

# Neutrino Oscillation Physics Discussions

---

Mu-Chun Chen  
University of California at Irvine

NuFact 2012, Williamsburg, July 27, 2012

# Where Do We Stand?

- Exciting Time in  $\nu$  Physics: recent hints of large  $\theta_{13}$  from T2K, MINOS, Double Chooz, and Daya Bay
- Latest 3 neutrino global analysis (including recent results from reactor experiments):

$$P(\nu_a \rightarrow \nu_b) = |\langle \nu_b | \nu, t \rangle|^2 \simeq \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4E} L \right)$$

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

Parameter	Best fit	$1\sigma$ range	$2\sigma$ range	$3\sigma$ range
$\delta m^2 / 10^{-5} \text{ eV}^2$ (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2 \theta_{12} / 10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2 / 10^{-3} \text{ eV}^2$ (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2 / 10^{-3} \text{ eV}^2$ (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2 \theta_{13} / 10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2 \theta_{13} / 10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2 \theta_{23} / 10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2 \theta_{23} / 10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
$\delta / \pi$ (NH)	1.08	0.77 – 1.36	—	—
$\delta / \pi$ (IH)	1.09	0.83 – 1.47	—	—

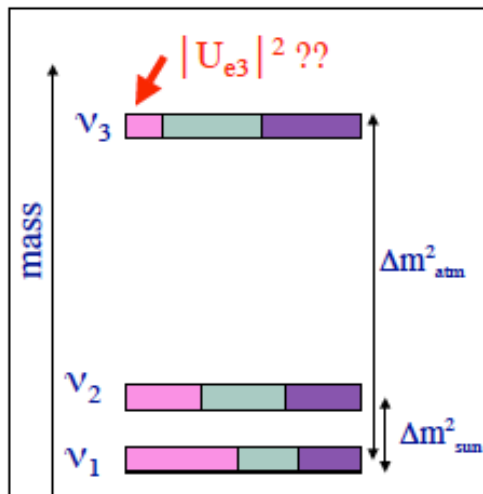
Cautions!! Different global fit analyses assume different error correlations among experiments  $\Rightarrow$  different results

# Where Do We Stand?

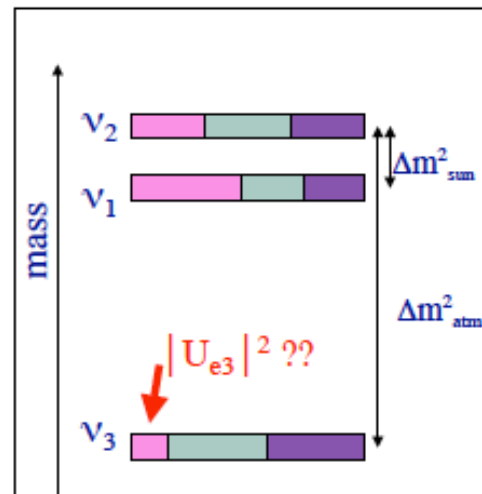


- The known knowns:

normal hierarchy:



inverted hierarchy:



## What's Next?

Reactor Exp: Double Chooz, Daya Bay, Reno  
 Long Baseline Exp: MINOS, NOvA, T2K, LBNE...

## The known unknowns:

- How small is  $\theta_{13}$ ? ( $\nu_e$  component of  $\nu_3$ )
- $\theta_{23} > \pi/4$ ,  $\theta_{23} < \pi/4$ ,  $\theta_{23} = \pi/4$ ?  
 ( $\nu_3$  composition of  $\nu_{\mu,\tau}$ )
- neutrino mass hierarchy ( $\Delta m_{13}^2$ )?
- CP violation in neutrino oscillations?
- Majorana vs Dirac?

## The unknown unknowns



# Theoretical Challenges

---

## (i) Absolute mass scale: Why $m_\nu \ll m_{u,d,e}$ ?

- seesaw mechanism: most appealing scenario  $\Rightarrow$  Majorana
  - GUT scale (type-I, II) vs TeV scale (type-III, double seesaw)
- TeV scale new physics (extra dimension,  $U(1)$ )  $\Rightarrow$  Dirac or Majorana

## (ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- seesaw doesn't explain entire mass matrix w/ 3 fairly large mixing angles
- neutrino anarchy: no parametrically small number Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003)
  - near degenerate spectrum, large mixing
  - predictions strongly depend on choice of statistical measure
- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - mixing determined by dynamics of underlying symmetry
  - for leptons only (normal or inverted)
  - for quarks and leptons: quark-lepton connection  $\leftrightarrow$  GUT (normal)

# Questions to be discussed

---

- precision in oscillation parameters needed to distinguish different models
  - theoretical frameworks; testable predictions
  - robustness of the model predictions?
    - theoretical uncertainties

⇒ Talk by Michael Ratz
- constraining the three neutrino paradigm
  - implications for new interactions

⇒ Talk by Jacobo Lopez-Pavon

# Small Neutrino Mass: Seesaw Mechanism

Yanagida, 1979; Gell-Mann, Ramond, Slansky, 1979;  
Mohapatra, Senjanovic, 1981

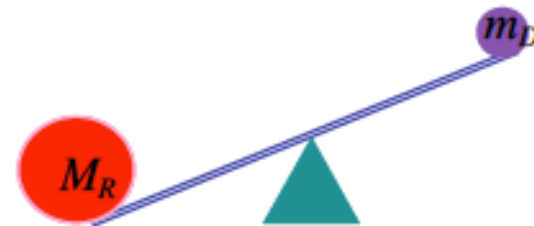
- Mixture of light fields and heavy fields

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$\nu_R$ : sterile (singlet under ALL gauge groups in SM)

$\nu_R \nu_R$  mass term allowed

- Smallness of neutrino masses suggest a high mass scale



- Diagonalize the mass matrix:

$$m_\nu \sim m_{\text{light}} \sim \frac{m_D^2}{M_R} \ll m_D$$

$$m_{\text{heavy}} \sim M_R$$

For  $m_{\nu_3} \sim \sqrt{\Delta m_{\text{atm}}^2}$

If  $m_D \sim m_t \sim 180 \text{ GeV}$

→  $M_R \sim 10^{15} \text{ GeV (GUT !!)}$

# Grand Unification

- Motivations:

- Electromagnetic, weak, and strong forces have very different strengths

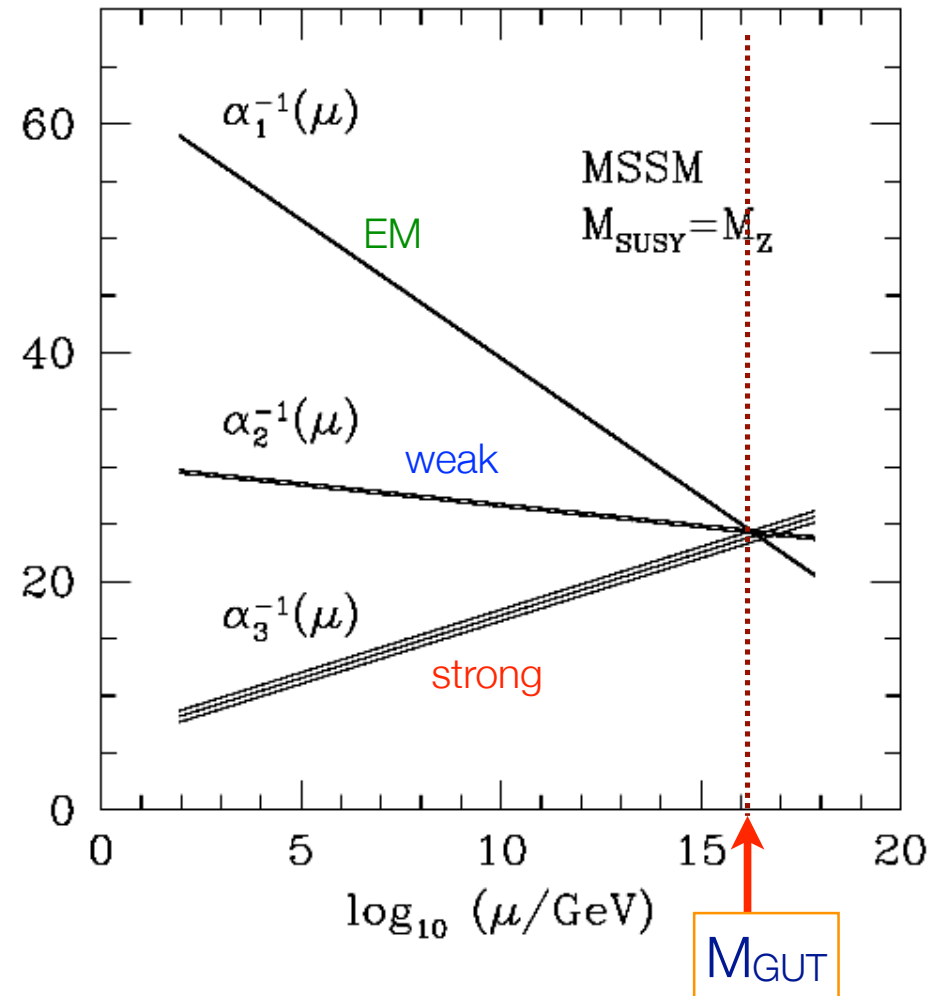
- But their strengths become the same at  $10^{16}$  GeV if there is supersymmetry

- To obtain

$$m_\nu \sim (\Delta m^2_{\text{atm}})^{1/2}, m_D \sim m_{\text{top}}$$

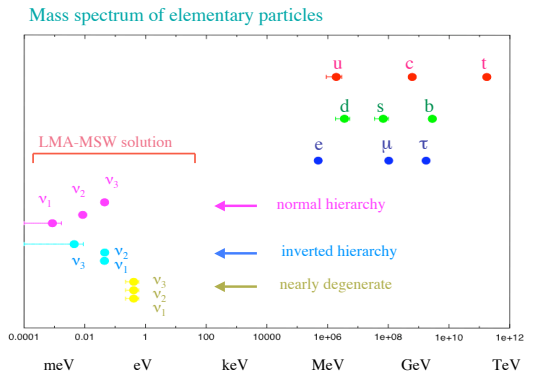
$$M_R \sim 10^{15} \text{ GeV}$$

coupling constants run!



# Origin of Mass Hierarchy and Mixing

- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- **No fundamental reason can be found in the framework of SM**
- less ambitious aim  $\Rightarrow$  reduce the # of parameters by imposing symmetries
  - **SUSY Grand Unified Gauge Symmetry**
    - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
      - one set of Yukawa coupling for a given GUT multiplet  $\Rightarrow$  intra-family relations
    - seesaw mechanism naturally implemented
    - proton decay, leptogenesis, LFV charged lepton decay
  - **Family Symmetry**
    - relate Yukawa couplings of different families
      - inter-family relations  $\Rightarrow$  further reduce the number of parameters

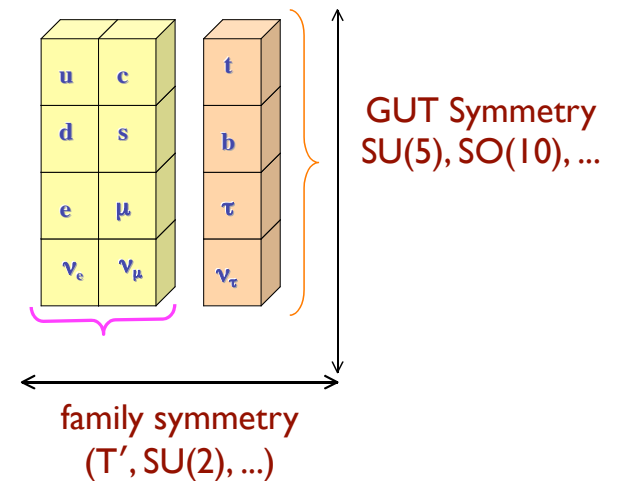


**$\Rightarrow$  Experimentally testable correlations among physical observables**



# Origin of Mass Hierarchy and Mixing

- Several models have been constructed based on
  - GUT Symmetry  $[SU(5), SO(10)] \oplus$  Family Symmetry  $G_F$
- Family Symmetries  $G_F$  based on continuous groups:
  - $U(1)$
  - $SU(2)$
  - $SU(3)$
- Recently, models based on discrete family symmetry groups have been constructed
  - $A_4$  (tetrahedron)
  - $T'$  (double tetrahedron)
  - $S_3$  (equilateral triangle)
  - $S_4$  (octahedron, cube)
  - $A_5$  (icosahedron, dodecahedron)
  - $\Delta_{27}$
  - $Q_4$



Motivation: Tri-bimaximal (TBM) neutrino mixing

# Tri-bimaximal Neutrino Mixing

---

- Neutrino Oscillation Parameters

$$P(\nu_a \rightarrow \nu_b) = |\langle \nu_b | \nu, t \rangle|^2 \simeq \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2}{4E} L \right)$$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Latest Global Fit ( $3\sigma$ )

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

$$\sin^2 \theta_{atm} = 0.386 \quad (0.331 - 0.637) \quad \sin^2 \theta_{\odot} = 0.307 \quad (0.259 - 0.359)$$

$$\sin^2 \theta_{13} = 0.0241 \quad (0.0169 - 0.0313)$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2 \theta_{atm, TBM} = 1/2 \quad \sin^2 \theta_{\odot, TBM} = 1/3$$

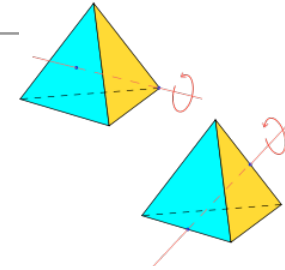
$$\sin \theta_{13, TBM} = 0.$$

- Leading Order: TBM (from symmetry) + Corrections/contributions (dictated by symmetry)

# An Example: a SUSY SU(5) x T' Model

M.-C.C, K.T. Mahanthappa  
Phys. Lett. B652, 34 (2007)

- Double Tetrahedral Group T'
  - may arise from extra dimensions
- Symmetries  $\Rightarrow$  9 parameters in Yukawa sector  $\Rightarrow$  22 physical observables
  - neutrino mixing angles from group theory (CG coefficients)
  - TBM: misalignment of symmetry breaking patterns
    - neutrino sector:  $T' \rightarrow G_{TST_2}$ , charged lepton sector:  $T' \rightarrow G_T$
  - GUT symmetry  $\Rightarrow$  deviation from TBM related to quark mixing  $\theta_c$
- **complex CG's of T'  $\Rightarrow$  Novel Origin of CP Violation**
  - CP violation in both quark and lepton sectors entirely from group theory
  - connection between leptogenesis and CPV in neutrino oscillation



M.-C.C, K.T. Mahanthappa,  
Phys. Lett. B681, 444 (2009)

# Predictions: a SUSY SU(5) x T' Model

- Charged Fermion Sector (7 parameters)

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g + (1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u \rightarrow V_{cb}$$

$$M_d, M_e^T = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & (1,-3)c & 0 \\ b & b & 1 \end{pmatrix} y_b v_d \phi_0 \rightarrow V_{ub}$$

spinorial representations  $\Rightarrow$  complex CGs  
 $\Rightarrow$  CPV in quark sector

quark CP phase:  $\gamma = 45.6$  degrees

Georgi-Jarlskog relations at GUT scale  
 $\Rightarrow V_{d,L} \neq I$

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha} \sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

$$m_d \simeq 3m_e \quad m_\mu \simeq 3m_s$$

SU(5)  $\Rightarrow M_d = (M_e)^T$   
 $\Rightarrow$  corrections to TBM related to  $\theta_c$

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3} \sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3} \theta_c$$

# Predictions: a SUSY SU(5) x T' Model

- Neutrino Sector (2 parameters):  $M_{RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} S_0$       $M_D = \begin{pmatrix} 2\xi_0 + \eta_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0 + \eta_0 \\ -\xi_0 & -\xi_0 + \eta_0 & 2\xi_0 \end{pmatrix} \zeta_0 \zeta'_0 v_u$

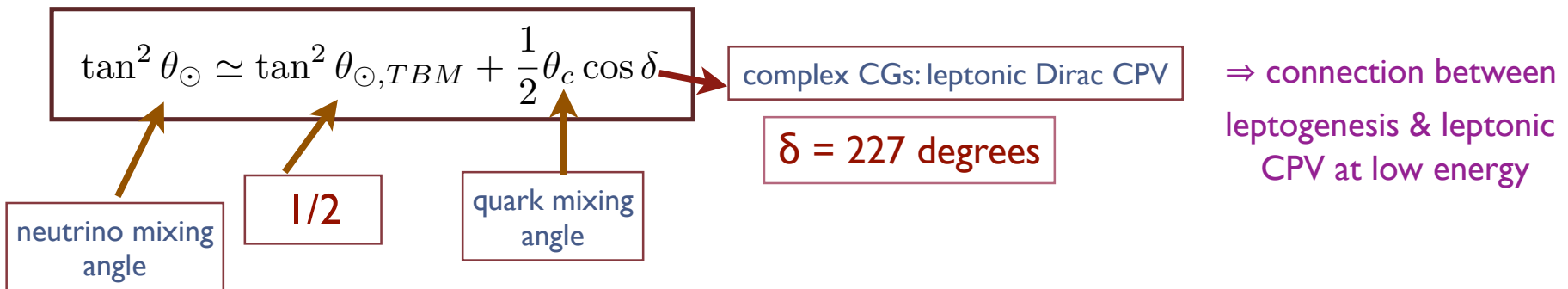
- Seesaw mechanism:  $U_{TBM}^T M_\nu U_{TBM} = \text{diag}((3\xi_0 + \eta_0)^2, \eta_0^2, -(-3\xi_0 + \eta_0)^2) \frac{(\zeta_0 \zeta'_0 v_u)^2}{s_0 \Lambda}$

- Prediction for MNS matrix:

$$U_{MNS} = V_{e,L}^\dagger U_{TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$\theta_{13} \simeq \theta_c/3\sqrt{2}$

CGs of SU(5) & T'



- sum rule among absolute masses:

normal hierarchy predicted

$$m_2^2 - m_1^2 = (\eta_0^4 - (3\xi_0 + \eta_0)^4) \frac{(\zeta_0 \zeta'_0 v_u)^2}{S_0} > 0$$

$$m_3^2 - m_1^2 = -24\eta_0 \xi_0 (9\xi_0^2 + \eta_0^2) \frac{(\zeta_0 \zeta'_0 v_u)^2}{S_0}$$

# Sum Rules: Quark-Lepton Complementarity

## Quark Mixing

mixing parameters	best fit	$3\sigma$ range
$\theta_{23}^q$	$2.36^\circ$	$2.25^\circ - 2.48^\circ$
$\theta_{12}^q$	$12.88^\circ$	$12.75^\circ - 13.01^\circ$
$\theta_{13}^q$	$0.21^\circ$	$0.17^\circ - 0.25^\circ$

## Lepton Mixing

mixing parameters	best fit	$3\sigma$ range
$\theta_{23}^e$	$42.8^\circ$	$35.5^\circ - 53.5^\circ$
$\theta_{12}^e$	$34.4^\circ$	$31.5^\circ - 37.6^\circ$
$\theta_{13}^e$	$5.6^\circ$	$\leq 12.5^\circ$

- **QLC-I**  $\theta_c + \theta_{\text{sol}} \cong 45^\circ$  Raidal, '04; Smirnov, Minakata, '04  
(BM)  $\theta_{23}^q + \theta_{23}^e \cong 45^\circ$

**measuring leptonic mixing parameters to the precision of those in quark sector**

- **QLC-II**  $\tan^2 \theta_{\text{sol}} \cong \tan^2 \theta_{\text{sol,TBM}} + (\theta_c / 2) * \cos \delta_e$   
(TBM)  $\theta_{13}^e \cong \theta_c / 3\sqrt{2}$  Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

- testing sum rules: a *more* robust way to distinguish different classes of models

**need improved  $\delta\theta_{12}$  measurement**

# Other Possibilities: Beyond TBM

- **Tri-bimaximal Mixing Accidental or NOT?** Albright, Rodejohann (2009); Abbas, Smirnov (2010)

- **Dodeca Mixing Matrix from  $D_{12}$  Symmetry**

J. E. Kim, M.-S. Seo, (2010)

leading order:

$$\theta_c = 15^\circ, \theta_{\text{sol}} = 30^\circ, \theta_{\text{atm}} = 45^\circ$$

breaking of  $D_{12}$  :

$$\theta_c = 15^\circ \rightarrow 13.4^\circ$$

$$\theta_{\text{sol}} = 30^\circ + O(\epsilon), \theta_{13} = O(\epsilon)$$

$$V_{\text{PMNS}} = U_l^\dagger U_\nu = \begin{pmatrix} \cos \frac{\pi}{6} & \sin \frac{\pi}{6} & 0 \\ -\frac{1}{\sqrt{2}} \sin \frac{\pi}{6} & \frac{1}{\sqrt{2}} \cos \frac{\pi}{6} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \sin \frac{\pi}{6} & \frac{1}{\sqrt{2}} \cos \frac{\pi}{6} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_c + \theta_{\text{sol}} = 45^\circ \text{ (not from GUT symmetry)}$$

**deviations correlated**

- **Golden Ratio for solar mixing angle**

$$\tan^2 \theta_{\text{sol}} = 1/\Phi^2 = 0.382, \text{ (1.4}\sigma \text{ below best fit)}$$

$$\Phi = (1 + \sqrt{5}) / 2 = 1.62$$

Datta, Ling, Ramond, '03;

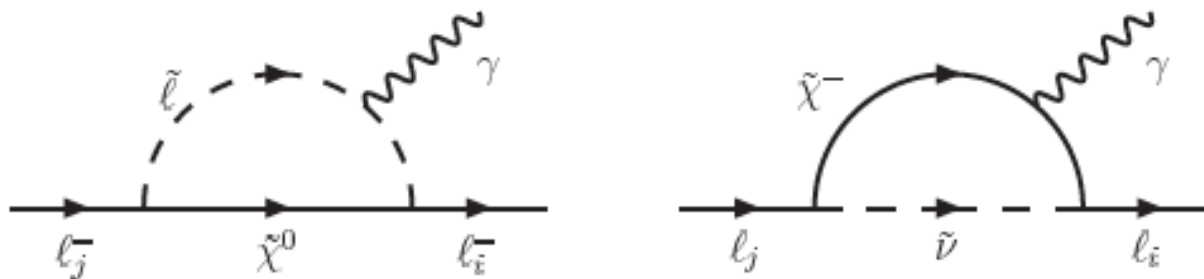
Z2 x Z2 or A5: Kajiyama, Raidal, Strumia, '07; ...

D10: Adulpravitchai, Blum, Rodejohann, '09; ...

- **prediction for  $\theta_{13}$  model/parameter dependent**

# Correlations: Charged Lepton Flavor Violation

- SUSY GUTs: Lepton flavor violating charged lepton decays



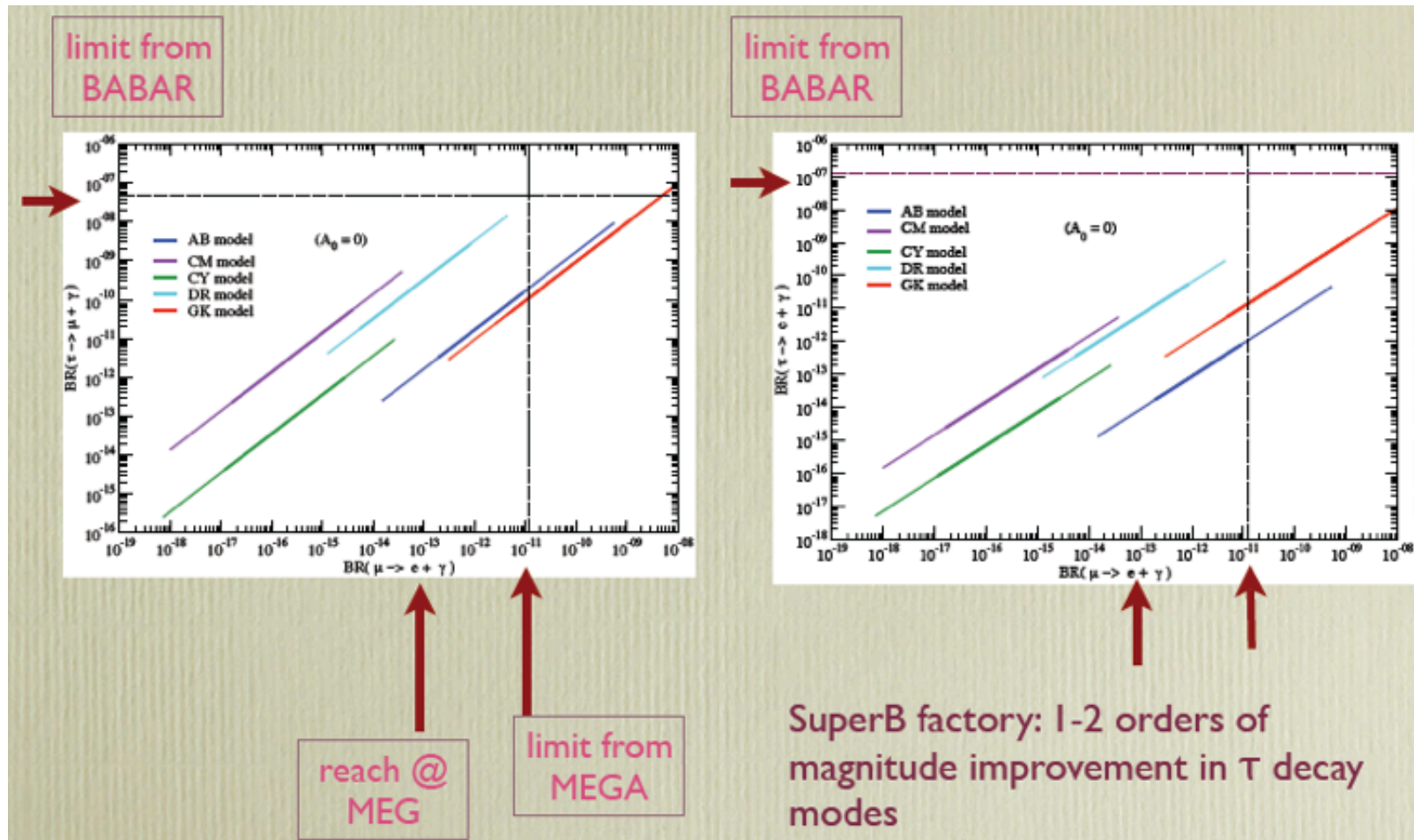
- ▶ individual branching fraction: strong dependence on soft SUSY parameters
- ▶ correlations between branching fractions: strong dependence on flavor structure



# Correlations: Charged Lepton Flavor Violation

- five viable SUSY SO(10) models with dark matter constraints:

C.H. Albright, M.-C.C (2008)

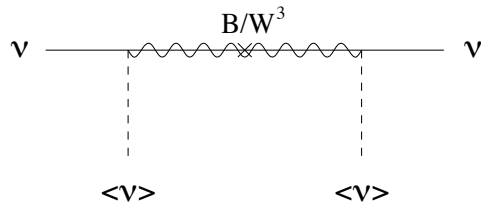


# Correlations: Sparticle Decay and Mixing Angle

- MSSM with bi-linear R-Parity Violation Kaplan, Nelson, 1999

$$\mathcal{W}_R = \epsilon_i \hat{L}_i \hat{H}_u$$

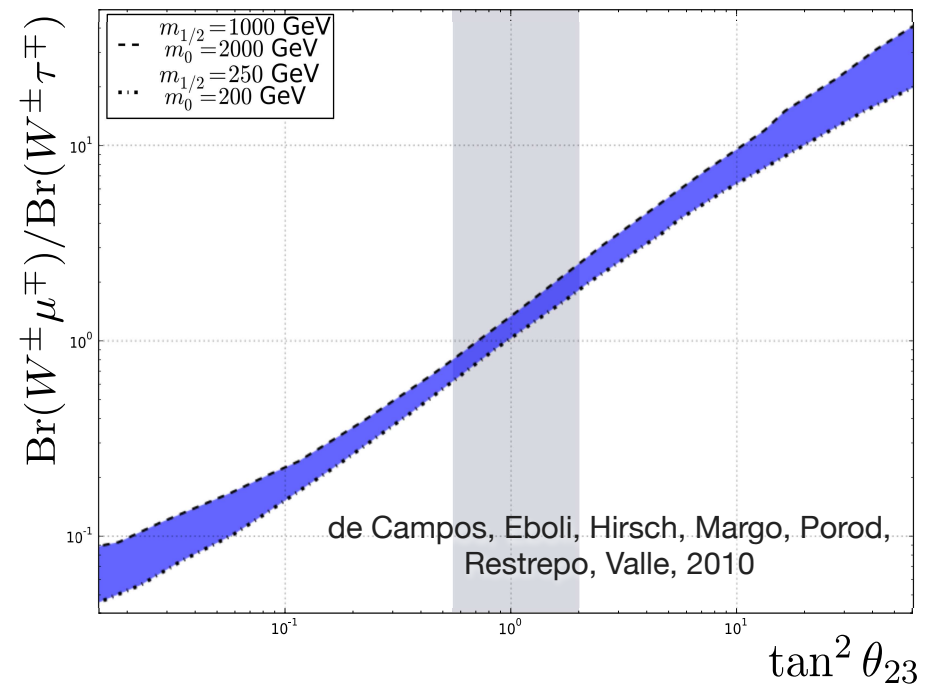
- mass generation for  $\Delta m_{\text{atm}}^2$ :



- mixing angle  $\leftrightarrow$  neutralino decay:

Mukhopadhyaya, Roy, Vissani, 1998

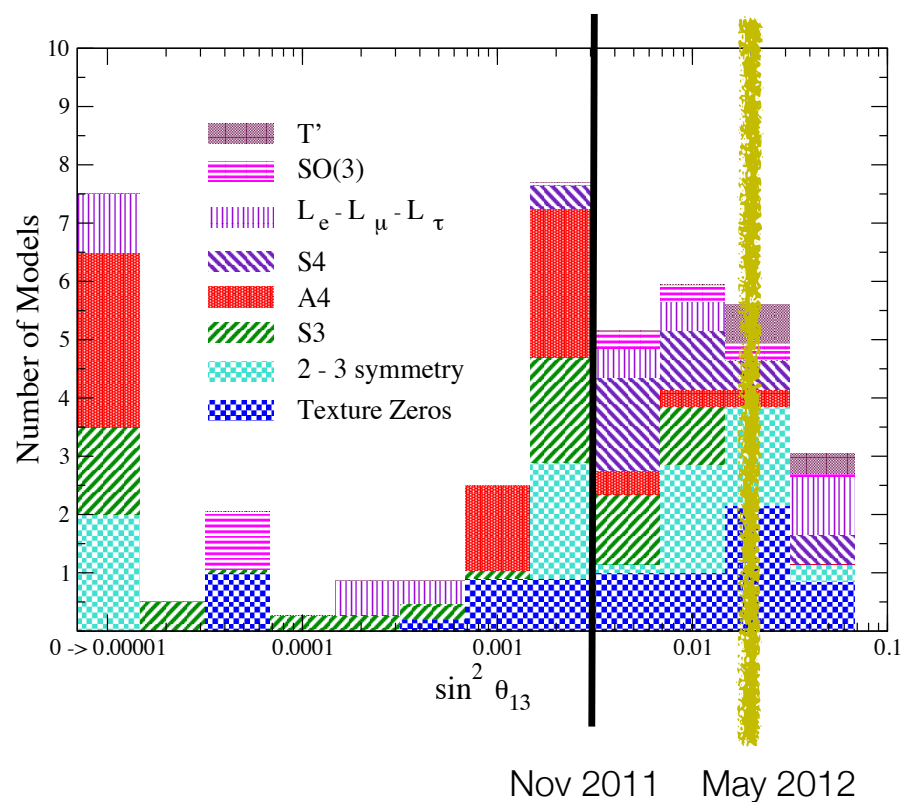
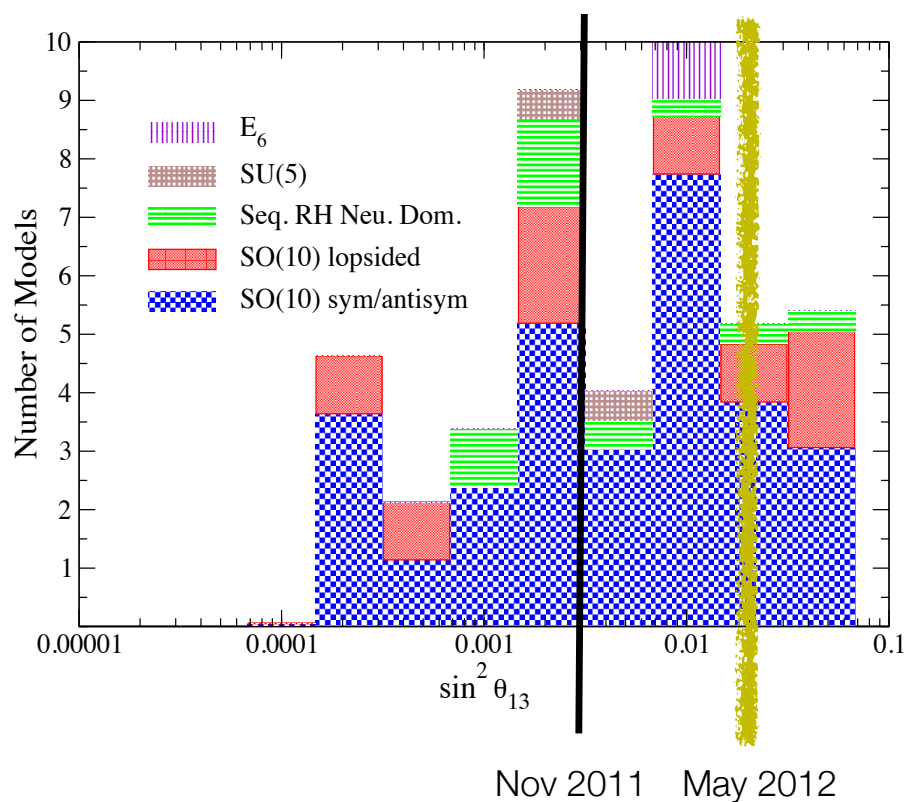
$$\tan^2 \theta_{\text{atm}} \simeq \frac{BR(\tilde{\chi}_1^0 \rightarrow \mu^\pm W^\mp)}{BR(\tilde{\chi}_1^0 \rightarrow \tau^\pm W^\mp)}$$



# Conclusions

- precise measurements of oscillation parameters important for pinning down underlying new physics

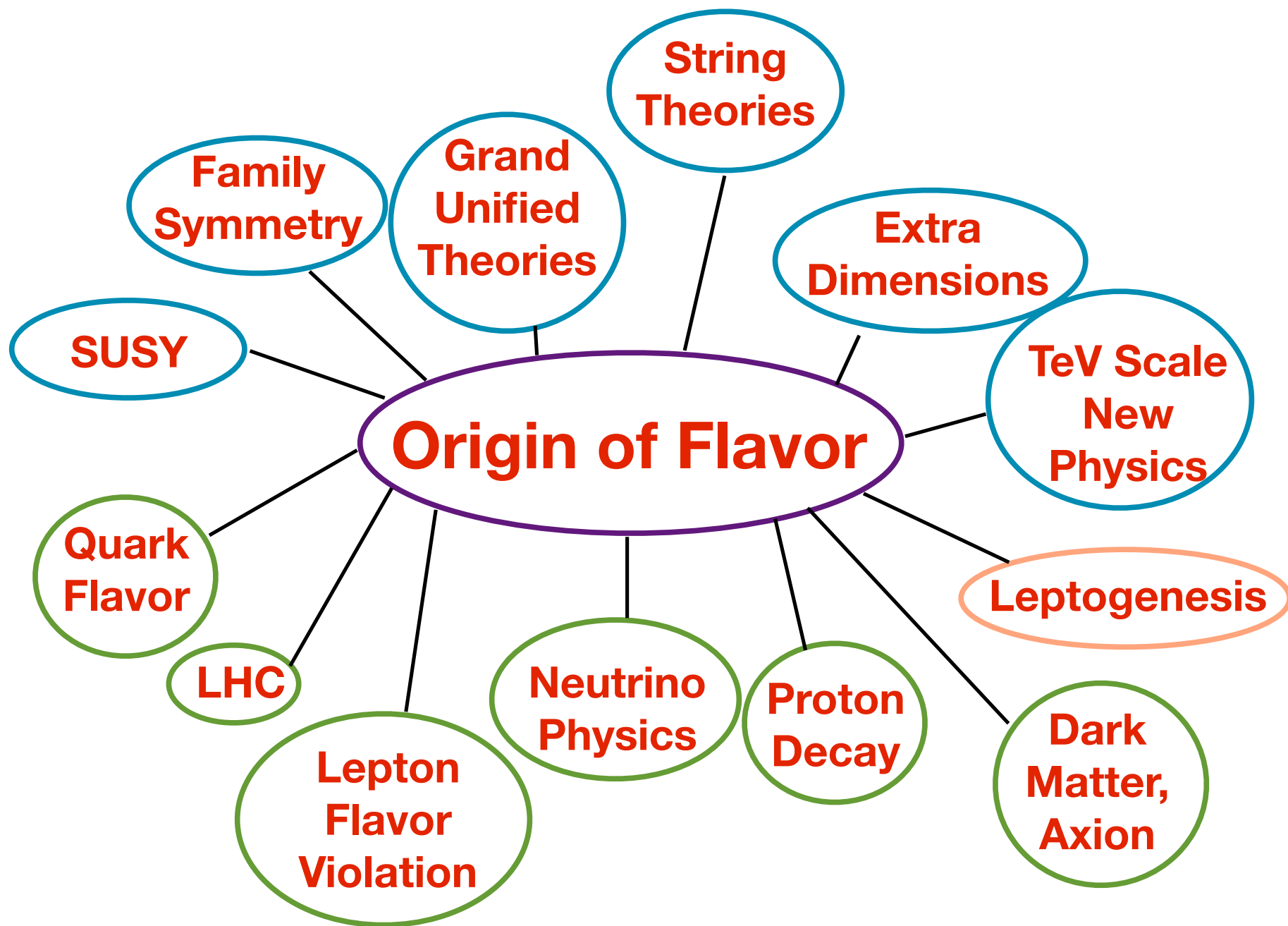
C.H. Albright (2009); C. H. Albright, M.-C. C (2006)



# Conclusions

---

- we are not just testing a number, but rather a paradigm like in the case of CKM matrix
- Testing correlations: robust way to distinguish different classes of models
  - correlations among neutrino mixing parameters
  - sum rules among quark and lepton mixing parameters
  - correlations among other flavor violating processes



# Discussion

---

Q: What accuracy do we need in oscillation parameters in order to distinguish different models?

Q: What precisions can be achieved experimentally in measuring  $\theta_{13}$ ,  $\theta_{23}$ ,  $\theta_{12}$ ?

# Discussion

---

Q: Is Tri-bimaximal mixing pattern still viable?

Q: What precisions can be achieved experimentally in measuring  $\theta_{13}$ ,  $\theta_{23}$ ,  $\theta_{12}$ ? to exclude TBM pattern?

Q: Can we ever exclude it?

# Discussion

---

Q: What precision do we need in CP phase?

Q: Given the large value of  $\theta_{13}$ , what precision can be achieved experimentally in measuring the CP phase?



# Discussion

---

Q: How seriously should we take the hints of sterile neutrinos (or something else) from LSND/MiniBooNE, reactor, radioactive source, or cosmological data?

Q: How far should we go and find out what is going on?

[B. Kayser @ GGI What's Nu?]

# Discussion

---

Q: In what sense do we need to constrain the three neutrino paradigm? What type of new physics can we constrain?

Q: What experimental precision do we need to establish three neutrino paradigm?