Differences in Quasi-Elastic Cross-Sections of Muon and Electron Neutrinos

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Motivation

- θ_{13} is large[1],[2]
- Current and future generations of neutrino experiments will look at oscillations between muon and electron neutrino and anti-neutrinos to:
  - Improve measurements of θ_{13}
  - Measure the CP violating parameter δ
  - Determine the neutrino mass hierarchy
- Differences in the electron and muon neutrino cross sections will affect the uncertainty of these measurements
- Quasi-elastic interaction dominates at low energies and is also used to normalize other cross sections

Sources of Difference and Uncertainties

- Kinematic Limits
- Axial Form Factor Contributions
- Pseudoscalar Form Factor Contributions
  - Pole mass uncertainty
  - Goldberger-Treiman Violation
- Second Class Current Contributions
  - Vector and Axial Form Factors
- Radiative Corrections

Quasi-Elastic Cross Section

- Equation as follows[3]:

$$\frac{d\sigma}{dQ^2}\left({}^{\nu n \to l^- p}_{\bar{\nu} p \to l^+ n}\right) = \left[A(Q^2) \mp B(Q^2)\frac{s-u}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right] \times \frac{M^2 G_F^2 \cos^2\theta_c}{8\pi E_\nu^2}$$

- Simple cross section assumes single nucleon interaction

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- Simple cross section assumes single nucleon interaction
- F^1_V, F^2_V measured in electron scattering experiments
- At low Q^2 (<1 GeV^2) $F^1_V, F^2_V \sim 1/(1+Q^2/m_V^2)^2$ - “Dipole Approximation”

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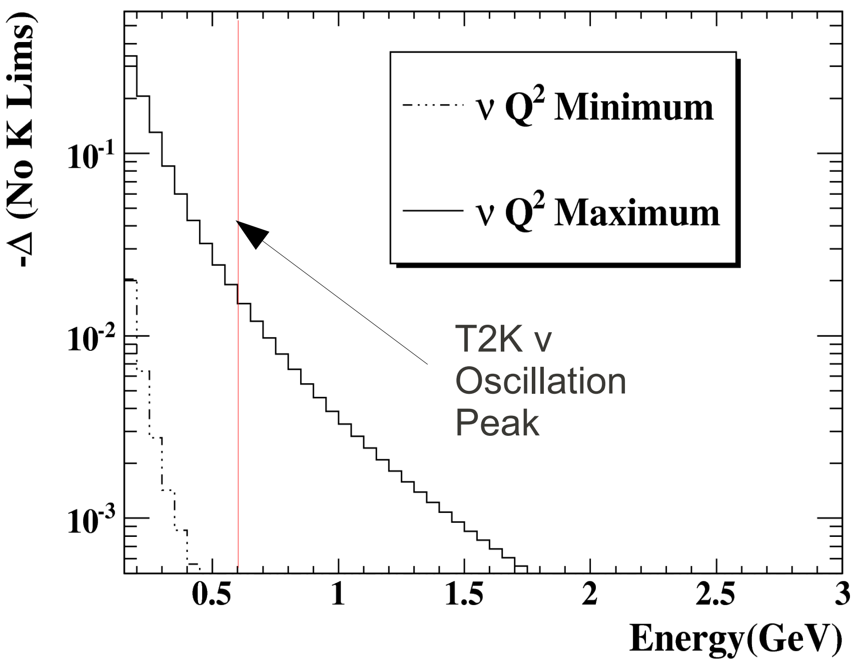
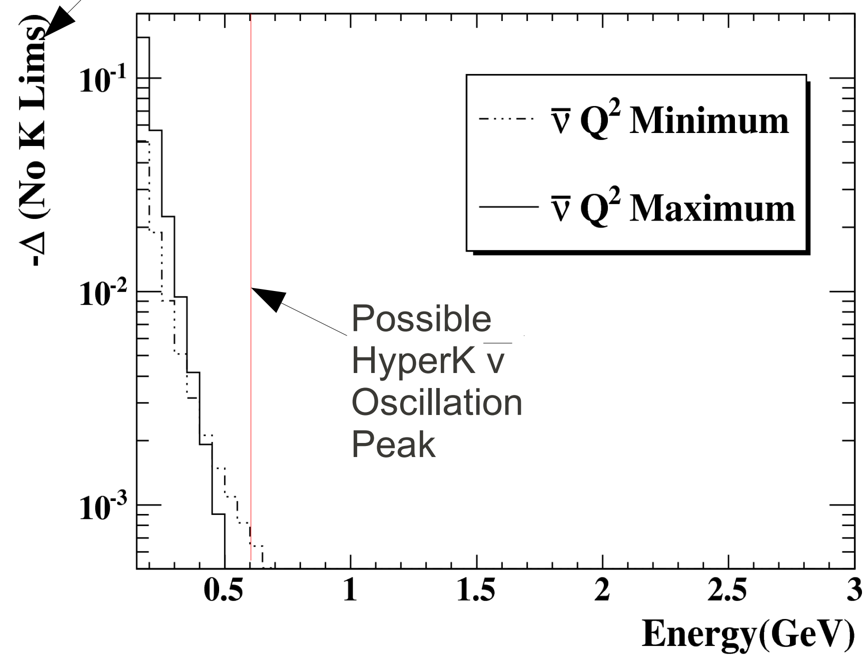
- At low Q^2(<1 GeV^2) F^1_V, F^2_V ~ $1/(1+Q^2/m_V^2)^2$ - “Dipole Approximation”
- Three axial and three vector form factors to parameterize
- F_A - Same model with m_A instead of m_V (no high Q^2 corrections studied)
- F_p, F^3_A and F^3_V terms are less well studied

Kinematic Limits

$$\Delta(E_\nu) \equiv \frac{\int dQ^2 \frac{d\sigma_\mu}{dQ^2} - \int dQ^2 \frac{d\sigma_e}{dQ^2}}{\int dQ^2 \frac{d\sigma_e}{dQ^2}}.$$





- Range of possible Q^2 values is larger for electron neutrinos, creating difference which is accounted for in all current generators
- The effect of the kinematic limits is larger at lower neutrino energies where limits make up more of the Q^2 range
- Effect at maximum is smaller for anti-neutrinos because electron anti-neutrino cross section is smaller at high Q^2

Lepton Mass in Bare Cross Section

- Contributions of various form factors affected by lepton mass, m:

$$A(Q^2) = \frac{m^2+Q^2}{4M^2}\left[\left(4+\frac{Q^2}{M^2}\right)|F_A|^2 - \left(4-\frac{Q^2}{M^2}\right)|F_V^1|^2 + \frac{Q^2}{M^2}\xi|F_V^2|^2\left(1-\frac{Q^2}{4M^2}\right) + \frac{4Q^2 ReF_V^{1*}\xi F_V^2}{M^2}\right.$$
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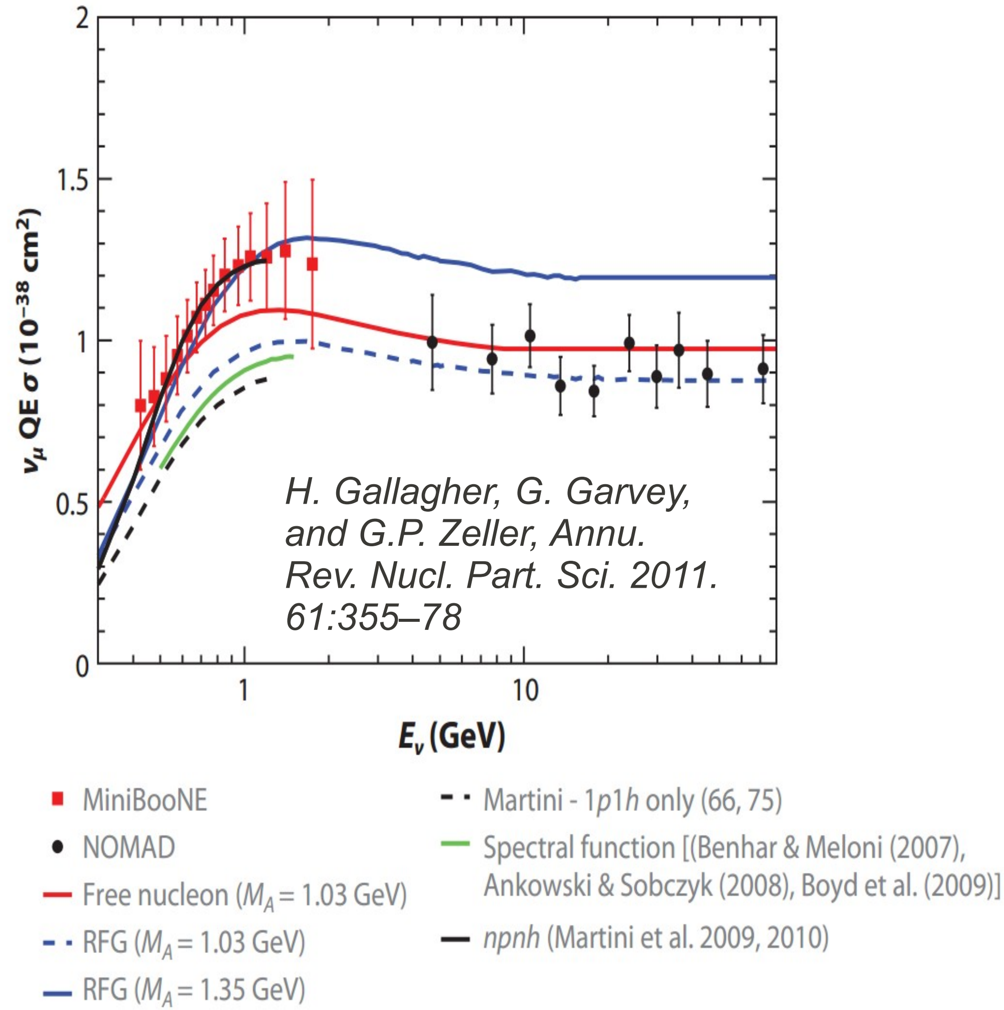
- All current neutrino event generators include mass terms with F^1_v, F^2_V, F_p and F_A
- Difference in Born cross section between the muon and electron neutrino case are caused completely by these mass terms
- For terms that exist only ~m^2/M^2 (where M is the nucleon mass), F_p and F^3_V, contribution to electron neutrino cross section is negligible

Uncertainty in F_A

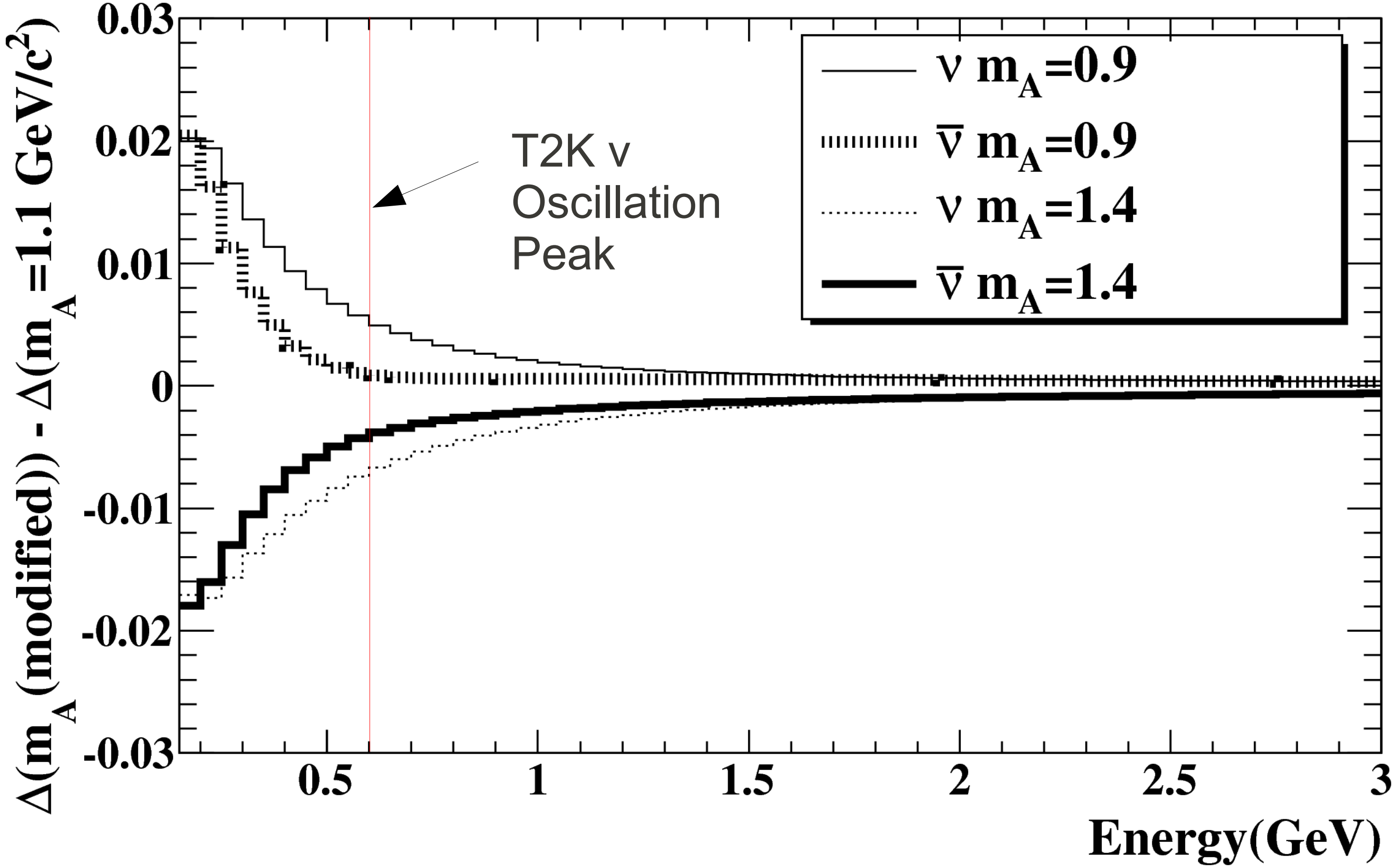
- Assume dipole approximation
- Large discrepancy for m_A in different neutrino experiments and pion electroproduction(ex. m^{avg}_A ~ 1.03[4], m^{π}_A~1.07[5], m_A ~ 1.35[6])
- Largest leading term uncertainty
- Uncertainty included in models
- Compare model with m_A = 0.9 and m_A = 1.4 to reference model with m_A = 1.1



Uncertainty in F_A cont.

$$\Delta(E_\nu) \equiv \frac{\int dQ^2 \frac{d\sigma_\mu}{dQ^2} - \int dQ^2 \frac{d\sigma_e}{dQ^2}}{\int dQ^2 \frac{d\sigma_e}{dQ^2}}.$$

Y axis is percentage difference in Delta between modified and reference model



- Large variation at low energy predominately from effects in Q^2 regions at kinematic boundaries

Calculating F_p

- From PCAC get relationship:

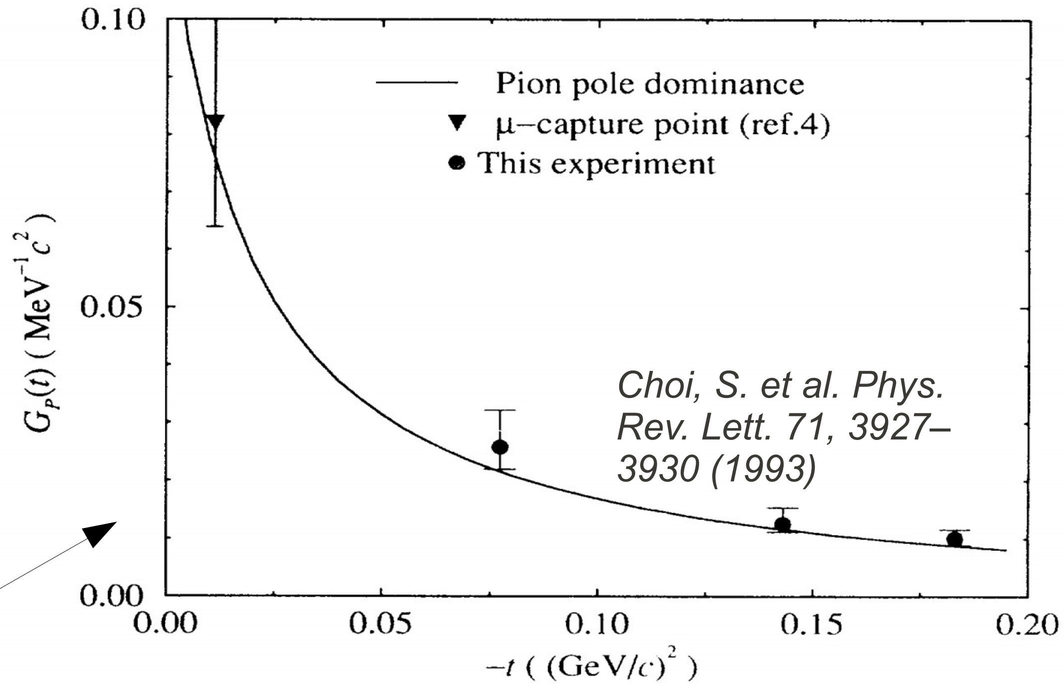
$$F_p(Q^2) = \frac{-2 M_n F_A(0)}{Q^2} \left(\frac{g_\pi(Q^2)}{g_\pi(0)\left(1+\frac{Q^2}{m_\pi^2}\right)} - \frac{F_A(Q^2)}{F_A(0)} \right)$$

???

- Where $g_\pi(Q^2)$ is the pionic form factor.
- Goldberger Treiman[7]: $f_\pi g_\pi(Q^2) = M_n F_A(Q^2)$
- Assume true for all Q^2
- Gives following relationship:

$$F_P(Q^2) = \frac{2M^2 F_A(Q^2)}{M_\pi^2 + Q^2}$$

Uncertainty in F_p



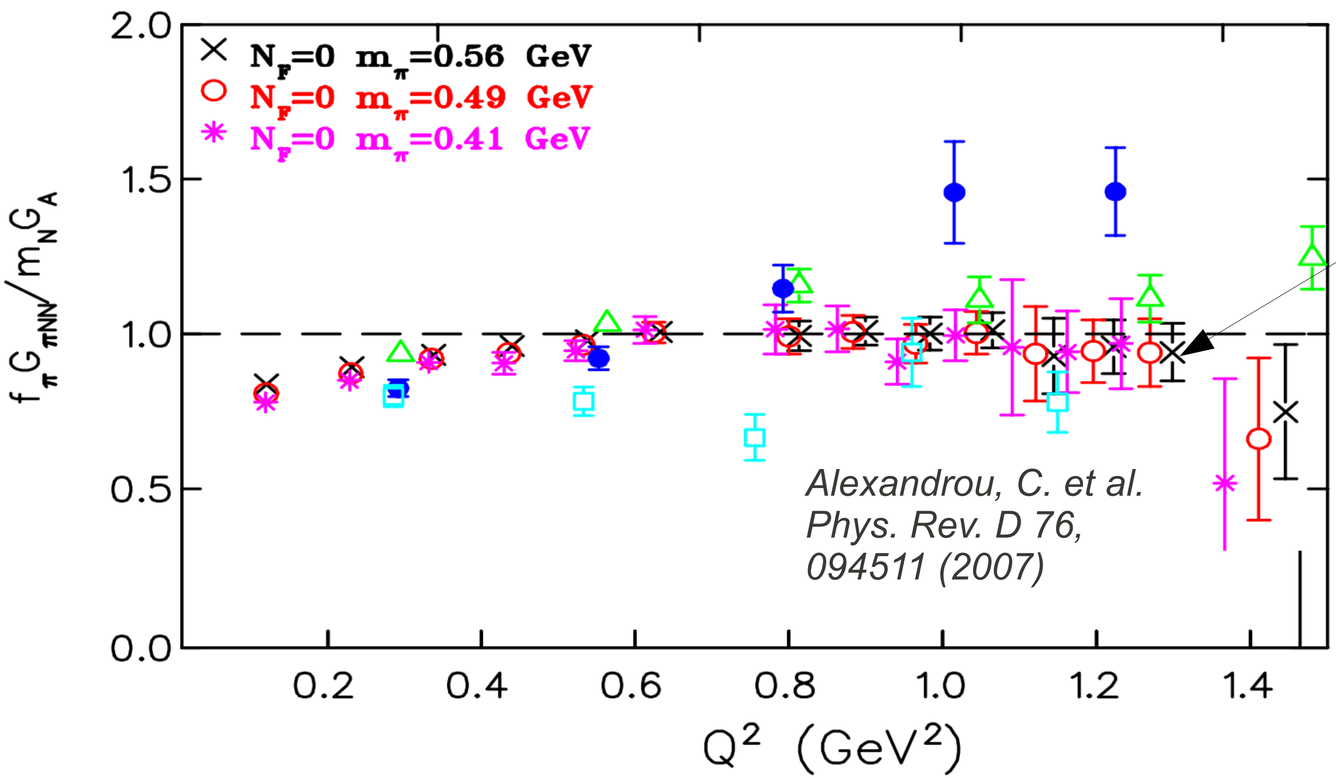
- F_p measured from pion electroproduction in range 0.05 to 0.2 GeV/c^2
- Uncertainties limit pole mass(assumed to be M_π) to range 0.6 M_π to 1.5 M_π

$$F_P(Q^2) = \frac{2M^2 F_A(Q^2)}{M_\pi^2 + Q^2}$$

- These uncertainties are not taken into account in current models

Uncertainty in F_p cont.

- Goldberger-Treiman violation of ~1-6%[8],[9] measured at Q^2=0
- Theoretical predictions suggest this may disappear at higher Q^2
- Model simply as 3% variation in $F_P(0)$
- Uncertainty not included in current models



Lattice QCD Prediction - Overestimates violation at low Q^2, predicts G-T Violation-->0 at high Q^2

Uncertainty in F_p cont.

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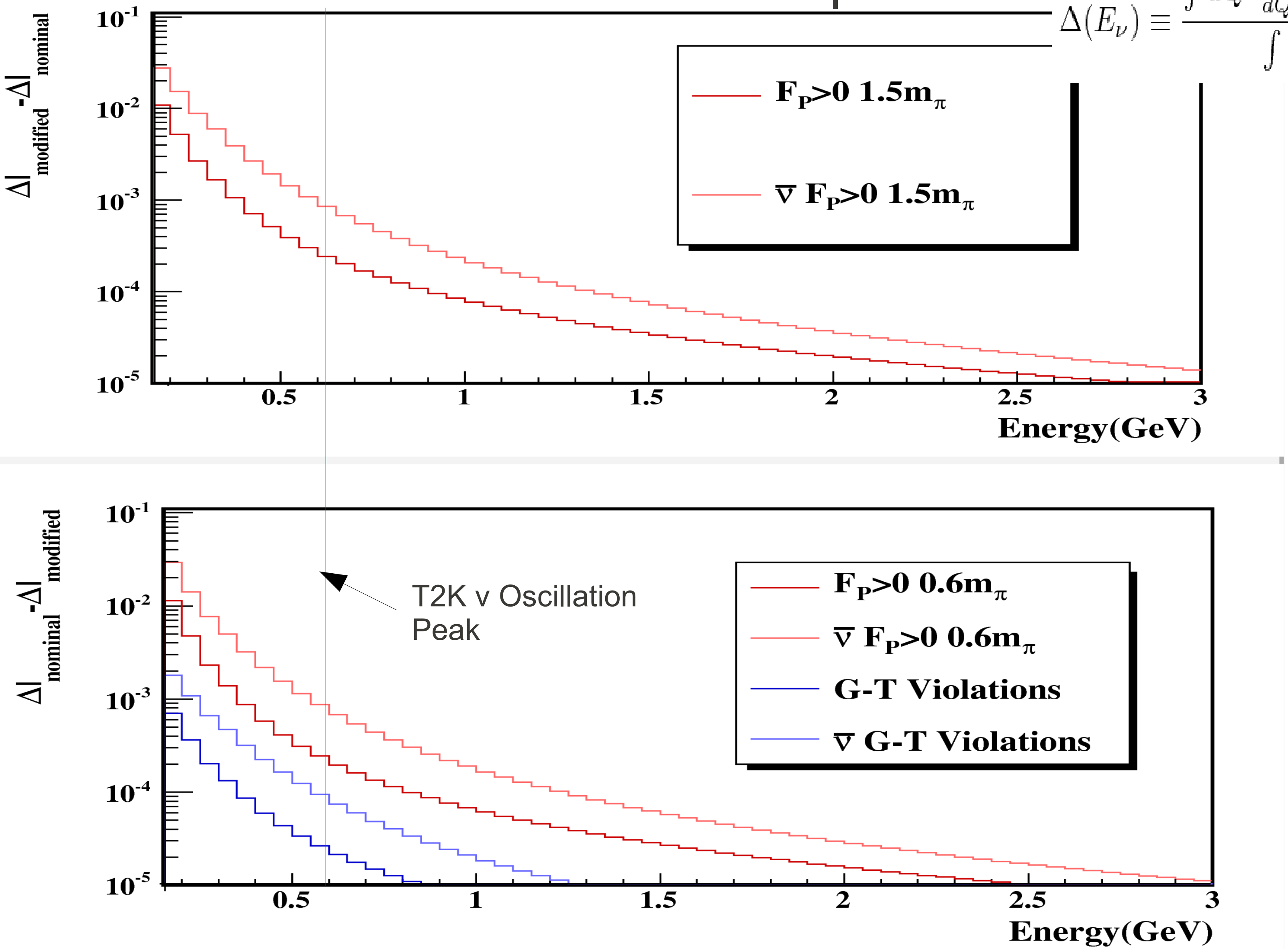


- All effects are small compared to neglecting F_p (~0.1-2% effect at reference)
- Even with exaggerated model, G-T violation effect is small

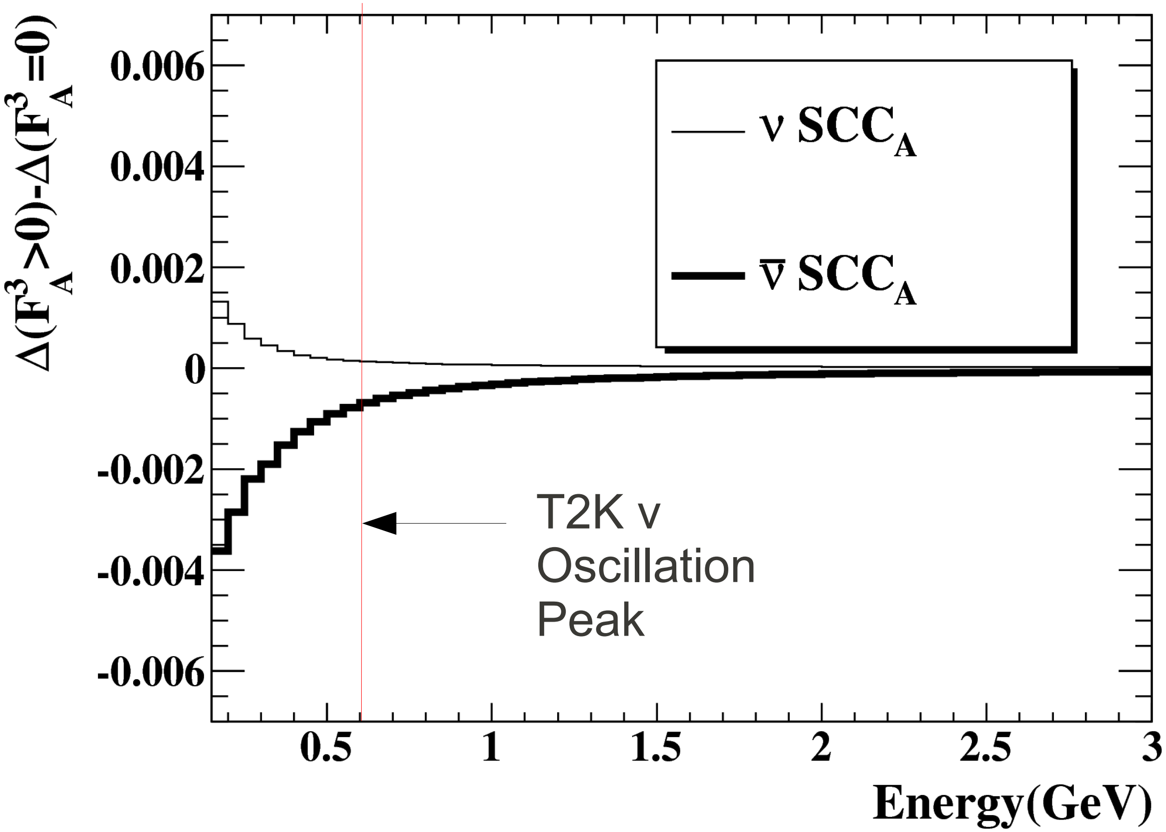
Second Class Currents

- G parity is basically an assertion that both T and C are conserved by the hadron current
- Second class current terms do not conserve G parity
- F^3_A and F^3_V are the form factors of the SCCs
- Non-zero F^3_V effect on CVC not seen in electron hadron scattering
- Constraints primarily from beta decay experiments at $Q^2 = 0$
- Calculations assume dipole form for Q^2 dependence

Uncertainty in F^3_A

- "KDR parameterization"[10], constrains $F^3_A(0)$ from:
  - Single nucleon form factor
  - Two nucleon mechanisms
  - Meson exchange currents
- Beta decay experiments use mirror nuclei, which swap n↔p
- Combine results to improve uncertainty[11] (A=8,12,20)
- $F^3_A(0)/F_A(0)$ ~ 0.1, consistent with no effect

Uncertainty in F^3_A cont.



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- Due to strong constraints, possible differences from $F^3_A(0)$ are very small

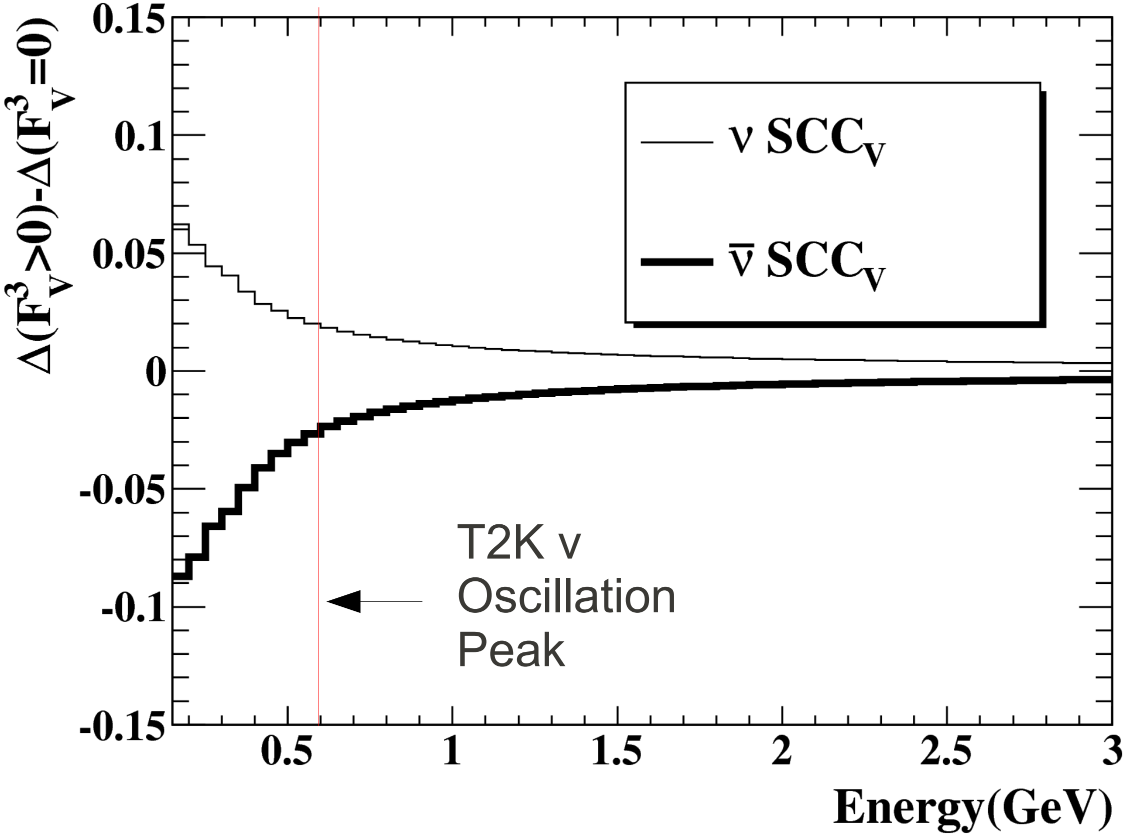
Uncertainty in F^3_V

- F^3_V less well studied than F^3_A
- Beta decay experiments[12] constrain:

$F^3_V(0) / F^1_V(0) \sim 2 \pm 2.4$ - Huge!

- Muon capture[13] , (anti-)neutrino cross sections[14] also sensitive
  - Current measurements require additional assumptions
- Poor constraint creates potentially large uncertainty
- Uncertainty is not included in current models

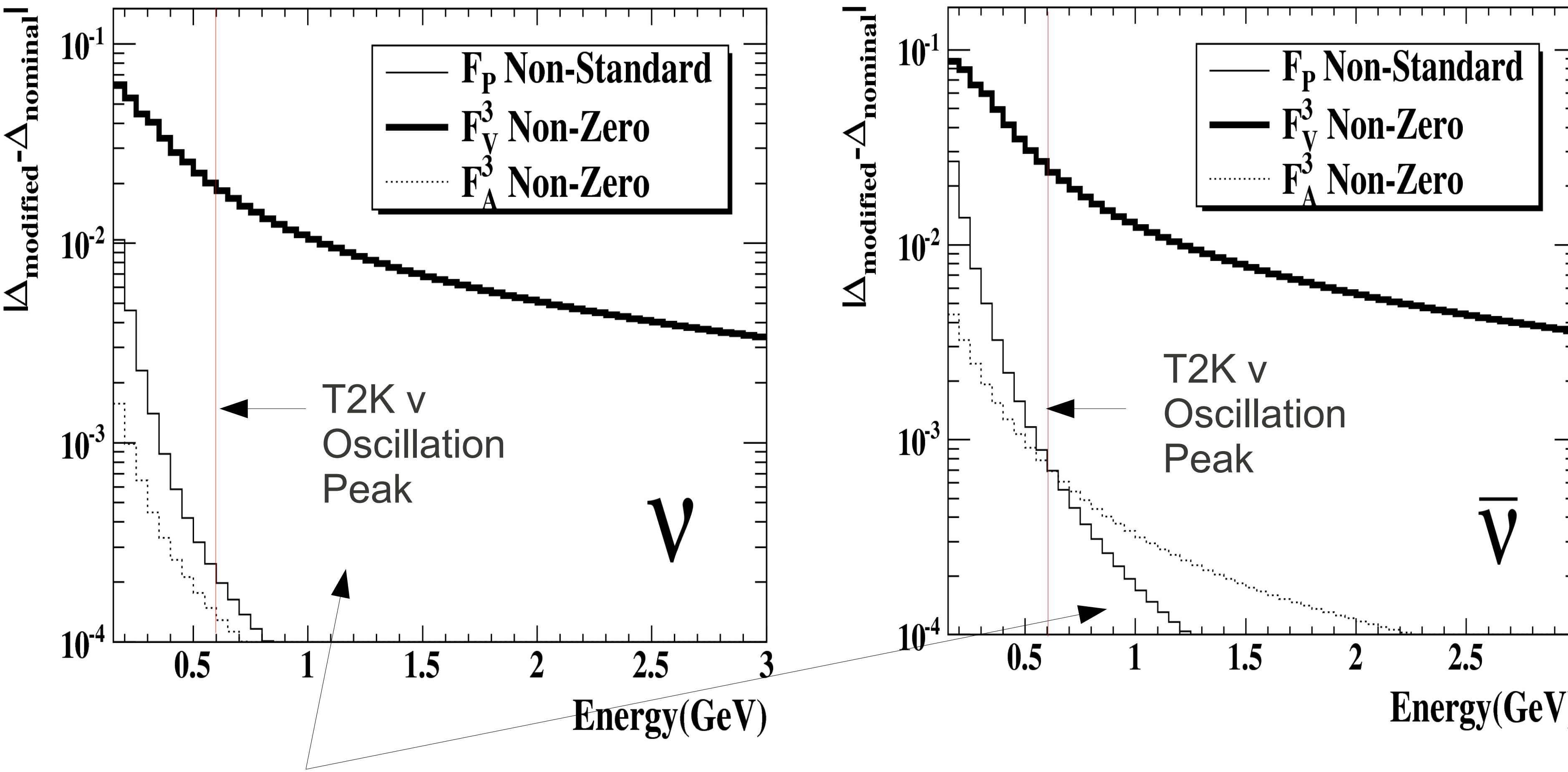
Uncertainty in F^3_V cont.



$$\Delta(E_\nu) \equiv \frac{\int dQ^2 \frac{d\sigma_\mu}{dQ^2} - \int dQ^2 \frac{d\sigma_e}{dQ^2}}{\int dQ^2 \frac{d\sigma_e}{dQ^2}}.$$

- With current limits on F^3_V at reference have difference of ~2%

Summary of Non-Included Effects



Vector Second Class Current has largest possible effect due to being poorly constrained

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Summary of Non-Included Effects cont.

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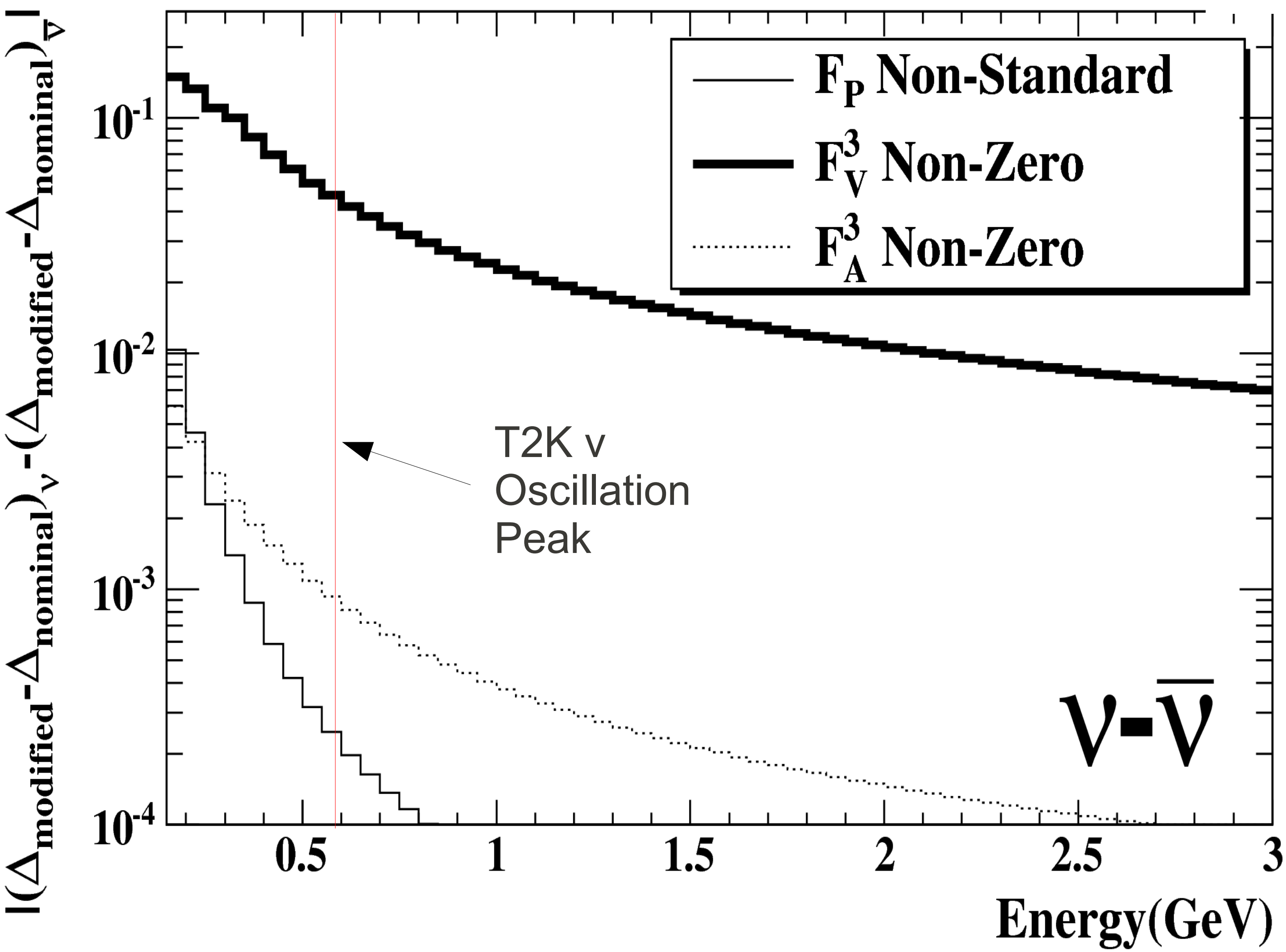
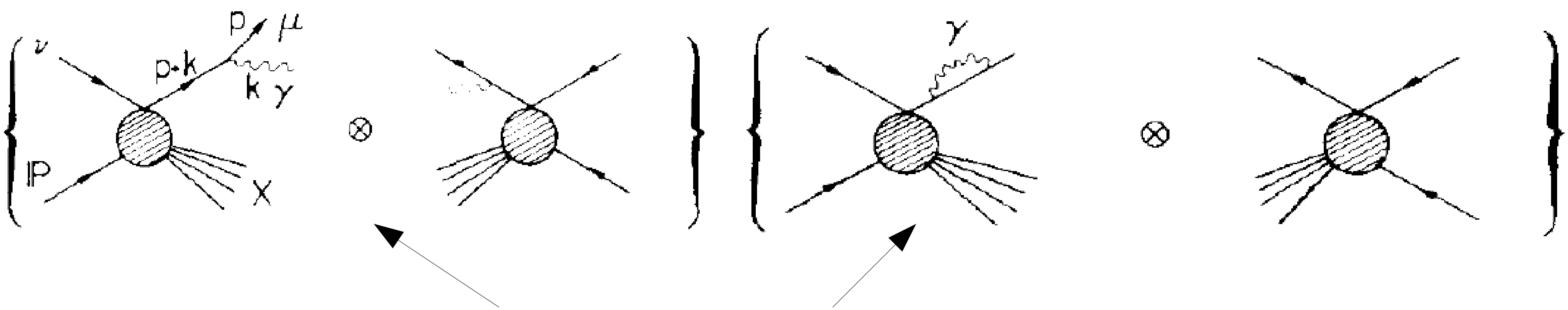


Difference between neutrino and anti-neutrino show possible contributions to CP violation uncertainties

Radiative Corrections

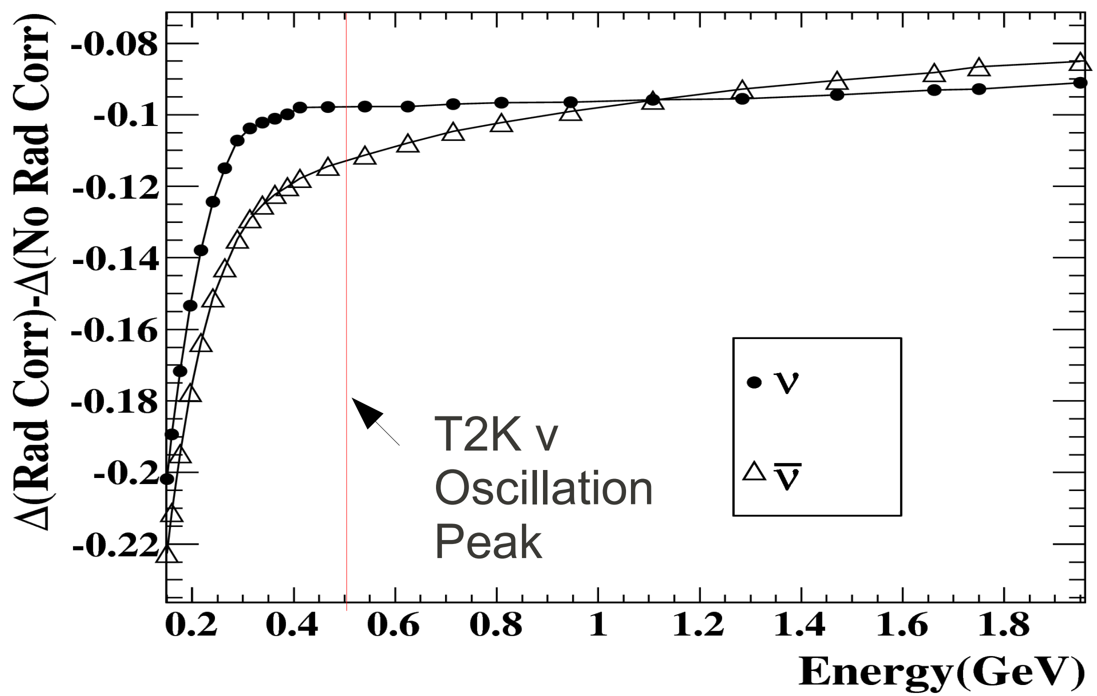
- No complete calculation for this energy region exists
- Experimental issue: Energy from radiated photons will be included for electron neutrino interactions but not for muon neutrino interactions
- Use leading log method (up to log(Q/m), where Q is the energy scale of the interaction process)[15]



- Only calculate “lepton leg” terms

Radiative Corrections cont.

- Correction from simple method seems extremely large



- Criticisms of this method say that Wγ exchange with the lepton legs will cancel some or all of the effects seen
- Full calculation needed
- Important to add this correction to current neutrino generators, if only to correct reconstruction issues

Effects at Various Energies

Effect ↓ Experiment(Oscillation Peak) →		Cern-Frejus[16] (260 MeV)	T2K[18](600 MeV)	NOvA[17](2 GeV)
F_A	ν	2 %	1 %	0 %
	$\bar{\nu}$	2 %	0.5 %	0 %
F_p	ν	0.5 %	0 %	0 %
	$\bar{\nu}$	1.5 %	0 %	0 %
F^3_A	ν	0 %	0 %	0 %
	$\bar{\nu}$	0.5 %	0 %	0 %
F^3_V	ν	5.5 %	2 %	0.5 %
	$\bar{\nu}$	8.5 %	3.5 %	0.5 %
Rad. Cor.	ν	10 %	10 %	9 %
	$\bar{\nu}$	13.5 %	11.5 %	8.5 %

- Lower energy, higher effect
- Vector SCC and Radiative Corrections may affect even NOvA

Conclusions

- Muon and electron neutrino cross section uncertainties affect mixing angle, CP violation and the mass hierarchy measurements
- Contributions come from multiple sources, some of which are currently modeled and some of which are not:
  - Kinematic limit has consistently large effect, but is modeled
  - Uncertainty in F_A contributes only ~1-2% to lower energy experiments
  - Non-Standard effects can contribute two to three times as much
  - From simple calculation, radiative corrections may have non-trivial contribution to cross section difference which should be understood
- Summary: To improve uncertainty must improve constraints and understand all sources of error

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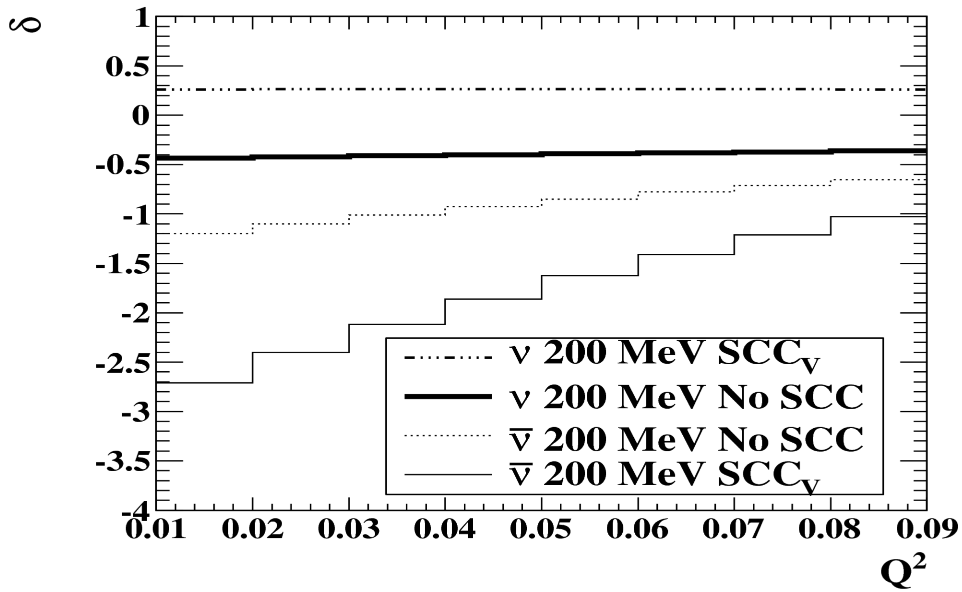
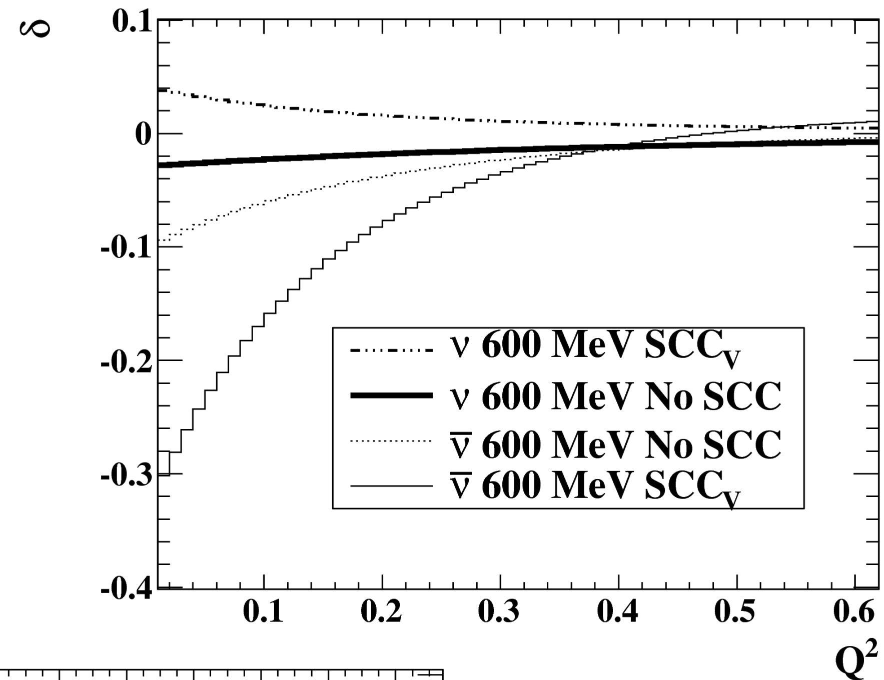
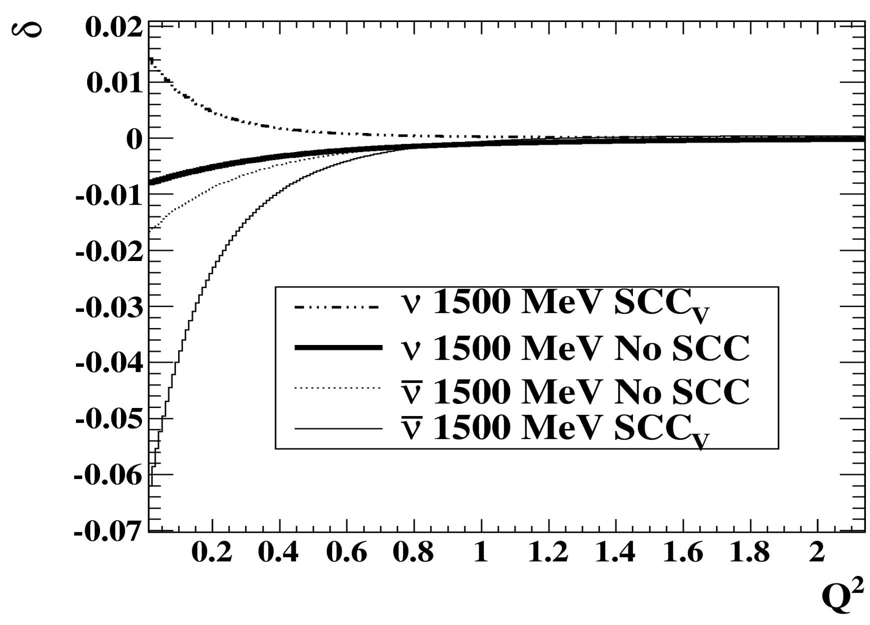
Backup Slides

F^3_V w/ Varied Q2

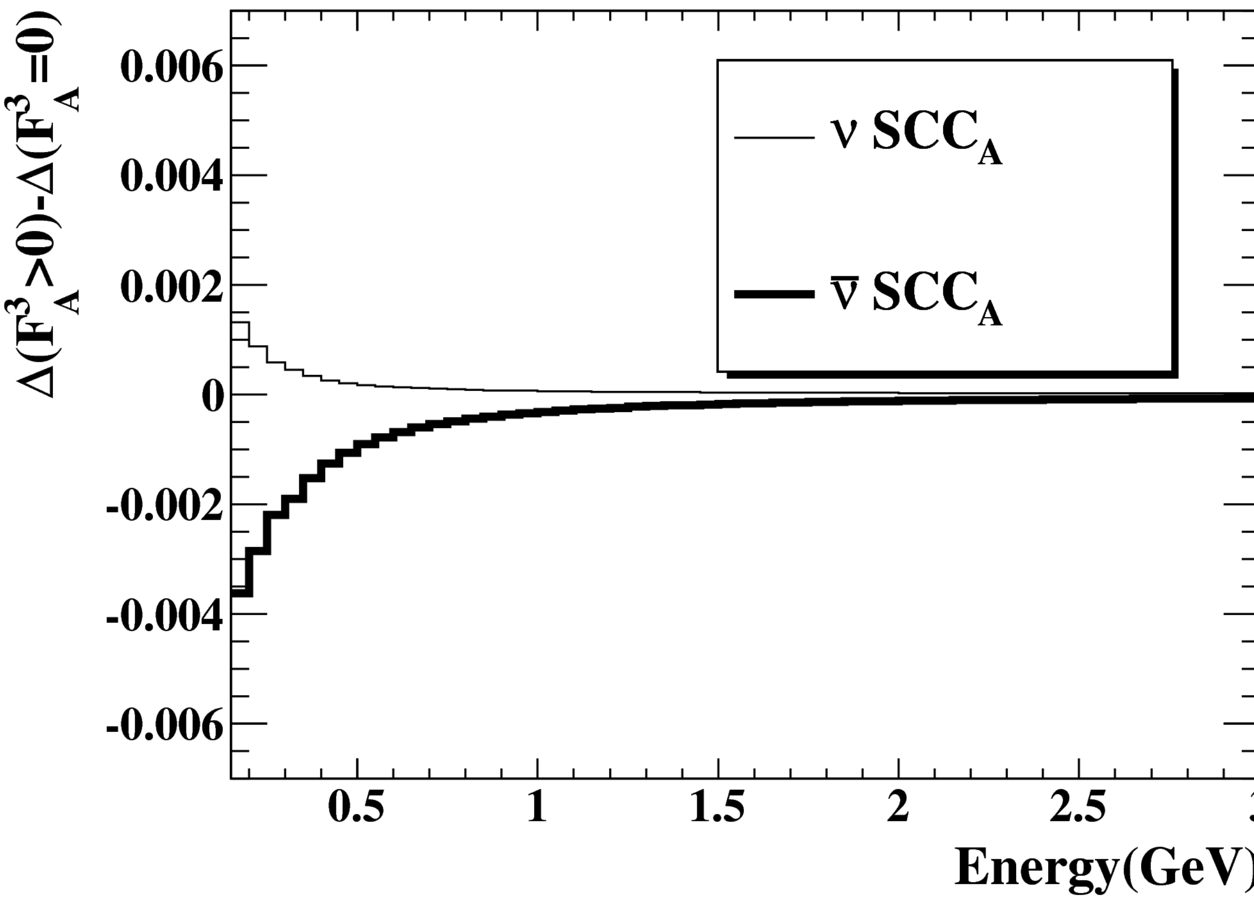




$$\delta(E_\nu, Q^2) \equiv \frac{\frac{d\sigma_\mu}{dQ^2} - \frac{d\sigma_e}{dQ^2}}{\int dQ^2 \frac{d\sigma_e}{dQ^2}}$$



F^3_A Muon Neutrino Difference



$$\Delta_\ell(E\nu) \equiv \frac{\int dQ^2 \frac{d\sigma_\ell}{dQ^2} - \int dQ^2 \frac{d\sigma_\ell^{ref}}{dQ^2}}{\int dQ^2 \frac{d\sigma_\ell^{ref}}{dQ^2}}$$