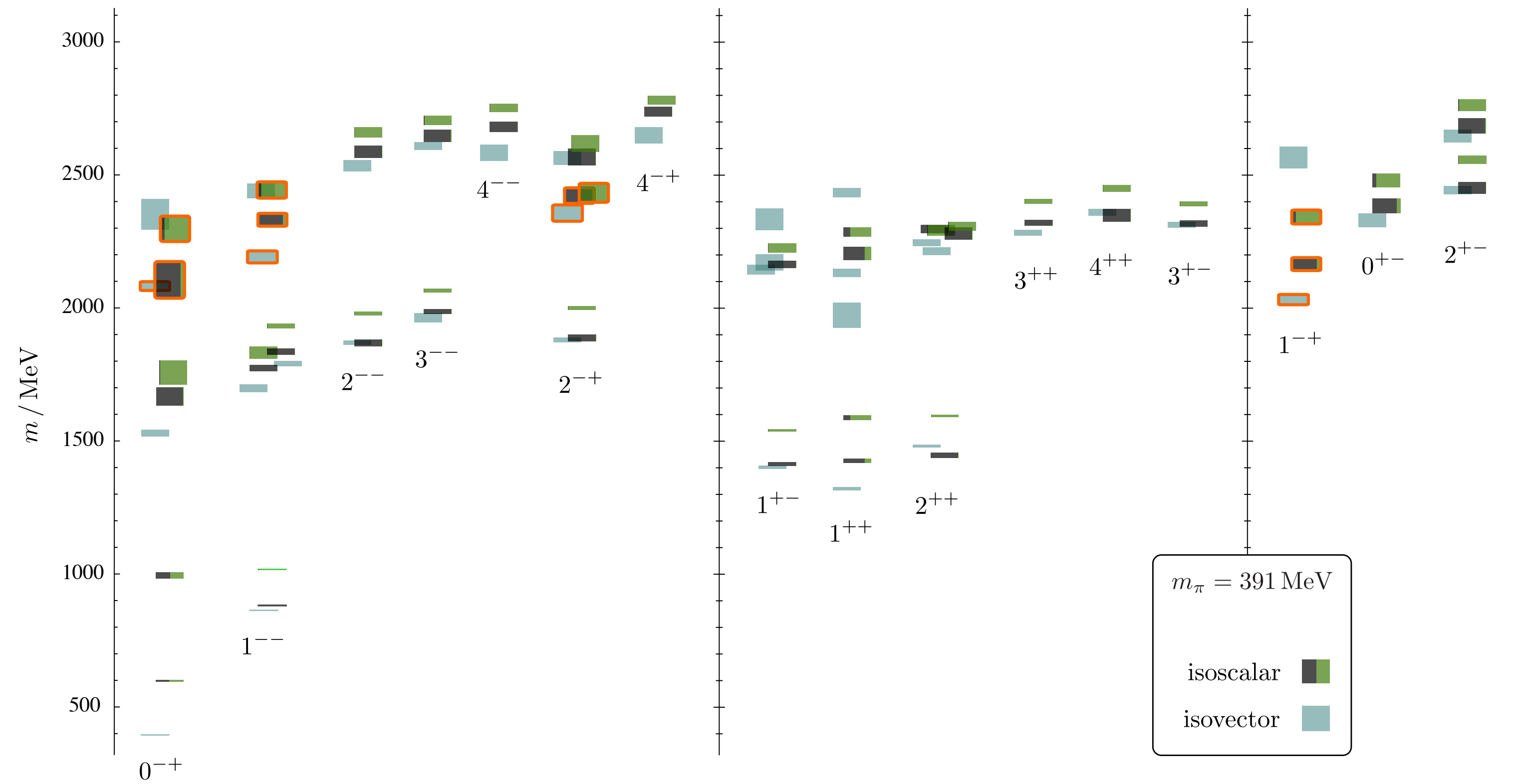
two-flavor: η, η'

simple operators
limited kinematics

new three-flavor calculation

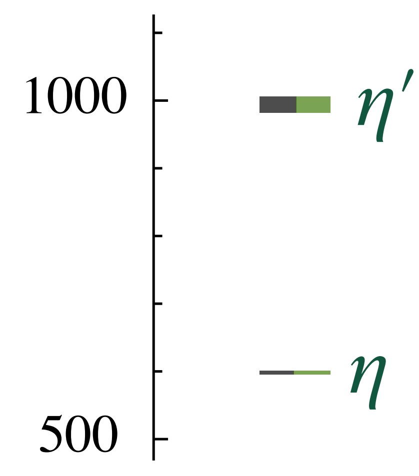
$m_\pi \sim 391$ MeV

PRD 88 094505 (2013)



basis of fermion bilinears

$\bar{u}\Gamma u + \bar{d}\Gamma d$
$\bar{s}\Gamma s$
$\Gamma = \{\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\}$



strong admixtures of hidden-light and hidden strange

lattice QCD approach

previous state-of-the-art

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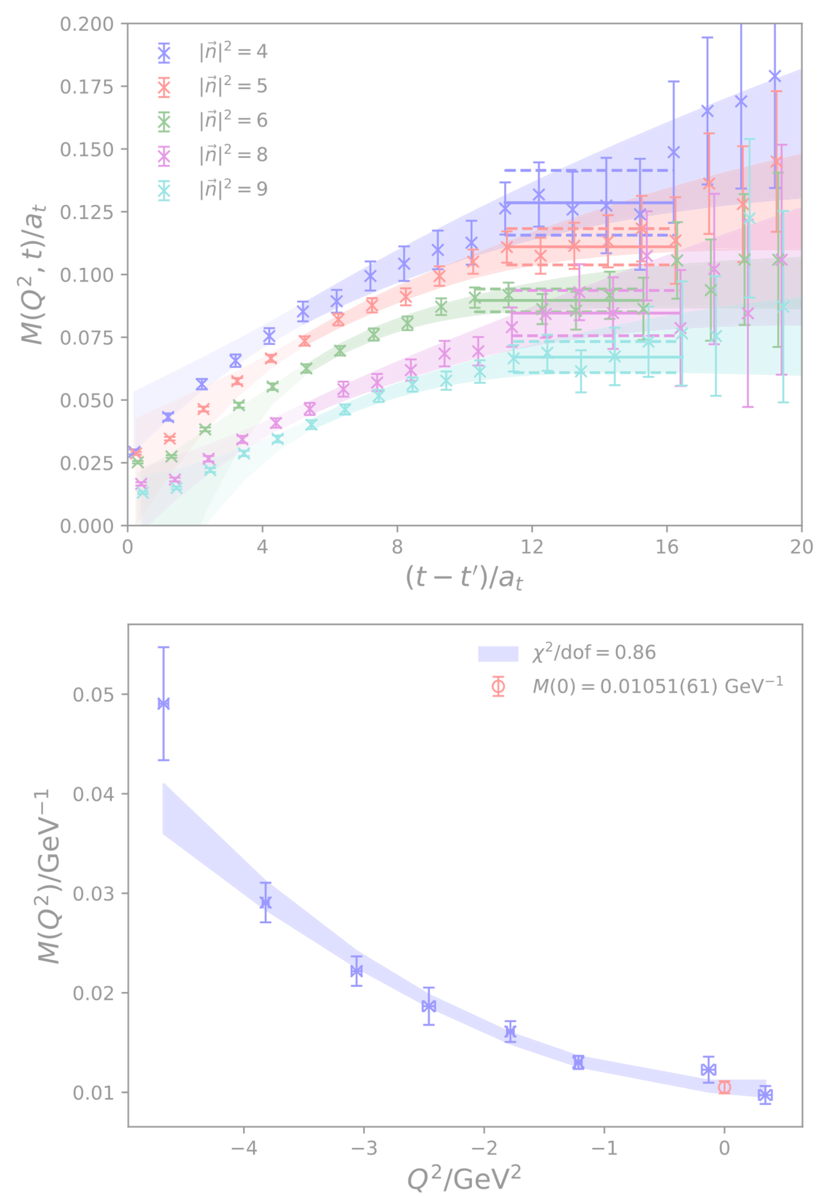

two-flavor: η, η'

simple operators
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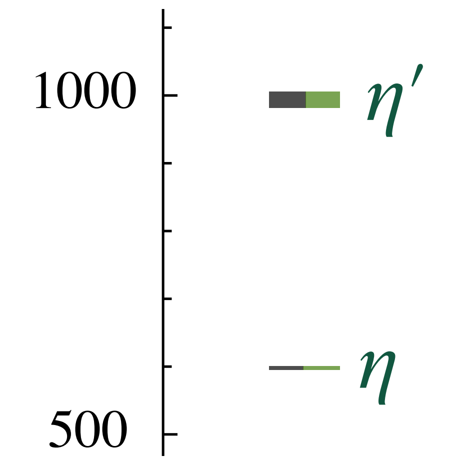
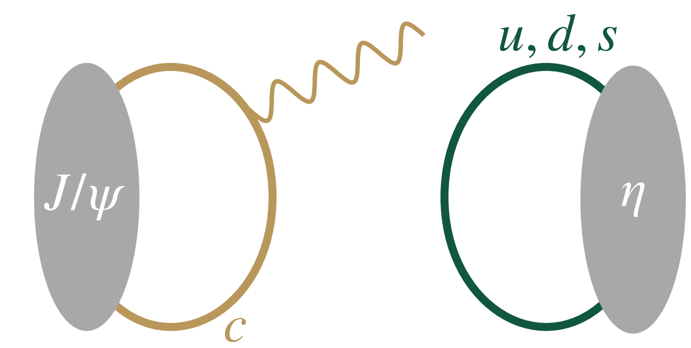


three–point functions with **optimized operators**

$$\langle 0 \,|\, \Omega_\eta(\Delta t)\; j(t)\; \Omega_\psi^\dagger(0) \,|\, 0 \rangle$$

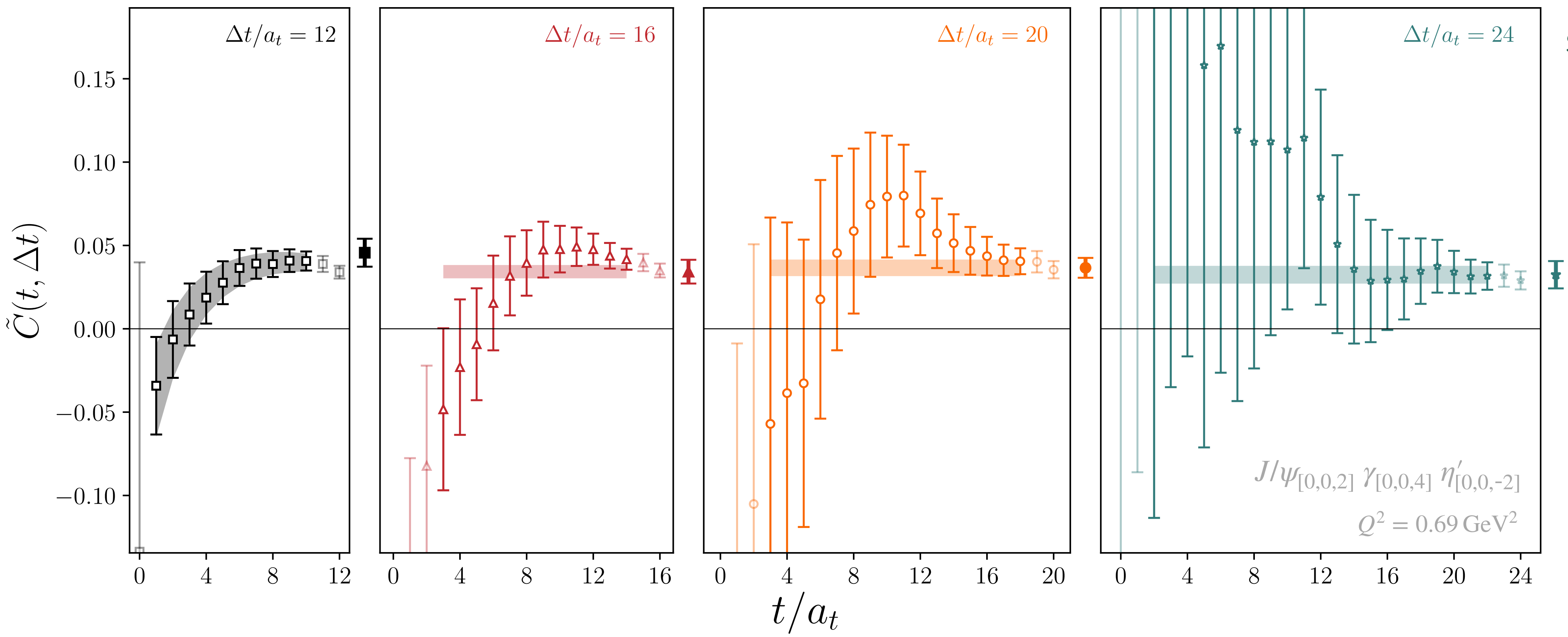
disconnected diagrams



need **large three-momentum** to access $Q^2 \approx 0$

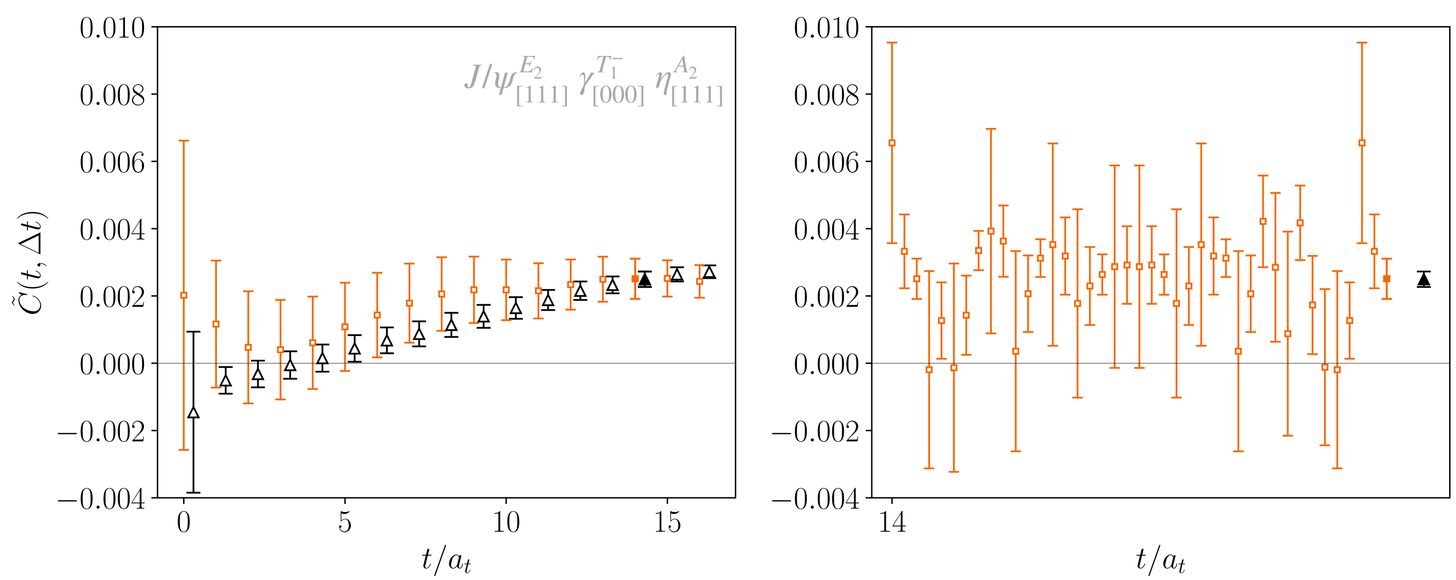
(lattice momentum comes in integer multiples of $\frac{2\pi}{L}$)

three–point functions

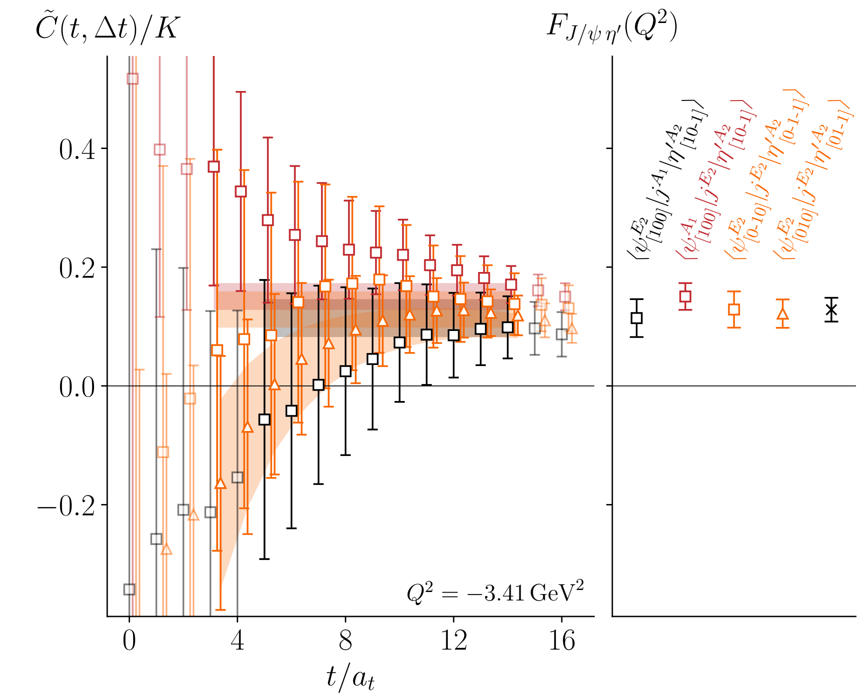


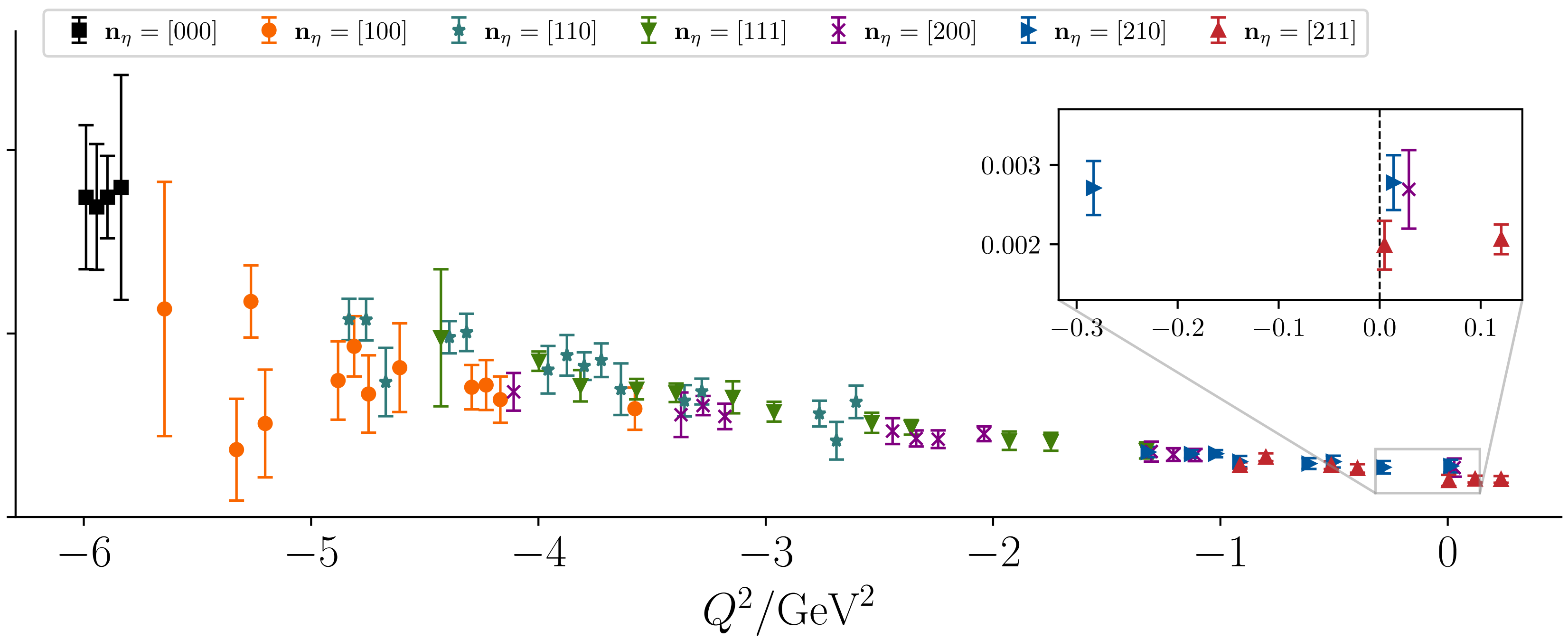
source–sink dependence under control

average over lattice–equivalent kinematics improves signals

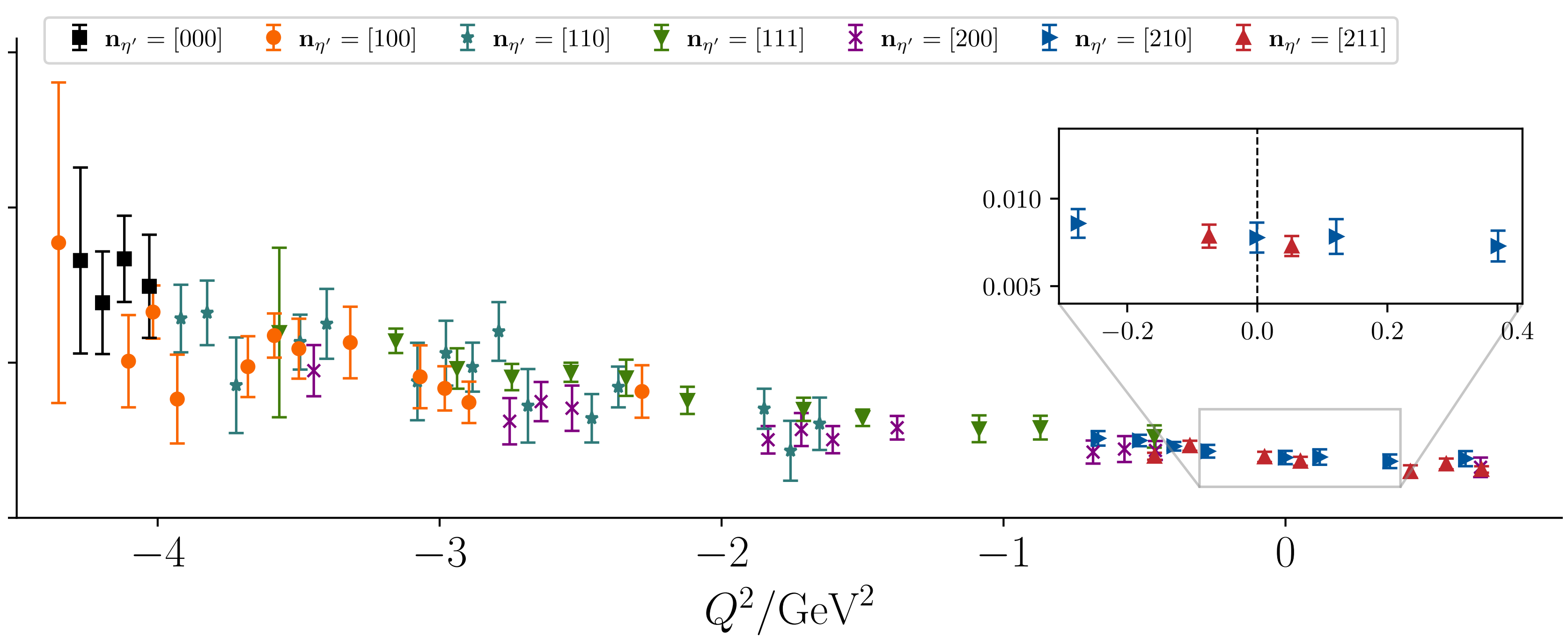


no significant Lorentz symmetry breaking





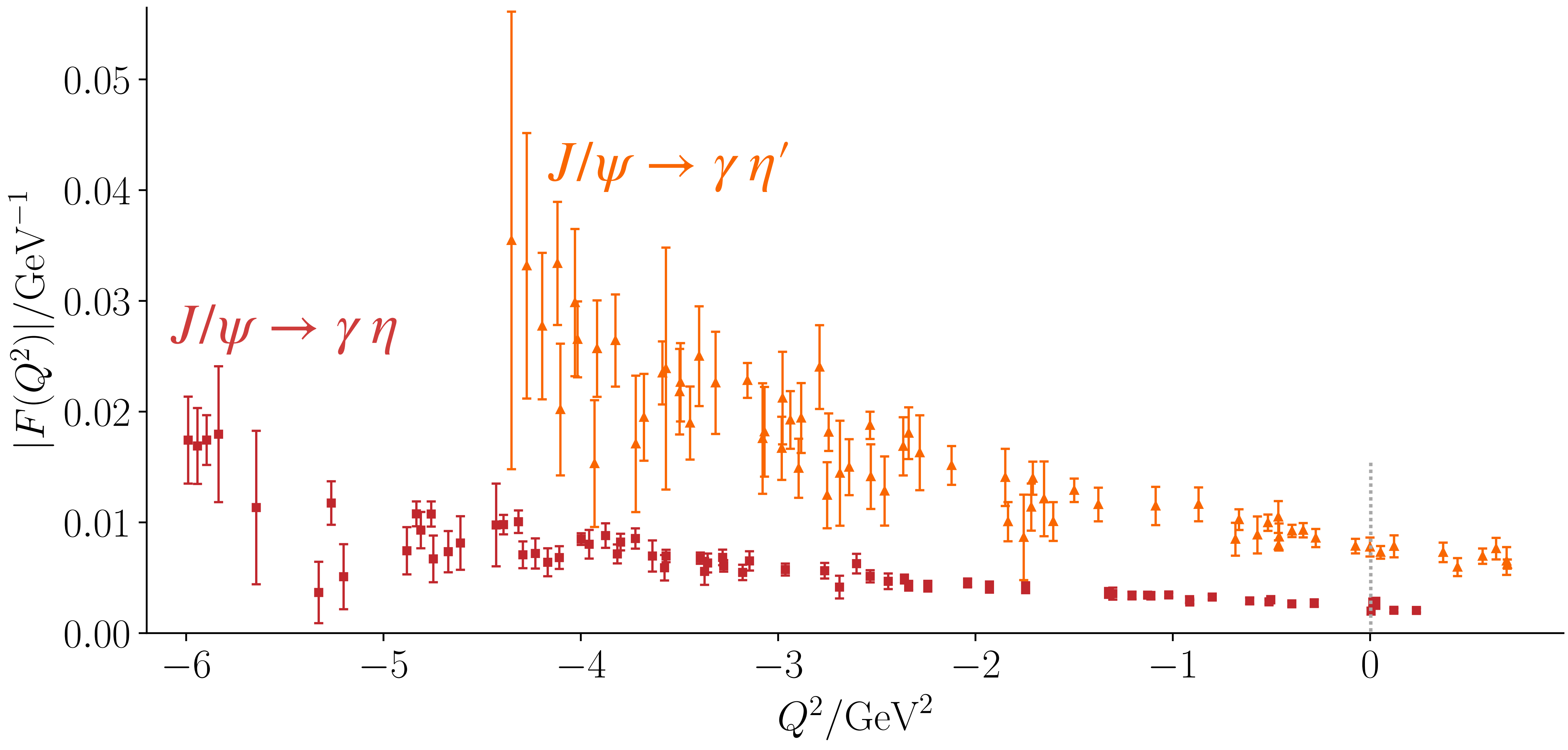
$J/\psi \to \gamma\, \eta$



$J/\psi \to \gamma\, \eta'$

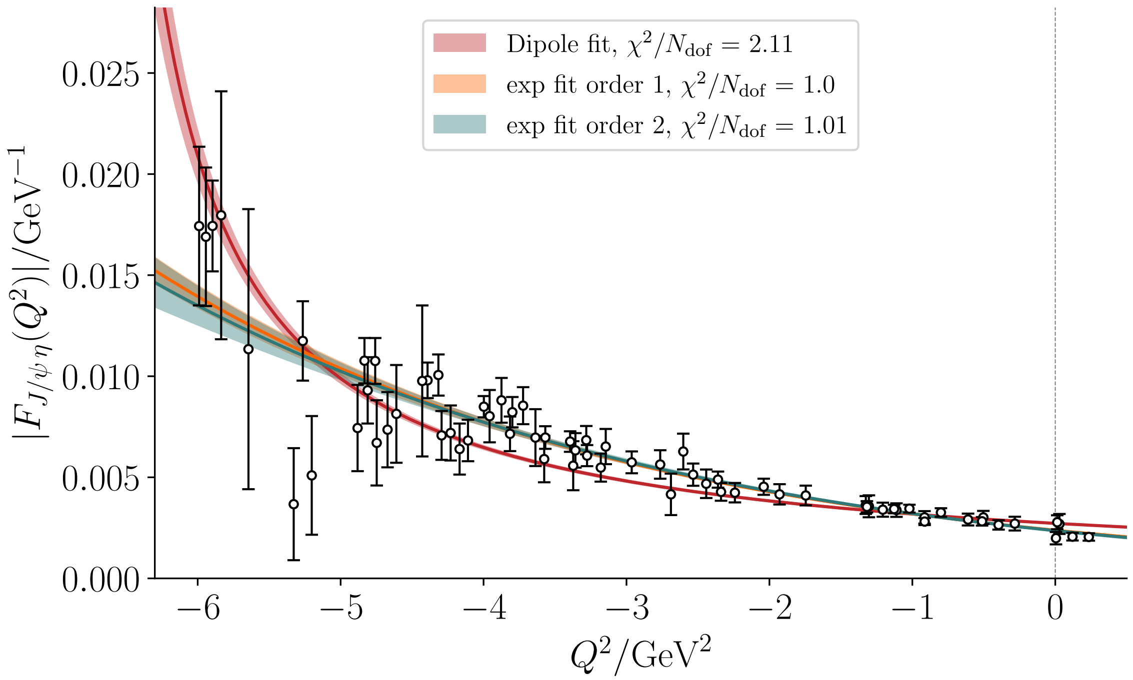
transition form–factors

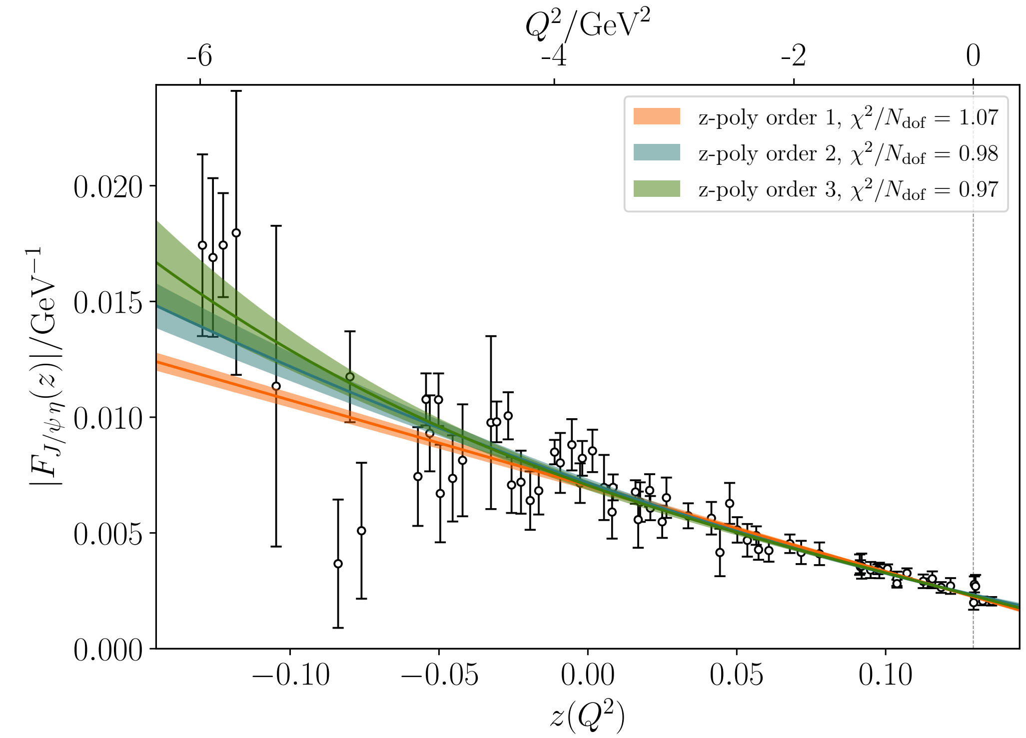
$\langle \eta(\mathbf{p}') \,|\, j^{\mu}_{\mathrm{em}}(0) \,|\, J/\psi(\mathbf{p},\lambda) \rangle = \epsilon^{\mu\nu\rho\sigma}\, p'_{\nu}\, p_{\rho}\, \epsilon_{\sigma}(\mathbf{p},\lambda)\, F(Q^2)$



describing Q^2 dependence

$J/\psi \to \gamma\,\eta$





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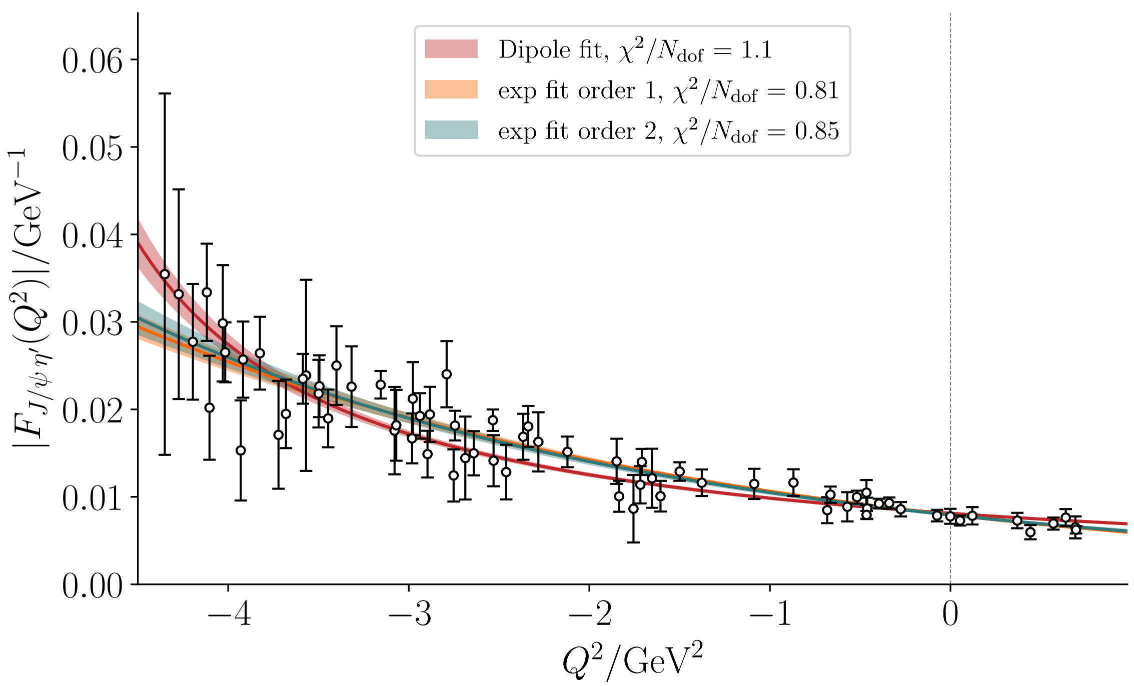
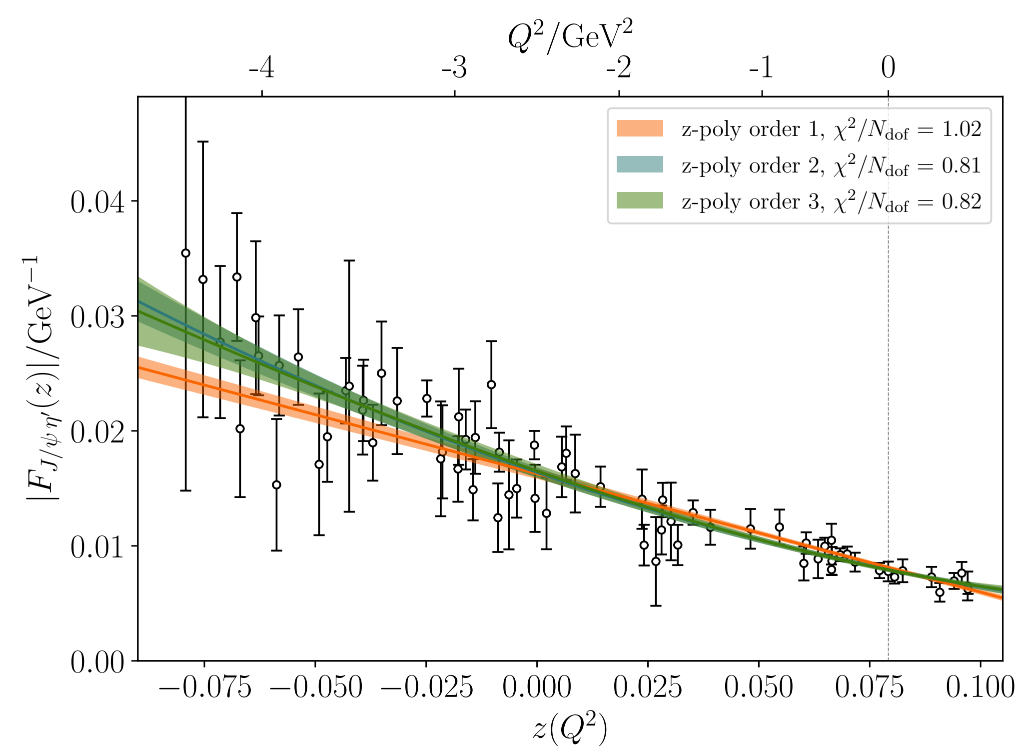


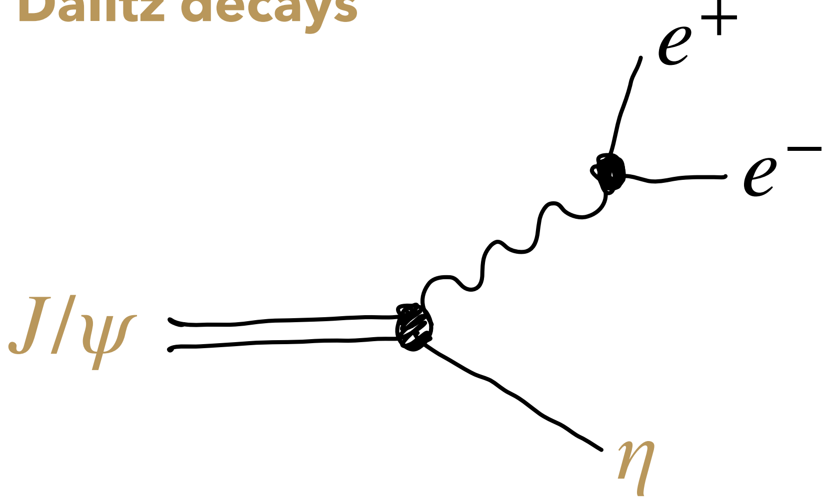


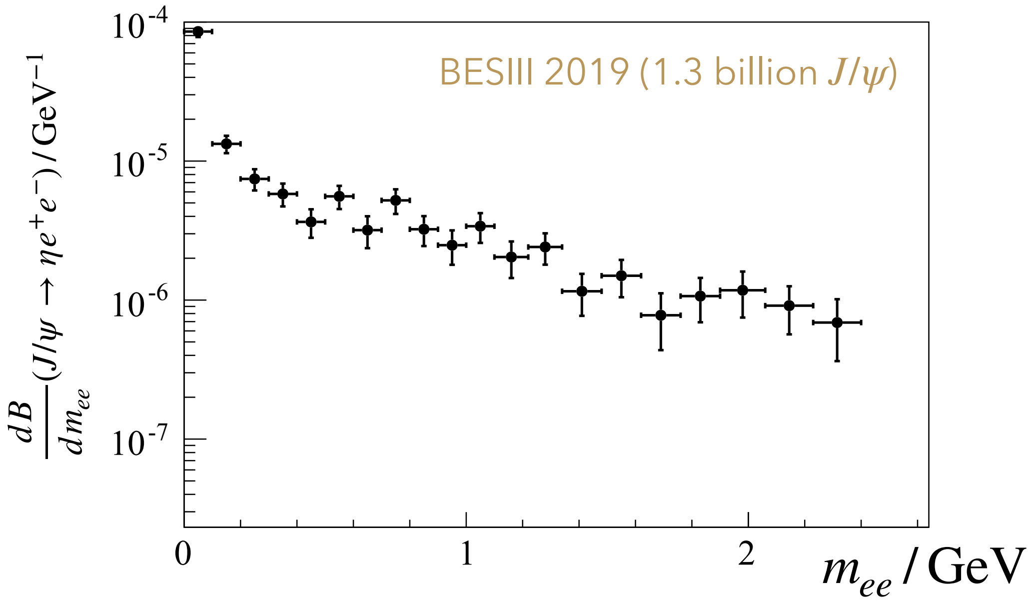
conformal mapping

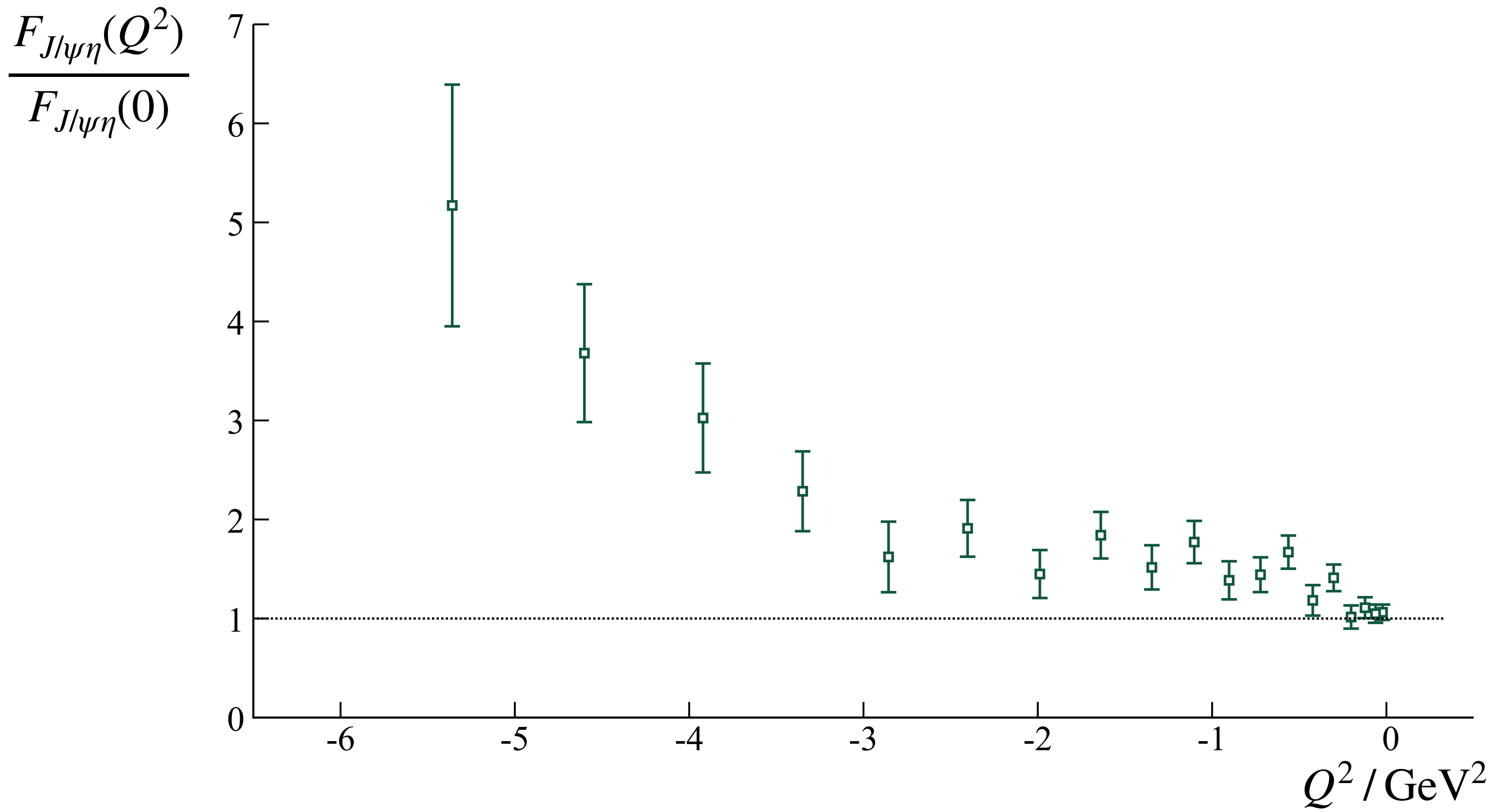
$$z(Q^2) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}} \qquad F(Q^2) = \sum_{n=0}^{k} a_n z^n$$

is the **timelike** virtuality accessible experimentally ?

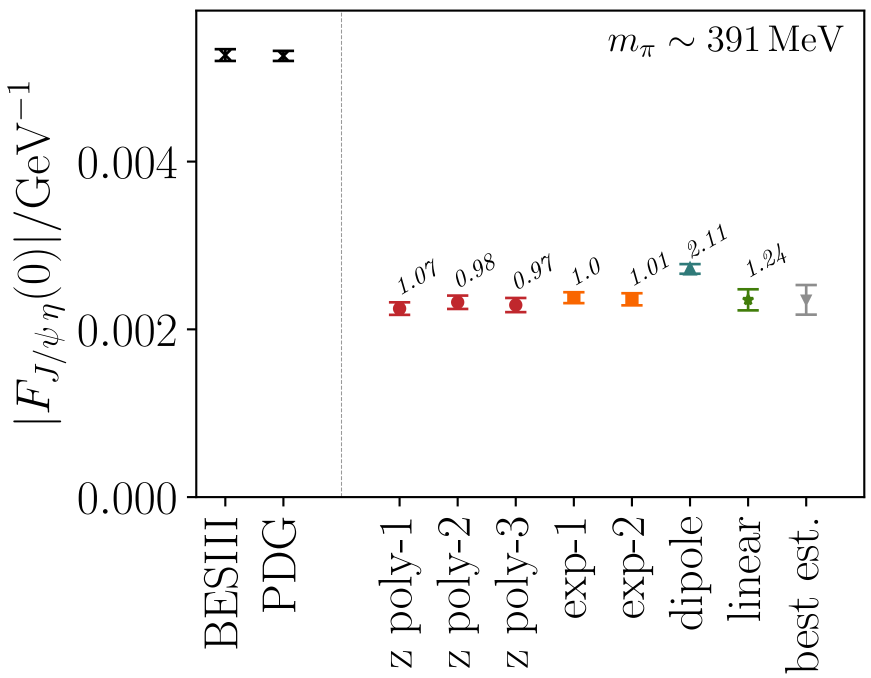
yes, in **Dalitz decays**



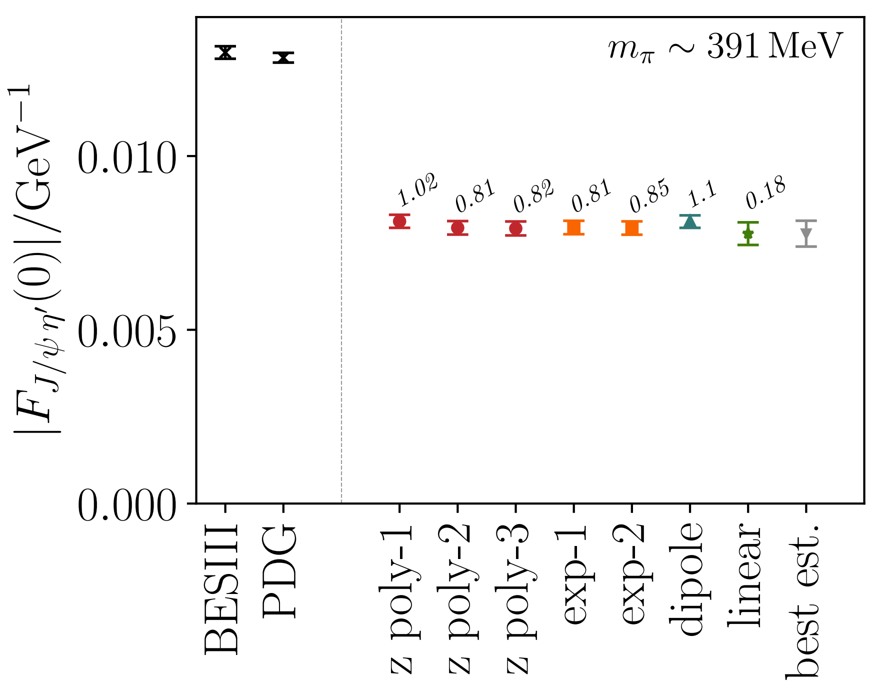




$J/\psi \to \gamma\,\eta$



$J/\psi \to \gamma\,\eta'$



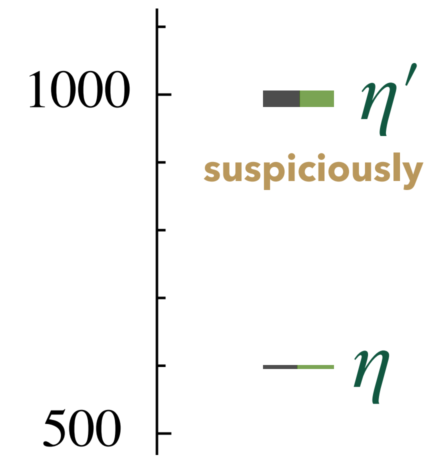
good statistical precision

controllable systematics under control

significant discrepancy with experiment

quark mass dependence ? $m_\pi \sim 391\,\mathrm{MeV}$

role of gluon field topology ? $U(1)_A$

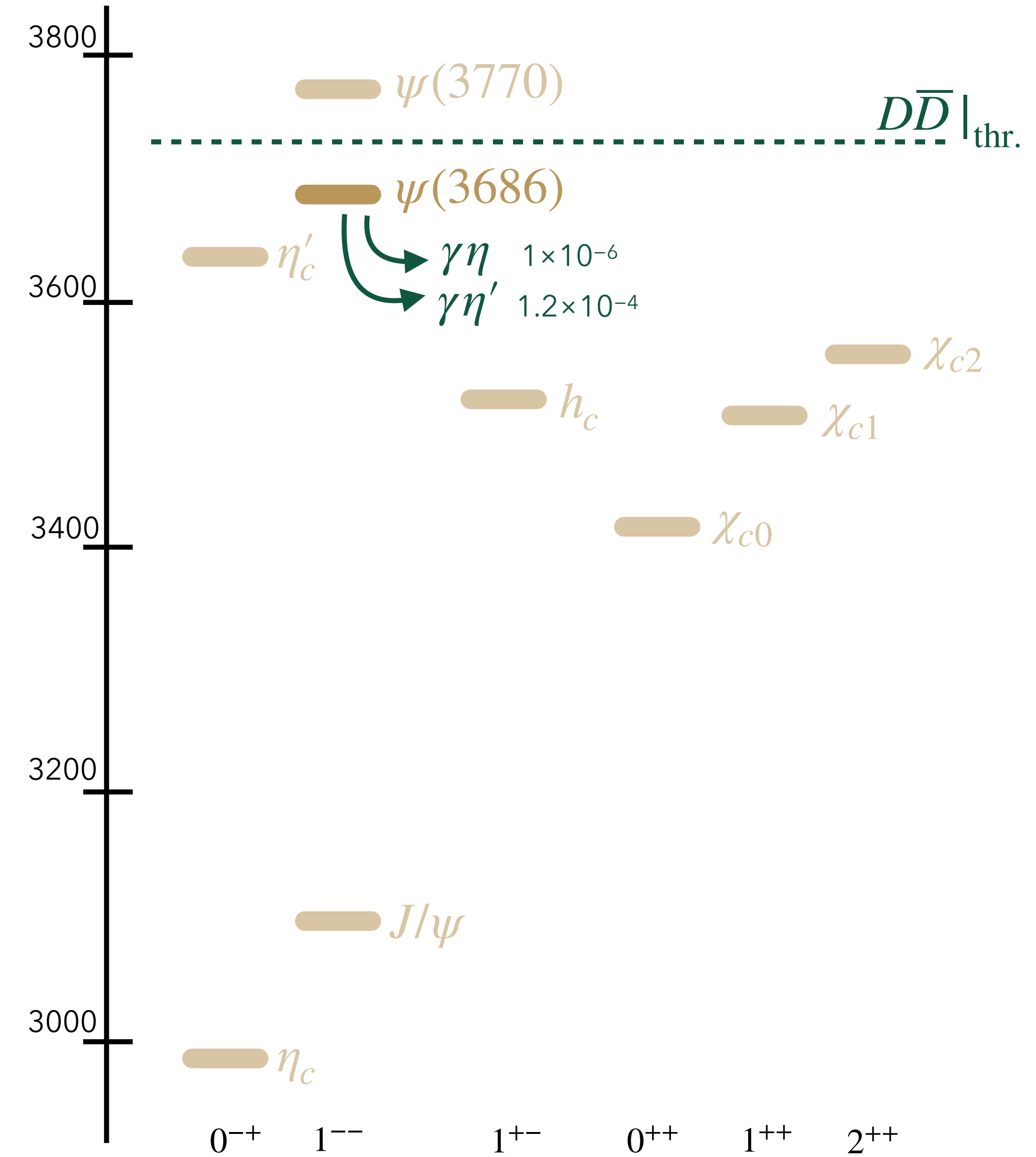


suspiciously light?

future: try same technology on different lattices

other charmonia ?

BESIII also has **3 billion** $\psi(3686)$



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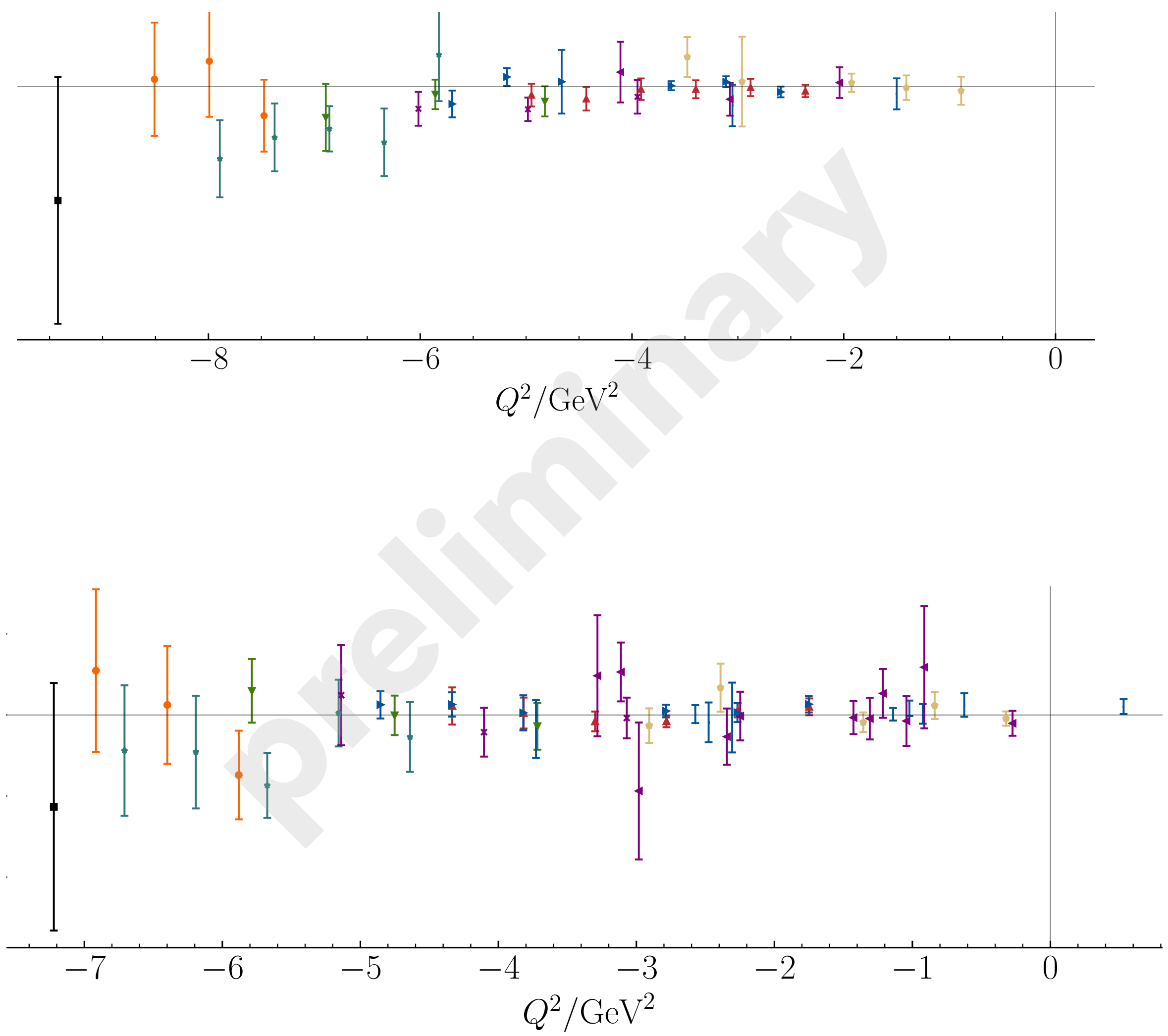

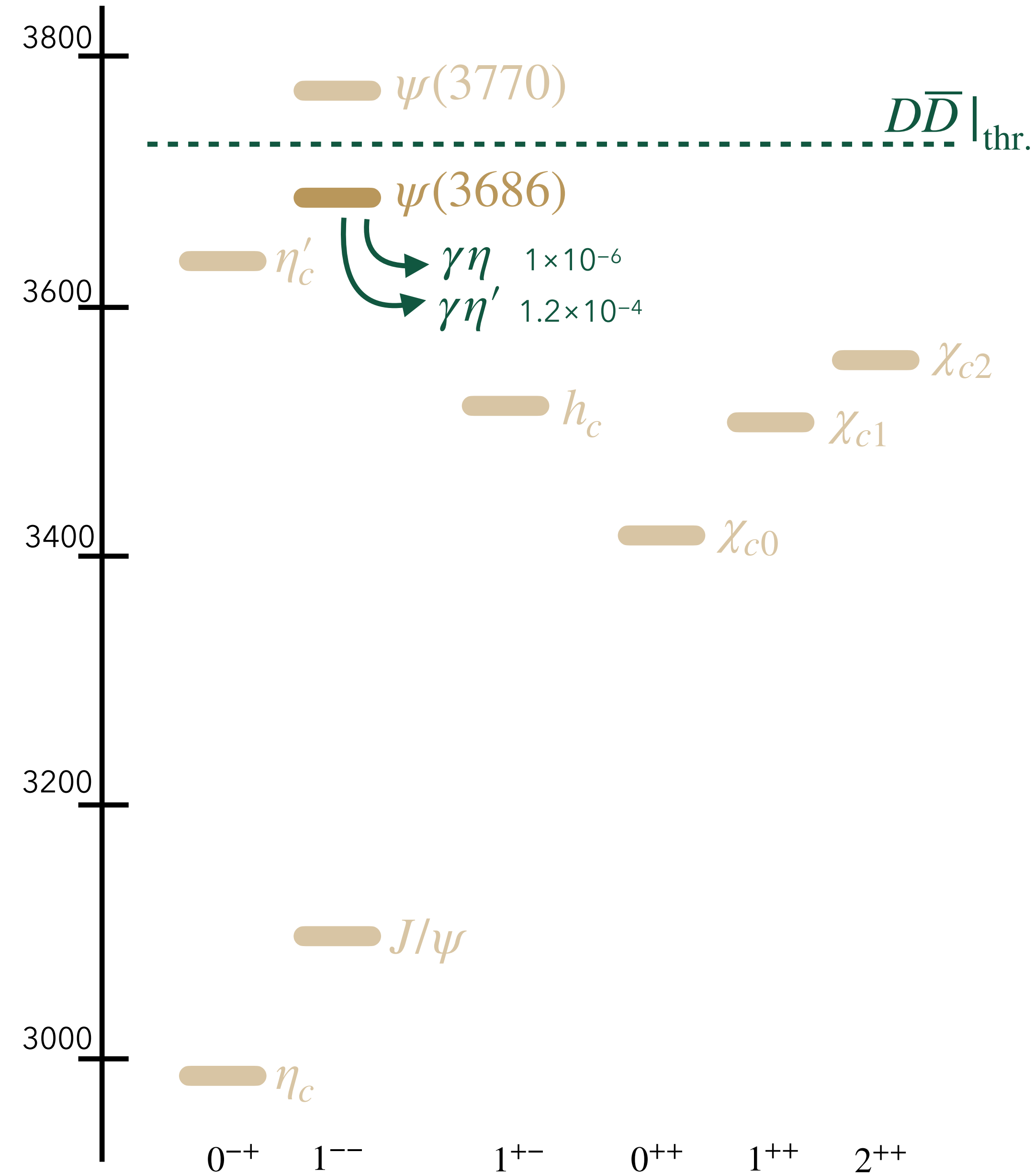

lattice statistics not currently enough to resolve these very weak decays

other charmonia ?

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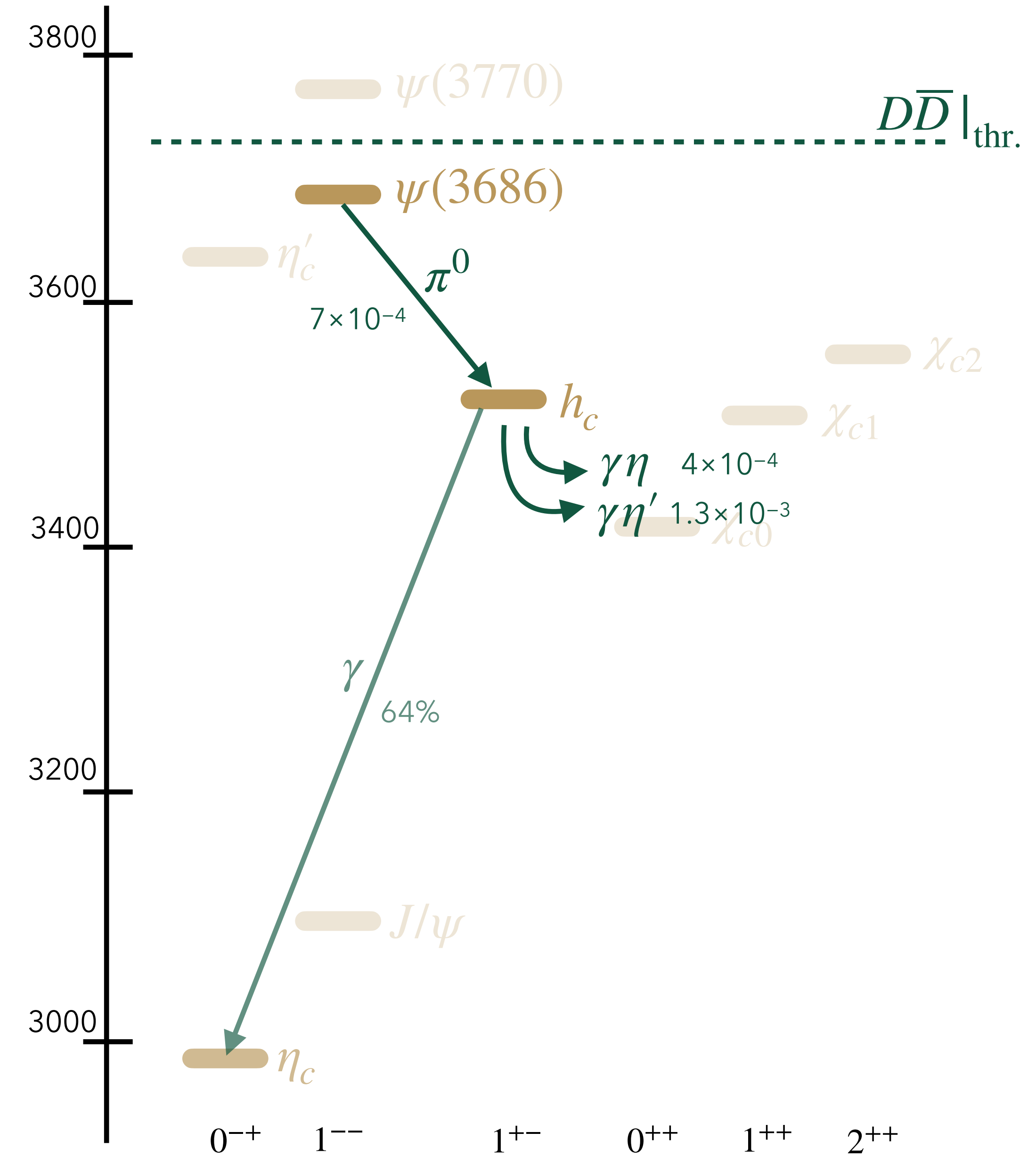
have produced a significant number of h_c



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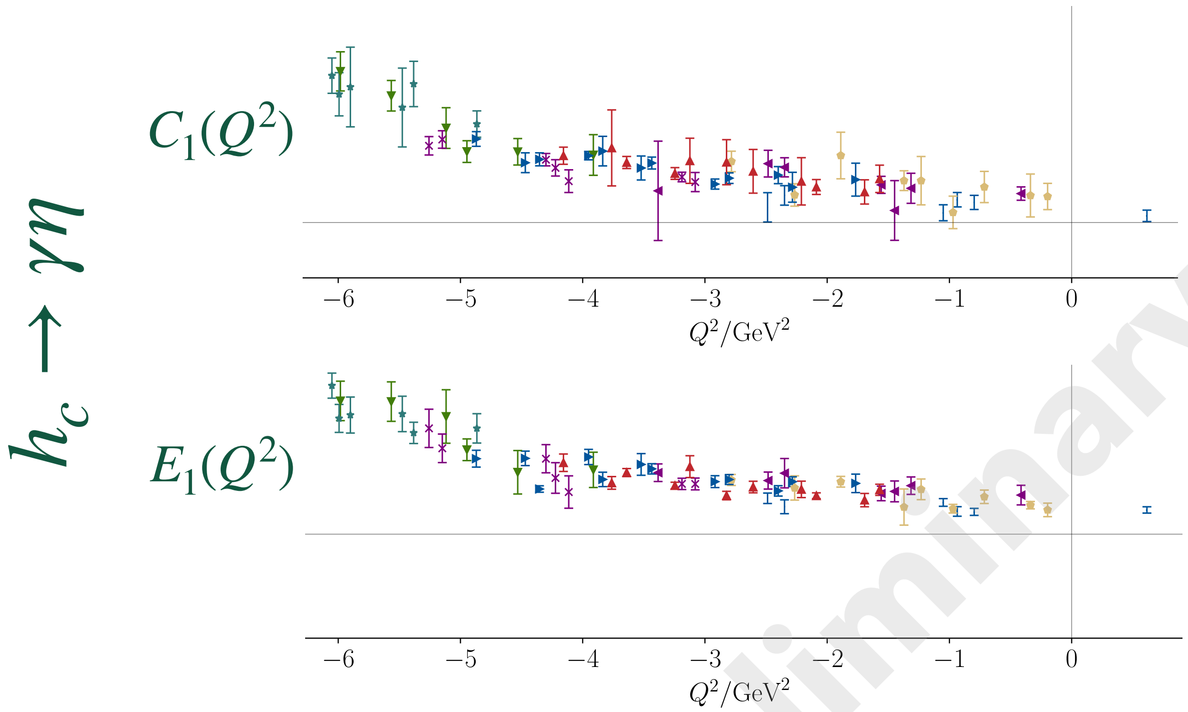
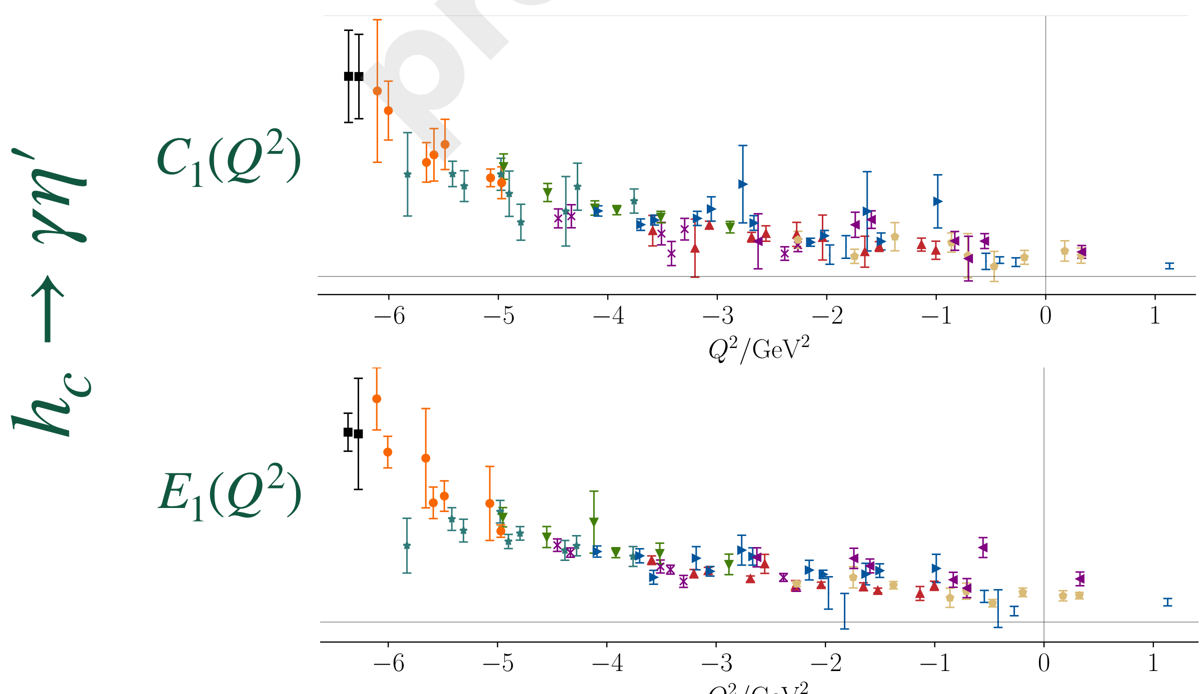
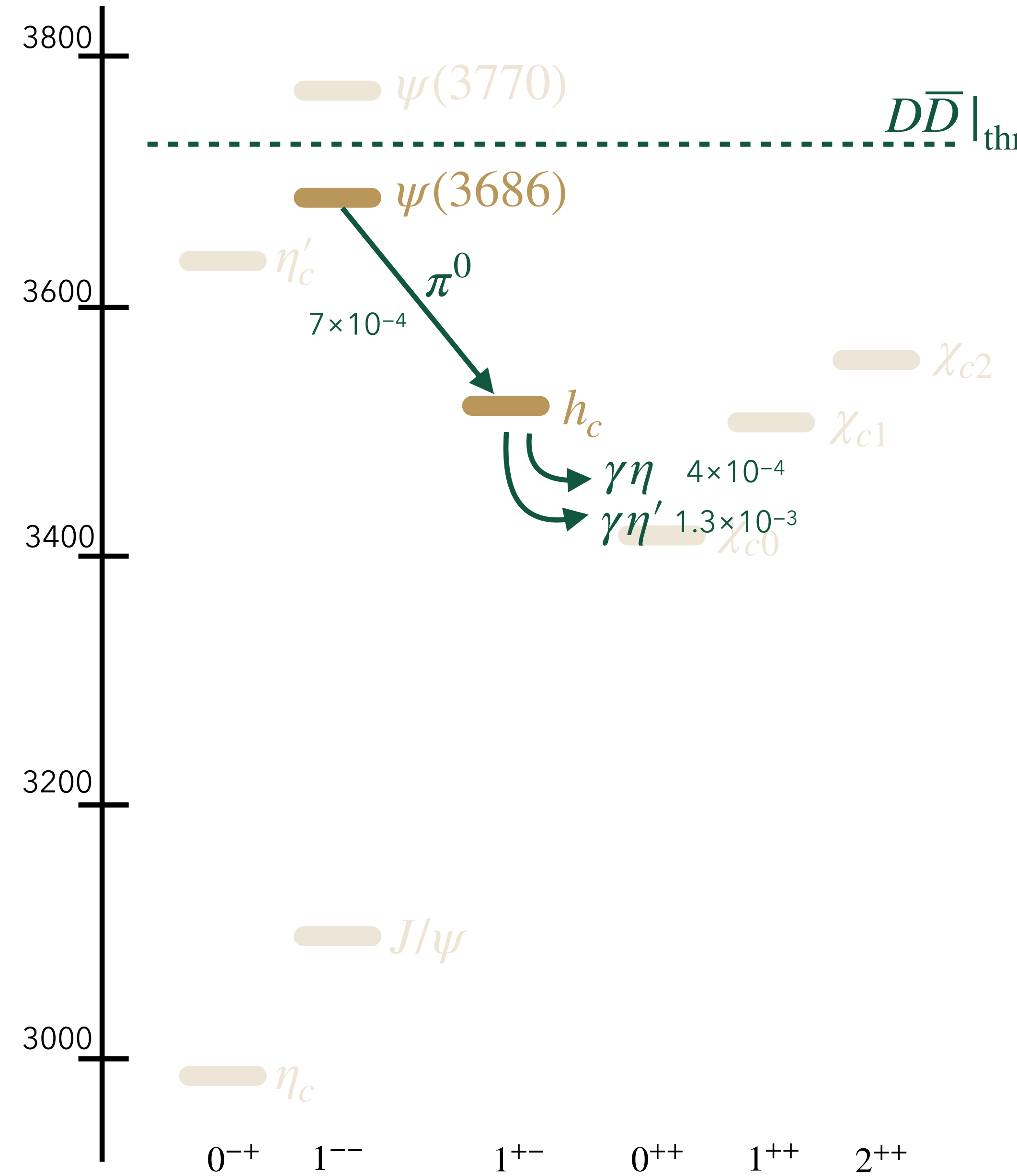


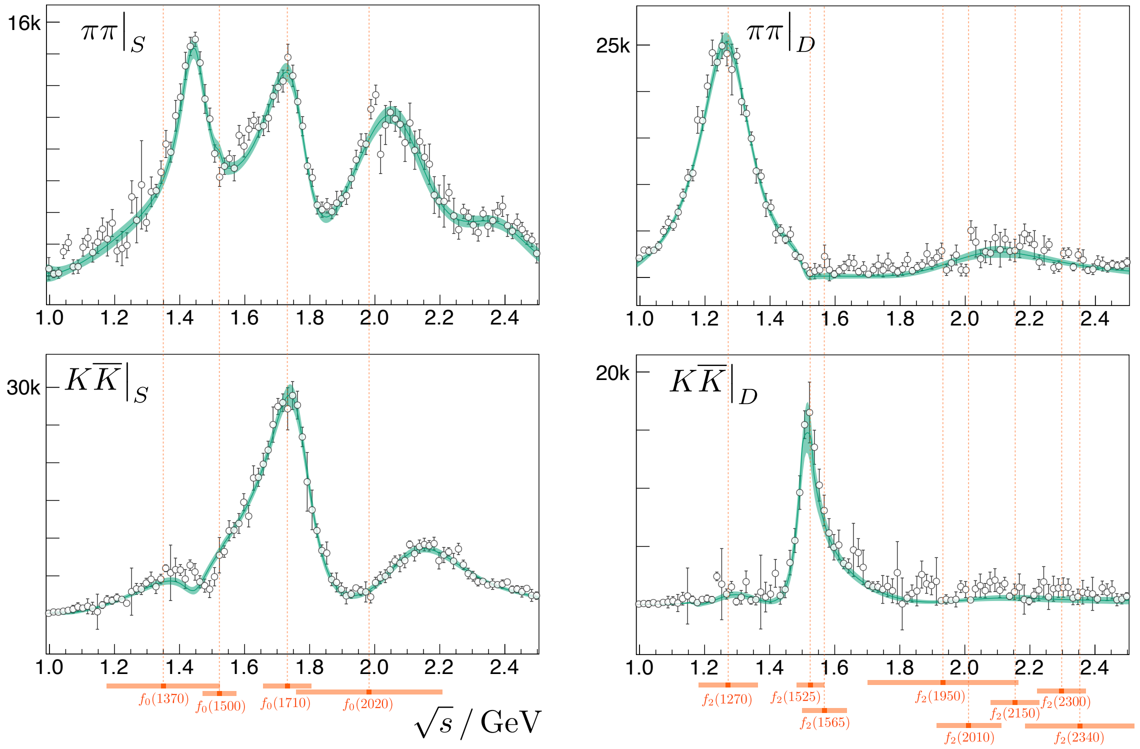




extensions of the technology

compute scalar and tensor **resonance** production

$$J/\psi \to \gamma\,(\pi\pi, K\overline{K}, \ldots)$$



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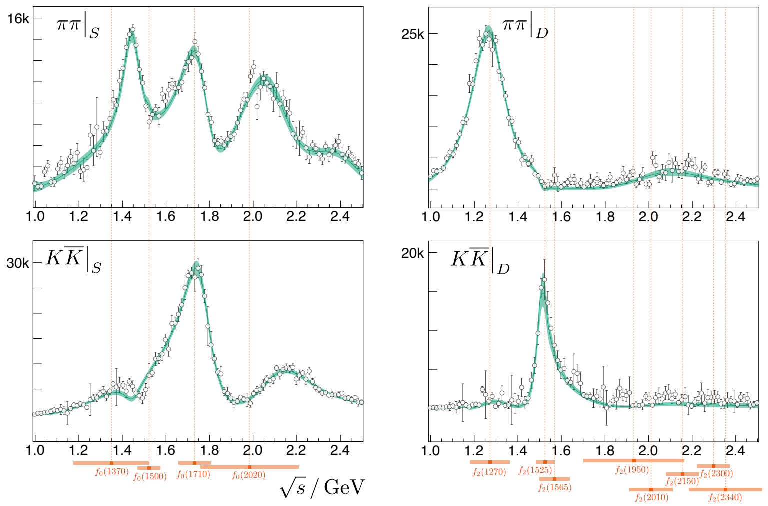


f_0, f_2 resonances previously studied in coupled-channel scattering using lattice QCD finite-volume techniques

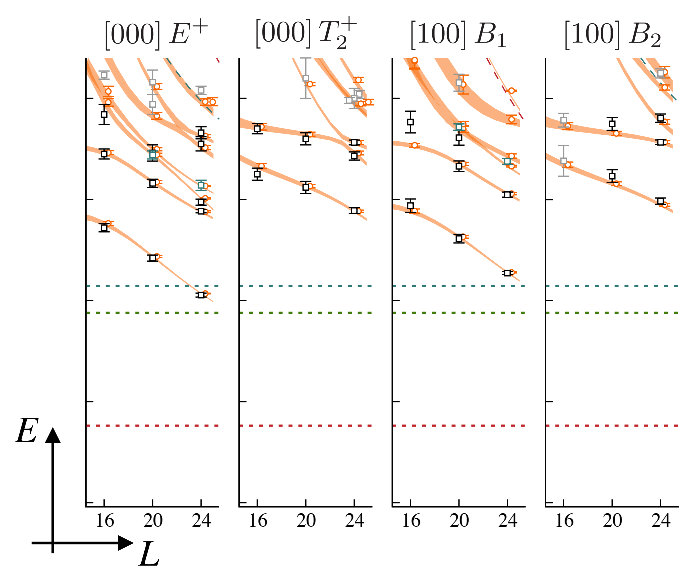
PHYSICAL REVIEW D **97,** 054513 (2018)

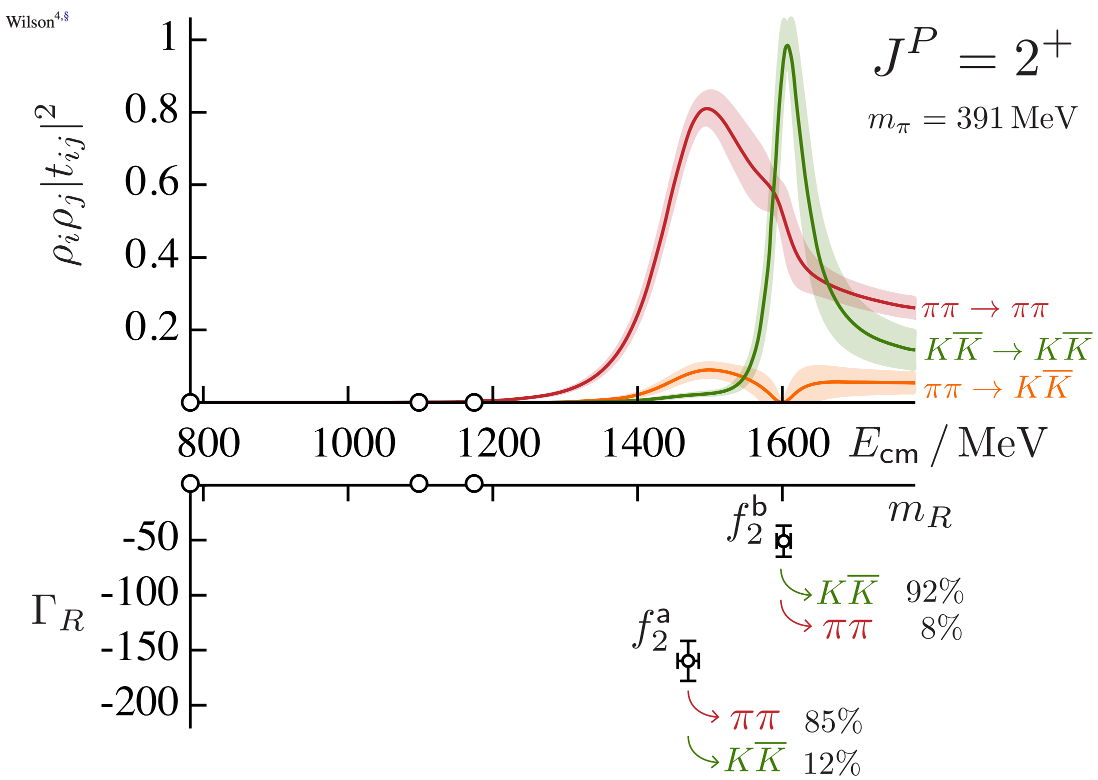
Editors' Suggestion

Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]

(for the Hadron Spectrum Collaboration)



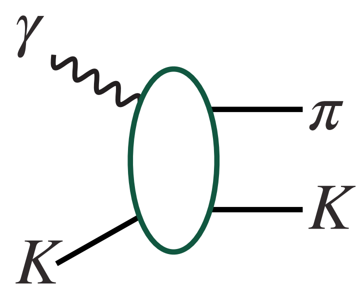


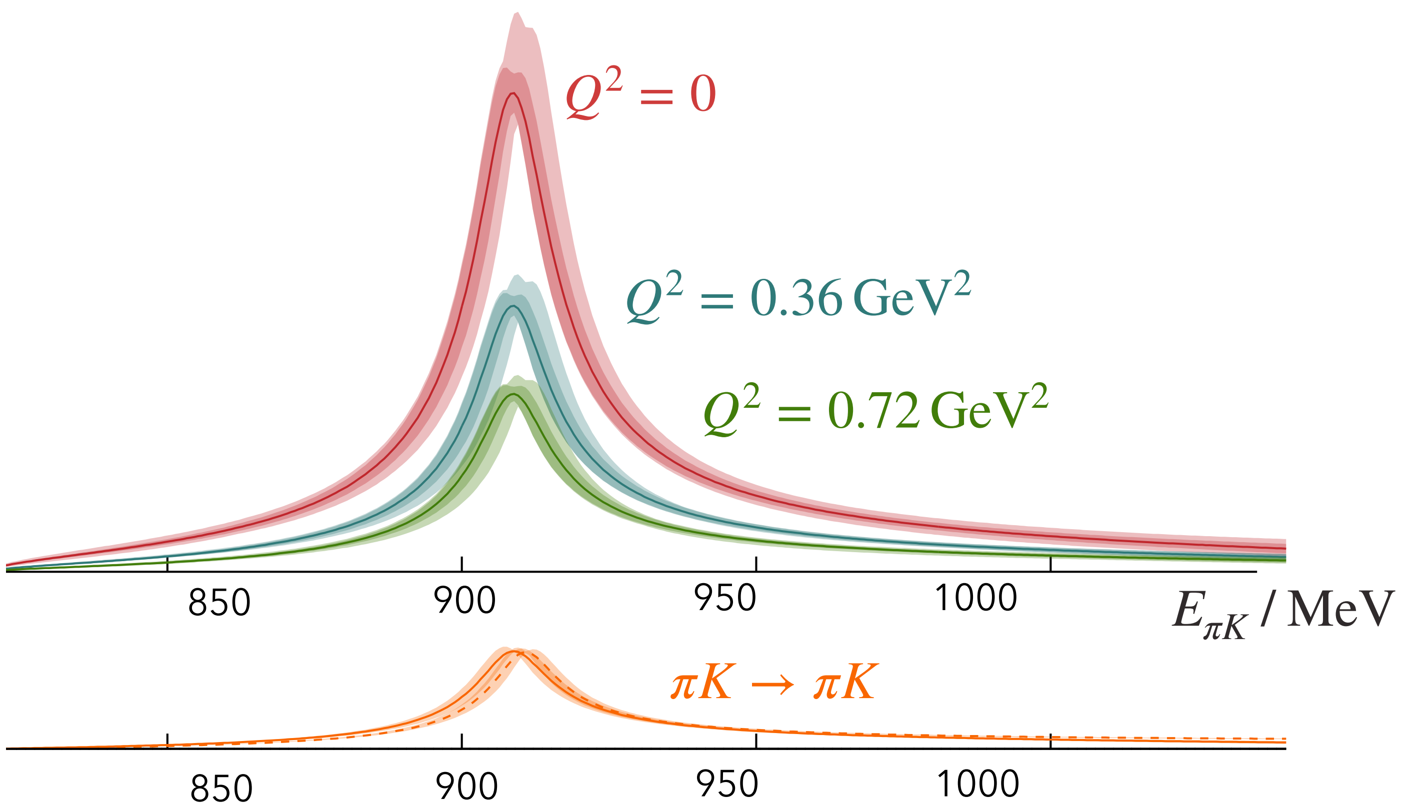
build up systematically toward computation of $J/\psi \to \gamma\,\eta\eta'$ and other final states

coupling resonances to currents

PRD 106 114513 (2022)

$\gamma K \to \pi K$



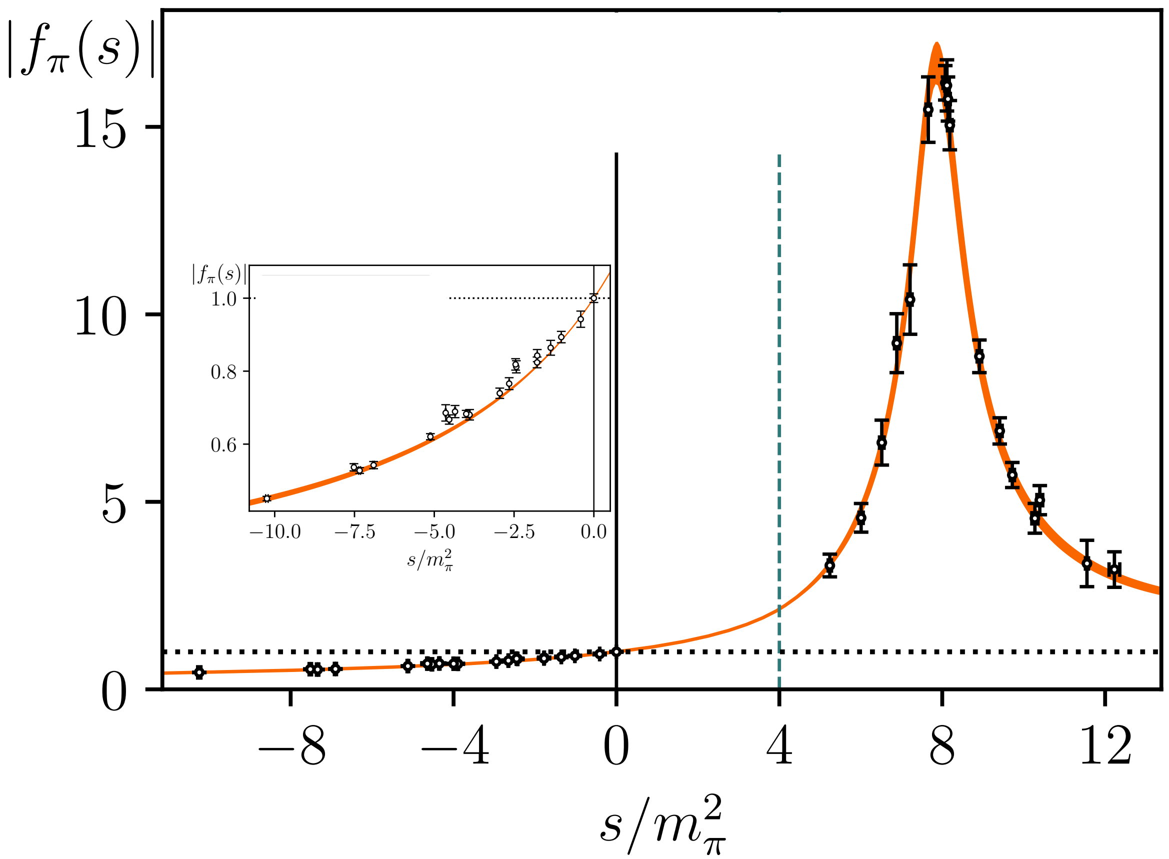


$J^P = 1^- \; K^*$ resonance appears

$m_\pi \sim 283$ MeV

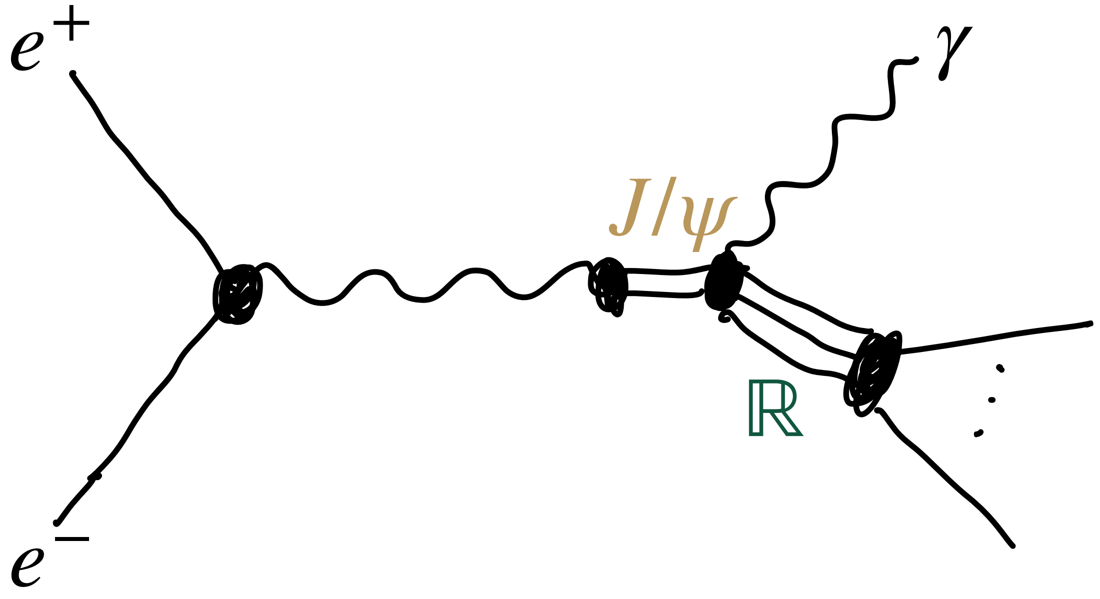
PRD 110 094505 (2024)

$e^+e^- \to \pi\pi$
& $e^-\pi \to e^-\pi$



$J^P = 1^- \; \rho$ resonance appears

$m_\pi \sim 283$ MeV



PHYSICAL REVIEW LETTERS **135,** 161904 (2025)

η and η' Production in J/ψ Radiative Decays from Quantum Chromodynamics

PHYSICAL REVIEW D **112,** 074505 (2025)

η and η' meson production in J/ψ radiative decays from lattice QCD

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hadspec
hadron spectrum collaboration
hadspec.org

kinematic structure of the matrix elements

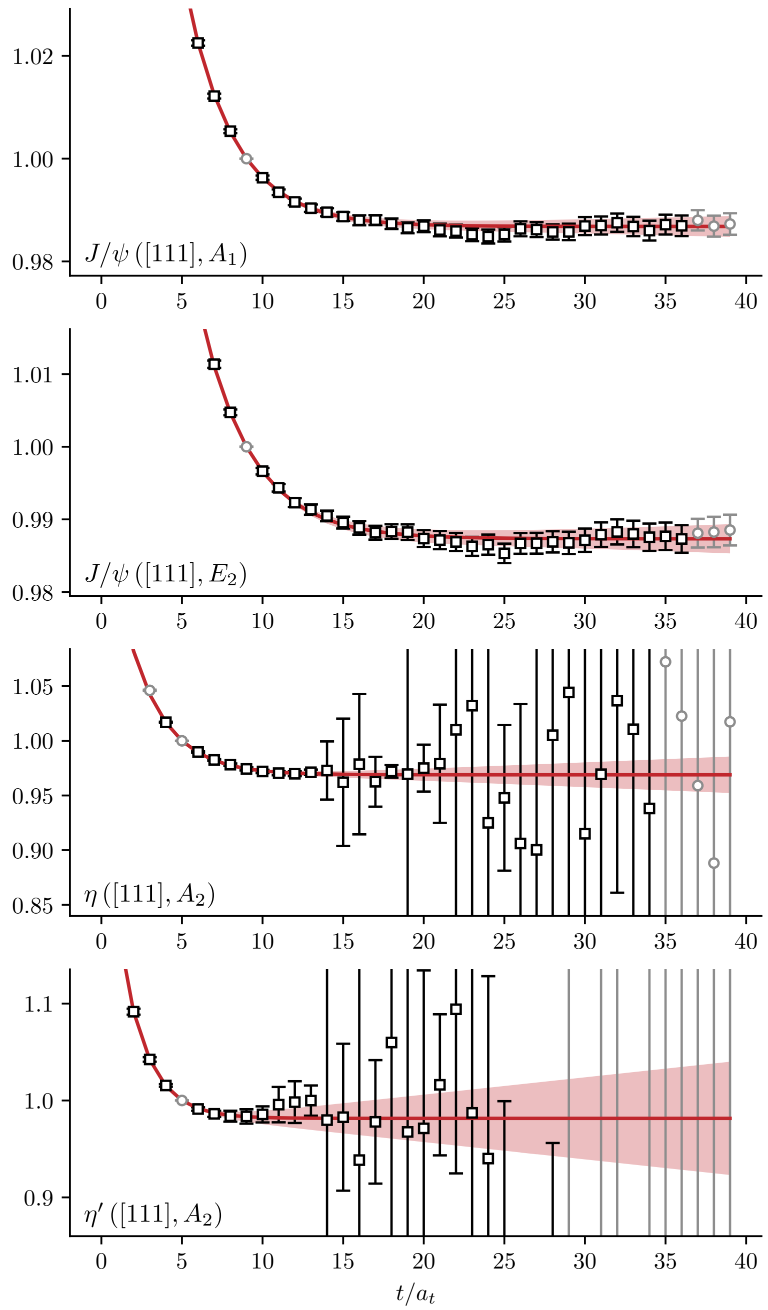
$$\langle \eta(\mathbf{p}') \,|\, j^{\mu}_{\mathrm{em}}(0) \,|\, J/\psi(\mathbf{p},\lambda) \rangle = \epsilon^{\mu\nu\rho\sigma}\, p'_{\nu}\, p_{\rho}\, \epsilon_{\sigma}(\mathbf{p},\lambda)\; F(Q^2)$$

$$\Gamma(J/\psi \to \gamma\eta) = \tfrac{4}{27}\alpha\, |\,\mathbf{q}\,|^{3}\, |\, F(0)\, |^{2}$$

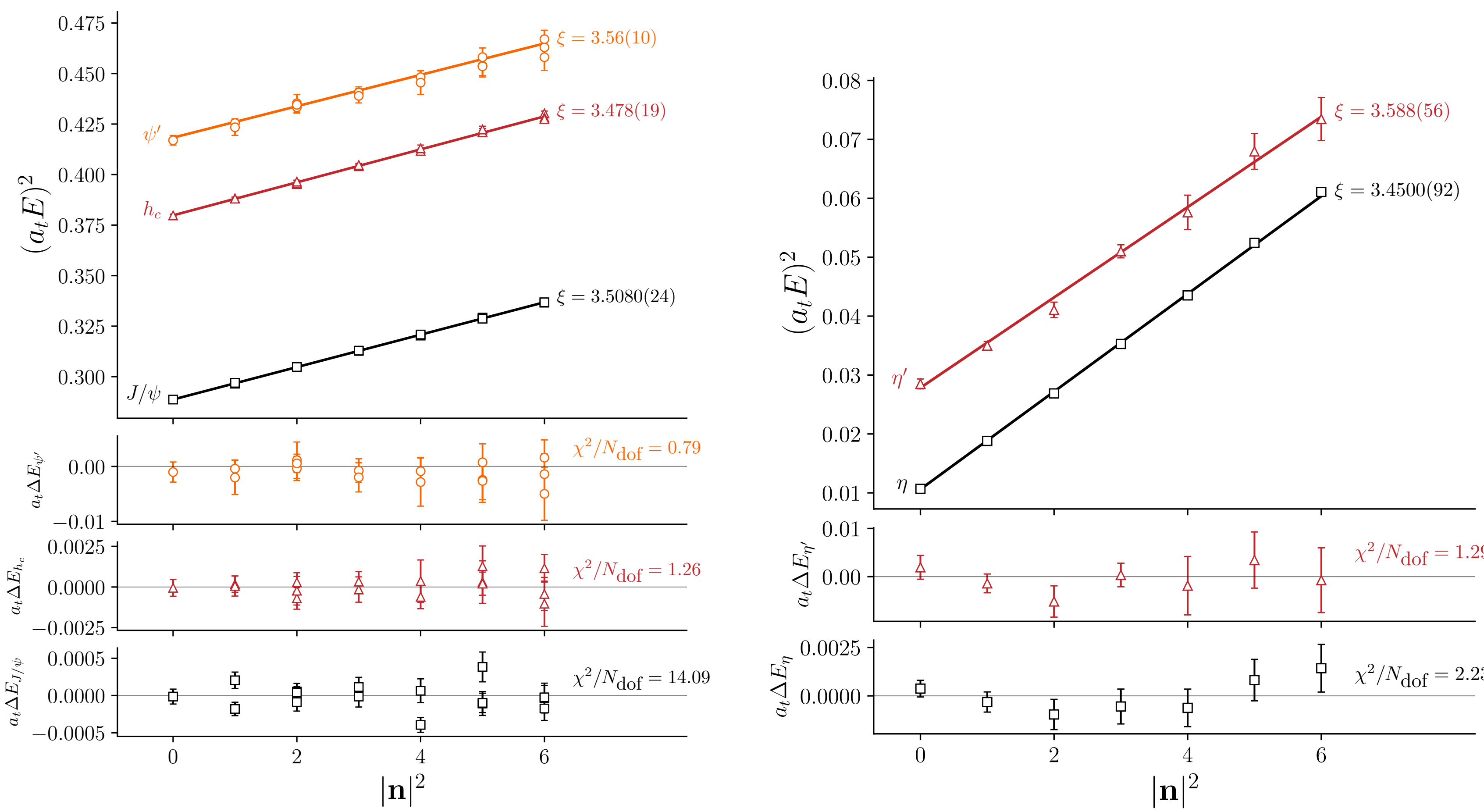
stable hadrons

$\mathbf{n}\,\Lambda$	dim.	helicity content	J^{PC} content
$[000]\ T_1^-$	3	–	$\mathbf{1}^{--}, 3^{--}\ldots$
$[100]\ A_1$	1	$0, \pm4\ldots$	$0^{+-}, \mathbf{1}^{--}, 2^{+-}, 3^{--}\ldots$
$[100]\ E_2$	2	$\pm1, \pm3\ldots$	$\mathbf{1}^{--}, 1^{+-}, 2^{--}, 2^{+-}, 3^{--}\ldots$
$[110]\ A_1$	1	$0, \pm2, \pm4\ldots$	$0^{+-}, \mathbf{1}^{--}, 2^{--}, 2^{+-}, 3^{--}\ldots$
$[110]\ B_1$	1	$\pm1, \pm3\ldots$	$\mathbf{1}^{--}, 1^{+-}, 2^{--}, 2^{+-}, 3^{--}\ldots$
$[110]\ B_2$	1	$\pm1, \pm3\ldots$	$\mathbf{1}^{--}, 1^{+-}, 2^{--}, 2^{+-}, 3^{--}\ldots$
$[111]\ A_1$	1	$0, \pm3\ldots$	$0^{+-}, \mathbf{1}^{--}, 2^{+-}, 3^{--}\ldots$
$[111]\ E_2$	2	$\pm1, \pm2, \pm4\ldots$	$\mathbf{1}^{--}, 1^{+-}, 2^{--}, 2^{+-}, 3^{--}\ldots$
$[200]\ A_1$	1	$0, \pm4\ldots$	$0^{+-}, \mathbf{1}^{--}, 2^{+-}, 3^{--}\ldots$
$[200]\ E_2$	2	$\pm1, \pm3\ldots$	$\mathbf{1}^{--}, 1^{+-}, 2^{--}, 2^{+-}, 3^{--}\ldots$

principal correlators

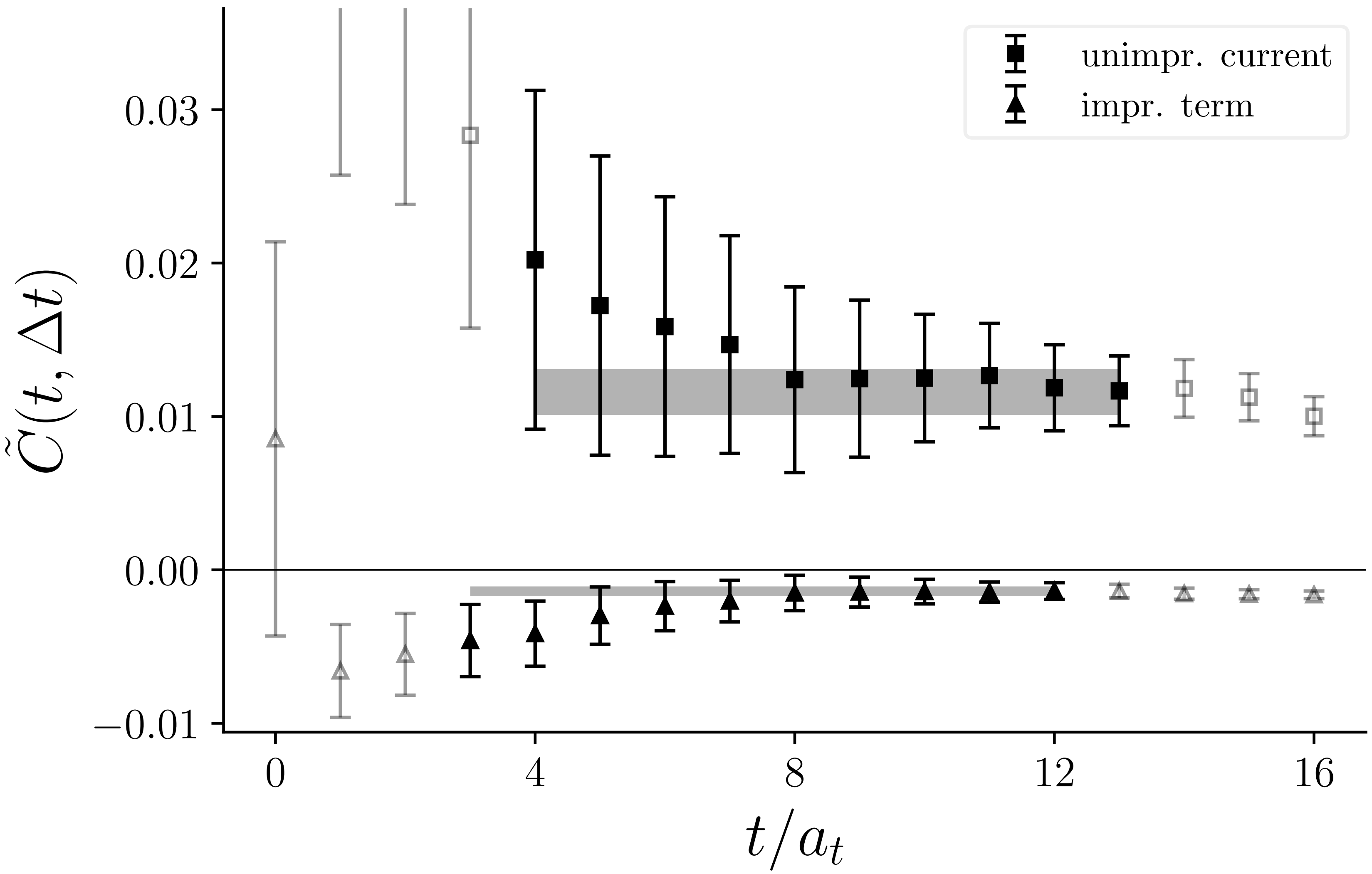


dispersion relations



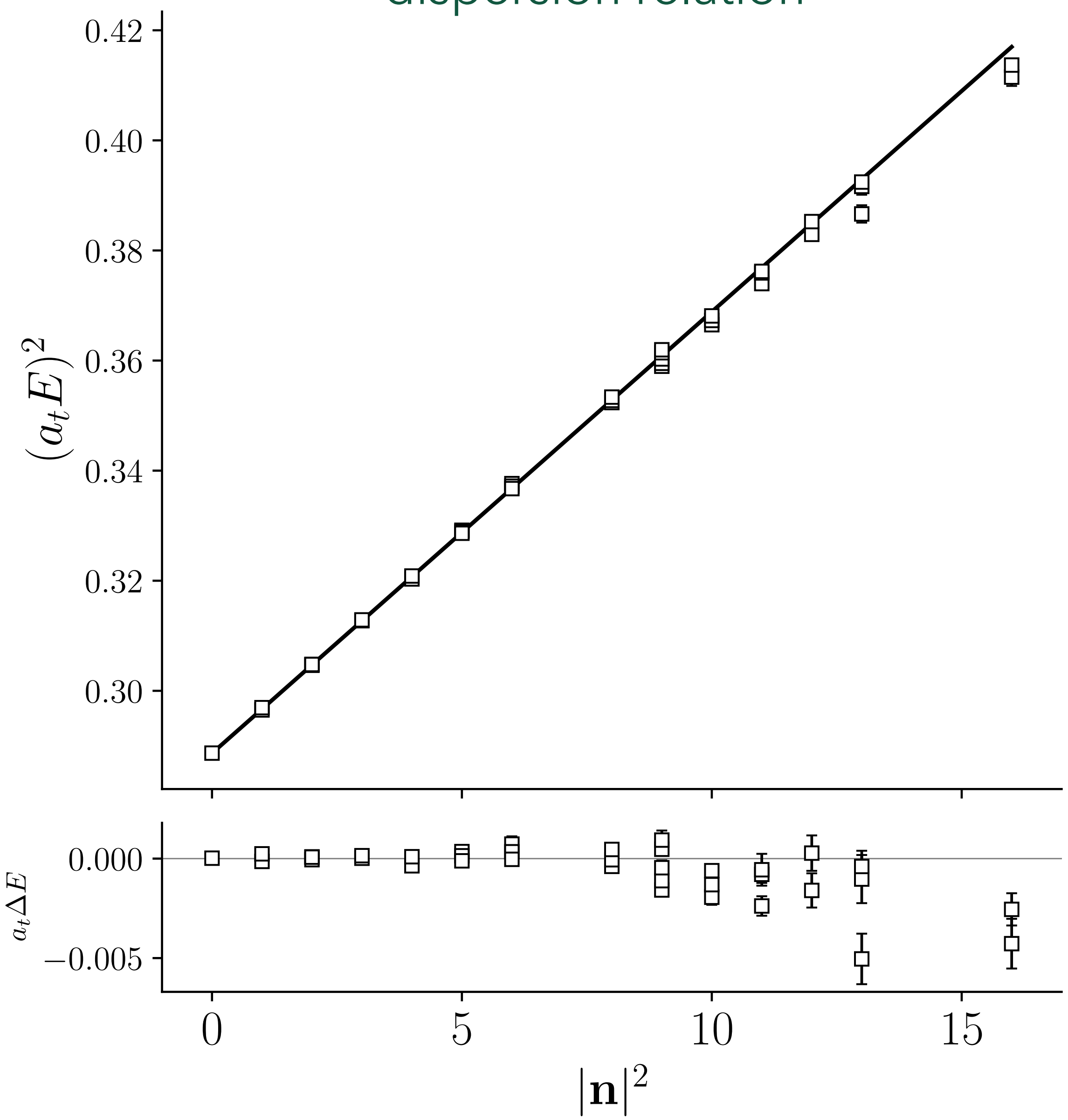
anisotropic current improvement

$$j_k = Z_V^s\left(\bar\psi\gamma_k\psi \;+\; \tfrac{1}{4}(1-\xi)\, a_t\partial_0(\bar\psi\sigma_{0k}\psi)\right)$$

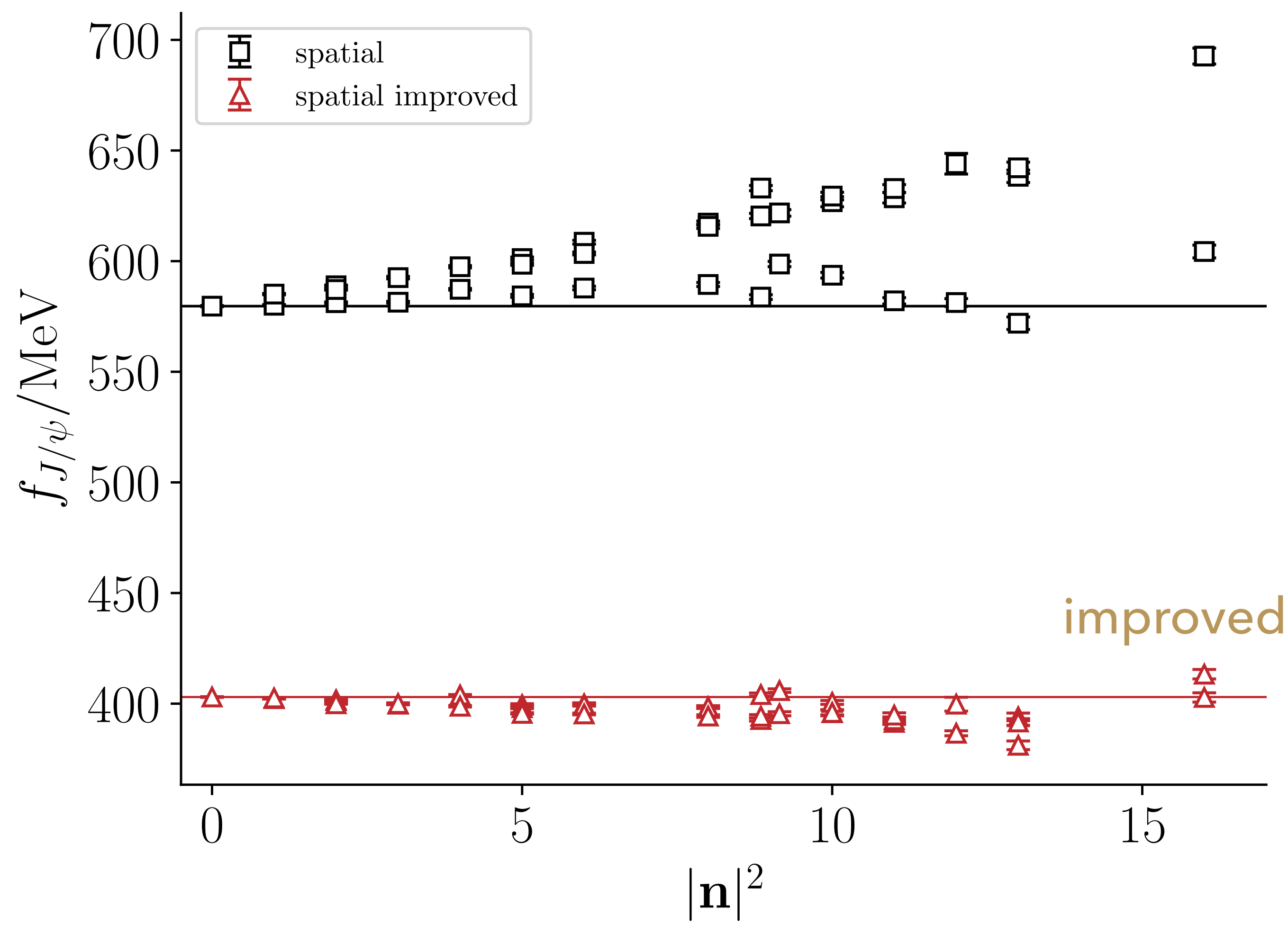


J/ψ at high momentum

dispersion relation



vector decay constant

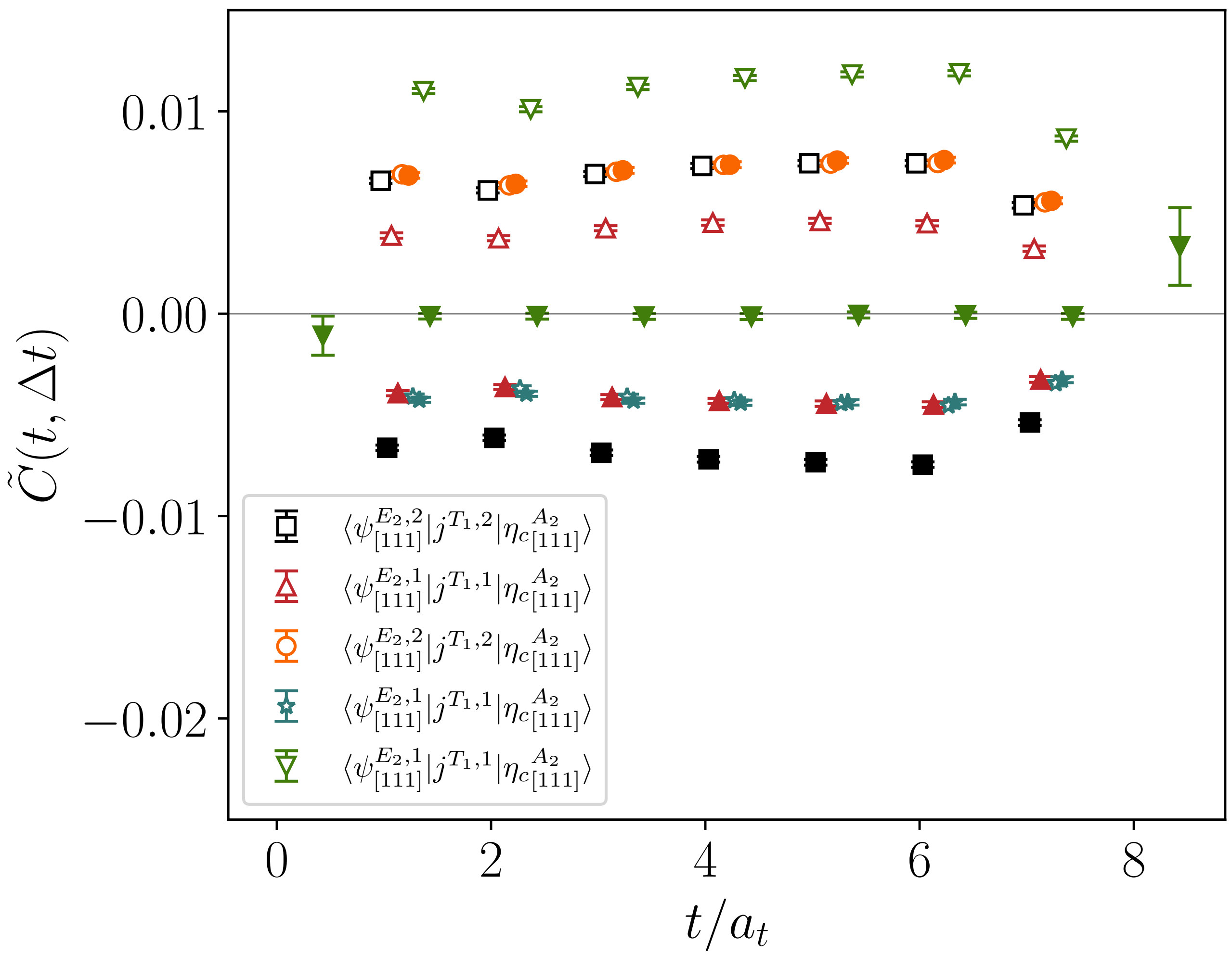


modest discretization errors ?

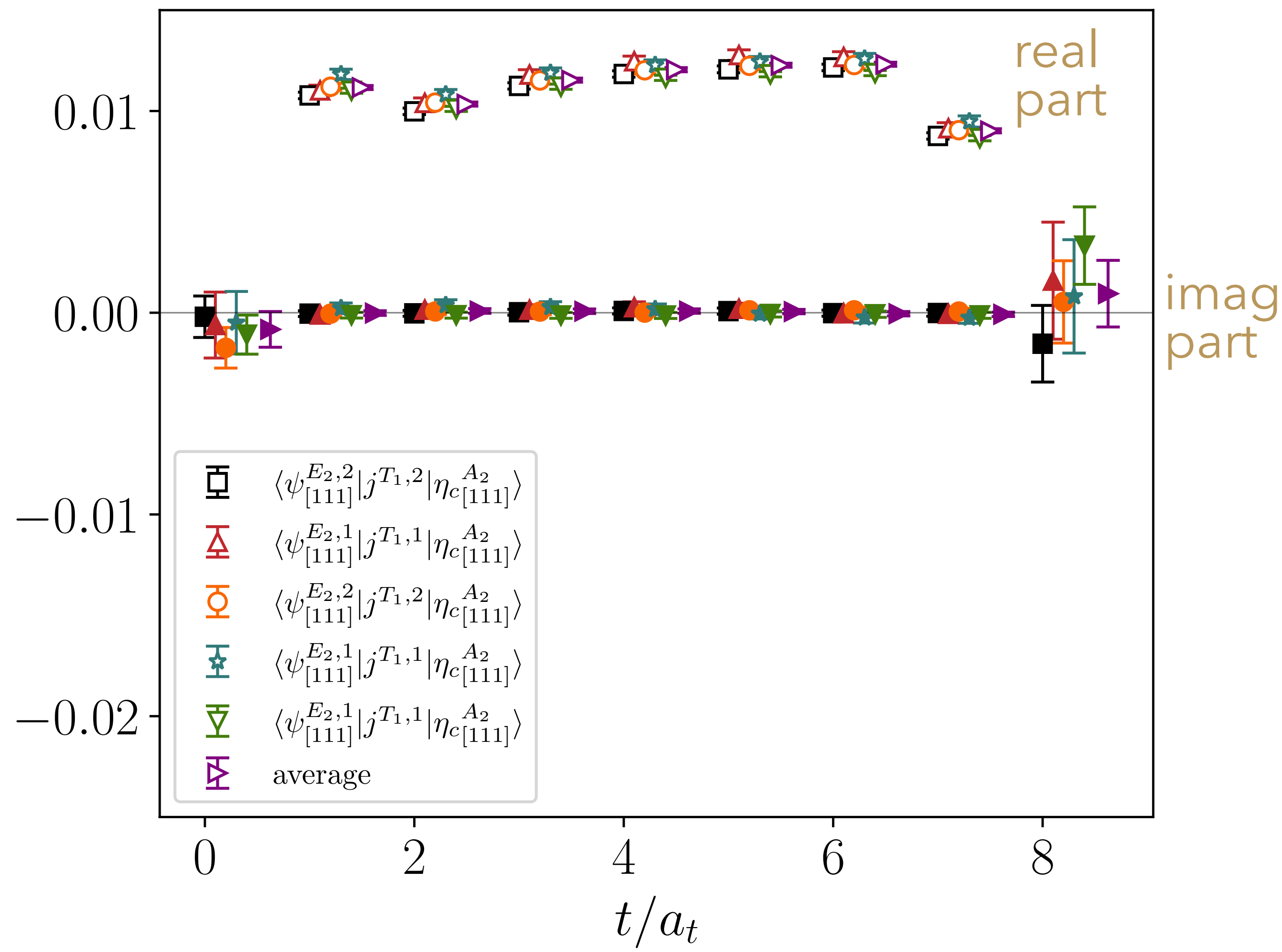
$$|f_{J/\psi}^{\text{expt}}| = 415(5)\,\text{MeV}$$

Wigner-Eckart – $J/\psi \to \gamma\eta_c$ example

‘raw’ correlators

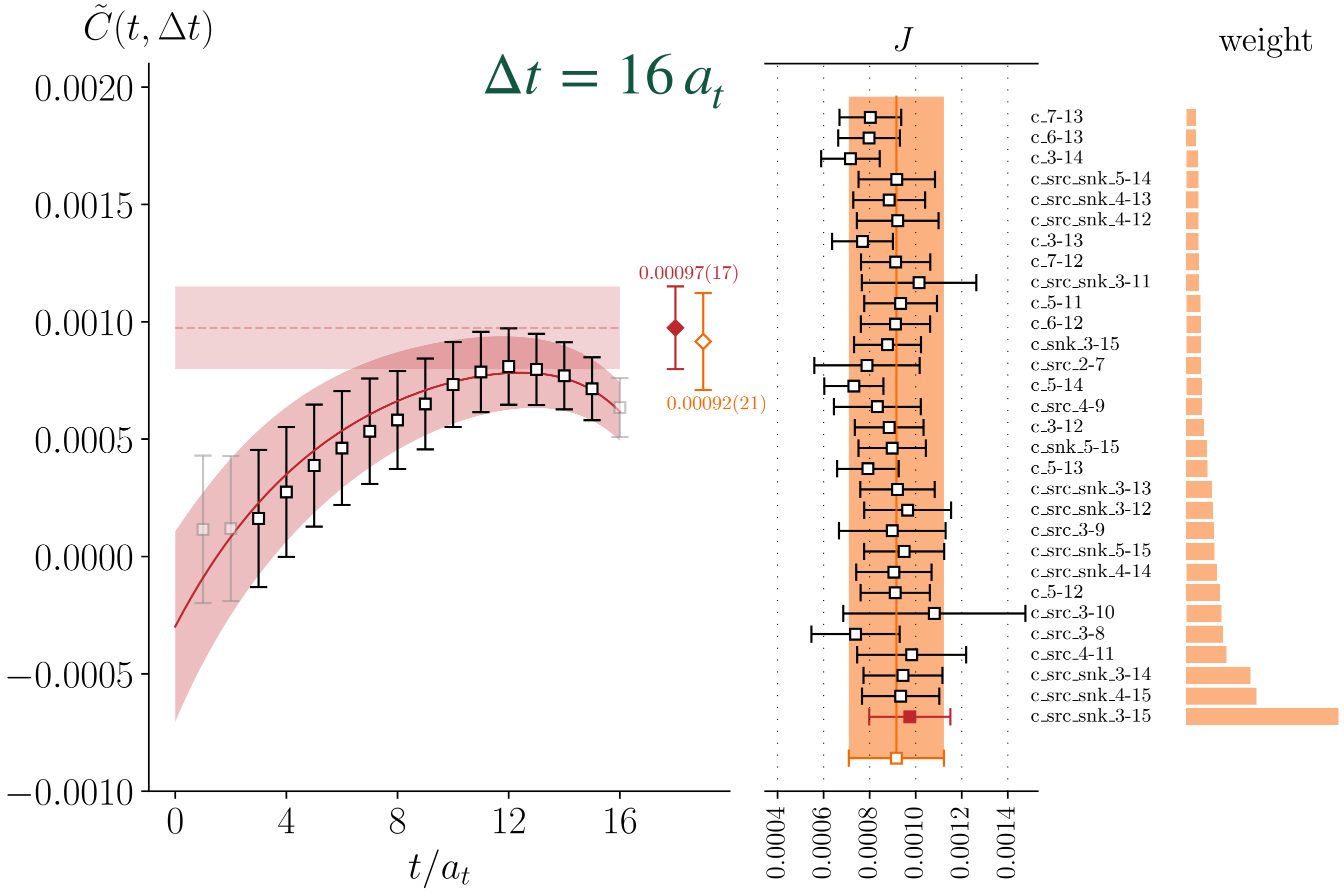


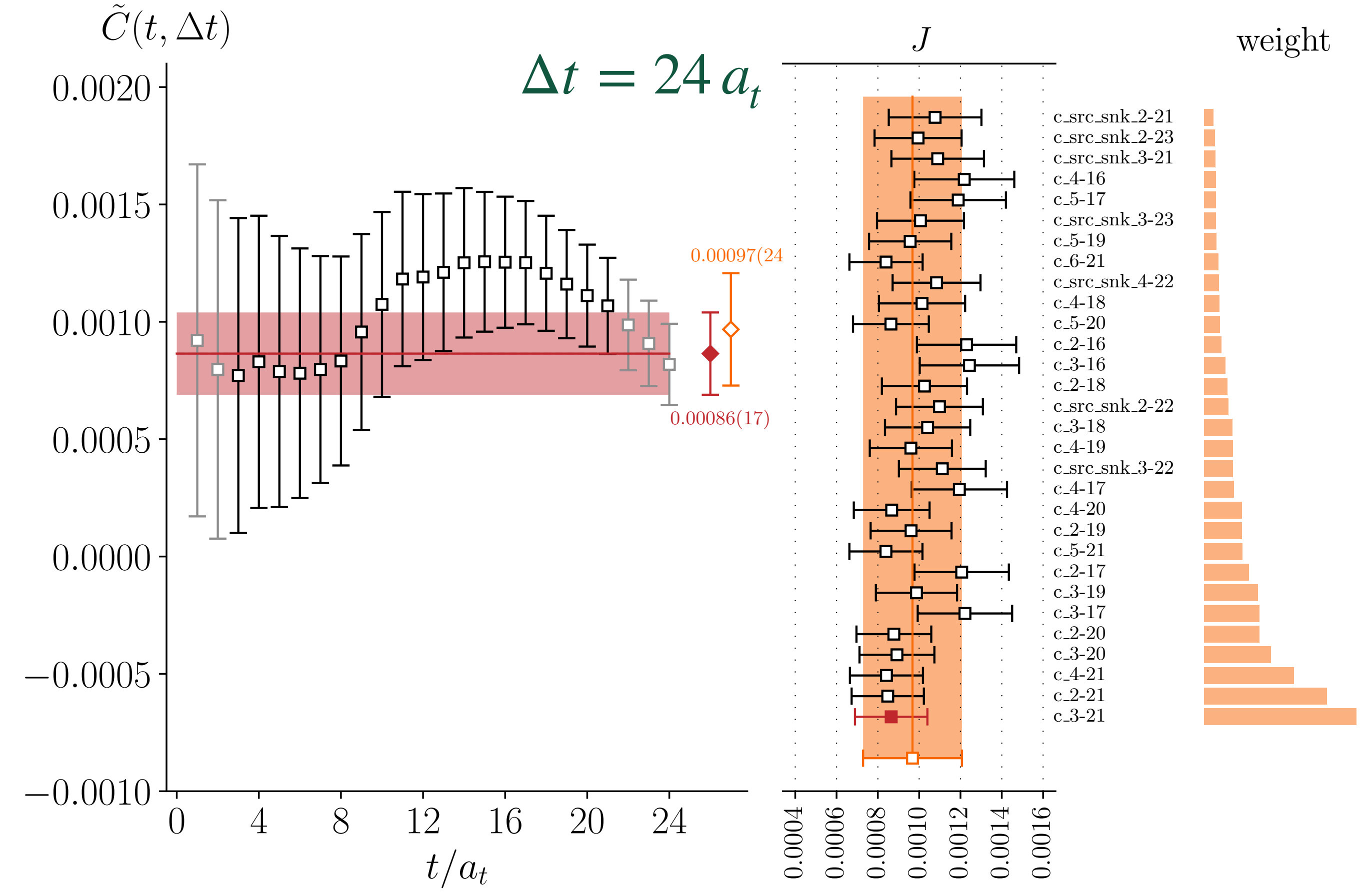
after Wigner-Eckart



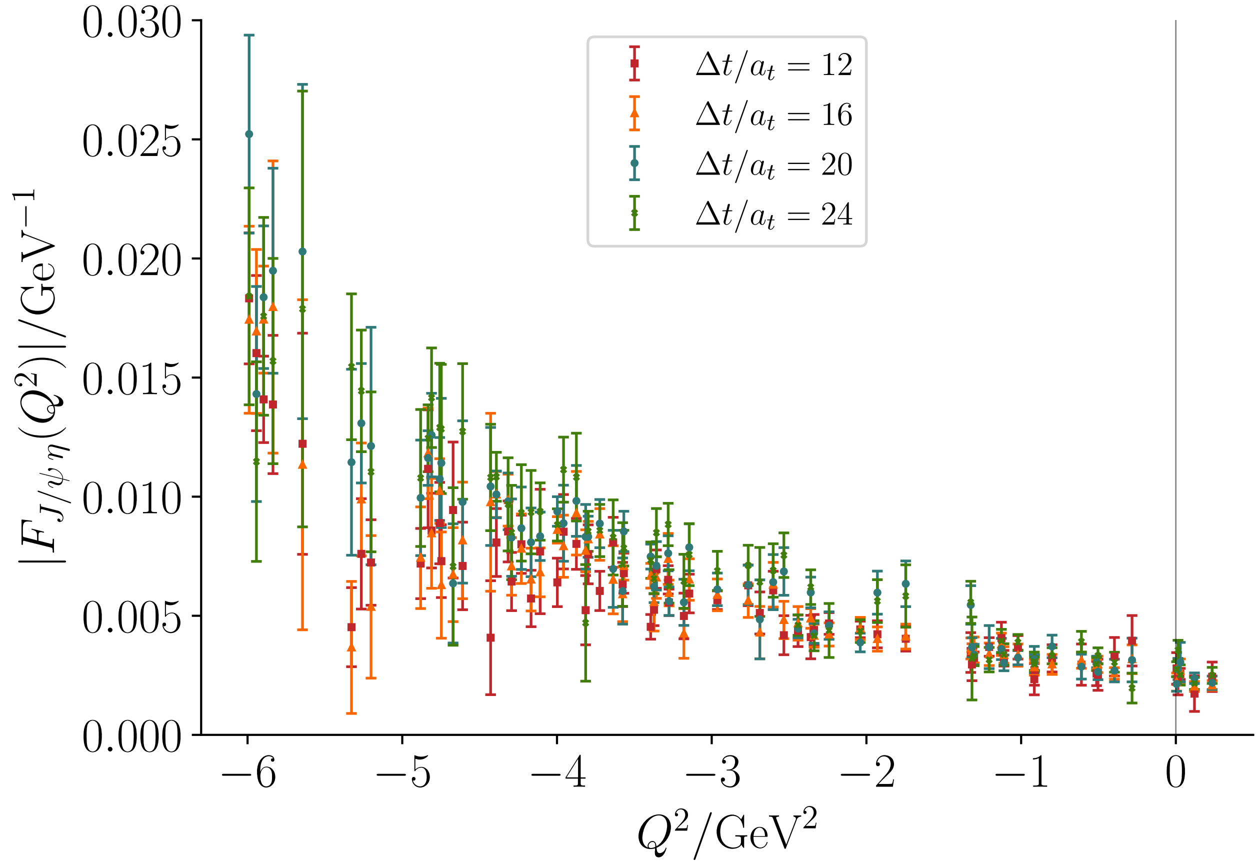
use of (exact) lattice symmetries to bring all correlators into same reference orientation

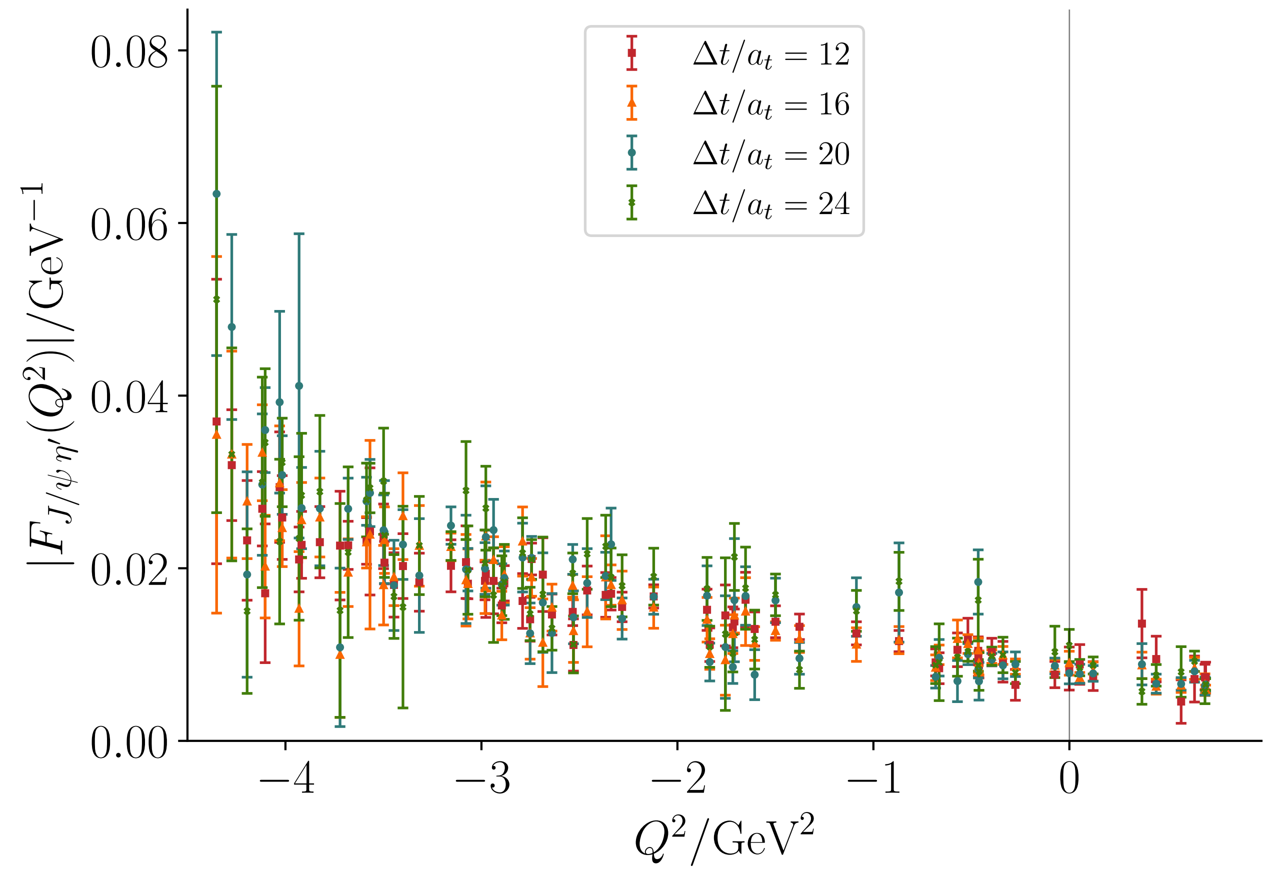
timeslice fitting with 'model averaging'

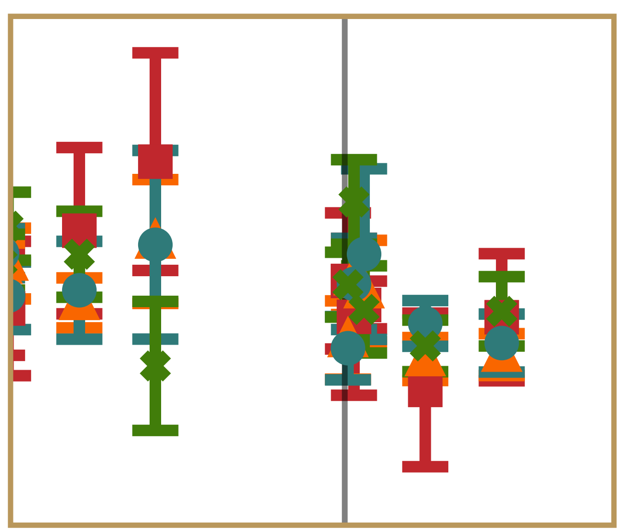


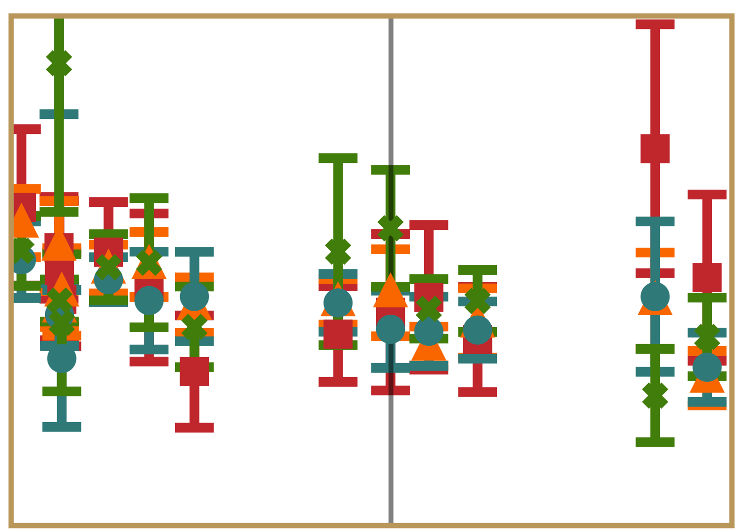


Δt dependence



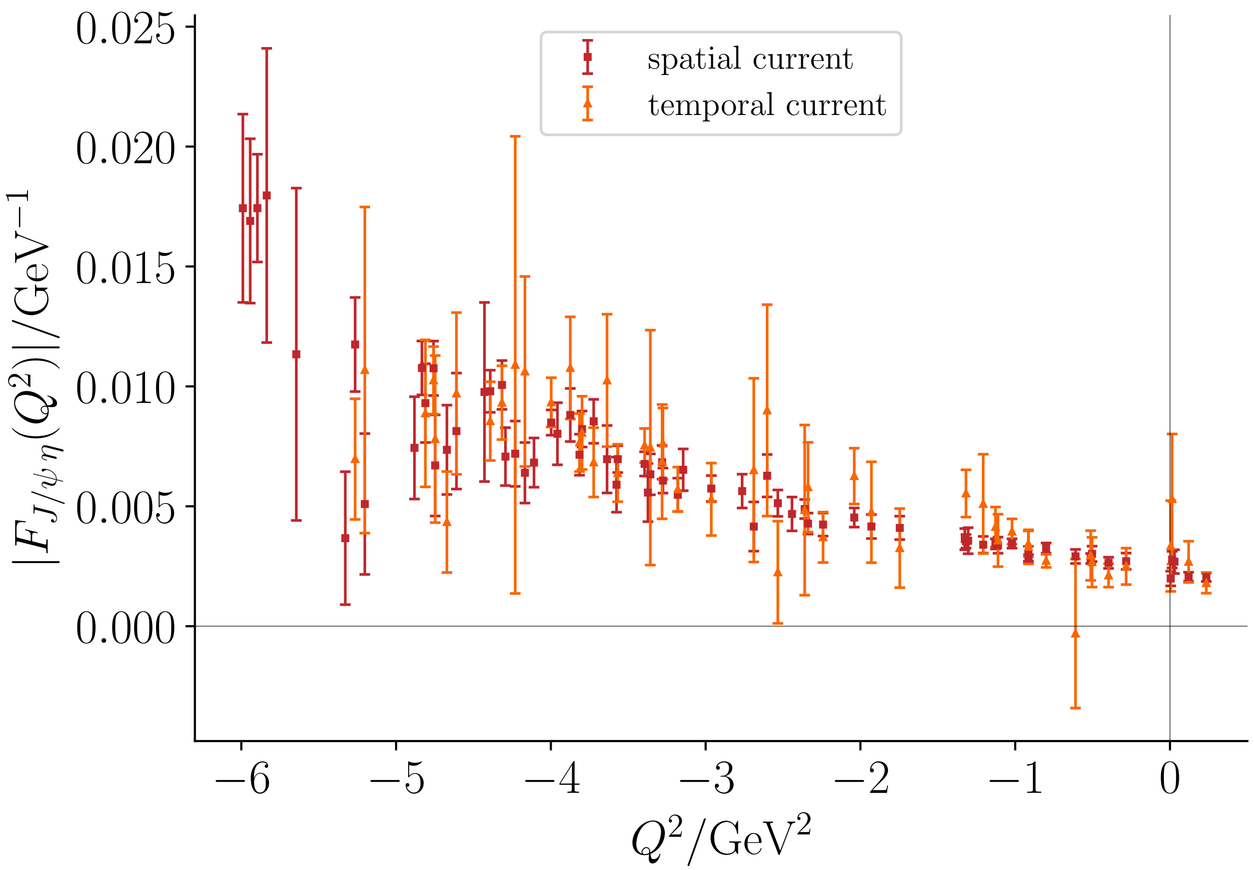


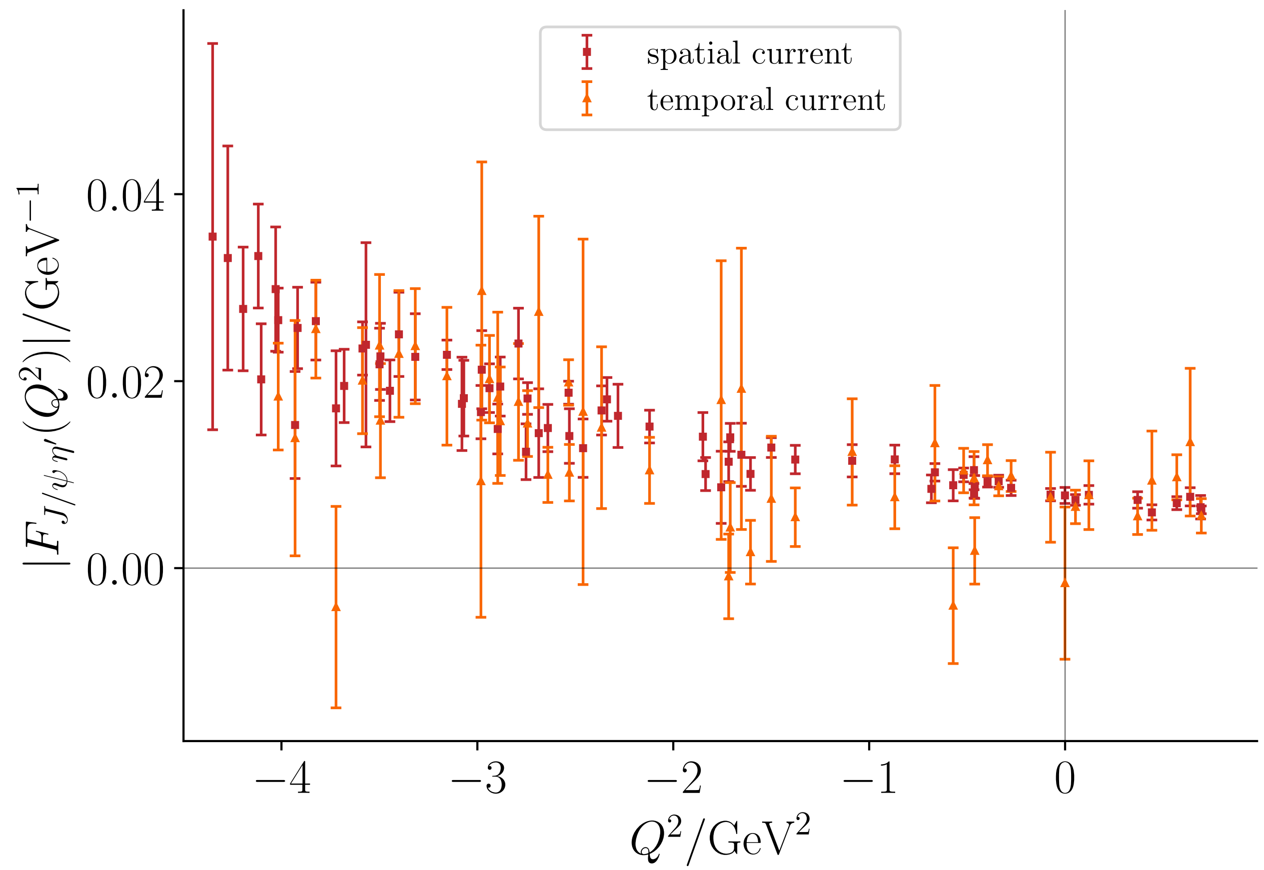




no real systematic dependence upon Δt observed

improved temporal current





many fewer possible 'rotations' to average over

axial – pseudoscalar multipole decomposition

$$\langle \eta(\mathbf{p}') \,|\, j^{\mu}_{\text{em}}(0) \,|\, h_c(\mathbf{p}, \lambda) \rangle = E_1(Q^2) \, \frac{1}{\Omega(Q^2)} \left[\epsilon^\mu - \epsilon \cdot p' \, (p^\mu \, p \cdot p' - p'^\mu \, m_h^2) \right]$$

$$+ \; C_1(Q^2) \frac{m_h}{\Omega(Q^2)} \, \epsilon \cdot p' \left[p^\mu (p \cdot p' - m_\eta^2) + p'^\mu (p \cdot p' - m_h^2) \right]$$

corresponds to helicity amplitudes

$$A_{\lambda=+1, \lambda_\gamma=+1} = A_{\lambda=-1, \lambda_\gamma=-1} = -\, E_1(Q^2),$$ transverse electric dipole

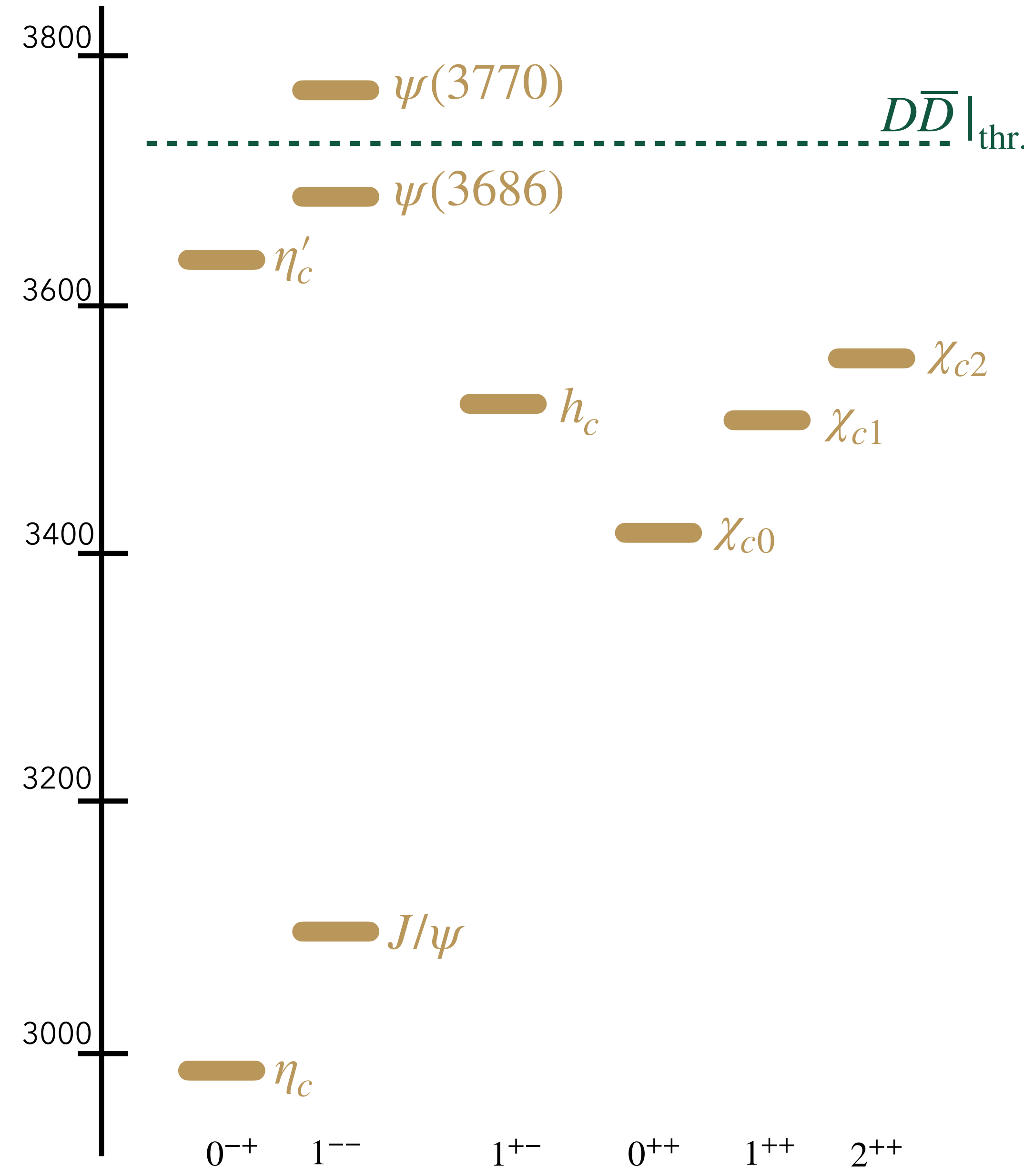
$$A_{\lambda=0, \lambda_\gamma=0} = -\sqrt{-Q^2} \cdot C_1(Q^2)$$ longitudinal, doesn't contribute for real photons

radiative decay width

$$\Gamma(h_c \to \gamma \eta^{(\prime)}) = \frac{4}{27} \alpha \frac{|\,\mathbf{q}\,|}{m_{h_c}^2} \left| E_1(0) \right|^2$$

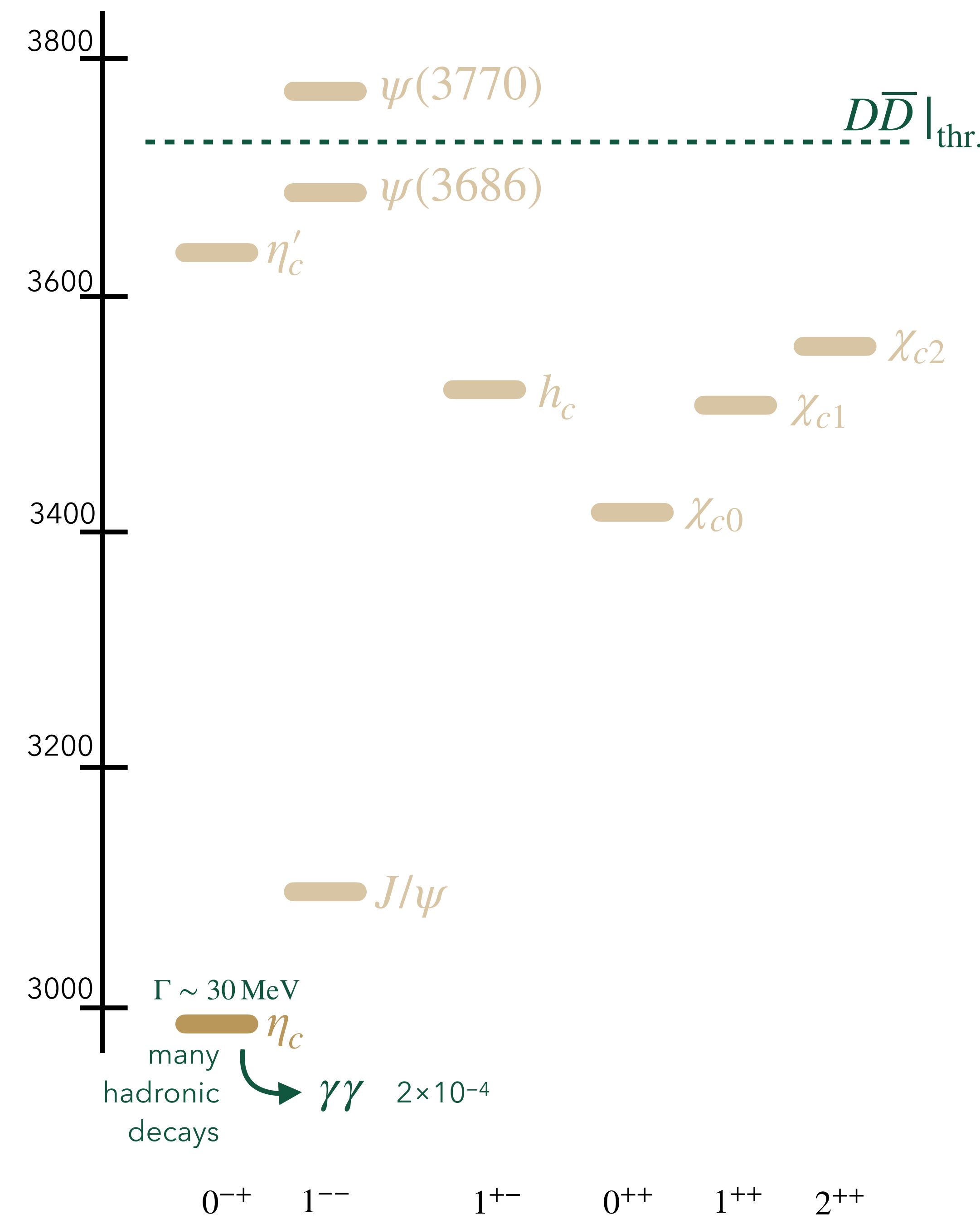
both form-factors contribute in Dalitz decays ($m_{\ell\ell}^2 = q^2 = -\,Q^2$)

$$\frac{d\Gamma}{dq^2 \, d\cos\theta_\ell^\star} = \frac{1}{27} \alpha^2 \frac{|\,\mathbf{q}\,|}{m_{h_c}^2} \, \frac{|\,\mathbf{p}_\ell^\star\,|}{\sqrt{q^2}} \, \frac{1}{q^2} \left[\left(\left(1 + \tfrac{4m_\ell^2}{q^2}\right) |\,E_1\,|^2 + q^2 \,|\,C_1\,|^2 \right) + 4 \frac{|\,\mathbf{p}_\ell^\star\,|^2}{q^2} \left(|\,E_1\,|^2 - q^2 \,|\,C_1\,|^2 \right) \cos^2\theta_\ell^\star \right]$$

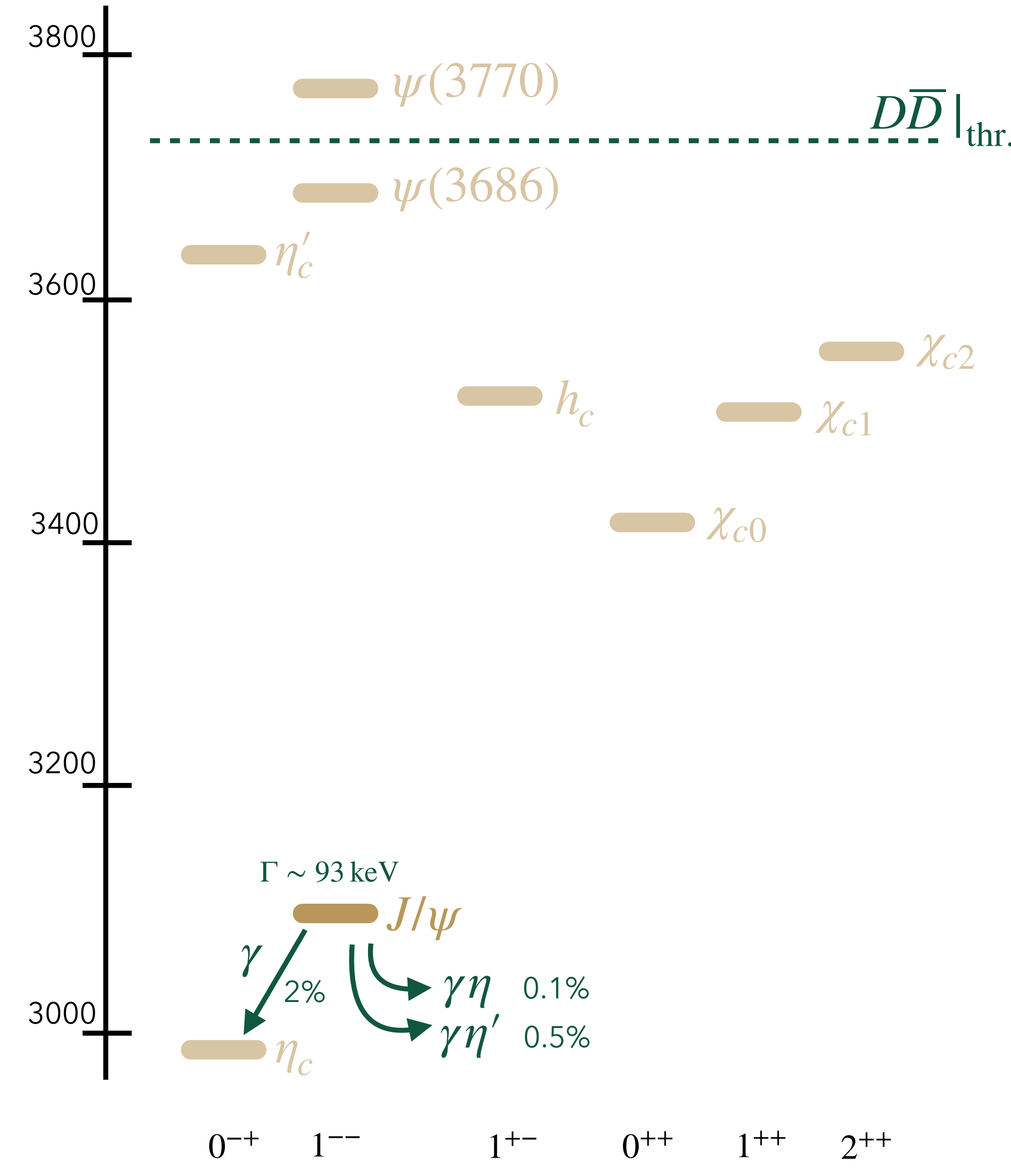


3800
3600
3400
3200
3000
$\psi(3770)$
$D\overline{D}|_{\text{thr.}}$
$\psi(3686)$
η_c'
χ_{c2}
h_c
χ_{c1}
χ_{c0}
J/ψ
η_c
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}

charmonia below $D\overline{D}$ threshold



charmonia below $D\overline{D}$ threshold



η' volume dependence is also a little suspicious