Progress towards Parametric-Resonance Ionization Cooling in the Twin Helix Channel

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Overview

- Muon Cooling Channels Issues
- Basics of Parametric Resonance Ionization Cooling (PIC)
- Implementation of PIC in the Twin Helix Channel
- Linear Modeling in COSY Infinity
- Progress towards Aberration Correction and Optimization
- Conclusions and Future Work

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Muon Cooling Channel Issues

- Muons are tertiary particles produced with large phase space volume
- Cooling required for many applications
- Short muon lifetime
- Muon cooling channels can have complicated fields with non-linear effects

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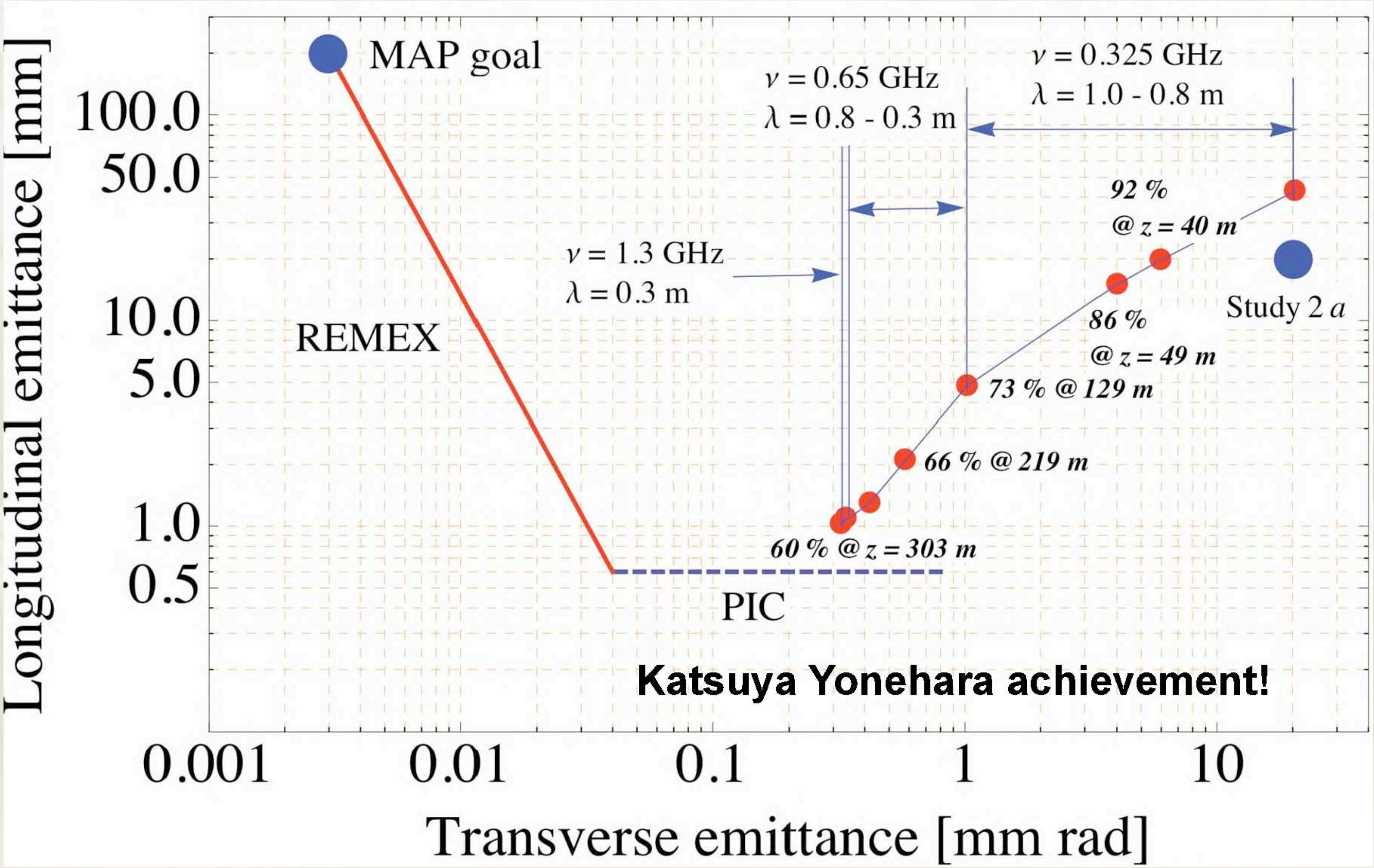
Emittance Evolution Plot



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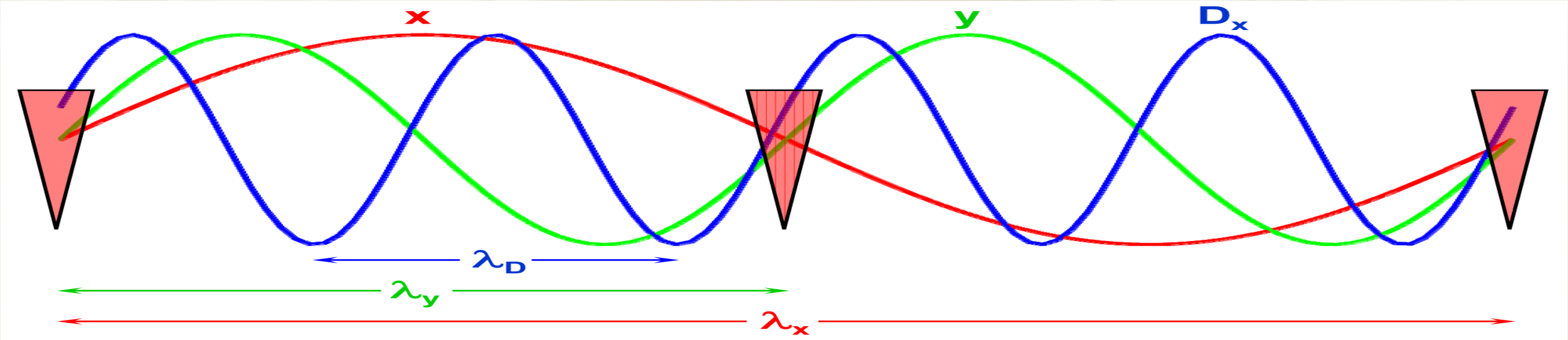
The figure is courtesy of K. Yonehara

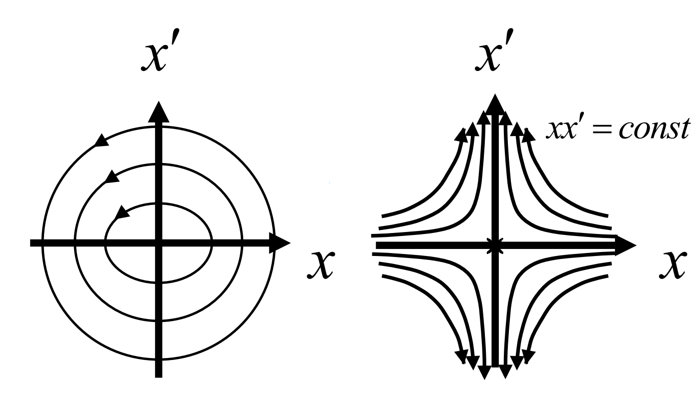
How PIC works

- Correlated optics maintains a stable reference orbit where the betatron tunes in the horizontal and vertical planes are integer multiples of the dispersion function for the system



Ex. $\lambda_y = 2\lambda_x = 4\lambda_D$

- ½ integer resonances are induced causing muons to follow a hyperbolic trajectory
- Wedge absorbers limit angular divergence via ionization cooling while RF cavities are used to maintain the reference momentum



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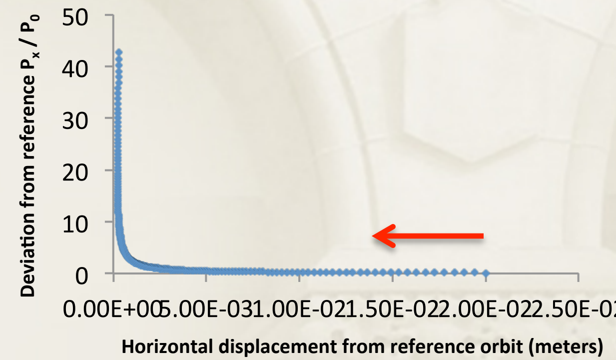
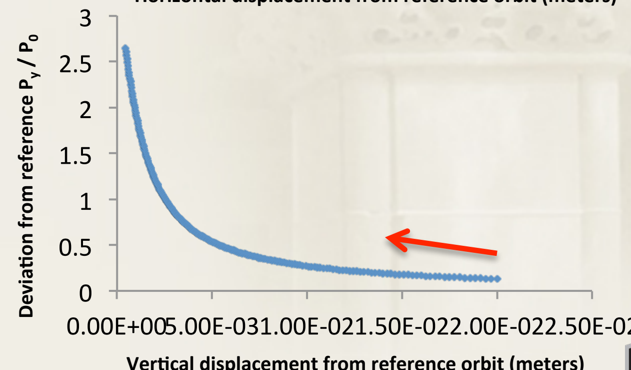
PIC in the Twin-Helix Channel

- One example implementing PIC criteria is the twin-helix channel
- The basic twin-helix channel:
  - A pair of helical dipole harmonics of equal field gradient and equal but opposite helicities
  - A continuous quadrupole field is superimposed to maintain the correlated optics conditions
  - Creates a periodic focal point in both x and y
- Parametric lenses induce PIC resonance condition: Two pairs of helical quadrupole harmonics - one pair for the horizontal and the other for the vertical plane
- Ionization cooling elements:
  - Beryllium wedge absorbers placed every other period at the focal point
  - RF cavities are placed 3 cms after each absorber
  - Dispersion at the focal points is small (to minimize heating from stochastic effects), but non-zero (to allow for emittance exchange)
- REMEX could be accomplished in same channel by reversing wedge angle

The basic twin-helix channel with parametric lenses is simulated in COSY Infinity without wedge absorbers or energy restoring RF cavities

250 MeV/c μ^- launched offset in both planes from the reference orbit by 2 cm and 130 mrad and tracked every 2 dipole periods





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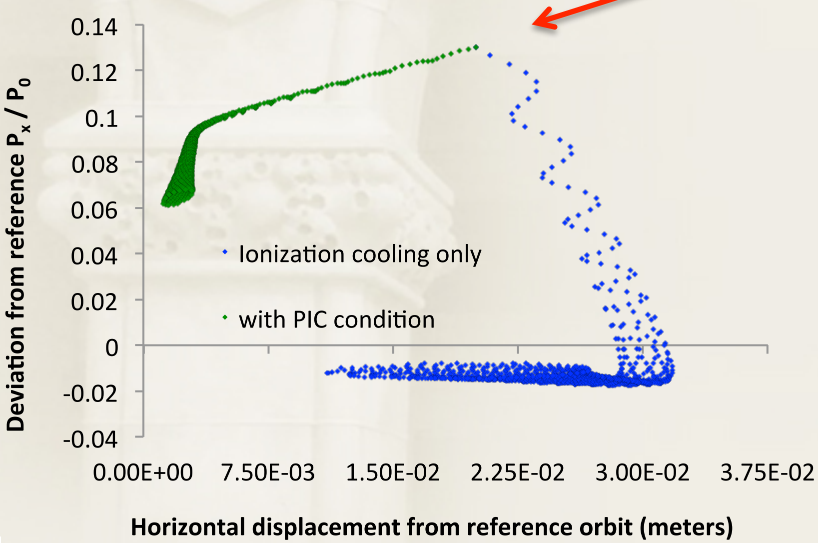
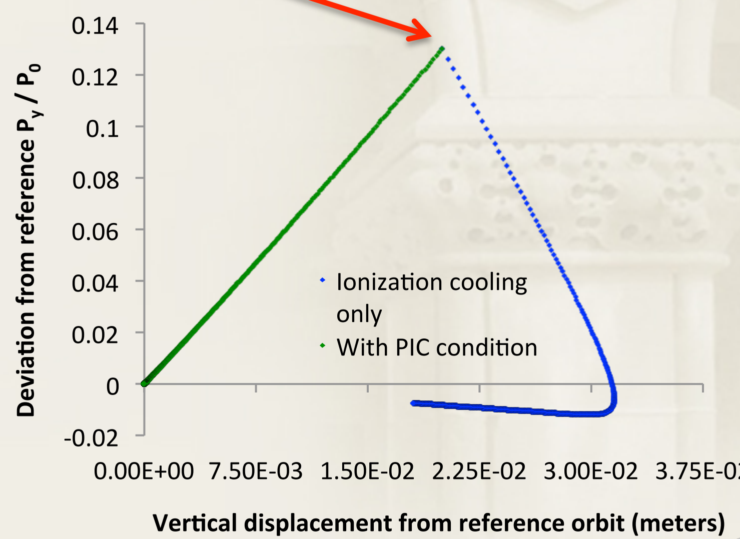
Ionization Cooling and PIC

Linear simulation in COSY Infinity of the basic twin-helix channel with wedge absorbers and energy-restoring RF cavities is simulated with and without parametric lenses

250 MeV/c μ^- launched offset from the reference orbit by 2 cm and 130 mrad in both planes and tracked in the center of each wedge





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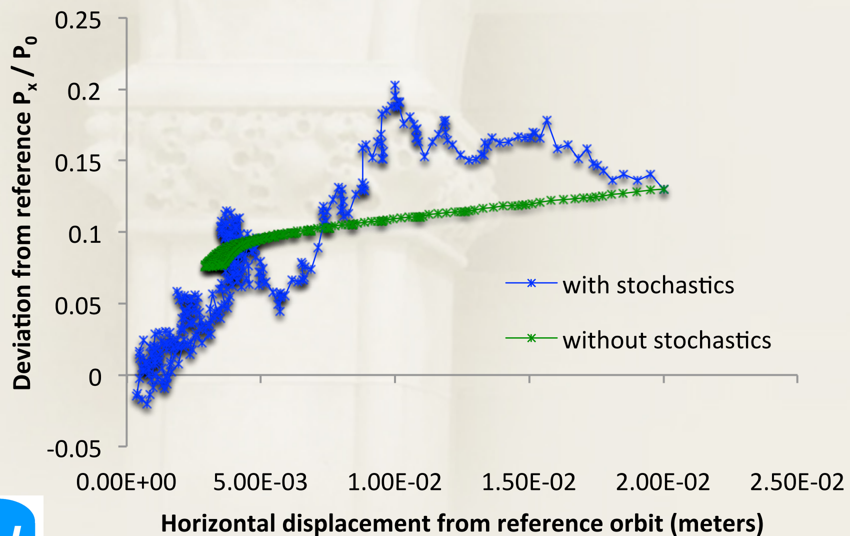
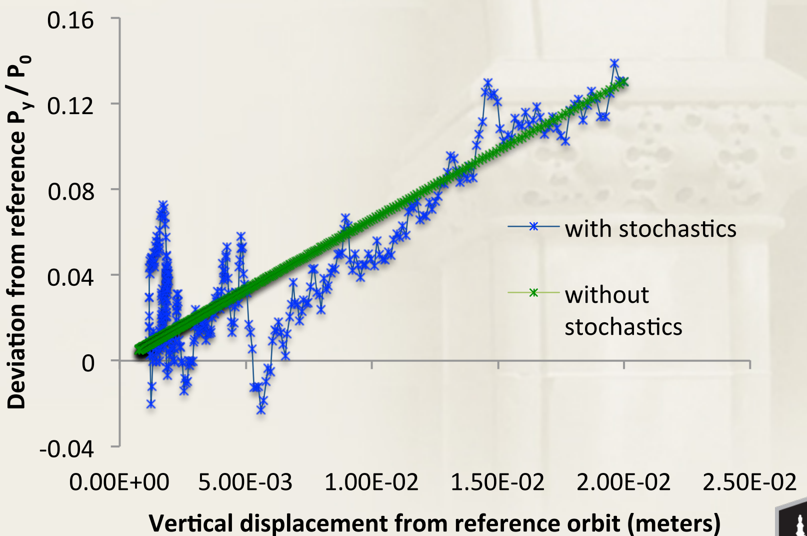
Stochastic Effects for Single Particle

Linear simulation in COSY Infinity of the full twin-helix channel (wedge absorbers, energy-restoring RF cavities and parametric lenses) with and without the stochastic effects of multiple scattering and energy straggling

250 MeV/c μ^- launched offset in both planes from the reference orbit by 2 cm and 130 mrad tracked in the center of each wedge





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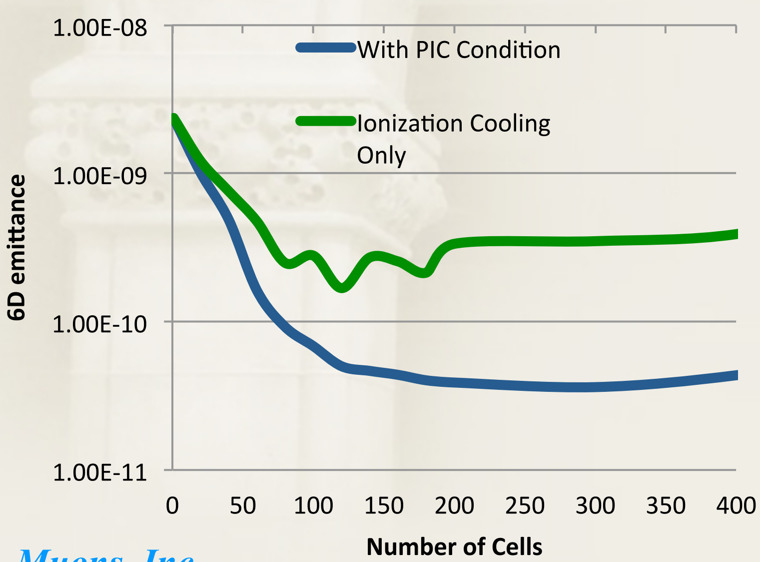
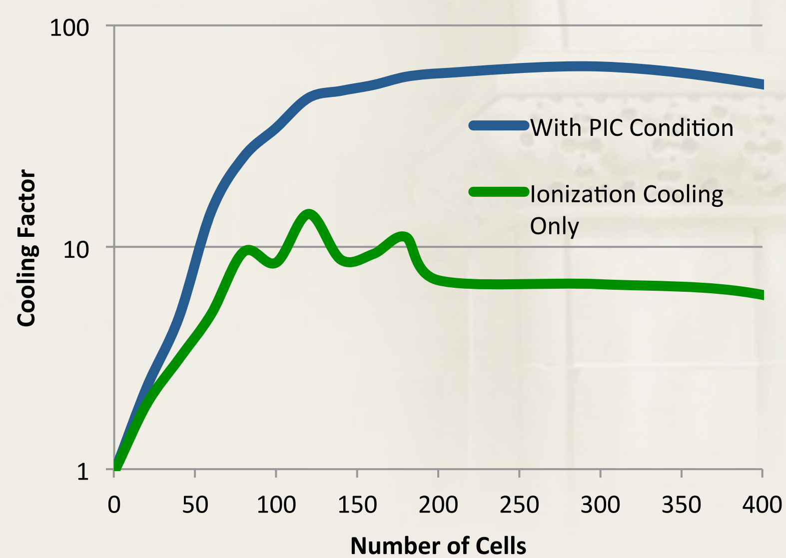
Cooling Factor Measurements

A distribution of test particles in the COSY Infinity simulation of the linear channel with stochastic effects. The initial distribution uses a sigma of 2 cm in offsets, 130 mrad in angles, and 1% spread in energy from the reference particle. The distribution is also spread over a bunch length of ± 3 cms relative to the reference particle.

Comparison of cooling factor (ratio of initial to final 6D emittance) with and without the strong focusing from PIC resonance condition is consistent with theory indicating improved in cooling by about a factor of 10





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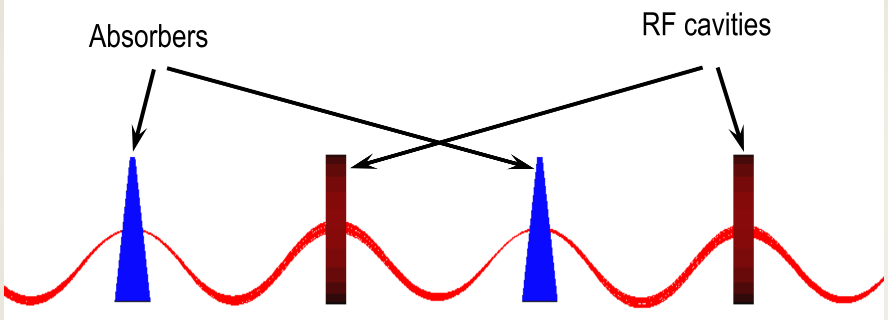
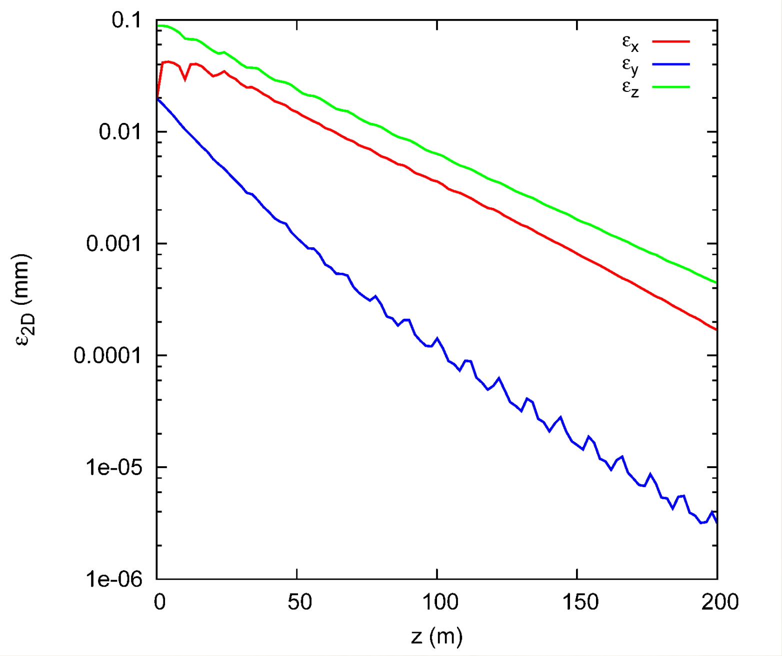
Cooling Simulations in G4Beamline

- Twin helix system incorporating wedge absorbers and RF modeled in G4beamline
- Helical quadrupole pair to excite parametric resonance in each plane
- Magnets scaled for dipole harmonic period of 20 cm.





Simulation in G4 Beamline of the uncorrected 20 cm period channel without stochastic effects

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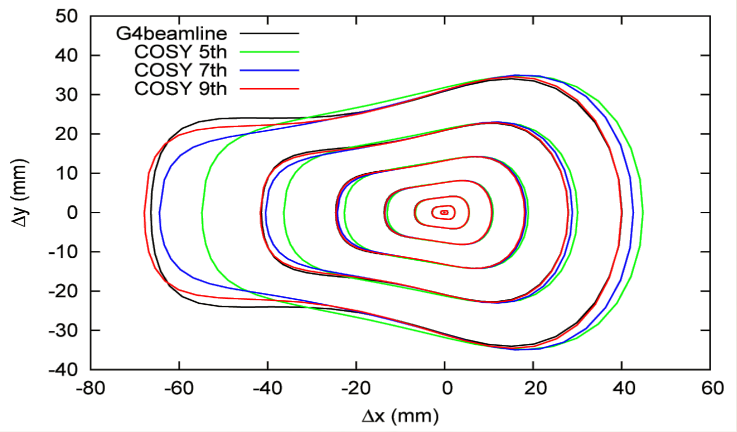
Evaluating Aberrations and Effects

- Linear Model sets a baseline for aberration corrections
  - Shows "perfect" aberration correction
- Aberrations can be studied through maps
  - Shows which aberrations at are largest
  - Aberration study can be done order by order
  - Demonstrates sensitivity of optics due to initial conditions
- Aberration effects can be evaluated by order
  - Shows when optics results converge
  - Correcting lower order aberrations can correct dependent higher order aberrations as well

Aberrations affecting spot size for the λ_D=20 cm. twin-helix > 10^{-3} at 2^{nd} and 3^{rd} order	
(x|aa)	0.0015
(x|aδ)	0.0021
(x|aaa)	-0.0178
(x|abb)	-0.0061
(y|aab)	0.0061
(y|bbb)	0.0012



monochromatic point source, ±160 mrad, 2 helix periods

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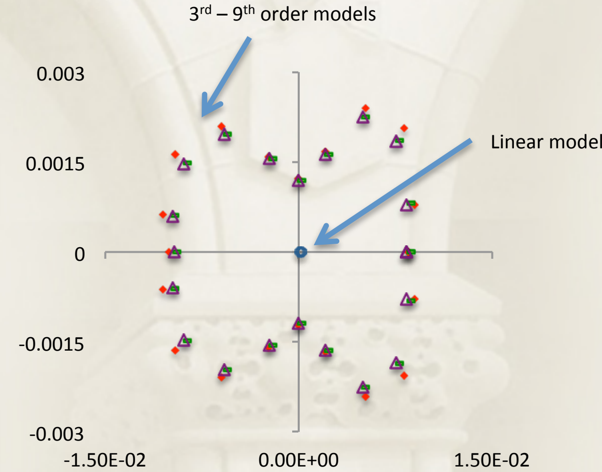
Impact of Nonlinear Aberrations

Simulation codes such as COSY Infinity can be used to calculate aberration coefficients and show impact on the channels optics:

Second Order Aberration Map

-0.5980590E-04	0.9763552E-05	0.000000	0.000000	-0.5401955E-04	2000000
[illegible]	0.7760842E-02	0.000000	0.000000	0.2169705E-03	1100000
0.1995436E-02	-0.3226334E-02	0.000000	0.000000	-0.1005011E-01	0200000
0.000000	0.000000	-0.1046741E-03	-0.7584133E-04	0.000000	1010000
0.000000	0.000000	0.3367245E-04	-0.3062197E-01	0.000000	0110000
-0.2356046E-03	-0.2986482E-04	0.000000	0.000000	-0.2483222E-03	0020000
0.000000	0.000000	-0.1126418E-05	0.6798941E-02	0.000000	1001000
0.000000	0.000000	-0.4565814E-03	-0.4007927E-02	0.000000	0101000
0.3271621E-04	-0.6803578E-02	0.000000	0.000000	-0.1972945E-03	0011000
0.2292975E-03	-0.1728703E-03	0.000000	0.000000	-0.3555218E-02	0002000
[illegible]	0.7349571E-03	0.000000	0.000000	0.7267789E-06	1000010
0.2061992E-02	-0.7999127E-03	0.000000	0.000000	0.2934764E-03	0100010
0.000000	0.000000	0.2216899E-04	0.3354879E-02	0.000000	0010010
0.000000	0.000000	-0.7371019E-03	-0.1125724E-02	0.000000	0001010
-0.2045517E-04	0.1356720E-03	0.000000	0.000000	-0.3927054E-04	0000020



Spot size after 20 cells for monochromatic point source, θ = 50 mrad,

Largest 2nd order aberrations effecting horizontal position are (x|aa) and (x|aδ)

Since muon beams can have large initial angular and energy spreads, these aberrations might dramatically impact final beam spot size

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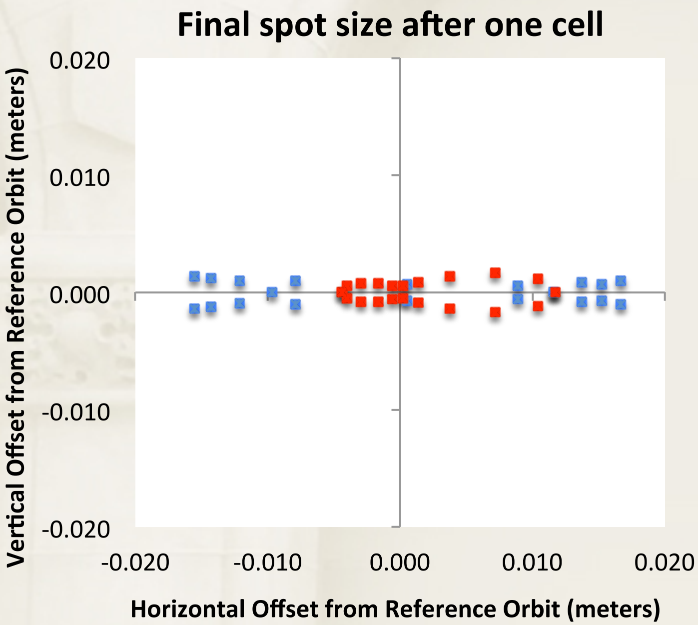
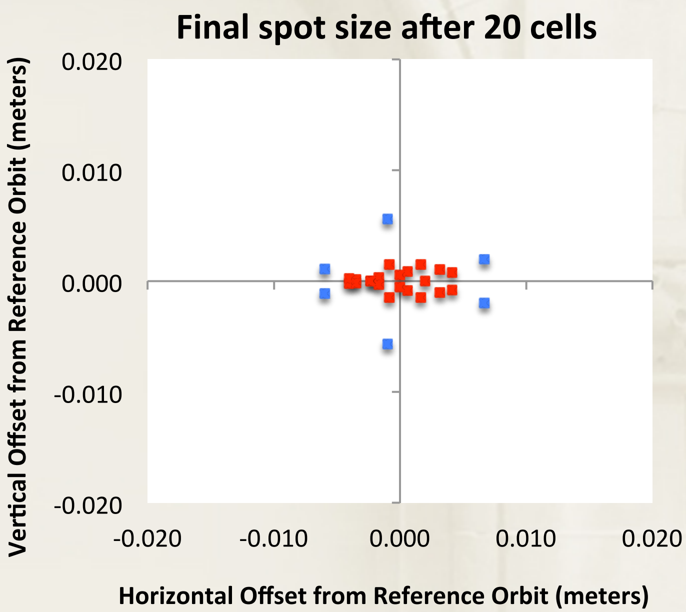
Progress on Aberration Correction

- Correction with straight octopole, pair of sextupole helical harmonics, 2 pairs of octopole helical harmonics
- 2 pairs of quadrupole helical harmonics added to maintain correlated optics
- 250 MeV/c muon cone (+/- 100 mrad) from point source



7th Order Simulation in COSY Infinity
Blue – Uncorrected Red - Corrected



3th Order Simulation in COSY Infinity
Blue – Uncorrected Red - Corrected

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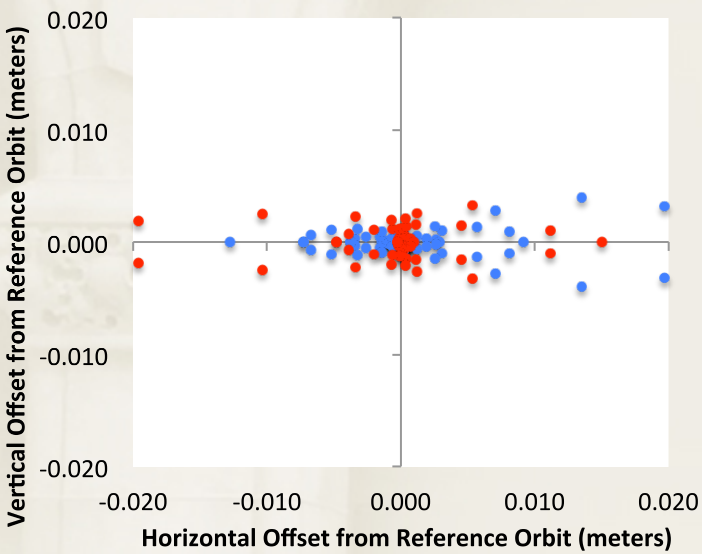
Progress on Aberration Correction

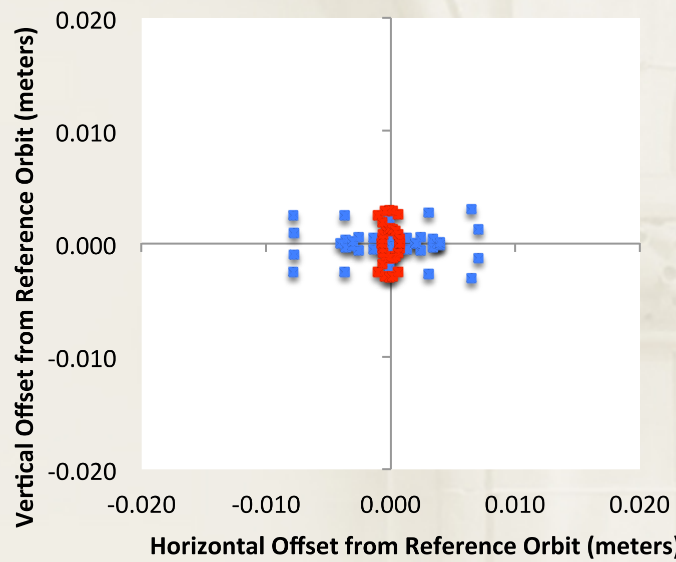
- Correction to 9th order with pair of sextupole helical harmonics and 3 pairs of octopole helical harmonics
- 2 pairs of quadrupole helical harmonics added to maintain correlated optics
- 250 MeV/c muon cones (+/- 120 mrad with 30 mrad steps) from point source

Final spot size after 1 cell
30 to 120 mrad cones



9th Order Simulation in COSY Infinity
Blue – Uncorrected Red - Corrected

Final spot size after 20 cells
Only 30 and 60 mrad cones



9th Order Simulation in COSY Infinity
Blue – Uncorrected Red - Corrected

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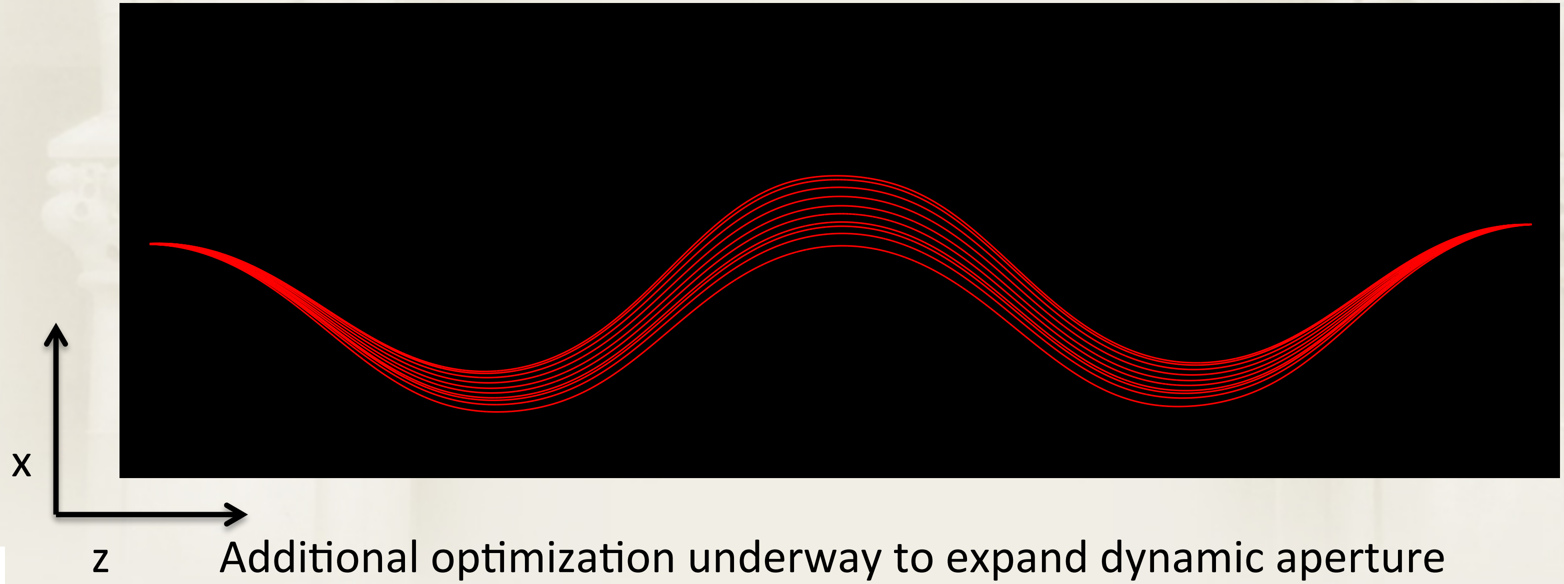
Progress on Aberration Correction

- Correction to 9th order with pair of sextupole helical harmonics and 3 pairs of octopole helical harmonics
- 2 pairs of quadrupole helical harmonics added to maintain correlated optics
- 250 MeV/c muons from point source with horizontal angle +/- 60 mrad from reference orbit

G4Beamline Simulation of horizontal motion thru 1 cell (2 dipole periods)



Additional optimization underway to expand dynamic aperture and combine simulations with stochastic effects

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Summary and Future Plans

- Cooling channel design with twin-helix invention being developed
- Basic model with wedge absorbers and rf cavities is in place and simulated
- Linear modeling with stochastic processes consistent with theory
- Dynamics with parametric resonance understood
- Cooling simulations initiated
- Study of aberration compensation is underway
- Next steps
  - Use COSY Infinity to analyze and correct aberrations to acceptable level
  - Consider coupling resonance to reduce number of compensation conditions
  - Demonstration of an optimized system with aberrations corrected including nonlinear effects and stochastic processes
  - Compare performances of conventional ionization cooling and PIC
  - Start addressing engineering aspects

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Back Up Slides

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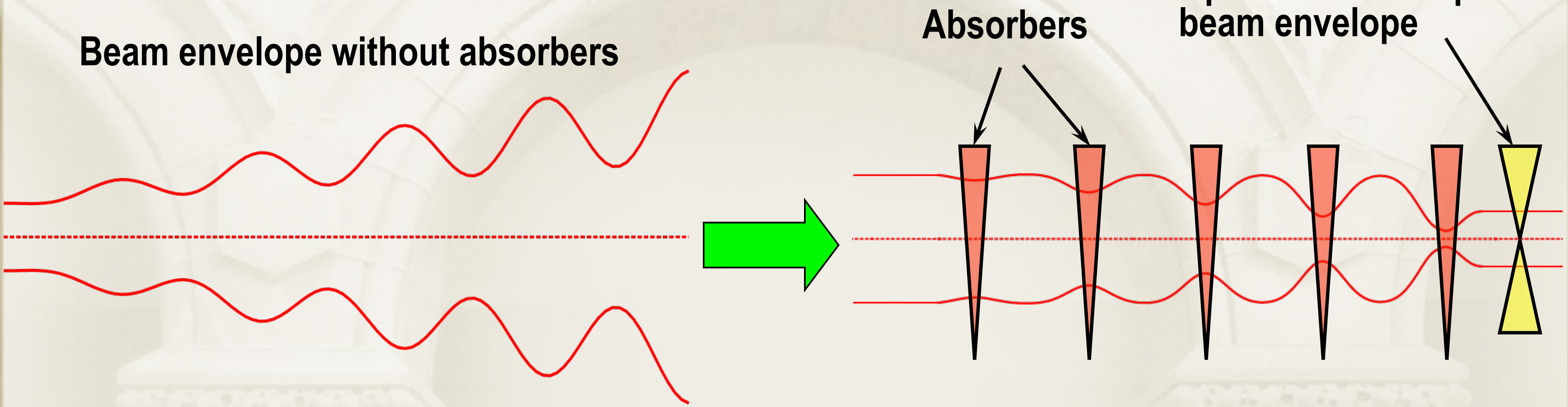
PIC Principles

- Resonant dynamics: angular spread grows while beam size shrinks
- Absorbers keep angular spread finite



- Equilibrium angular spread and beam size at absorber

$$\theta_a^2 = \frac{3}{2}\frac{(Z+1)}{\gamma\beta^2}\frac{m_e}{m_\mu} \qquad \sigma_a = \frac{1}{2\sqrt{3}}\theta_a w$$

- Equilibrium emittance

$$\varepsilon_n = \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w$$ (a factor of $\frac{\pi}{\sqrt{3}}\frac{w}{\lambda} = \frac{\pi}{2\sqrt{3}}\frac{\gamma'_{acc}}{\gamma'_{abs}}$ improvement)

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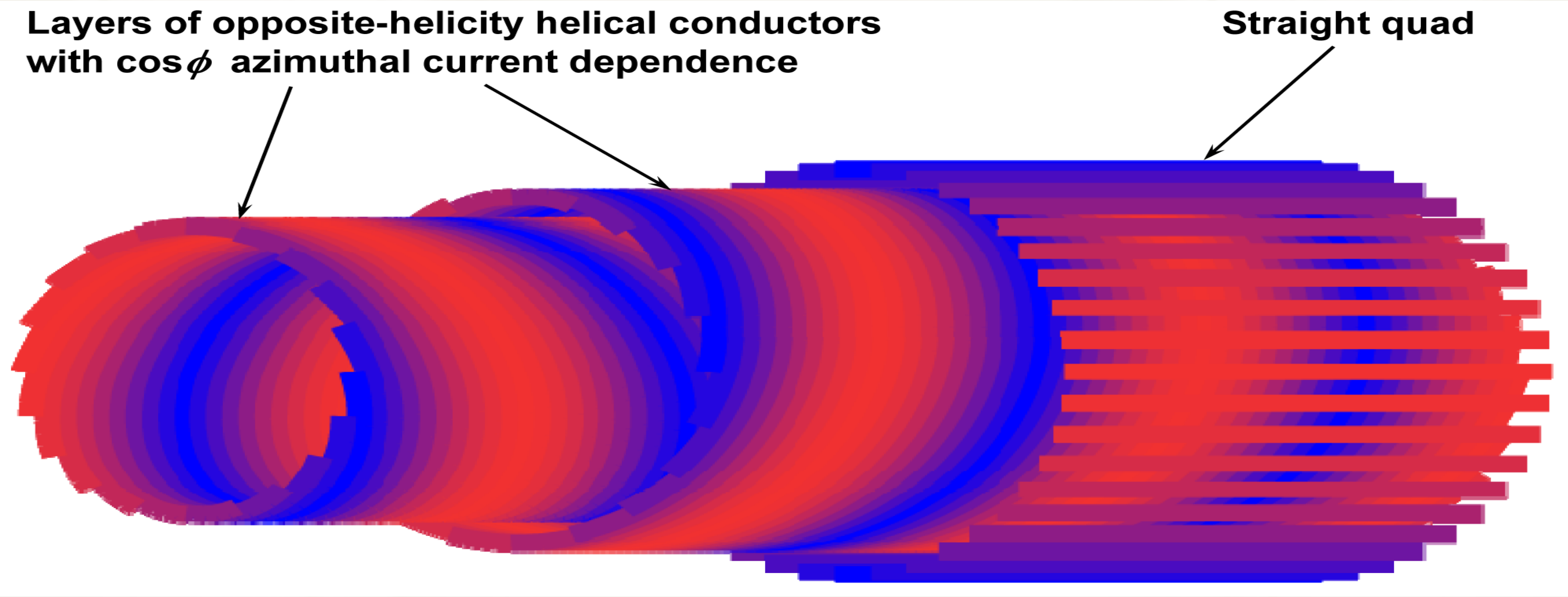
Implementation of Twin-Helix

- One proposed conceptual drawing of implementation of twin-helix channel using a combination of 2 helical conductor layers and a straight quadrupole.
- Colors indicate current variation in the conductors



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Beam Optics Overview

- Define Particle Coordinates:

$$\vec{z}\left(x,a,y,b,l,\delta\right)$$

- Taylor Map Calculates Change in Coordinates:

$$\vec{z}_f = \left(M + N\right)\vec{z}_i$$

Linear terms can be represented as a matrix

$$\begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ l_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} x|x & x|a & 0 & 0 & x|l & x|\delta \\ a|x & a|a & 0 & 0 & a|l & a|\delta \\ 0 & 0 & y|y & y|b & 0 & 0 \\ 0 & 0 & b|y & b|b & 0 & 0 \\ l|x & l|a & 0 & 0 & l|l & \delta|l \\ \delta|x & \delta|a & 0 & 0 & l|\delta & \delta|\delta \end{pmatrix} \circ \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ l_i \\ \delta_i \end{pmatrix}$$

Non-linear terms can be calculates as a Taylor series expansion in terms of the initial particle coordinates:

$$x_f = \left(x\,|\,xx\right)x_i^2 + \left(x\,|\,xa\right)x_i a_i + \left(x\,|\,aa\right)a_i^2 + ...$$
$$+(x\,|\,xxx)x_i^3 + (x\,|\,xxa)x_i^2 a_i + ...$$

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Uses of Transfer Maps

Terms in the linear map contain crucial optical information about your beam channel

Magnification terms

(x|a) = 0 for point to point imaging
(a|x) = 0 for parallel to parallel imaging

$$\begin{pmatrix} x|x & x|a & 0 & 0 & x|l & x|\delta \\ a|x & a|a & 0 & 0 & a|l & a|\delta \\ 0 & 0 & y|y & y|b & 0 & 0 \\ 0 & 0 & b|y & b|b & 0 & 0 \\ l|x & l|a & 0 & 0 & l|l & \delta|l \\ \delta|x & \delta|a & 0 & 0 & l|\delta & \delta|\delta \end{pmatrix}$$

Determinant of the matrix will be less than one for a system with cooling in 6-D phase space

Non-zero for dispersion

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Nonlinear Optics

- Chromatic Aberrations:
  - energy dependent
  - ex. (x|ad) is a second order aberration dependent on initial angle and energy spread relative to reference particle orbit
- Geometric Aberrations:
  - angular and position dependent only
  - ex. (x|xxx) is the third order aberration dependent on the cube of initial position offset from the reference orbit

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Addition of Stochastic Effects

- Tracking using transfer maps: $\vec{z}_f = \ldots \mathrm{M} \bullet \mathrm{M} \bullet \mathrm{M} \bullet \vec{z}_i$
- Adding stochastics "map": $\vec{z}_f = \ldots \Sigma \bullet \mathrm{M} \bullet \Sigma \bullet \mathrm{M} \bullet \Sigma \bullet \mathrm{M} \bullet \vec{z}_i$
- Stochastic "map" defined to produce these results:

$$\begin{pmatrix} x_f = x_i \\ a_f = a_i + \Delta(scattering) \\ y_f = y_i \\ b_f = b_i + \Delta(scattering) \\ l_f = l_i \\ \delta_f = \delta_i + \Delta(straggling) \end{pmatrix}$$

Multiple scattering modeled using PDG formula RMS98

$$\theta_{scatter} = \frac{13.6 MeV}{\beta c p} \sqrt{\frac{z}{\chi_0}} \left(1 + .028 \ln\left(\frac{z}{\chi_0}\right)\right)$$

Energy straggling modeled using Bohr approximation

$$\Omega^2_{straggling}\left[KeV^2\right] \approx .26 Z_{absorber}\, z N_t \left[10^{18} atoms / cm^2\right]$$

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