Measuring Neutrino Flux at Low Energies: The Low ν Method (for $E_\nu > 0.4$ GeV)

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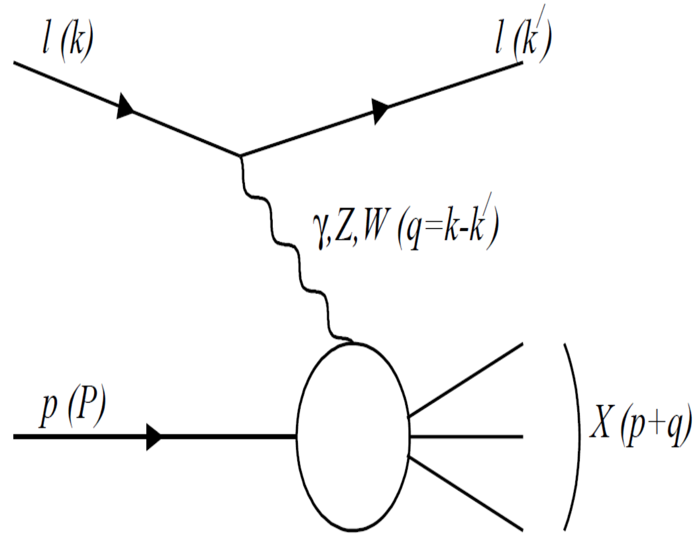
Plenary talk Nufact 2012
Thursday July 26. 2012
35min +5 min

$$
\begin{aligned}
Q^2 &= -q^2 = -(k-k')^2 > 0, \\
s &= (P+k)^2, \\
W^2 &= (P+q)^2, \\
x &= \frac{Q^2}{2P \cdot q}, \\
y &= \frac{P \cdot q}{P \cdot k}, \\
\nu &= \frac{P \cdot q}{M}.
\end{aligned}
$$



ν = the energy of the final state hadrons in the lab (Ehad) i.e. the energy of the particles that recoil against the final state lepton (Erecoil)

See lectures of neutrino physics

https://dl.dropbox.com/u/7144095/Perdue-INSS2012-NuXS-L1-v2.pdf // 19 MB, lecture 1

https://dl.dropbox.com/u/7144095/Perdue-INSS2012-NuXS-L2-v3.pdf // 59 MB(!), lecture 2,

References on low ν method.

For High Neutrino Energies (E>30 GeV, 1997)

S.R. Mishra (CCFR) , in Proceedings of the Workshop on Hadron Structure Functions and Parton Distributions, ed. by D. Geesaman et al.

W. Seligman, Ph.D. thesis (CCFR) Columbia, University, 1997, Nevis

For Intermediate Neutrino Energies (E>3.5 GeV, 2009)

D. Battacharya, PhD. Thesis (MINOS) , U. Pittsburgh (2009)

P. Adamson et al. (MINOS), Phys. Rev**. D 81**, 072002 (2010)

For Low Neutrino Neutrino Energies (E>0.4 GeV, 2012)

A. Bodek, U. Sarica, D. Naples, L. Ren, "Methods to Determine Neutrino Flux at Low Energies:Investigation of the Low ν Method". Eur.Phys.J. **C72**, 1973 (2012) arXiv:1201.3025 [hep-ex] (for Eν > 0.7 GeV)

A. Bodek, U. Sarica, K.S. Kuzmin, V.A. Naumov. "Extraction of Neutrino Flux with the Low ν Method at MiniBooNE Energies". to be published in proceedings of CIPANP 2012, St. Petersburg (2012). arXiv:1207.1247 [hep-ex] (for Eν > 0.4 GeV)

The following two methods were used in the determination of the neutrino flux in high energy (E_ν) neutrino beams.

1. *All experiments use the Standard Technique which is to model the distribution of pions and kaons produced by incident proton beam in the target. Then, tracking the pions and kaons though the Horn focusing magnetic fields, and modeling the* decays of pions *and kaons in the decay pipe. THIS METHOD TYPICALLY HAS errors of 15%.*

2. ***CCFR, NuTeV, MINOS also use the "low-v" method for the determination of the energy dependence of the relative neutrino and antineutrino flux and normalize to a well known cross section at high energy. This method reduces the systematic errors.***

Here v is the energy transfer to the target (sometimes called Ehad)

- *CCFR and NuTeV use this method for* E_ν *>30 GeV.*
- *MINOS uses this method for* E_ν *>3 GeV for neutrinos and* E_ν *>6 GeV for antineutrinos. The method only yields the realtive flux vs energy. Both experiment normalize to the world average total cross section at high energy.*

In this talk we investigate the use of the method down to E_ν ***=0.4 GeV which is in the MiniBooNE, MINERvA and T2K range of interest.***

The Low ν Method for High Energies

- At high E, cross section for DIS dominates.

Dominant constant term

Small terms

$$\frac{d\sigma}{d\nu} = A\left(1 + \frac{B}{A}\frac{\nu}{E} - \frac{C}{A}\frac{\nu^2}{2E^2}\right)$$

$$A = \frac{G_F^2 M}{\pi}\int_0^1 F_2(x)dx,$$

$$B = -\frac{G_F^2 M}{\pi}\int_0^1 \left(F_2(x) \mp xF_3(x)\right)dx,$$

$$C = B - \frac{G_F^2 M}{\pi}\int_0^1 F_2(x)\,\tilde{R}\,dx,$$

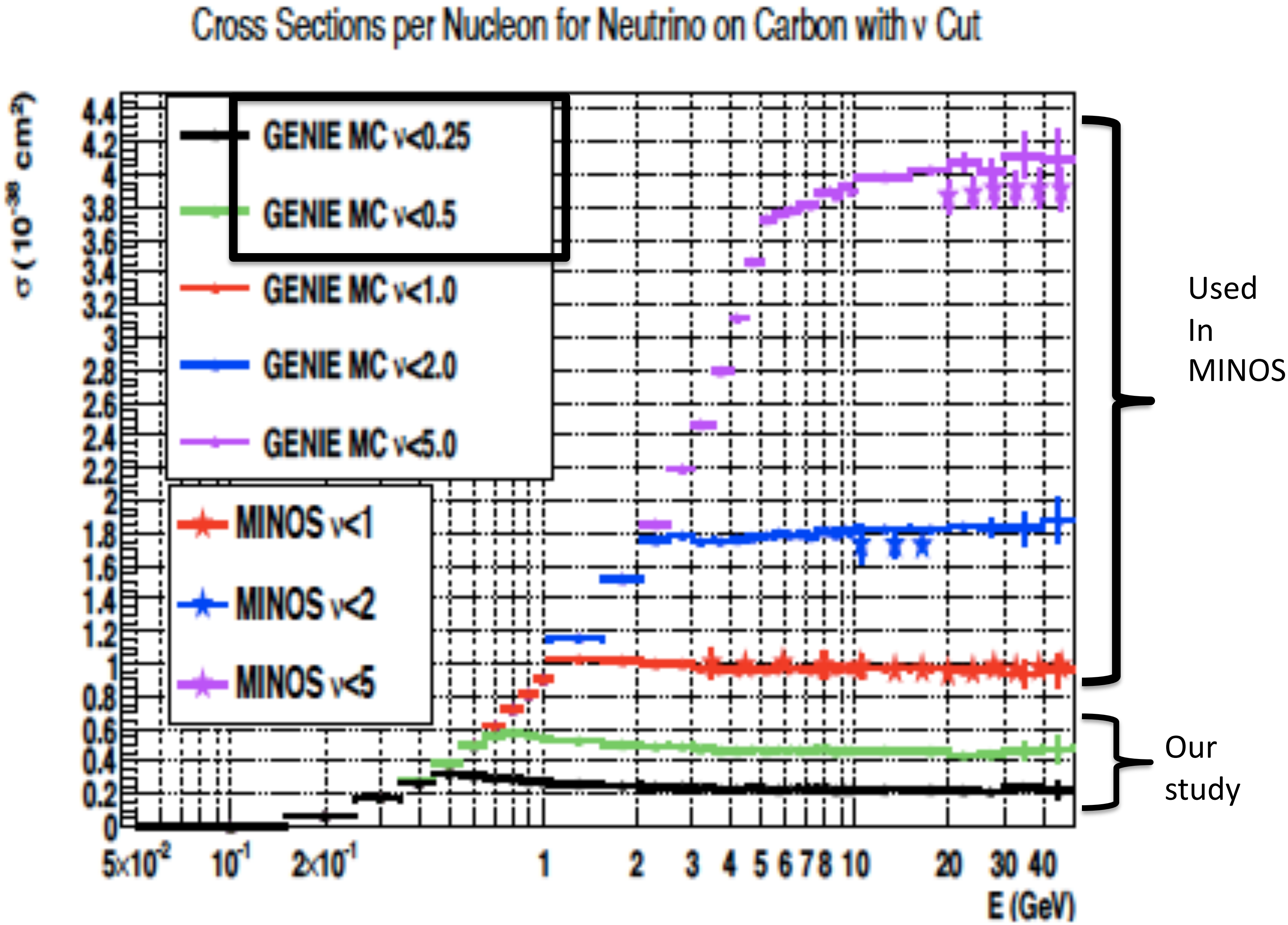
Multiply both sides by flux →

$$\frac{dN}{d\nu} = \phi A\left(1 + \frac{B}{A}\frac{\nu}{E} - \frac{C}{A}\frac{\nu^2}{2E^2}\right)$$ and

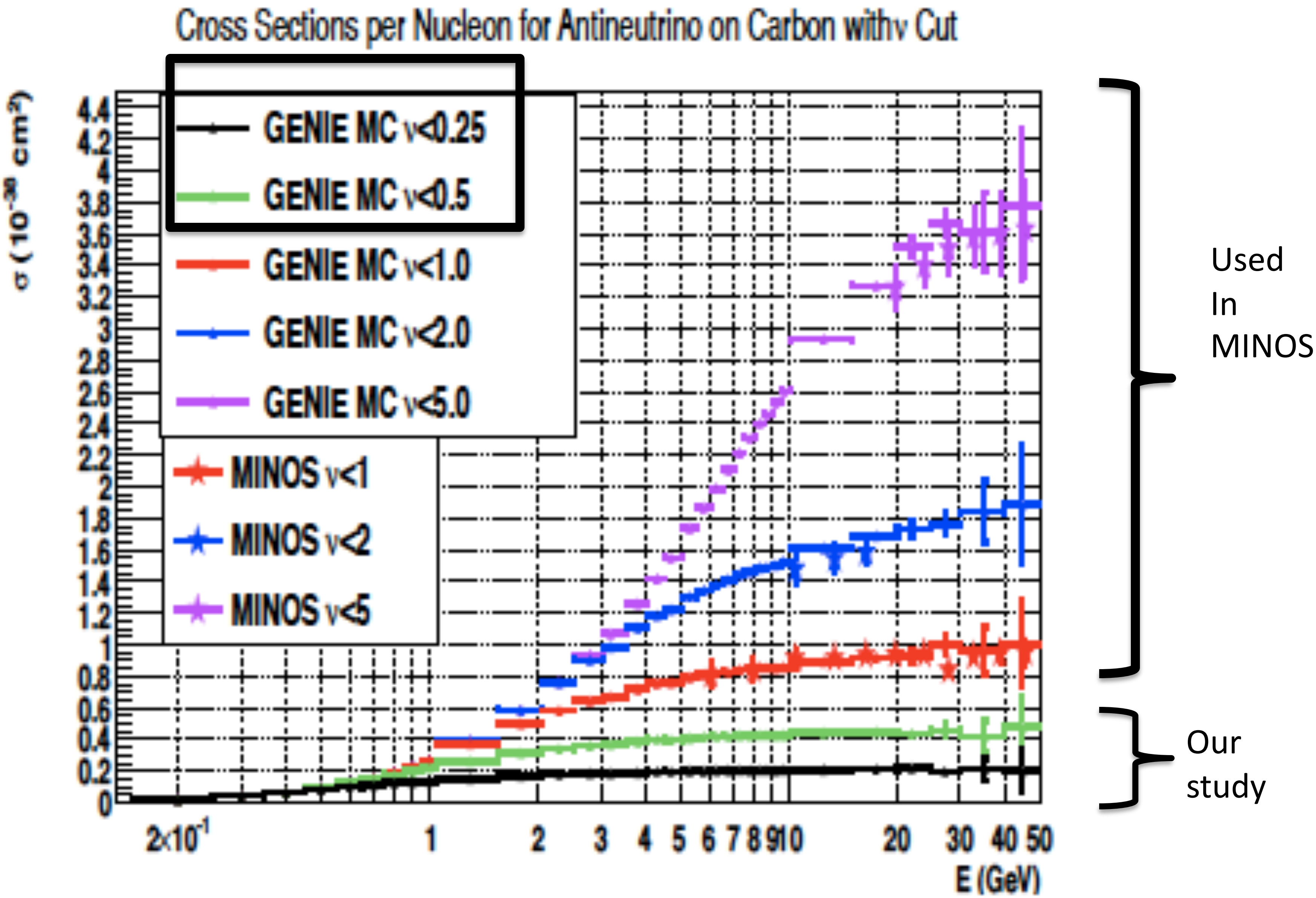
$$\tilde{R} = \left(\frac{1 + \frac{2Mx}{\nu}}{1 + R(x, Q^2)} - \frac{Mx}{\nu} - 1\right).$$

As ν→0, $\frac{dN}{d\nu} \approx \phi A$

"A term" only depends on F2, other terms are small at low ν. Correction terms proportional to ν/E depend on qbar (xF3) and R = σL/σT (2xF1)



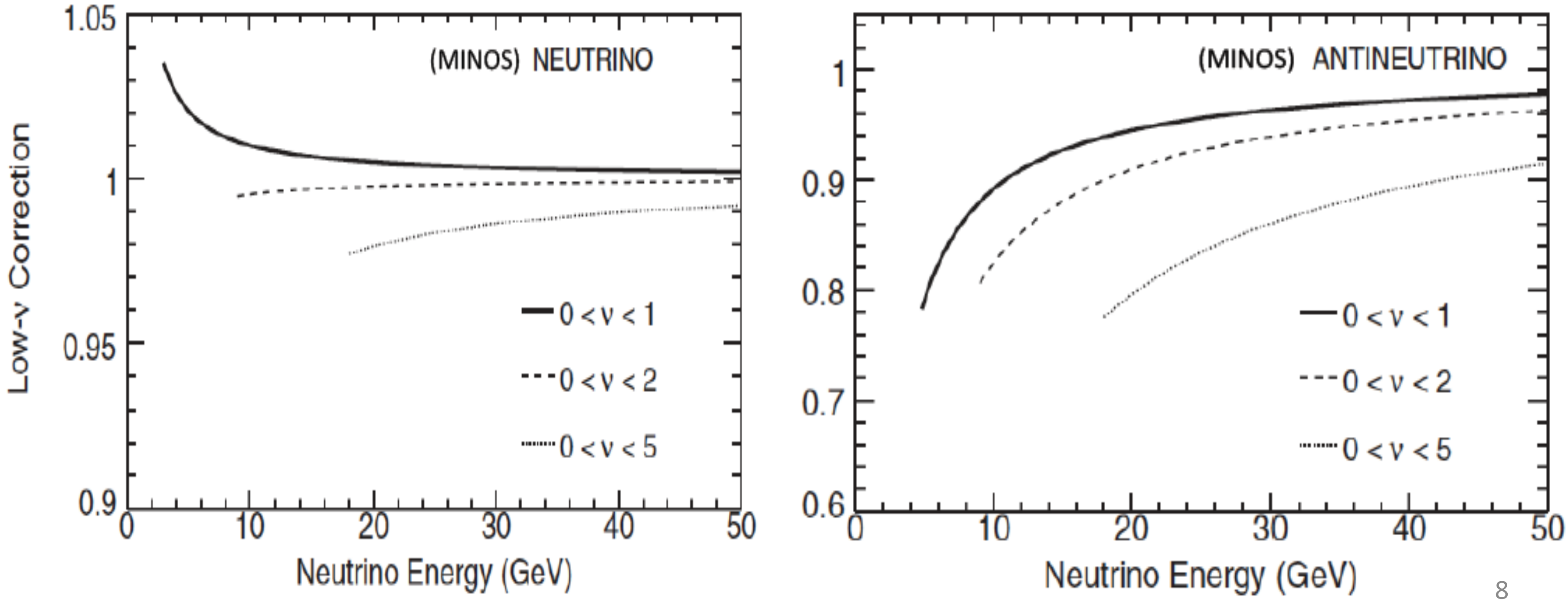
How constant are the low ν cross sections for neutrinos ?

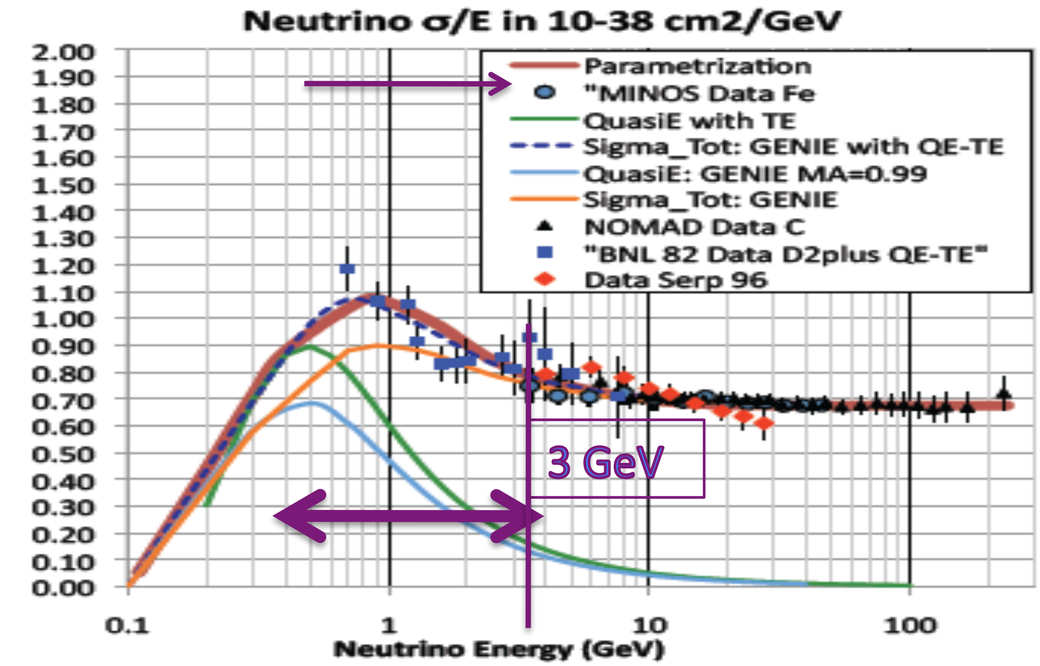


How constant are the low ν cross sections for anti-neutrinos ?

The low ν cross setion is approximately contant, with some small corrections. The method has been used by MINOS for neutrino energies greater than 3 GeV and antineutrino energies of 6 GeV

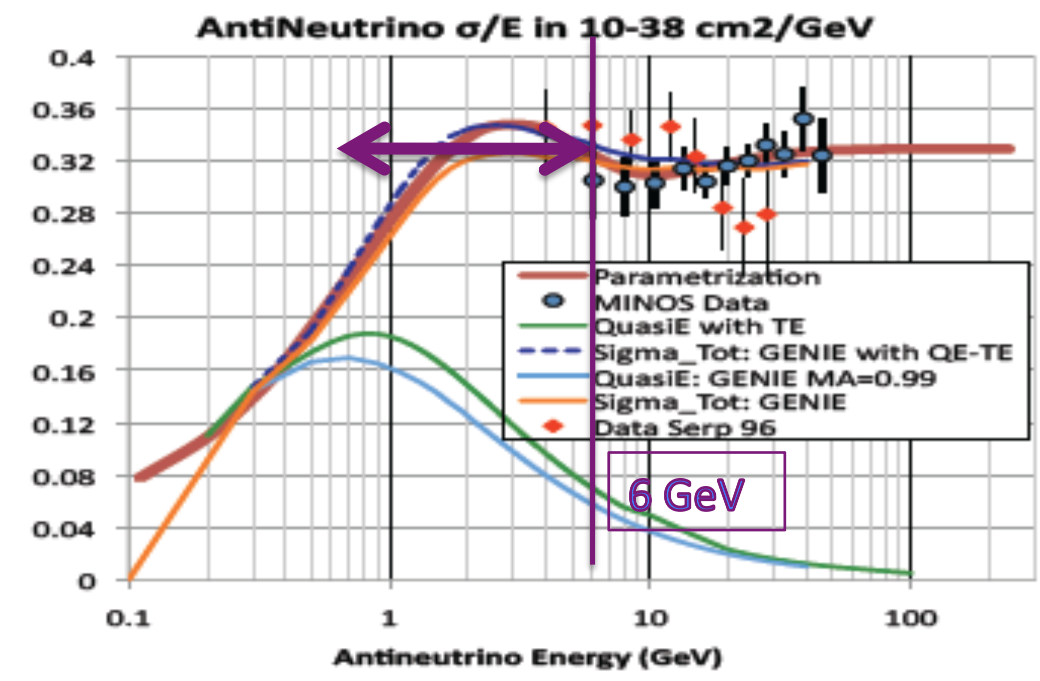
- The MINOS Collaboration suggested the number of low ν events should not exceed 60% of the total cross section
- However, this number should also be statistically significant.
- Hence, MINOS used ν<1 GeV for E>3 GeV for neutrinos and E>5 GeV for antineutrinos; ν<2 GeV and ν<5 GeV for E>9 GeV and E>18 GeV, respectively.





Total neutrino cross sections extracted by MINOS
For neutrino energies $\mathbf{E}_\nu$ >3 GeV using the low ν method

We investigate the uncertainties in the method for neutrino energies as low as $\mathbf{E}_\nu$ =0.4 GeV.



Total antineutrino cross sections extracted by MINOS for antineutrino energies $\mathbf{E}_\nu$ >6 GeV using the low ν method

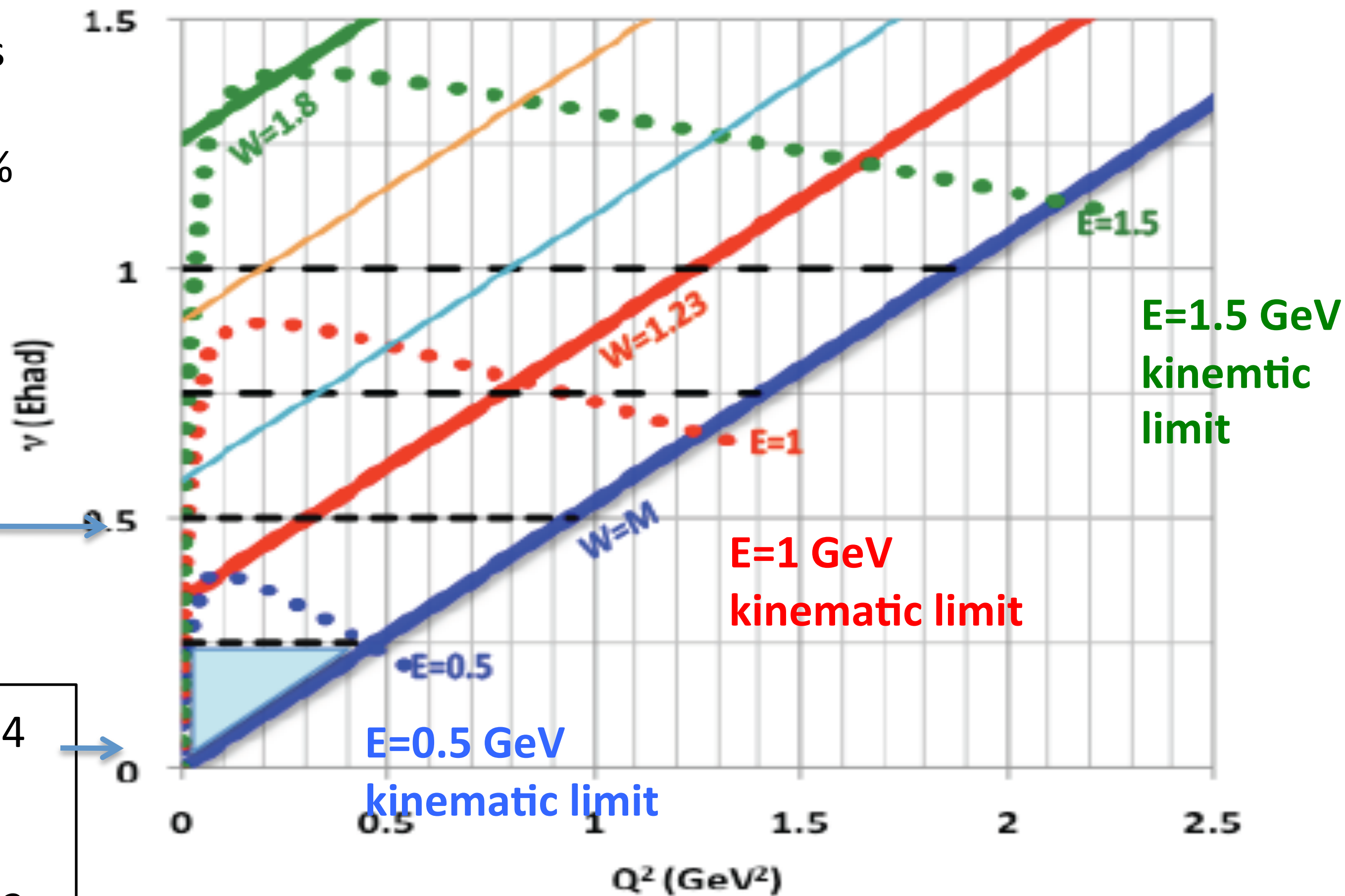
We investigate the uncertainties in the method for antineutrino energies as low as $\mathbf{E}_\nu$= 0.7 GeV.

Fraction of low ν events used to determine the flux must be less the 60% of the total number of events. The low ν cut must be less the 25% of the beam energy

ν <0.5 GeV means Q^2<1 GeV^2. , 2/3 QE events, 1/3 Delta

ν <0.2 GeV means Q^2<0.4 GeV^2 , all QE events

ν <0.1 GeV means Q^2<0.2 GeV^2, all QE events



Need to investigate uncertainties in modeling QE (and Delta production) processes on nuclear targets.

In Eur. Phys. J. C 72, 1973 (2012) we investigated:

ν <0.25 and ν <0.5 GeV for $\mathbf{E}_\nu$ >0.7 GeV.

Here, we investigate ν<0.2 and ν< 0.1 for $\mathbf{E}_\nu$ >0.4 GeV (see also arXiv:1207.1247)

The Low ν Method for Low Energies

- The low ν cross section is approximately constant, so it can be used to determine the flux.
- The question is, what are the model uncertainties of the energy dependent corrections when we use the method at very low energies (between 0.4 GeV and 5 GeV).
- We need to use events with very low ν (i.e. less than 0.10 , 0.20, 0.25, and 0.5 GeV).
- We studied all of the above low ν regions.

The low ν for low neutrino energies.
At low ν the cross section is still dominated by W_2

$$\frac{d\sigma^{ud}}{dQ^2 d\nu} = S_{cos}\frac{1}{2E^2}\mathcal{W}_1\left[Q^2+m_\mu^2\right]$$
$$+S_{cos}\mathcal{W}_2\left[(1-\frac{\nu}{E})-\frac{(Q^2+m_\mu^2)}{4E^2}\right]$$
$$+S_{cos}\mathcal{W}_3\left[\frac{Q^2}{2ME}-\frac{\nu}{4E}\frac{Q^2+m_\mu^2}{ME}\right]$$
$$+S_{cos}\mathcal{W}_4\left[m_\mu^2\frac{(Q^2+m_\mu^2)}{4M^2E^2}\right]$$
$$-S_{cos}\mathcal{W}_5\left[\frac{m_\mu^2}{ME}\right]$$

$$\sigma_{\nu cut}(E) = \int_{\nu_{min(E)}}^{\nu_{cut}} \frac{d^2\sigma}{dQ^2 d\nu} dQ^2 d\nu$$
$$= \sigma_{W_2} + \sigma_2 + \sigma_1 + \sigma_3 + \sigma_4 + \sigma_5,$$

= constant plus correction terms that depend on ν/E.

$$\sigma_2 = S_{cos}\int_{\nu_{min(E)}}^{\nu_{cut}}\left[-\frac{\nu}{E}-\frac{Q^2+m_\mu^2}{4E^2}\right]\mathcal{W}_2\, d\nu.$$ **known**

$$\sigma_1 = S_{cos}\int_{\nu_{min(E)}}^{\nu_{cut}} -\left[\frac{(Q^2+m_\mu^2)}{2E^2}\right]\mathcal{W}_1\, d\nu.$$ GE/GM

$$\sigma_3 = S_{cos}\int_{\nu_{min(E)}}^{\nu_{cut}}\left[\frac{Q^2}{2ME}-\frac{\nu}{4E}\frac{Q^2+m_\mu^2}{ME}\right]\mathcal{W}_3$$ **V-A interfence**

$$\sigma_4 = S_{cos}\int_{\nu_{min(E)}}^{\nu_{cut}}\left[m_\mu^2\frac{(Q^2+m_\mu^2)}{4M^2E^2}\right]\mathcal{W}_4\, d\nu.$$ Very small

$$\sigma_5 = S_{cos}\int_{\nu_{min(E)}}^{\nu_{cut}}\left[\frac{-m_\mu^2}{ME}\right]\mathcal{W}_5\, d\nu.$$ Very small

$$\sigma_{tot}(E) = \sigma_{W_2}(\infty)\,[f_C]$$

$$f_C = [f_{W2} + f_2 + f_1 + f_3 + f_4 + f_5]$$

$$f_{W2} = \frac{\sigma_{W2}}{\sigma_{W_2}(\infty)}(\approx 1)$$

$$f_2 = \frac{\sigma_2}{\sigma_{W_2}(\infty)}(= kinematic\ correction)$$

$$f_1 = \frac{\sigma_1}{\sigma_{W_2}(\infty)}(= important)$$

$$f_3 = \frac{\sigma_3}{\sigma_{W_2}(\infty)}(= important)$$

$$f_4 = \frac{\sigma_4}{\sigma_{W_2}(\infty)}(= very\ small)$$

$$f_5 = \frac{\sigma_4}{\sigma_{W_2}(\infty))}(= very\ small)$$

Correction terms that make the cross section deviate from a constant are proportional to ν /E

$$\sigma_{\nu cut}(E) = \sigma_{W_2}(\infty)\,[f_C],$$

$$f_C = [f_{W2} + f_2 + f_1 + f_3 + f_4 + f_5],$$

$$f_{W2} = \frac{\sigma_{W2}}{\sigma_{W_2}(\infty)} \approx 1,$$

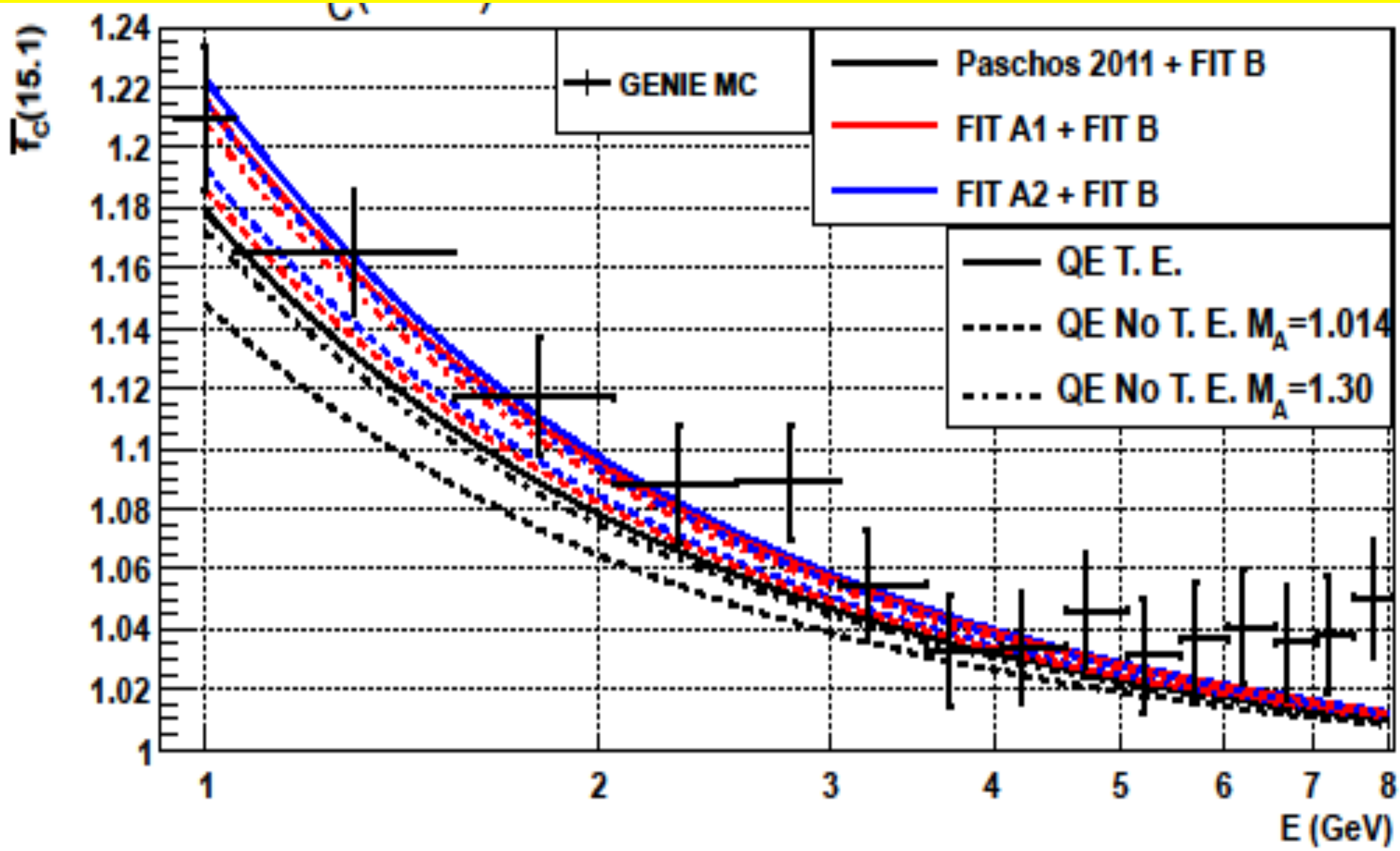
$$f_i = \frac{\sigma_i}{\sigma_{W_2}(\infty)}.$$

1. $f_{W2} = \frac{\sigma_{W2}}{\sigma_{W_2}(\infty)} \approx 1$ is well known and does not contribute to the uncertainty in f_C.
2. The energy dependent correction f_2 is explicit and therefore does not contribute to the uncertainty in f_C.
3. The contributions of f_4 and f_5 are small since they are proportional to the square of the muon mass, and therefore have a negligible contribution to the uncertainty in f_C. (Note that the vector parts of f_4 and f_5 are known very well since they can be expressed in terms of the vector parts of $\mathcal{W}_1$ and $\mathcal{W}_2$).
4. The only non-negligible uncertainty originates from the modeling of the contributions of f_1 and f_3 (primarily from f_3).

Lowest ν cut in MINOS was 1 GeV (poor calorimeter). For MINERvA we can go to lower hadron energies. Start with a ν cut of 0.5 GeV. 2/3 QE scattering, 1/3 Resonance

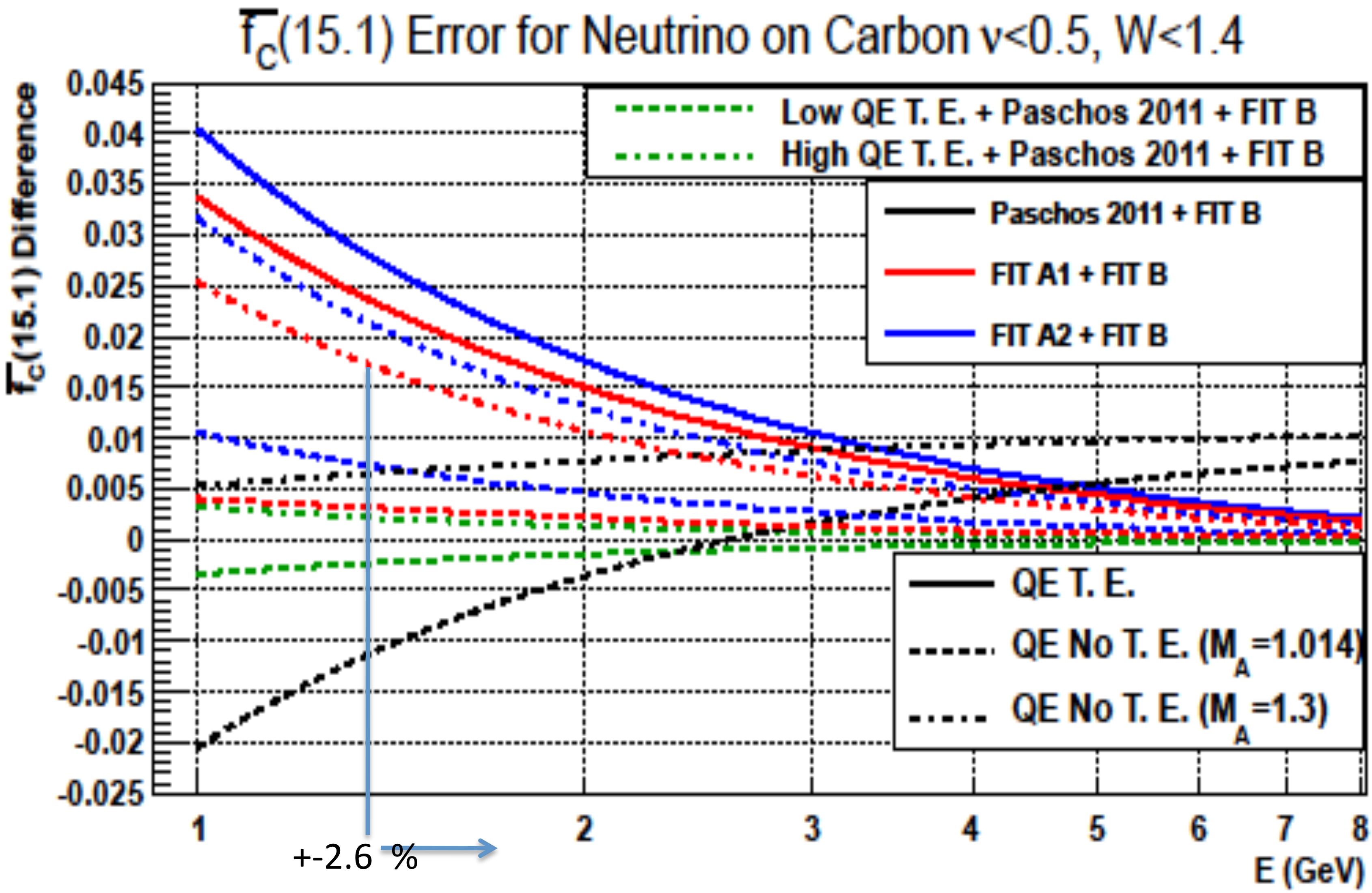
$$\ldots \nu_{max}\text{=0.5: } f_C \ldots$$

Relative low ν Neutrino cross section normalized to the cross section at 15.1 GeV



Uncertainty in the relative low ν cross section for neutrinos: from uncertainties in QE and Delta production cross sections and form factors: Use above 1.2 GeV

$$\ldots \nu_{max} = 0.5 \ldots$$



…ν_{max}=0.5: Conclusion

The error in fc is about than 2.6 % for neutrinos (above 1.2 GeV)
The error in fc is about than 1.4 % for antineutrinos (above 2 GeV)

For these energies, the fraction of $\nu<0.5$ GeV events is less than 60%).

For lower energies need to use ν cuts less than 0.5 GeV.

Note the number of event is proportional to the ν cut, so the number of events with $\nu<0.5$ GeV is twice the number of $\nu<0.25$ GeV).

Next: investigate $\nu<0.2$ and $\nu<0.1$ GeV cuts.

$$W_{1-Qelastic}^{\nu-vector} = \delta(\nu - \frac{Q^2}{2M})\tau|\mathcal{G}_M^V(Q^2)|^2$$

$$W_{1-Qelastic}^{\nu-axial} = \delta(\nu - \frac{Q^2}{2M})(1+\tau)|\mathcal{F}_A(Q^2)|^2$$

$$W_{2-Qelastic}^{\nu-vector} = \delta(\nu - \frac{Q^2}{2M})|\mathcal{F}_V(Q^2)|^2$$

$$W_{2-Qelastic}^{\nu-axial} = \delta(\nu - \frac{Q^2}{2M})|\mathcal{F}_A(Q^2)|^2$$

$$W_{3-Qelastic}^{\nu} = \delta(\nu - \frac{Q^2}{2M})|2\mathcal{G}_M^V(Q^2)\mathcal{F}_A(Q^2)|$$

where

$$\mathcal{G}_E^V(Q^2) = G_E^p(Q^2) - G_E^n(Q^2),$$

$$\mathcal{G}_M^V(Q^2) = G_M^p(Q^2) - G_M^n(Q^2).$$

and

$$|\mathcal{F}_V(Q^2)|^2 = \frac{[\mathcal{G}_E^V(Q^2)]^2 + \tau[\mathcal{G}_M^V(Q^2)]^2}{1+\tau}.$$

For ν <0.25 GeV , the uncertainties in the relative low ν cross section depends on uncertainties in the modeling of the QE cross section.

We investigate several models that fit either the MiniBooNE cross section or the free nucleon cross sections:

1. Free nucleon form factors with M_A varying between 1.014 and 1.3 GeV and a **dipole fit form** for F_A .

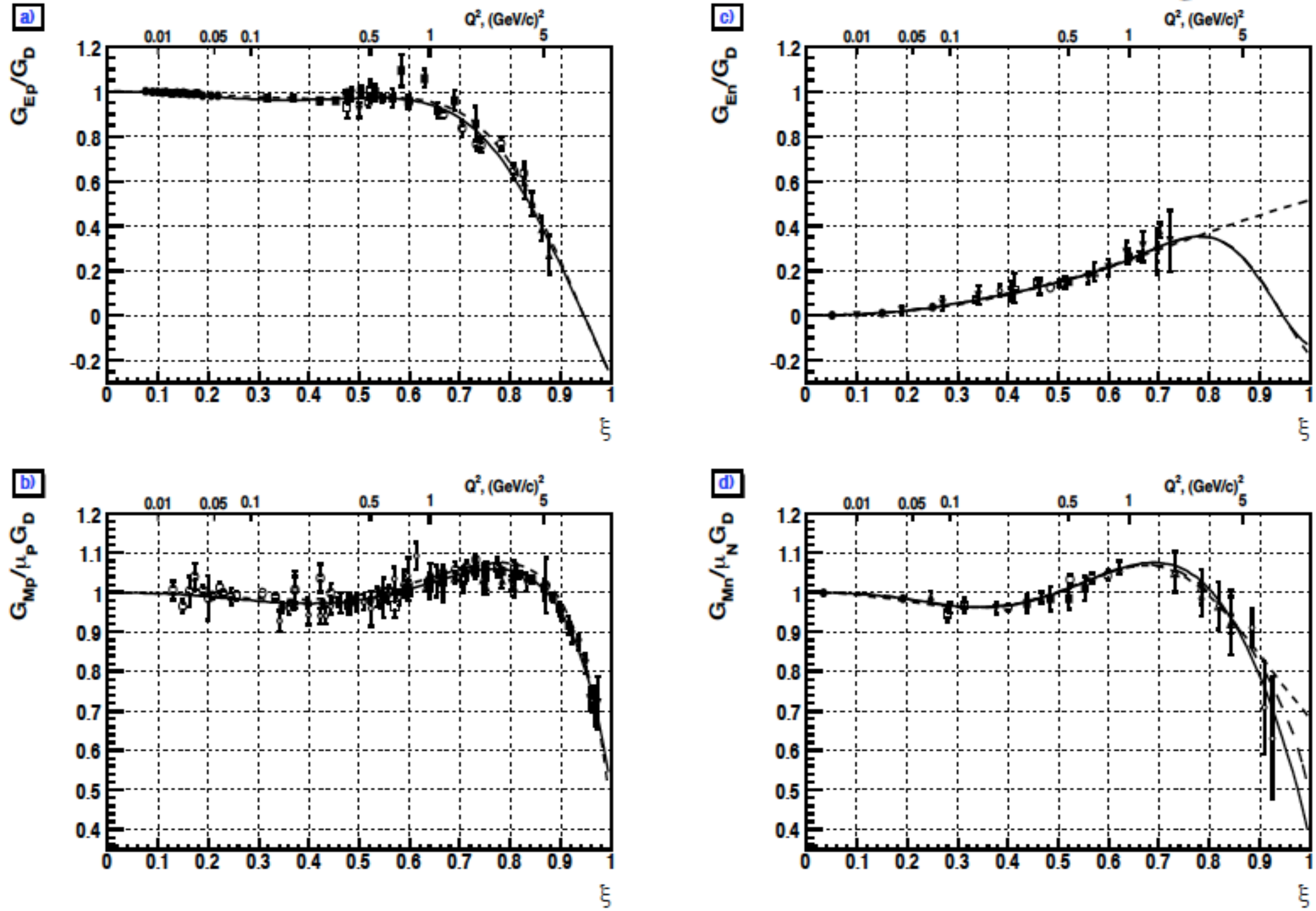
2. **Transverse enhancement of GMp and GMn** as observed in electron scattering on bound nucleons, and a **dipole form for F_A**

3. Transverse enhancement of GMp and GMn as observed in electron scattering on bound nucleons, and a **modified dipole form for F_A** fit to all free nucleon data (our best model).

We use BBBA form factors. Duality based parametrization of the ratio of free EM nucleon form factors to dipole (Gep, Gmp, Gen, Gmn) .

A. Bodek, S. Avvakumov, R. Bradford, and H. Budd: Vector and Axial Nucleon Form Factors:

Phys. J. C53, 349 (2008).

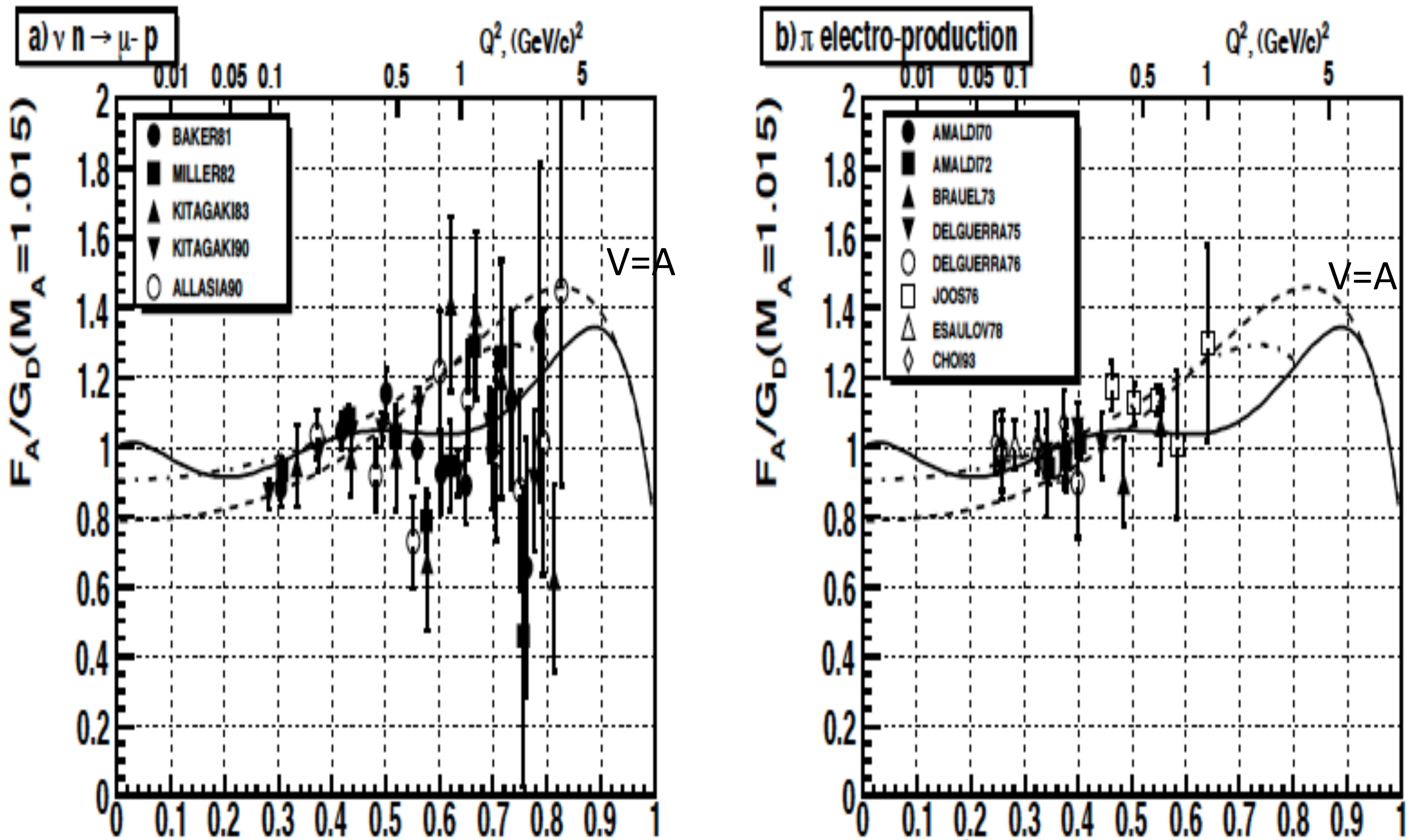


1. Ratios of G_{Ep} (a), G_{Mp}/μ_p (b), G_{En} (c) and G_{Mn}/μ_n (d) to G_D. The short-dashed line in each plot is the old Kelly

BBBA form factors are also available for the axial form factor. Duality base prametrization of the ratio of free nucleon F_A to dipole with M_A **=1.015 .**

A. Bodek, S. Avvakumov, R. Bradford, and H. Budd: Vector and Axial Nucleon Form Factors:

Phys. J. C53, 349 (2008).



Ratio of F_A **data on deuterium to dipole with** M_A **=1.015 (solid line=modified dipole)**

Fig. 3. (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G_D^A(Q^2)$[22]. (b) $F_A(Q^2)$ from pion electroproductio divided by $G_D^A(Q^2)$[22], corrected for for hadronic effects[12]. Solid line - duality based fit; Short-dashed line - $F_A(Q^2)_{A2=V2}$

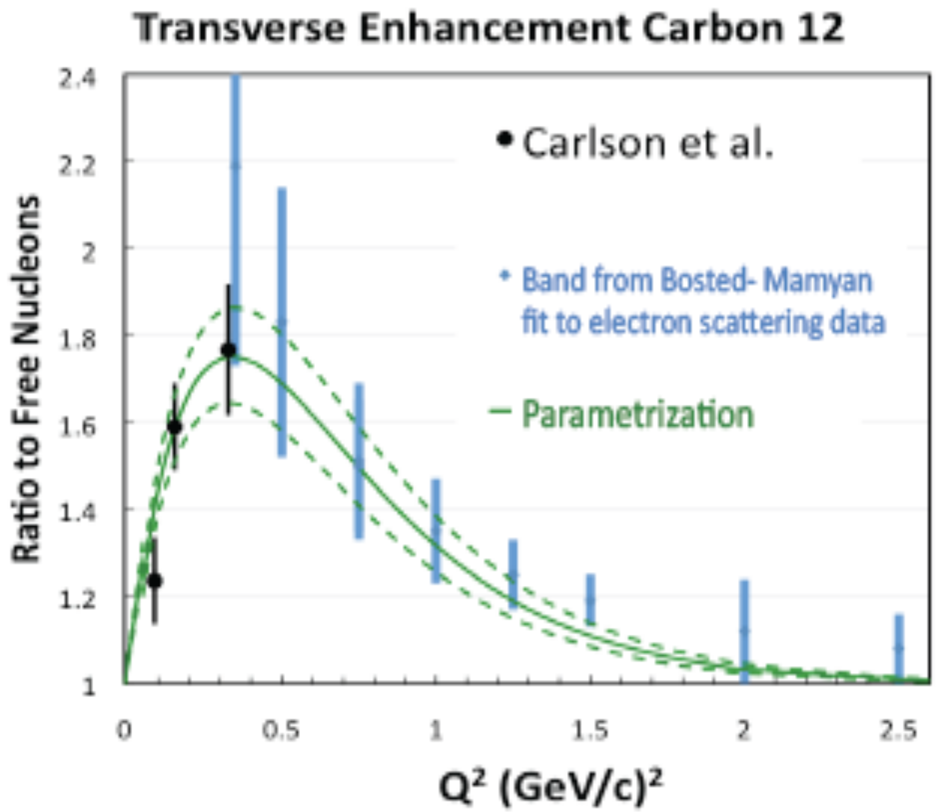


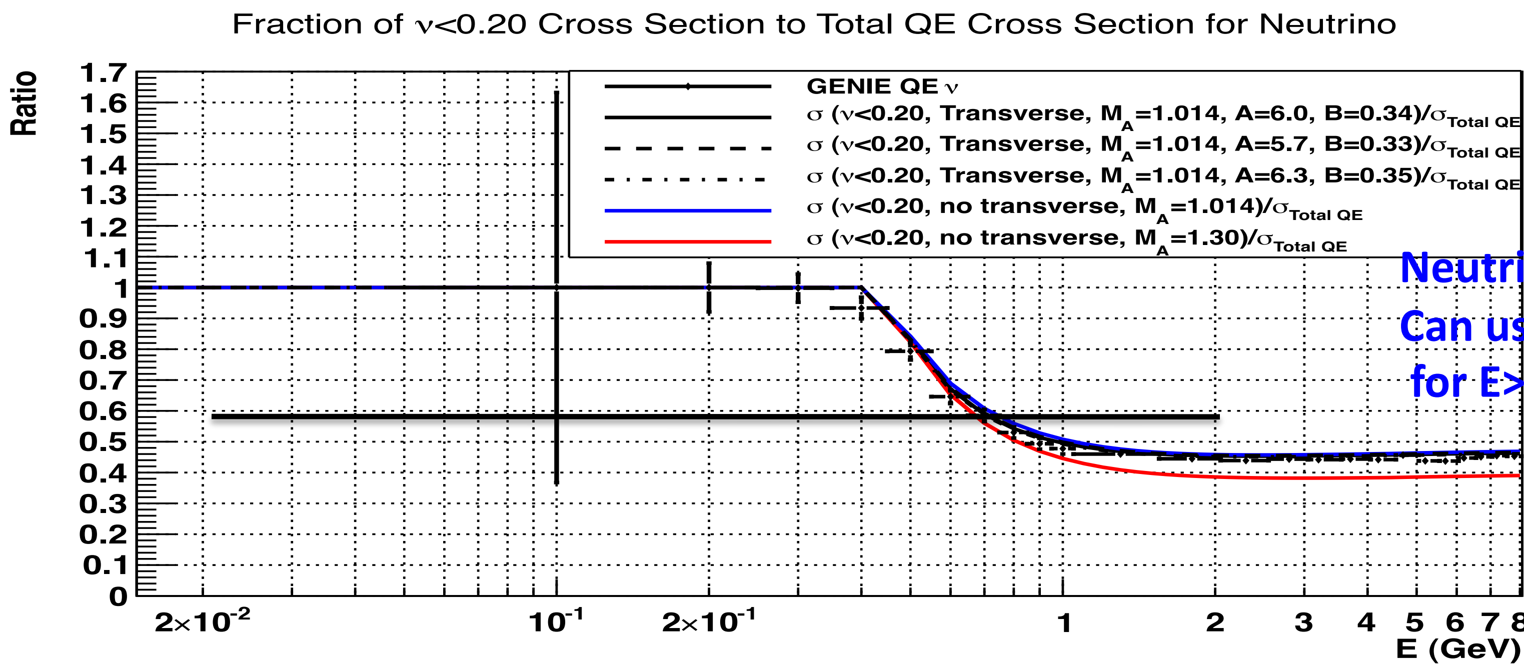
Fig. 7. The transverse enhancement ratio[14] ($\mathcal{R}_T$) as a function of Q^2. Here, $\mathcal{R}_T$ is ratio of the integrated transverse response function for QE electron scattering on nucleons bound in carbon divided by the integrated response function for independent nucleons. The black points are extracted from Carlson *et al*[16], and the blue bands are extracted from a fit[19] to QE data from the JUPITER[18] experiment (Jlab experiment E04-001). The curve is a fit to the data of the form $\mathcal{R}_T = 1 + AQ^2e^{-Q^2/B}$. The dashed lines are the upper and lower error bands (color online).

Nuclear Corrections from electron scattering data (BBC Parametrization of transverse enhancement of form factors in Carbon)

A. Bodek, H. Budd and E. Christy, Eur.Phys. J. C71, 172 (2011)

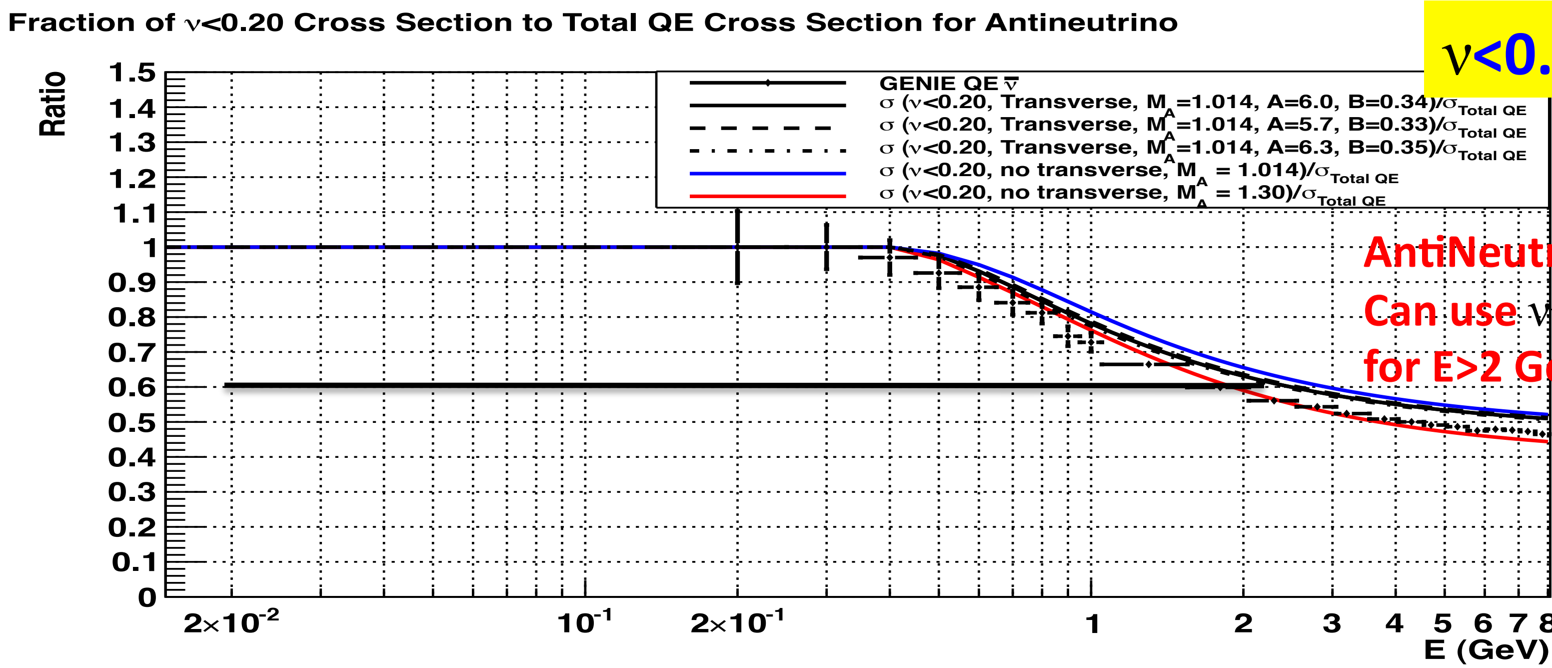
Use RT factor to enhance GMp and GMn in bound nucleons extracted from in electron scattering experiments on Carbon.

We will refer to TE cross sections with this correction as BBC form factors for a nucleus.
(i.e. BBBA times RT).



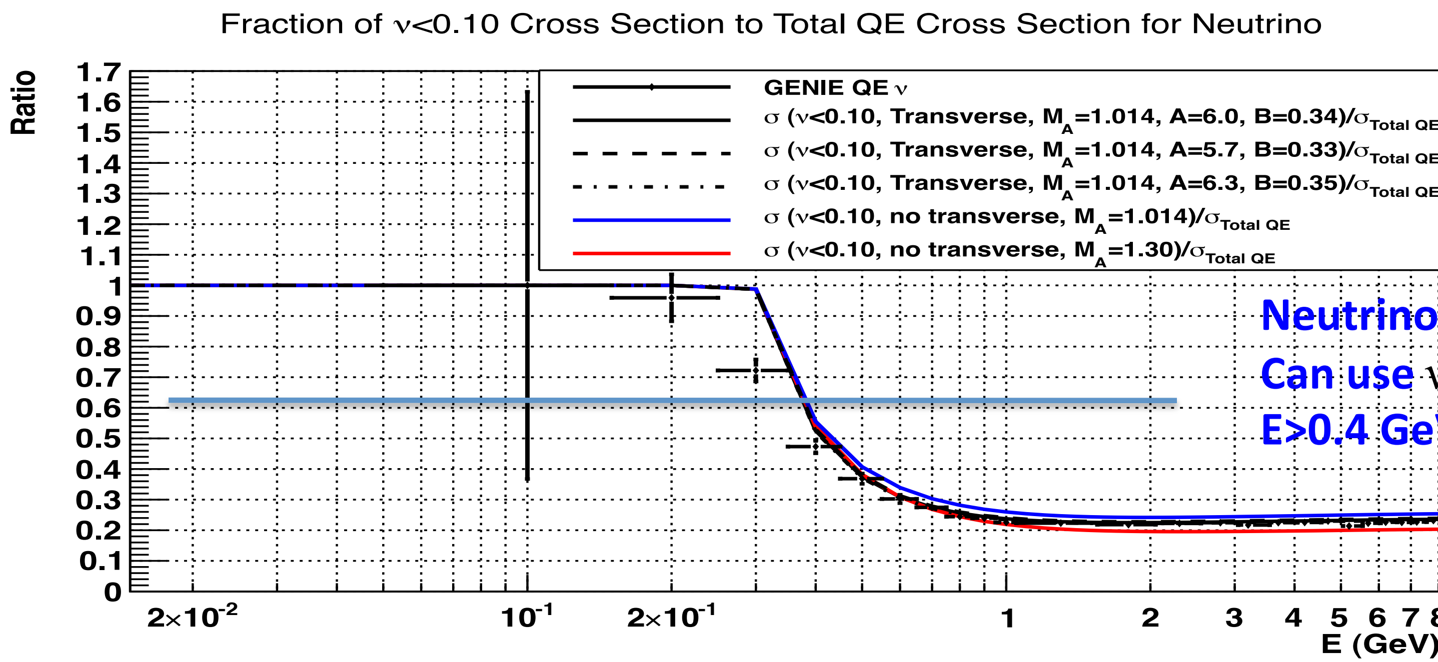
Neutrinos:
Can use ν<0.2 GeV
for E>0.7 GeV

ν<0.2 GEV



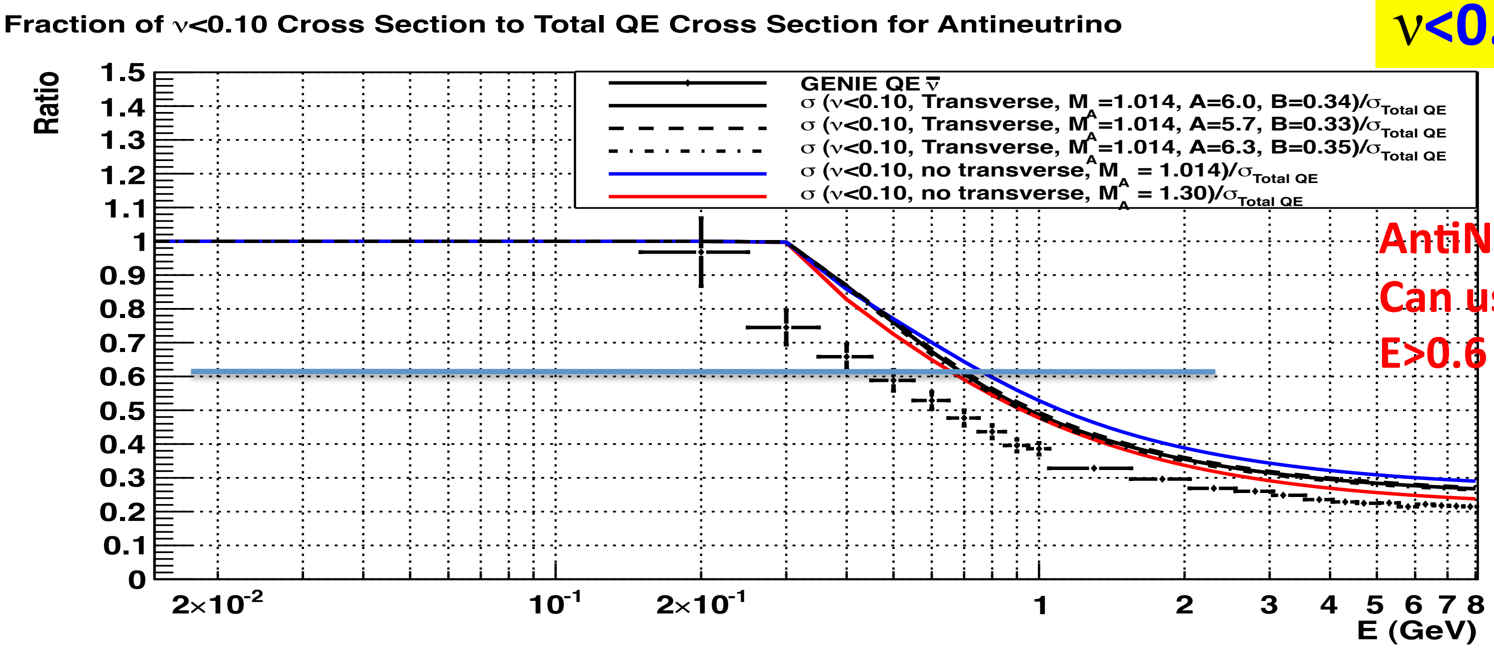
AntiNeutrinos:
Can use ν< 0.2 GeV
for E>2 GeV

Want fraction of events in the low ν flux sample to be less than 60% of the **QE cross section**



Neutrinos:
Can use ν<0.1 GeV for E>0.4 GeV

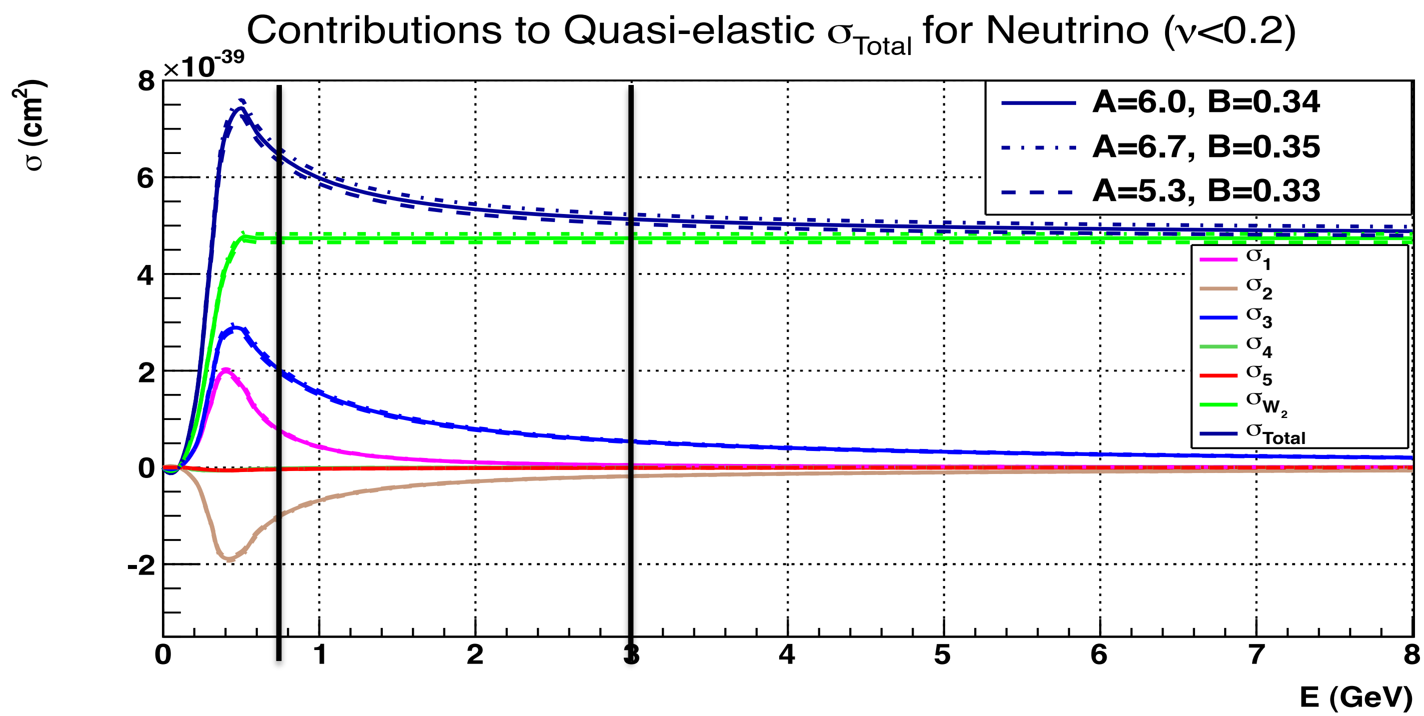
ν<0.1 GEV

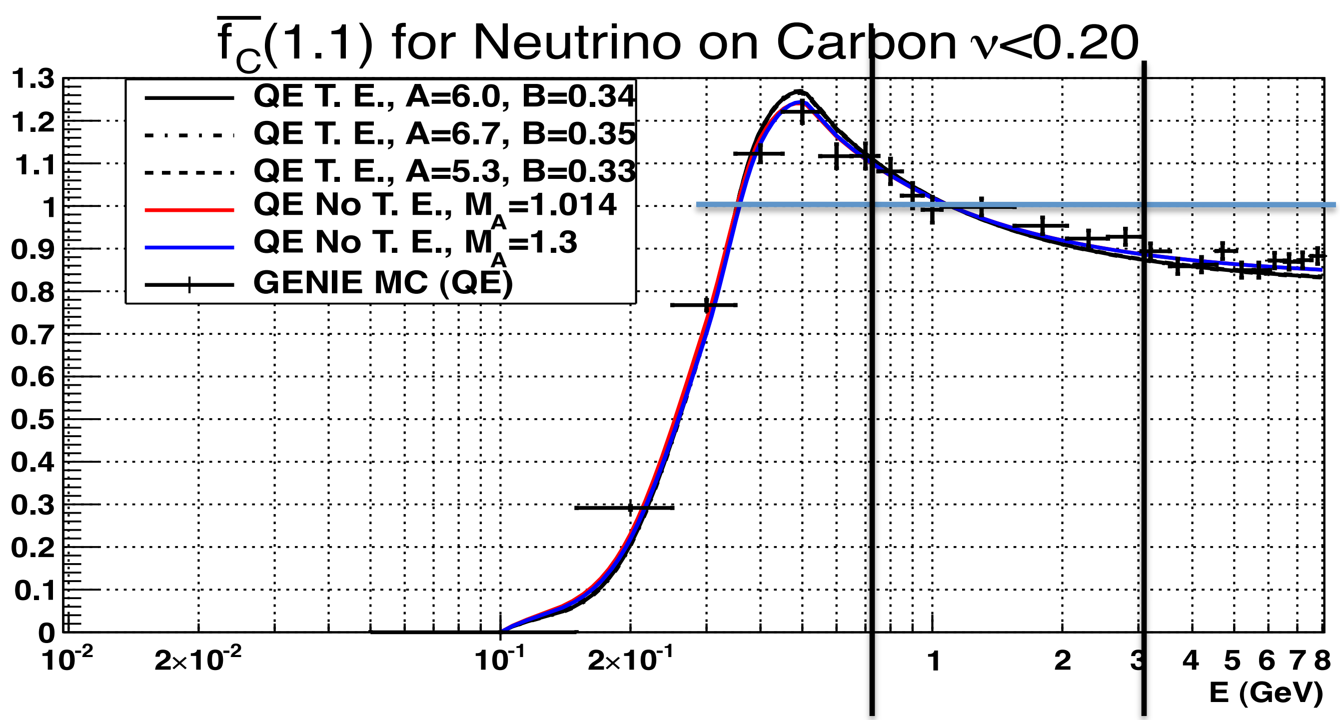


AntiNeutrinos:
Can use ν<0.1 GeV for E>0.6 GeV

GENIE cross section has no TE

Want fraction of events in the low νflux sample to be less than 60% of the **QE cross section**



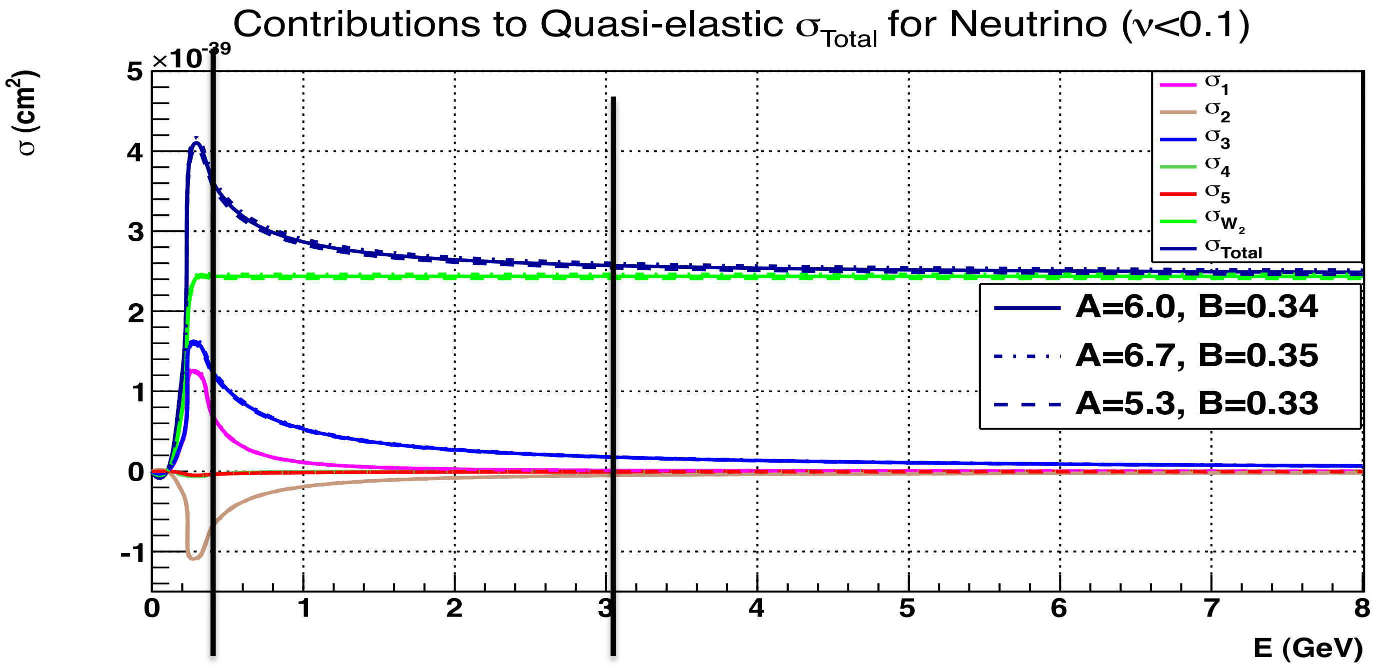


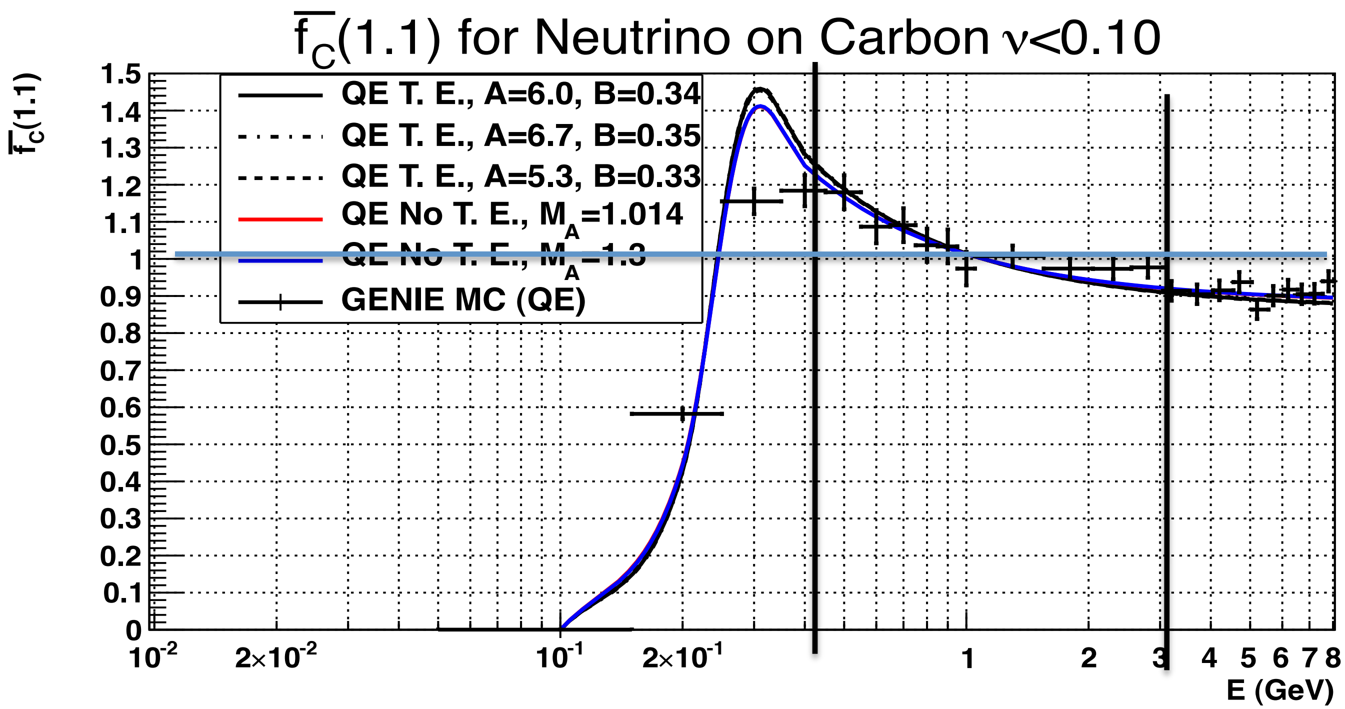
ν <0.2 GeV NEUTRINOS

BBC (TE) form factors with modified dipole F_A (M_A =1.014)

Normalize to the low ν cross section at E= 1.1 GeV and look at the relative low ν cross section to get the correction factor.

ν <0.2 can only be used down to E_ν=0.7 GeV.





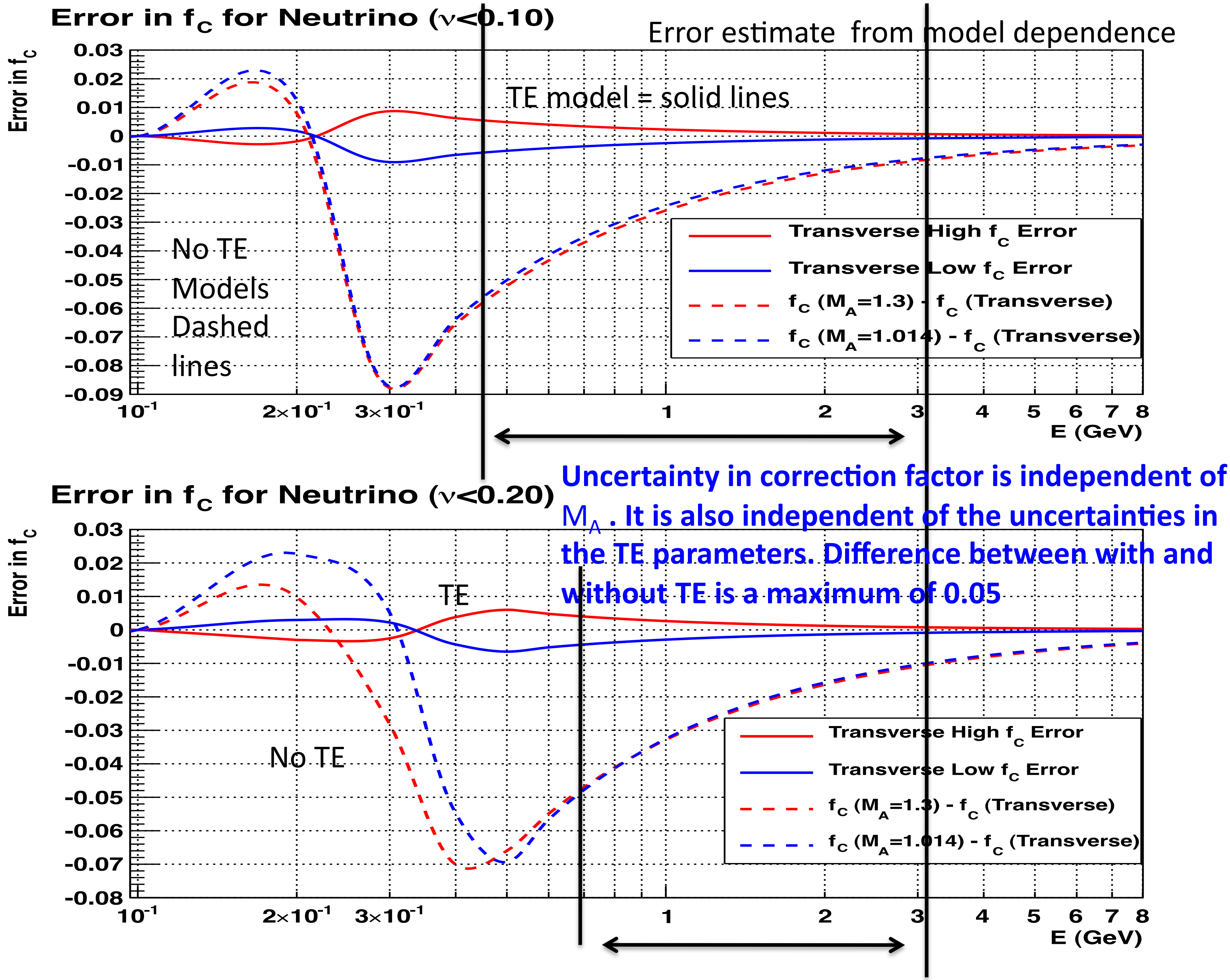
To go to lower energy need a lower ν cut.

ν <0.1 GeV Neutrinos

BBC (TE) form Factors with modified dipole F_A (M_A =1.014)

Normalize to the low ν cross section at E_ν=1.1 GeV and look at the relative low ν cross section to get the correction factor.

ν <0.1 GeV can be used down to E_ν=0.4 GeV.



Error in f_C for Neutrino (ν<0.10)
Error estimate from model dependence
TE model = solid lines
No TE
Models
Dashed
lines
Error in f_C
Transverse High f_C Error
Transverse Low f_C Error
f_C (M_A=1.3) - f_C (Transverse)
f_C (M_A=1.014) - f_C (Transverse)
E (GeV)
Uncertainty in correction factor is independent of M_A . It is also independent of the uncertainties in the TE parameters. Difference between with and without TE is a maximum of 0.05
Error in f_C for Neutrino (ν<0.20)
TE
No TE
Error in f_C
Transverse High f_C Error
Transverse Low f_C Error
f_C (M_A=1.3) - f_C (Transverse)
f_C (M_A=1.014) - f_C (Transverse)
E (GeV)

Conclusion 1

- **The energy dependence of the flux in low energy neutrino beams can be determined by using the low ν events with**

  ν<0.2 GeV for E_ν>0.7 GeV (for neutrino beams)
  ν<0.1 GeV for E_ν>0.4 GeV (for neutrino beams)

  ν<0.2 GeV for E_ν>2 GeV (for antineutrino beams)
  ν<0.1 GeV for E_ν>0.7 GeV (for antineutrino beams)

The uncertainty in the model dependent correction factors is small so any model which fits the data can be used.

(note the absolute flux cannot be determined with the low ν method, so it must be normalized at some energy).

- **The primary error will be experimental, i.e. statistical and systematic experimental error in the extracted number of low ν events as a function of neutrino energy, resolution smearing and unfolding.**

Example: Application to *MiniBooNE* *see* arXiv:1207.1247

MiniBooNE reconstructs $E\nu^{QE}$ by Assuming QE kinematics for the muon.

We extract $\nu = E\nu^{QE} - E\mu$

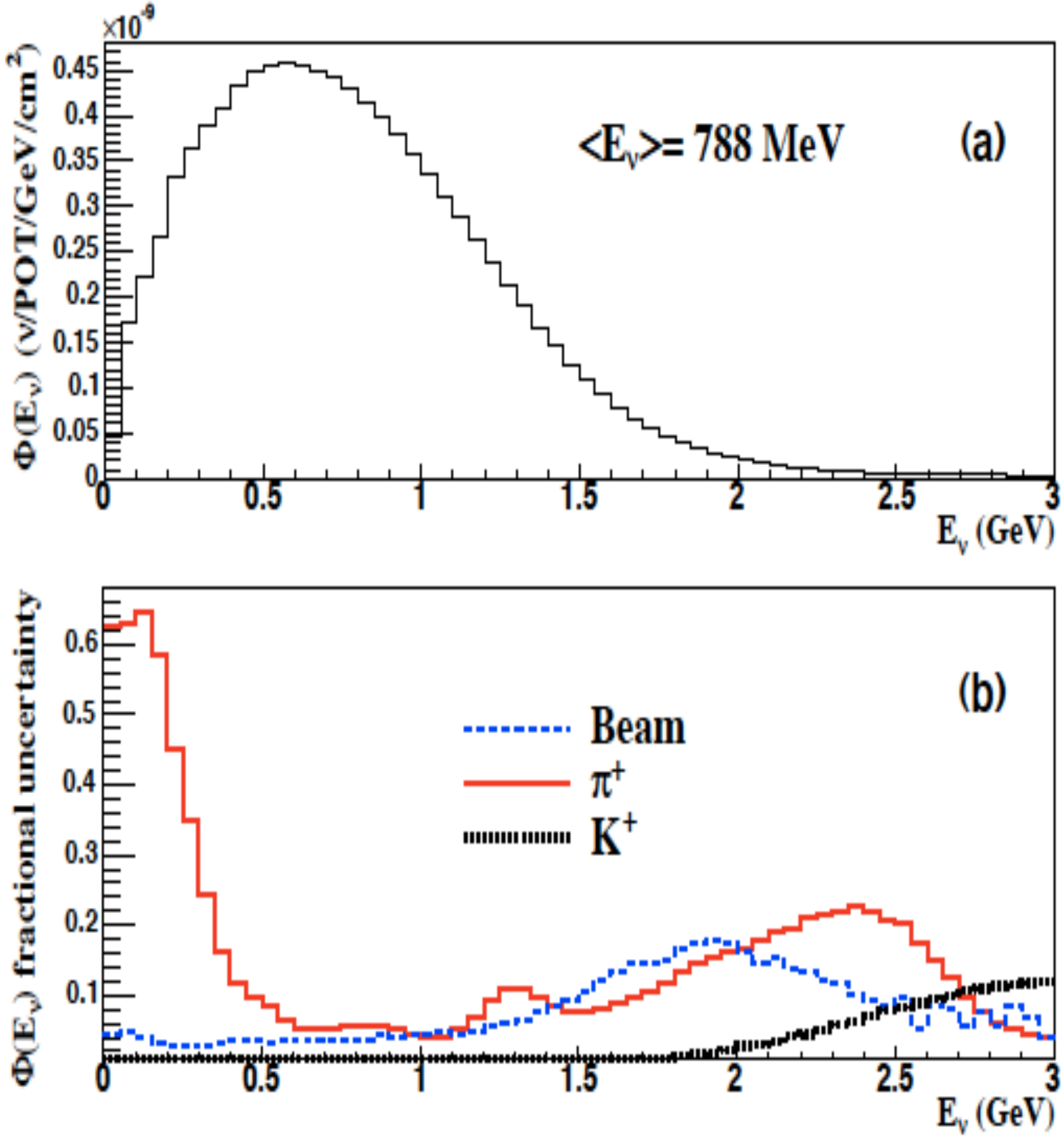
$$E_\nu^{QE} = \frac{2(M_n')E_\mu - ((M_n')^2 + m_\mu^2 - M_p^2)}{2 \cdot [(M_n') - E_\mu + \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu]}, \quad (1)$$

$$Q_{QE}^2 = -m_\mu^2 + 2E_\nu^{QE}(E_\mu - \sqrt{E_\mu^2 - m_\mu^2}\cos\theta_\mu), \quad (2)$$

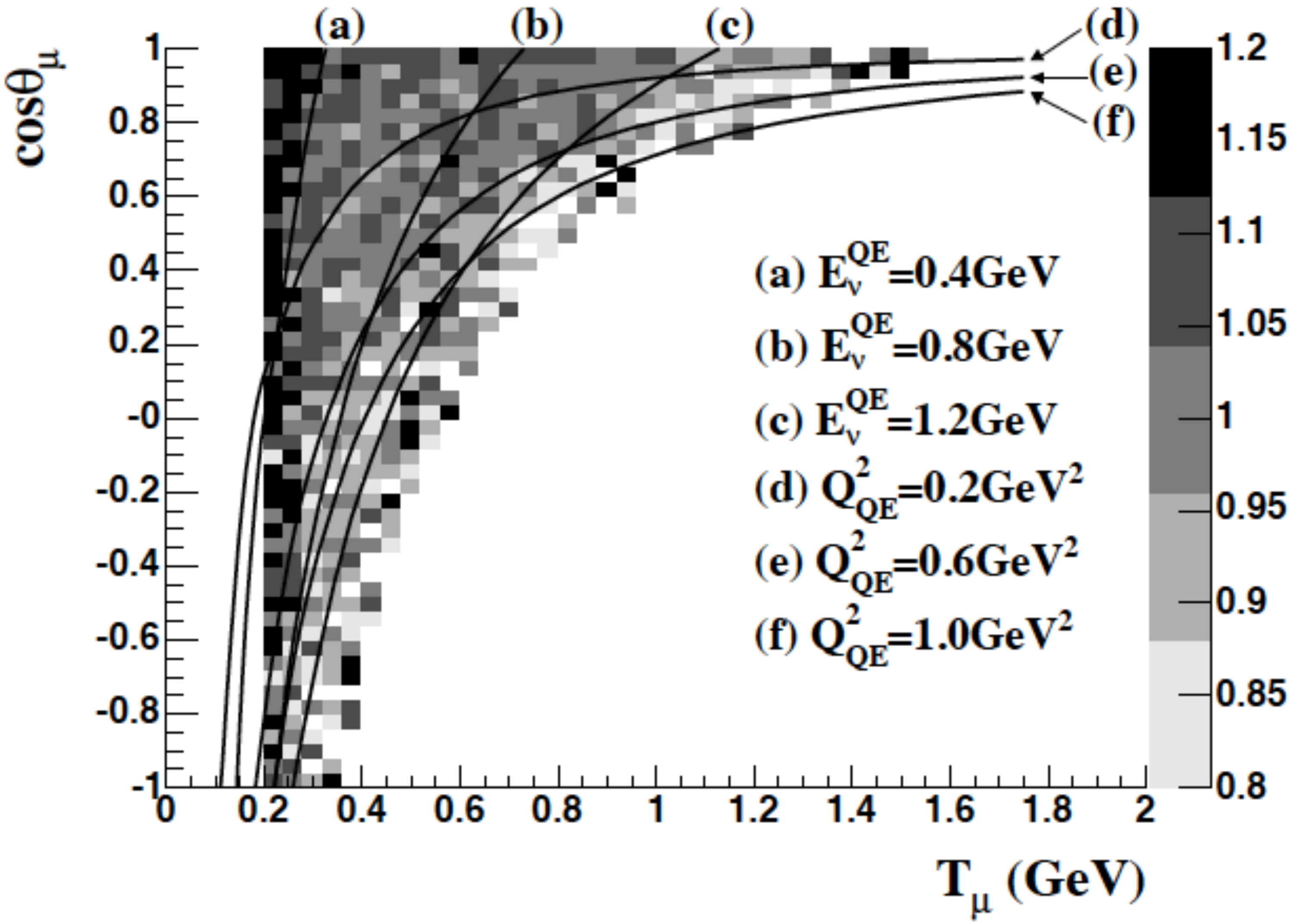
where $E_\mu = T_\mu + m_\mu$ is the total muon energy and M_n, M_p, m_μ are the neutron, proton, and muon masses. The adjusted neutron mass, $M_n' = M_n - E_B$, depends on the binding energy (or more carefully stated, the separation energy) in carbon, E_B, which for this analysis is set to 34 ± 9 MeV.

MiniBooNE removes pion production events and subtracts non QE background (e.g. Delta)

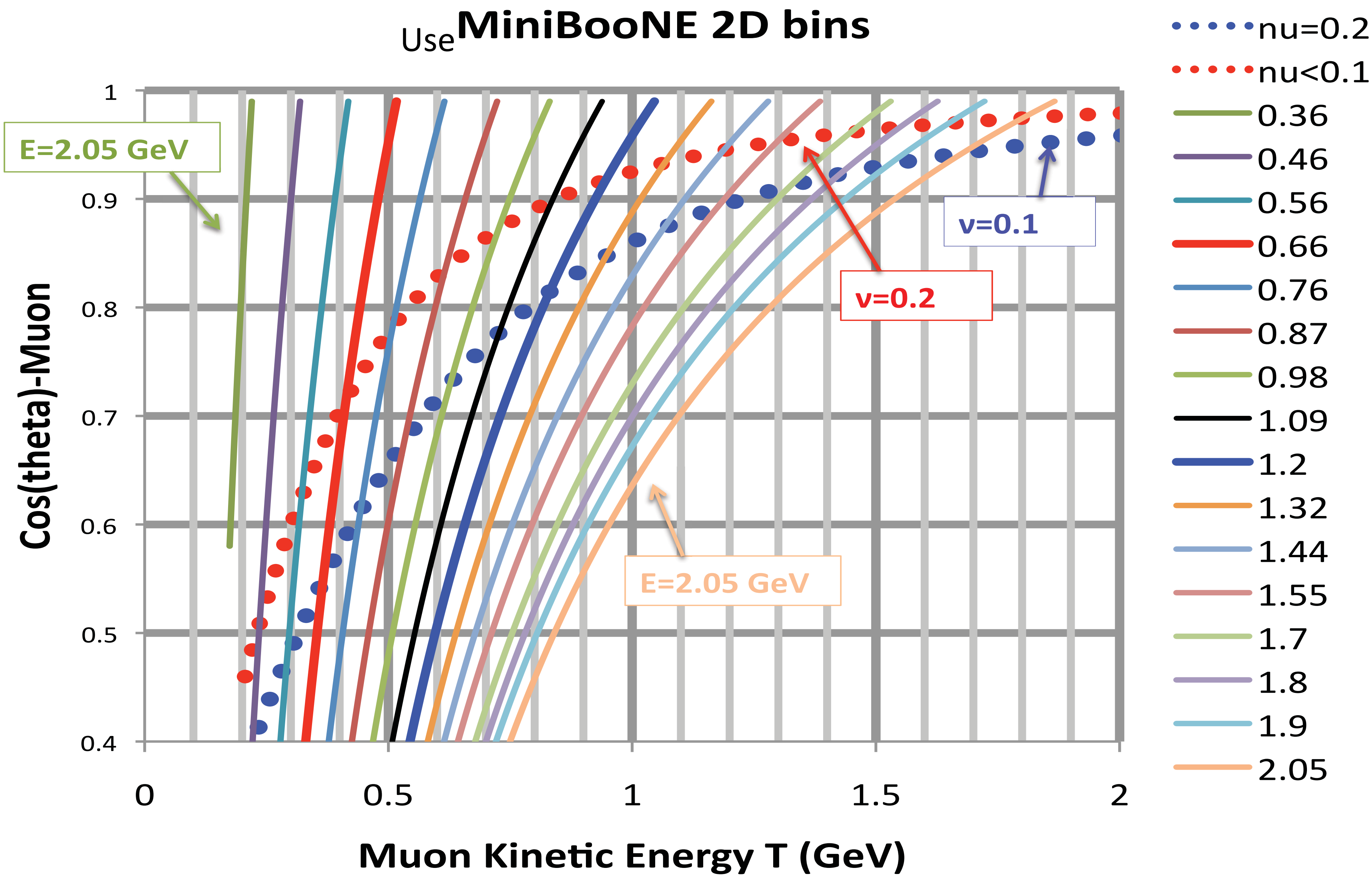
MiniBooNE flux and estimated fractional error



MiniBooNE published flux weighted cross section in Cosθ_μ and Muon Kinetic, T_μ bins.
We need to the data binned in $E\nu^{QE}$ and ν.

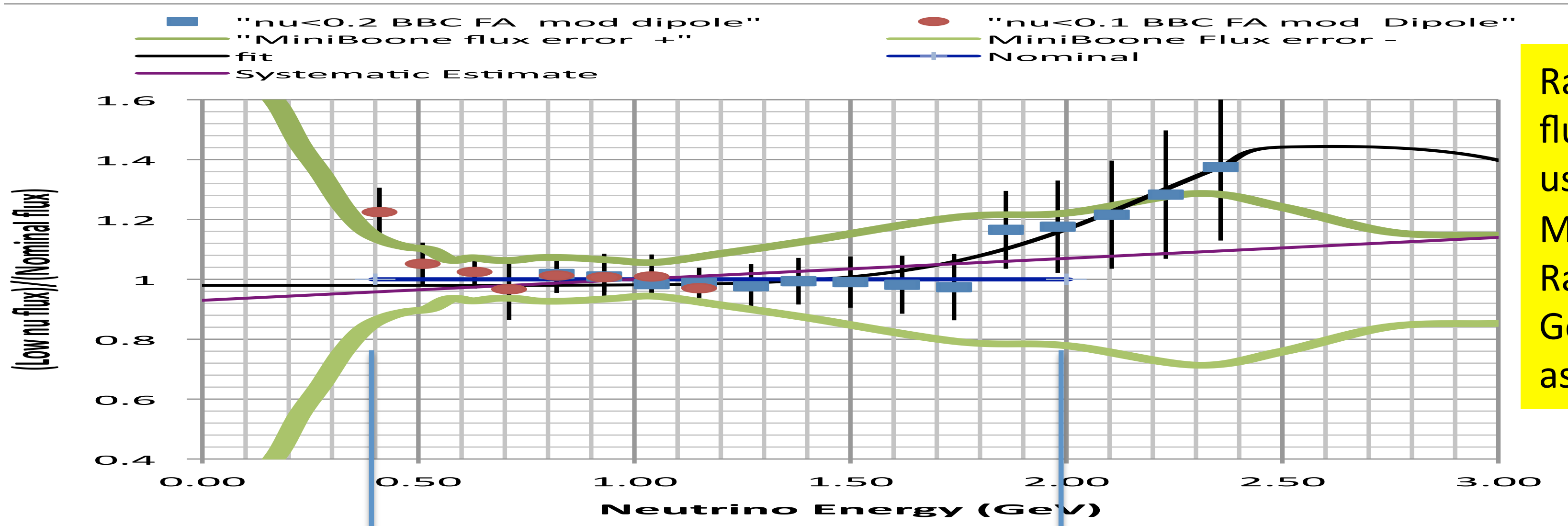


M_A =1.30 Ratio of data to model with M_A =1.3 (Plot from MiniBooNE paper



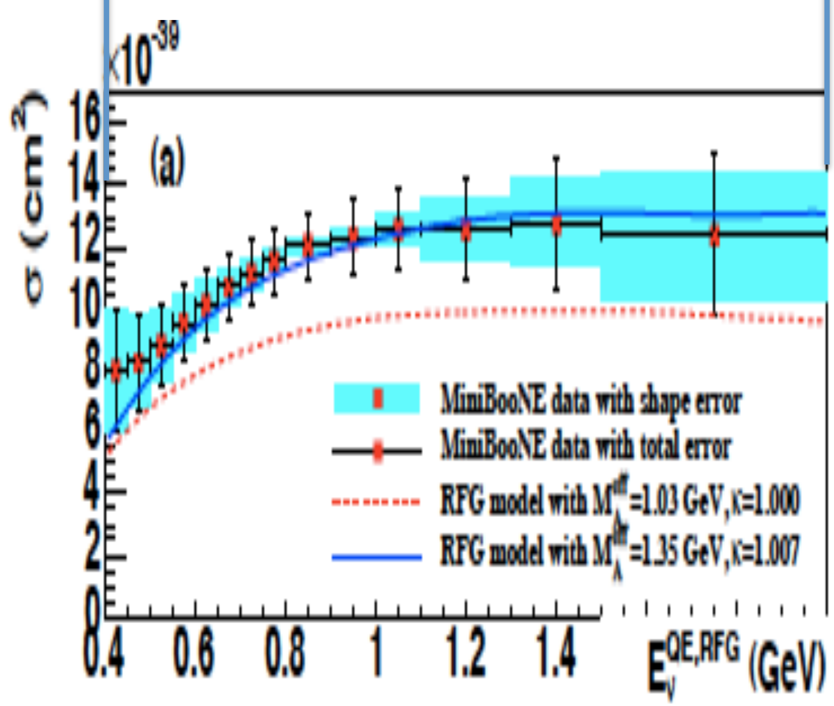
We use average of the ratio of data to model for bins that fall below $\nu < 0.1$ and $\nu < 0.2$ in bins of $E\nu^{QE}$ to extract the ratio of the flux from low ν events to published MiniBooNE flux.
The MiniBooNE binning of 0.1 in $\cos\theta_\mu$ is not ideal for this study. Bins of 0.05 in $\cos\theta_\mu$ would be better, or better yet bins in $E\nu^{QE}$ and ν. We use the published data as is a an example.

Simple test: Statistical errors only. **No unfolding for resolution in** $E\nu^{QE}$



Ratio of low ν flux to flux used by MiniBoNE. Ratio at 1.0 GeV is defined as 1.0

MiniBooNE unfolded QE cross section



Conclusion 2, a low ν analysis with $E\nu^{QE}$ **and ν bins (and accounting for systematic errors and unfolding) can provide and independent check and reduce the uncertainty on the energy dependence of the measured QE cross section.**

The extracted low ν flux is within the quoted systematic error in the flux.

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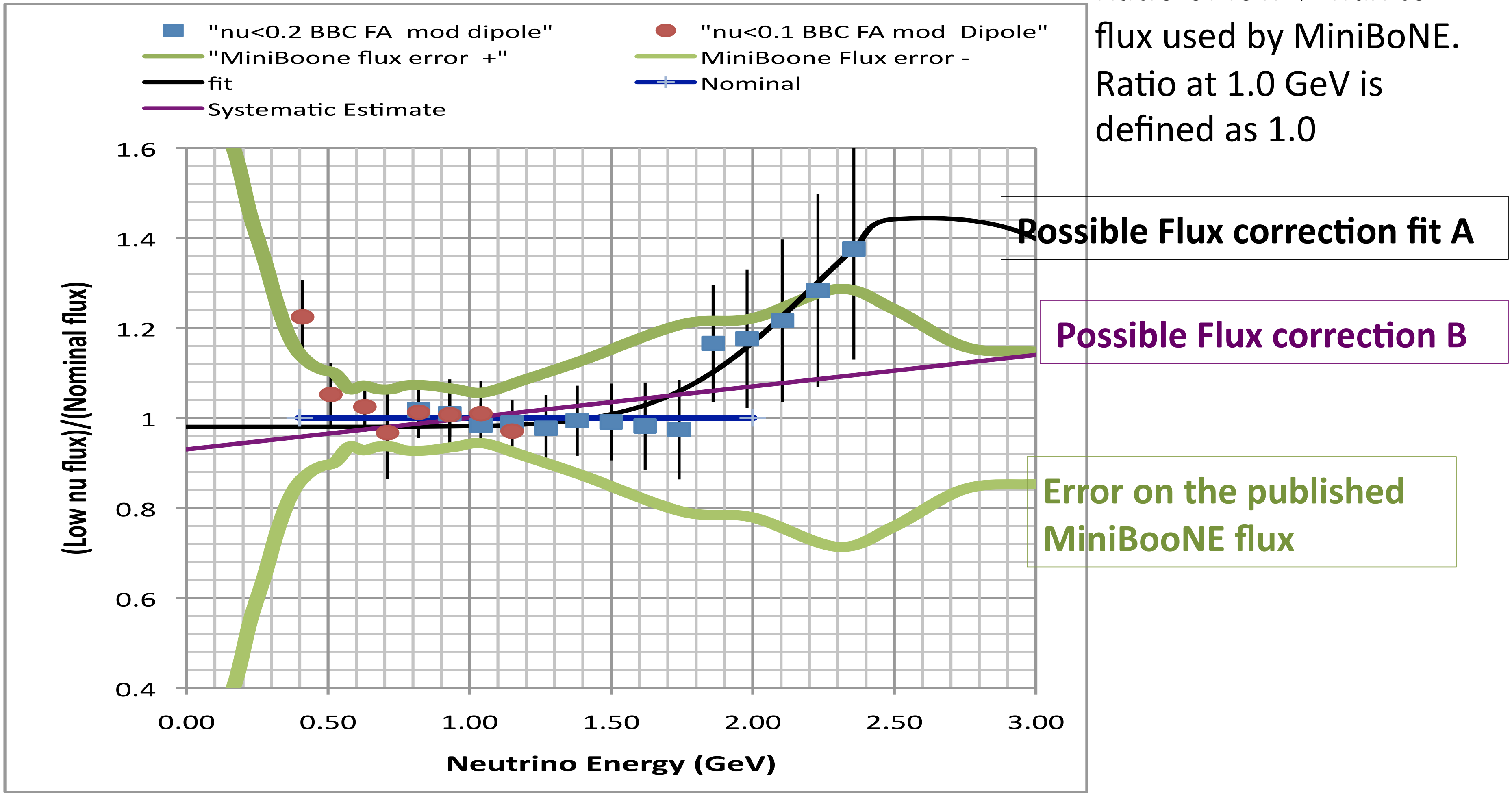
Next we test the effect on M_A

Statistical errors only. No unfolding for experimental resolution in neutrino energy.

Testing the effect on M_A

Ratio of low ν flux to flux used by MiniBoNE. Ratio at 1.0 GeV is defined as 1.0



Ratio of flux extracted from low ν events in *MiniBooNE* to the published flux is within the published *MiniBooNE* systematic errors on the flux. Next we test the effect of possible flux modifications (A or B) which are consistent with the low ν flux on the extraction of M_A

Fit to the double differential ($d^2\sigma/d\mathrm{Cos}\theta_\mu dE\mu$) cross sections (137 points) (2D)

M_A (in GeV)	N	χ^2/NDF
BBBA(07), dipole F_A		
1.348 \pm 0.018	0.994 \pm 0.009	39.798/135=0.295 (published flux)
1.347 \pm 0.018	0.994 \pm 0.009	42.709/135=0.316 (Flux correction A)
1.340 \pm 0.018	0.996 \pm 0.009	39.360/135=0.292 (Flux correction B)
BBBA(07), modified F_A (our best estimate of free nucleon form factors)		
1.299 \pm 0.016	0.994 \pm 0.009	30.434/135=0.225 (published flux)
1.288 \pm 0.016	0.991 \pm 0.009	42.939/135=0.318 (Flux correction A)
1.283 \pm 0.016	0.993 \pm 0.009	37.666/135=0.279 (Flux correction B)
BBC(11)(TE) , dipole F_A		
1.219 \pm 0.018	1.013 \pm 0.009	43.683/135=0.324 (published flux)
1.217 \pm 0.018	1.011 \pm 0.009	40.700/135=0.302 (Flux correction A)
1.206 \pm 0.018	1.012 \pm 0.009	44.562/135=0.330 (Flux correction B)
BBC(11) (TE), modified F_A (our best nuclear model based on electron scattering data)		
1.174 \pm 0.017	**1.010 \pm 0.009**	**37.190/135=0.276 (published flux)**
1.171 \pm 0.016	**1.008 \pm 0.009**	**35.453/135=0.263 (Flux correction A)**
1.162 \pm 0.016	**1.009 \pm 0.009**	**38.307/135=0.284 (Flux correction B)**

We let the absolute normalization float in the fit

Conclusion 3: Possible low ν flux modification do not change the extracted value of M_A. Expected since flux changes the energy dependence of cross section but not the Q^2 distribution

Conclusion 4: The extracted values of M_A with BBC (Transverse Enhancement and modified dipole) from the 2D fits is 1.17 with similar χ^2/DF as all other fits

With our best model, the MiniBooNE 2D data favors TE with modified dipole F_A , and M_A =1.17

2D fits to *MiniBooNE data*

Fit to the differential ($d\sigma/dQ^2$) cross sections (17 data points) (1D).

M_A (in GeV)	N	χ^2/NDF
BBA(07), dipole F_A		
1.410 \pm 0.040	**0.993 \pm 0.020**	11.764/15=0.784 (published flux)
1.403 \pm 0.040	0.991 \pm 0.020	12.045/15=0.803 (Flux correction A)
1.388 \pm 0.039	0.991 \pm 0.020	12.222/15=0.815 (Flux correction B)
BBBA(07), modified F_A (our best estimate of free nucleon form factors)		
1.330 \pm 0.036	0.986 \pm 0.021	5.611/15=0.374 (published flux)
1.324 \pm 0.036	0.984 \pm 0.021	5.674/15=0.378 (Flux correction A)
1.311 \pm 0.035	0.984 \pm 0.021	5.896/15=0.393 (Flux correction B)
BBC(11), dipole F_A		
1.170 \pm 0.034	1.050 \pm 0.022	19.714/15=1.314 (published flux)
1.165 \pm 0.034	1.048 \pm 0.022	18.537/15=1.236 (Flux correction A)
1.152 \pm 0.033	1.047 \pm 0.022	18.516/15=1.234 (Flux correction B)
BBC(11) (TE) , modified F_A (our best nuclear model based on electron data)		
1.110 \pm 0.031	**1.039 \pm 0.023**	**19.042/15=1.270 (published flux)**
1.106 \pm 0.031	**1.037 \pm 0.023**	**17.766/15=1.184 (Flux correction A)**
1.095 \pm 0.030	**1.036 \pm 0.023**	**17.711/15=1.181 (Flux correction B)**

Here also, possible low ν flux modification do not change the extracted value of M_A. Expected since flux changes the energy dependence of cross section but not the Q^2 distribution

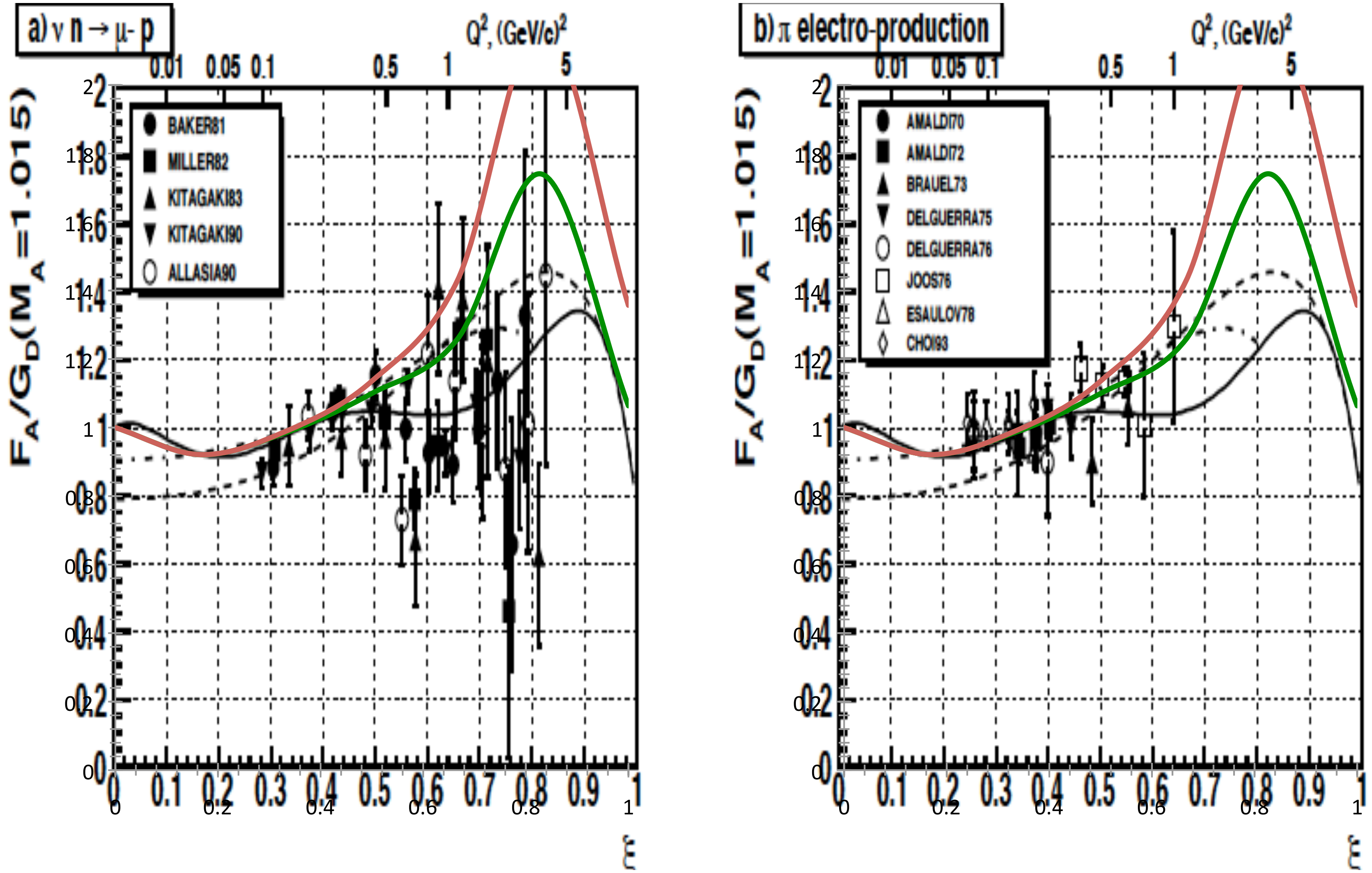
Conclusion 5: The extracted values of Ma with BBC (Transverse Enhancement and modified dipole) for the 1D fit is 1.10 with similar χ^2/DF as all other fits. With our best model, the MiniBooNE 1D Q^2 distribution favors TE with modified dipole F_A and M_A =1.10

We let the absolute normalization float

1D fits to *MiniBooNE* Q^2 distribution

Next: compare M_A =1.17 and M_A =1.10 with modified dipole with free nucleon data.

Ratio of F_A data on deuterium to dipole with M_A =1.015 GeV (solid line=modified dipole)



Orange=modified dipole with M_A =1.17, Green = modified dipole with M_A =1.10

Conclusion 6:

Modified dipole with M_A =1.10 is consistent with pion electroproduction data.

It is also consistent with neutrino data on deuterium for Q^2<0.5 GeV^2.

It is not consistent with the average of the neutrino deuterium data for Q^2>1 GeV.

Final Conclusion

- We can greatly reduce the systematic errors in the neutrino cross section for energies greater than 0.4 GeV by using the low ν method.

- The low ν method must be normalized to a cross section at some energy. MINERvA can extend the cross section measurements to overlap both MINOS and MiniBooNE, thus normalizing to known cross sections at high energies.

backup

Are there other Techniques to determine flux?
The following is a list of all the techniques

1. Modeling the distribution of pions and kaons produced by incident proton beam in the target. Then, tracking the pions and kaons though the Horn focussing magnetic fields, and modeling the decays of pions and kaons in the decay pipe. Standard method
2. Measuring the muon flux that exits the decay pipe and relating it to the neutrino flux.
3. Monitoring Inverse muon decay events ($\nu_\mu + e \to \mu^- + \nu_e$) in the detector.
4. Monitoring neutrino-electron scattering events ($\nu_\mu + e \to \nu_\mu + e$) in the detector.
5. The "low-ν" method for the determination of the energy dependence of the relative neutrino and antineutrino flux. Low ν method

1. In method 1, the differential cross sections for the production of pions and kaons by protons incident on a thick nuclear target must be known very well. In addition, the magnetic field of the horn focussing magnets must be modeled reliably.
2. In method 2, the response of the muon detectors at the end of the decay pipe must be very well understood (for absolute calibration of the neutrino flux). The response of the muon detectors is sensitive to δ rays. In addition, since the energy of the muons is not measured, it is difficult to determine the energy dependence of the neutrino flux.
3. In method 3, the threshold for the reaction $\nu_\mu + e \rightarrow \mu^- + \nu_e$ is about 12 GeV. Therefore, this method can only be used at higher energies. Unfortunately, this method cannot be used for the determination of the flux for antineutrinos. Inverse muon decay was used by NOMAD to constrain their neutrino flux at high energies.
4. In method 4, only the sum of the fluxes for neutrinos and antineutrinos can be measured. This is because calorimetric detectors such as MINERvA cannot determine the charge of final state electron in $\nu_\mu + e \rightarrow \nu_\mu + e$ events.

1. Modeling the distribution of pions and kaons

Modeling particle production. Standard method, errors of order 15% at present.

2. Measuring the muon flux that exits the decay pipe

Muon Monitors, were used at CERN, difficult technically. Installed in NUMI beam

3. Monitoring Inverse muon decay events

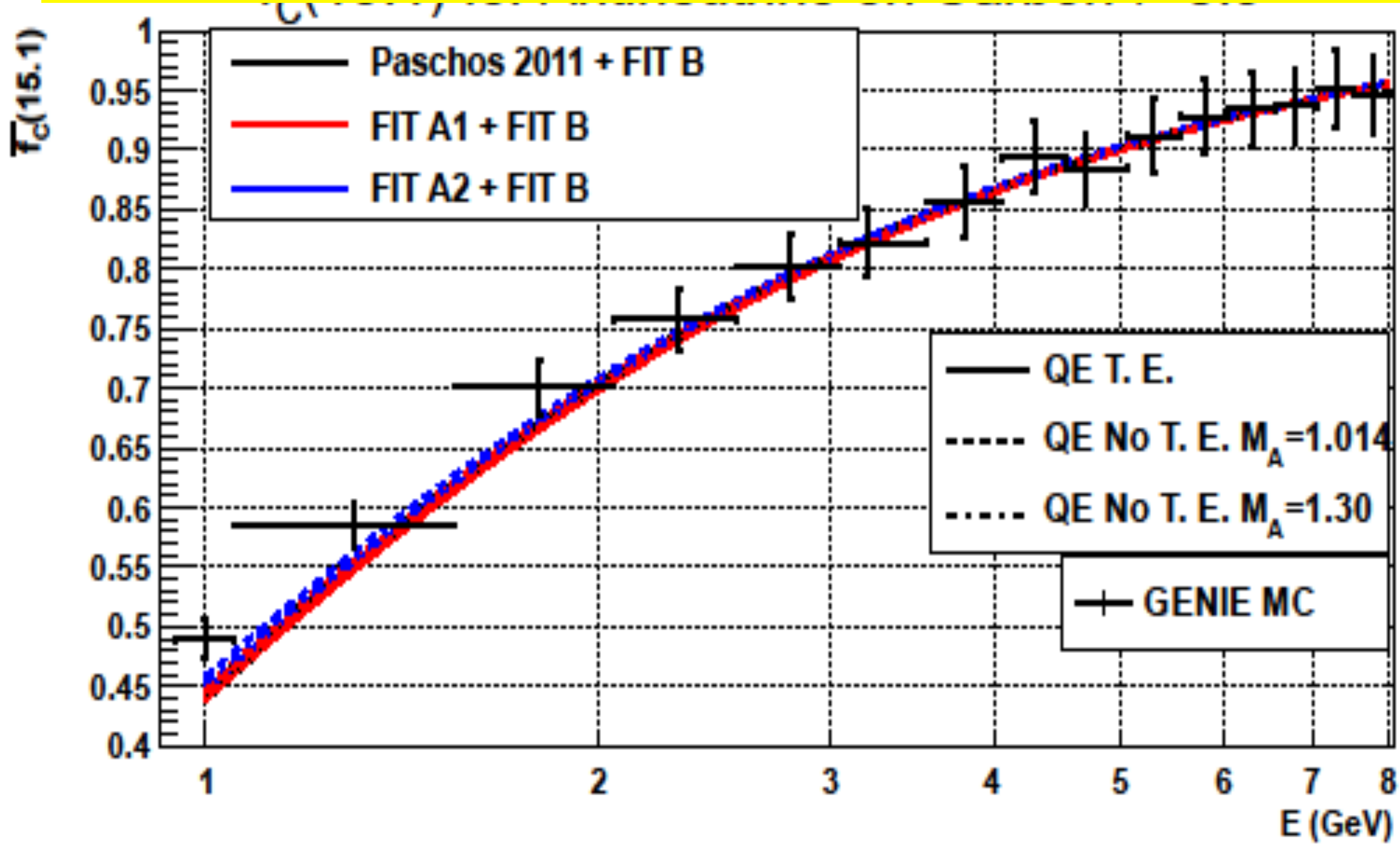
Inverse Muon decay. Statistically limited, Good only for E>12 GeV, not valid for antienutrinos

4. Monitoring neutrino-electron scattering events

Neutrino electron scattering Statistically limited, yields sum of neutrino and antineutrino flux

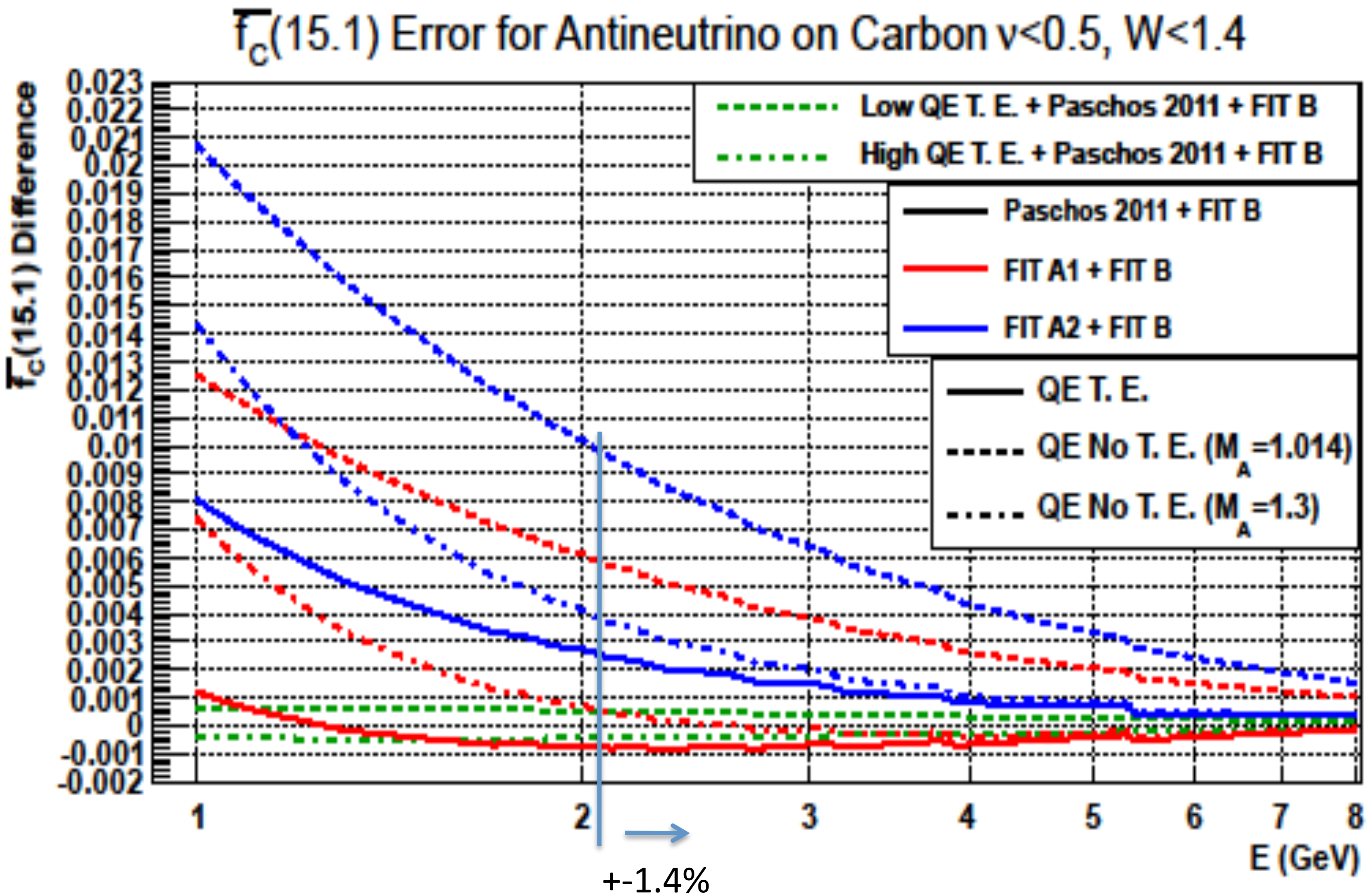
…ν_{max}=0.5: $\bar{f}_C$…

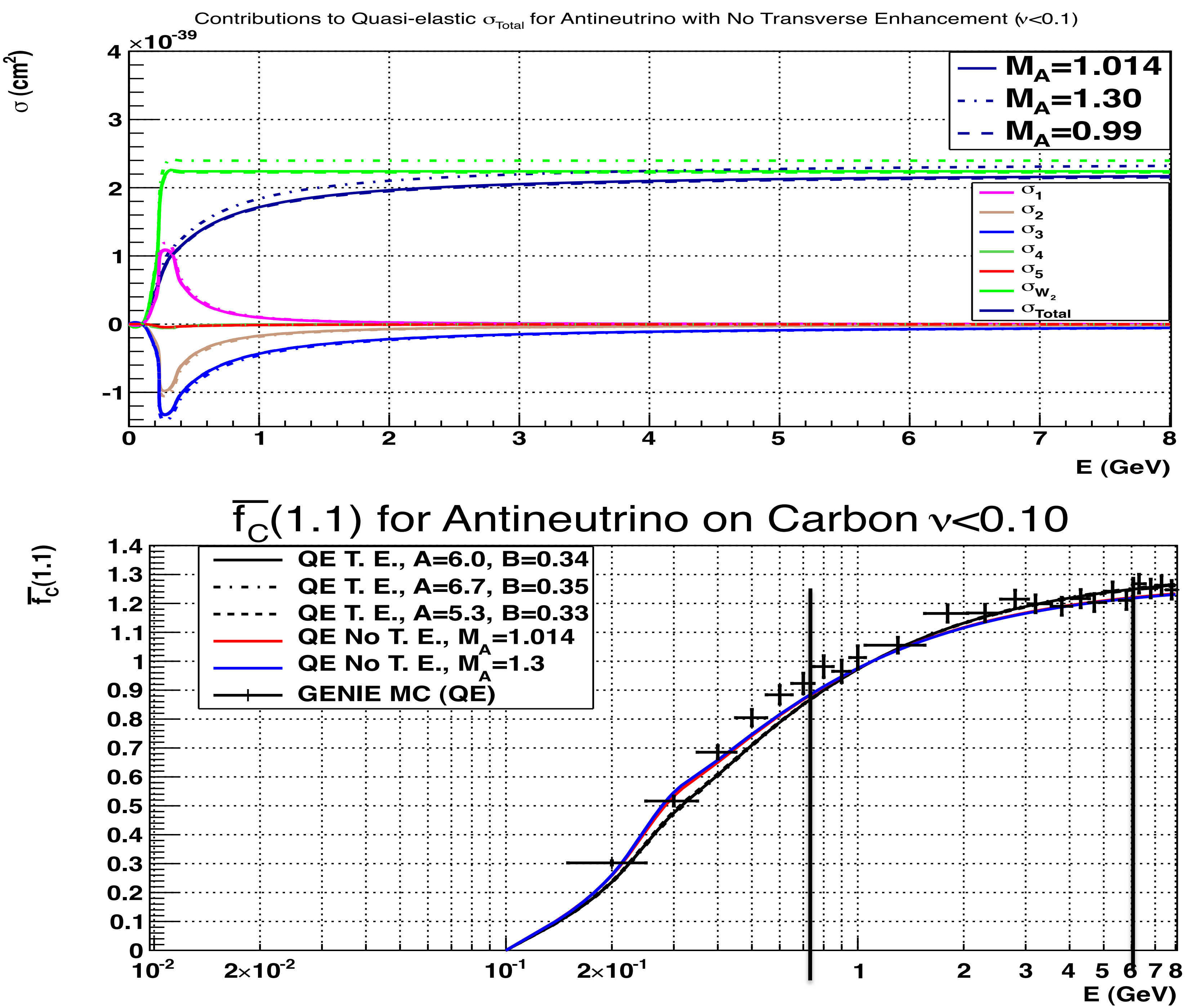
Relative low ν Anti-neutrino cross section normalized to the cross section at 15.1 GeV



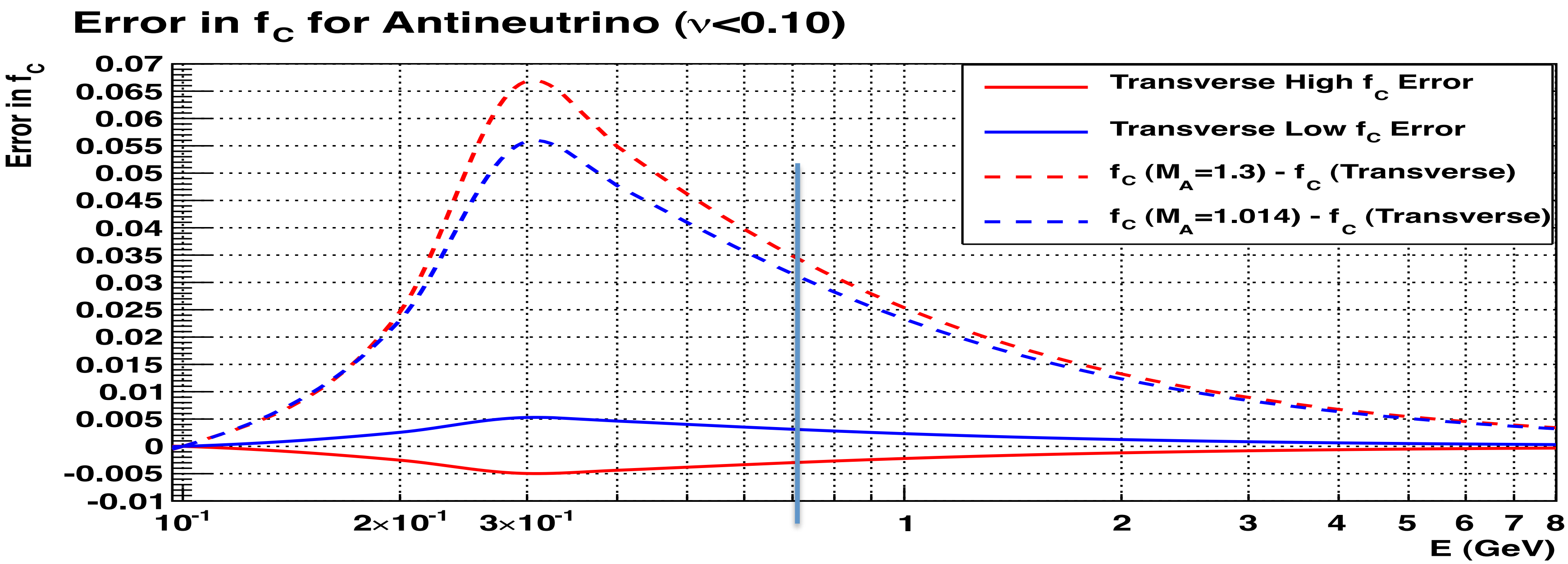
Uncertainty in the relative low ν cross section for antineutrinos neutrinos from uncertainties in QE and Delta production cross sections and form factors, Use above 2 GeV

$$\ldots \nu_{max}=0.5\ldots$$

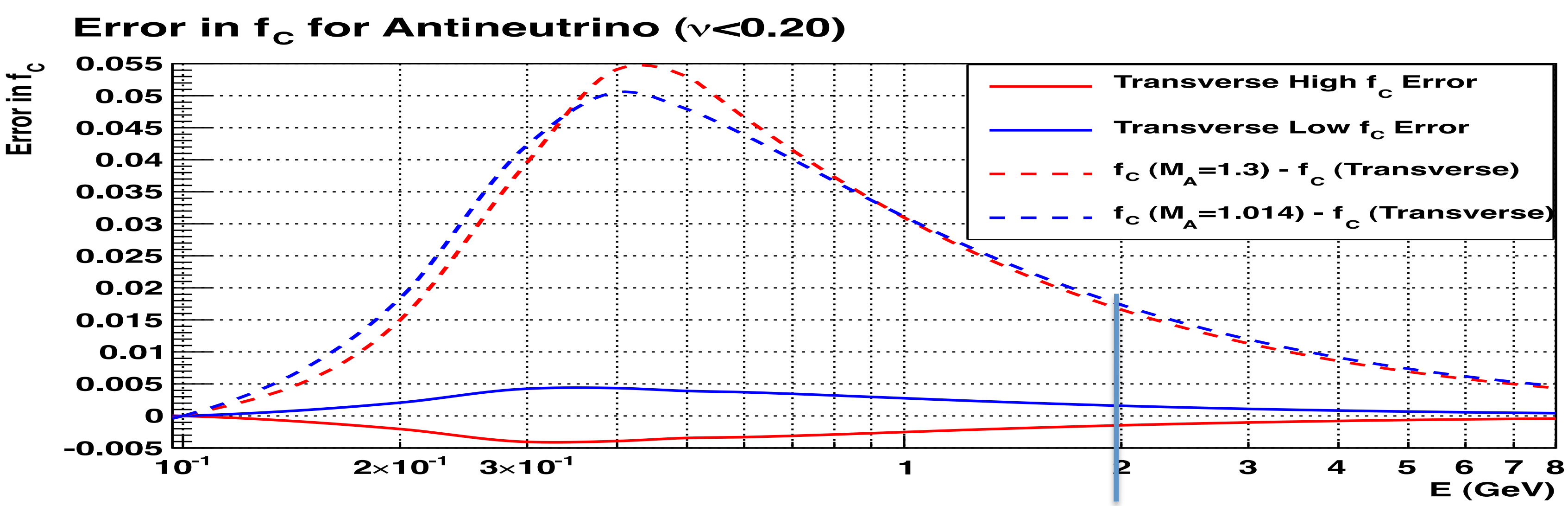


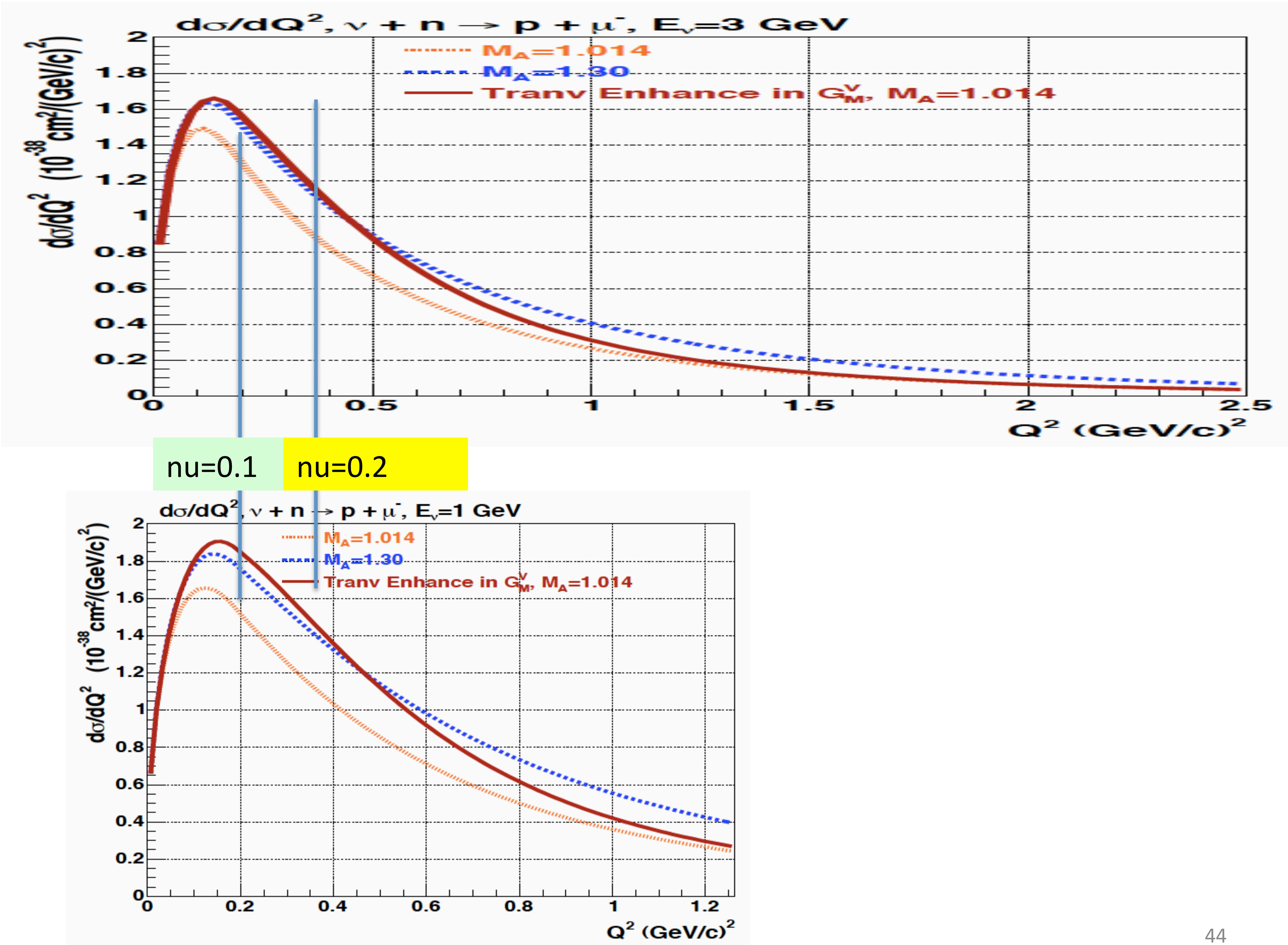


Contributions to Quasi-elastic σ_Total for Antineutrino with No Transverse Enhancement (ν<0.1)
σ (cm²)
×10⁻³⁹
M_A=1.014
M_A=1.30
M_A=0.99
σ_1
σ_2
σ_3
σ_4
σ_5
σ_W2
σ_Total
E (GeV)
f̄_C(1.1) for Antineutrino on Carbon ν<0.10
f̄_C(1.1)
QE T. E., A=6.0, B=0.34
QE T. E., A=6.7, B=0.35
QE T. E., A=5.3, B=0.33
QE No T. E., M_A=1.014
QE No T. E., M_A=1.3
GENIE MC (QE)
E (GeV)



Uncertainty in correction factor is independent of M_A . Also independent of the uncertainties in the TE parameters. Difference between with and without TE is a maximum of 0.035.




dσ/dQ², ν + n → p + μ⁻, E_ν=3 GeV
M_A=1.014
M_A=1.30
Tranv Enhance in G_M^V, M_A=1.014
dσ/dQ² (10⁻³⁸cm²/(GeV/c)²)
Q² (GeV/c)²
nu=0.1
nu=0.2
dσ/dQ², ν + n → p + μ⁻, E_ν=1 GeV
M_A=1.014
M_A=1.30
Tranv Enhance in G_M^V, M_A=1.014
dσ/dQ² (10⁻³⁸cm²/(GeV/c)²)
Q² (GeV/c)²